MUSTAFA HEKİMOĞLU

Spare Parts Management of Aging Capital Products

Spare Parts Management of Aging Capital Products

Spare Parts Management of Aging Capital Products

Reservedelenbeheer voor verouderende kapitaalgoederen

Thesis

to obtain the degree of Doctor from the
Erasmus University Rotterdam
by command of the
rector magnificus

Prof.dr. H.A.P. Pols

and in accordance with the decision of the Doctorate Board
The public defense shall be held on
Thursday 26 November 2015 at 09:30 hrs
by

Mustafa Hekimoğlu
born in İzmir, Turkey.

Erasmus University Rotterdam

Doctoral Committee

Promotor:	Prof.dr.ir. R. Dekker
Other members:	Prof.dr. G.J. van Houtum
	Prof.dr. W.H.M. Zijm
	Dr. R. Kuik
Copromotor:	Dr. E. van der Laan

Erasmus Research Institute of Management - ERIM
The joint research institute of the Rotterdam School of Management (RSM)
and the Erasmus School of Economics (ESE) at the Erasmus University Rotterdam
Internet: http://www.erim.eur.nl

ERIM Electronic Series Portal: http://repub.eur.nl/pub

ERIM PhD Series in Research in Management, 368
ERIM reference number: EPS-2015-368-LIS
ISBN 978-90-5892-416-2

Design: B&T Ontwerp en advies www.b-en-t.nl

This publication (cover and interior) is printed by haveka.nl on recycled paper, Revive®.
The ink used is produced from renewable resources and alcohol free fountain solution.
Certifications for the paper and the printing production process: Recycle, EU Flower, FSC, ISO14001.
More info: http://www.haveka.nl/greening

Acknowledgement

I started my PhD October, 2011 and studied for this dissertation for almost four years at Erasmus University. I would like to express my gratitude to the ones who contributed and helped me to grow.

First of all, I would like to thank my promoter Prof. Rommert Dekker who not only provides new ideas and insights in all of our meeting, but also conveys his experience and unique style in scientific inquiry in the field of management science. Thanks to him I made a significant progress in developing my approach to scientific research and communicate my outputs with academicians as well as managers.

Second, I am thankful to my daily supervisor Erwin van der Laan who accepted me to his supervision when i was in the middle of my PhD. During our meetings for more than two years, his different perspectives enlightened my ways and helped me reach meaningful and useful conclusions that are presented in different chapters of this dissertation.

Third, I am grateful for his helps and ideas to Prof. Alan Scheller-Wolf, whom I visited in the second year of my PhD. After my research visit, he continued to provide input to my research in the dual sourcing context. His comments and critics contributed to the quality of my entire PhD dissertation significantly.

Furthermore, I am indebted to my inner committee members Geert-Jan van Houtum, Roelof Kuik and Henk Zijm for their evaluation and contributions to this thesis.

Last but not least, I would like to thank employees of Fokker Services, namely Frans van Doorn, Sander Coolman, Matthijs Kloosterboer, Kaveh Alizadeh, Auke Nouwens, Vincent van Vliet, Marit Ubbink and Burak Çakır. More than four years ago I started to visit that company as a MPhil student and those people share their business problems and their insights and data with me. Observations that I made at Fokker Services will also be useful in my future research.

These professional contributions are useful only if they are complemented with warmth and support that only can be provided by the family. For her unconditional support and love, for bringing our daughter, Irmak Hekimoğlu, to this world and being extra patient while taking care of our daughter on her own in Turkey, I thank to my lovely wife Başak

Savun Hekimoğlu. Truly, she is the best thing that could have been happened to me in this life and I am blessed with having her and our daughter in my life. Also I am grateful to my mother for her support in all of my endeavors.

Also I would like to express my gratitude to SLF Research and PROSELO project by DINALOG which provided partial financial support and gave me opportunities to present my result to managers from different companies.

Mustafa Hekimoğlu
Rotterdam, July 2015

Table of Contents

Chapter 1

Introduction

A life cycle of capital products consists of different phases: design, introduction, growth, maturity, decline and out-of-production (Levitt, 1965; Wagner et al., 2012). In the first two stages of a life cycle, the installed base increases as Original Equipment Manufacturers (OEMs) produce and sell new products. Furthermore, the maturity phase is associated with replacement of old capital products and a decline in the OEM's production rate of the new ones. These two factors contribute to the capital product proceeding to the decline phase of its life cycle. The decline phase is usually associated with an end-of-production decision of the OEM: the capital product becomes *out-of-production*. It is not uncommon that a capital product is kept in operation for a long period of time after it becomes out-of-production. Even after end-of-production, OEMs desire to keep their capital products in operation as he has a service obligation and the after-sales services are a profitable revenue stream.To do so, they employ specific support programs to extend the economic life time of capital products as long as possible. 'FLYFokker Support Solutions' by Fokker Services or Saudi Aramco's investment decision for extending its oil extraction facilities (Dipaola and Okada, 2013) can be considered two good examples of such programs.

Spare parts are one of the main inputs of after-sales services. Naturally, the demand for spare parts changes according to the changing life cycle of the underlying capital products. In the introduction and growth phases, the number of products in use increases, which eventually stimulates the spare parts demand. After the maturity phase, the size of the installed base starts decreasing due to replacements of old products with new ones. This is so, because customers want to have access to the latest technology and because they may like to avoid the high maintenance costs that typically come with aging equipment. The declining number of capital products suppresses spare parts demand which may result in a gradual decay or even sudden death of spare parts demand. One possible consequence of this is obsolete inventory, which ties up a significant portion of working capital for OEMs.

To mitigate this problem, scholars proposed various methods to consider obsolescence risk in inventory control and demand forecasting. Note that demand risk for spare parts supply chains becomes more serious after the end-of-production date of capital products.

Furthermore, declining installed bases and dropping demand rates eventually elevate the *supply-side* risk for spare parts of aging capital products. Depending on a spare part's characteristics, this risk may appear as varying lead times, increasing minimum order quantities, or the permanent loss of suppliers. Since supply-side risk is the main focus of this thesis, we discuss causes, symptoms, their impact on OEMs and means of mitigation in the sections below.

1.1 Causes of Supply Risk

Problems in spare parts supply are caused by many different factors such as economic factors, raw material unavailability, process obsolescence and environmental regulations. Empirical evidence (Chapter 2) suggests that among these many different factors, the economic reasons are the main cause triggering supply-side problems for spare parts.

From the perspective of spare parts suppliers, production of components/parts for a capital product has different dynamics before and after an end-of-production announcement by an OEM. When a capital product is in production, its OEM keeps ordering from suppliers. Spare parts requirements for after-sales services can be satisfied easily, e.g. by picking spare parts from the production line Cohen et al. (2006). After the end-of-production announcement (by the OEM), spare parts demand might be a reliable stream of revenue or a factor crippling the capacity utilization of the supplier, depending on the size of the installed base.

For large installed bases, keeping a stock of spare parts and producing to maintain an inventory level might contribute to the profitability of spare part suppliers positively. If the installed base is small or gradually shrinking, keeping a spare parts inventory or even producing in make-to-order fashion might hurt the financial performance of a supplier due to non-moving inventory or decreasing capacity utilization because of set-ups. Eventually a spare parts supplier will seize its support for old capital products and will no longer accept orders from the OEM or other third-party service providers. This is called an *end-of-support* decision of the supplier. The rationality of this decision is further explained in Chapter 3.

Next to the economic factors, there may be other reasons for supply problems. *Raw material unavailability* might be due to changes in raw material markets or the loss of raw material suppliers. Losing a supplier might lead to serious and long term disruptions, depending on the uniqueness of the supplier. *Process obsolescence* stands for obsolete manufacturing processes or technologies which were available at the time of spare parts design. Over time advancement in manufacturing technologies and tooling may cause older methods to become obsolete. *Environmental factors* may introduce other risks. For instance, changes in regulations of a country or region may restrict the use of a specific raw material or manufacturing method. As another example, suppliers may go bankrupt due to financial problems.

1.2 Symptoms and Impact of Supply Risk

Supply risk might manifest itself in different forms, such as increasing mean and variance of the lead time, more frequent or longer supply disruptions, and increasing minimum order quantities. Each symptom has different effects on service rates and inventory related costs of the buyers. Increasing lead time duration and variability lead to stock-outs and lower service rates. Higher minimum order quantities yield larger batches which naturally increase inventory holding costs. Supply disruptions might be of a temporary or permanent nature. The effects of temporary disruptions might be mitigated by multi-sourcing or by capacity reservation contracts. Permanent loss of a supplier may require additional actions such as the development of a new supplier, licensing new part drawings, or changing the entire subsystem of a capital product. The resolution of a permanent loss of a supplier naturally depends on the underlying cause and may take up-to several years.

Recall that supply problems are a by-product of decreasing demand rates. As supply problems may lead to decreasing service rates, this may create extra motivation for customers to further replace their capital products: a vicious cycle. In our experience, supply problems are either overlooked or dealt with ad hoc manner by decision makers, which is another aggravating factor for supply risk in the service sector.

1.3 Dealing with Supply Problems

Supply problems can be treated with different approaches such as using advance warning signals, state-dependent inventory control policies, sourcing from multiple suppliers. capacity reservation or capacity flexibility agreements with suppliers, etc (Tomlin, 2006; Serel et al., 2001; Tsay, 1999). After conducting an empirical analysis regarding the extent and characteristics of the supply risk, one method or a combination of several methods should be utilized to mitigate the risk. Naturally, the method chosen should be appropriate for the issue at hand. For instance, *nonstationarity* of supply risk can be treated with state-dependent inventory policies, whereas dual sourcing may be found to be more preferable for lead time variability problems. Similarly, capacity flexibility of a reliable supplier is recognized as a useful approach for dealing with long and infrequent supply disruptions.

From another perspective, suitable supply risk mitigation methods can be characterized as either proactive or reactive. A proactive approach requires utilization of advance warning signals which are capable of detecting supply problems using various indicators, such as lead time, price, environmental conditions or even financial performance of a supplier. Such an advance warning system not only provides some extra time to an OEM for dealing with the problem before it occurs, it may also signal problematic suppliers. The latter property is especially crucial for OEMs who rely on a very large supplier base. The author's personal contacts with a Dutch OEM experiencing occasional supply disruption problems revealed that in absence of advance warning signals, the OEM has to rely on the supplier sending an end-of-support notification to which he can respond in a reactive manner. If the supplier does not send such a notice the company most likely discovers the disruption only after a spare part demand arrives and he tries to place a replenishment order.

1.4 Business Context

In this thesis we focus on supply problems for spare parts inventory control and service management. Research questions raised in each chapter of the thesis is taken from a Dutch Original Equipment Manufacturer, Fokker Services (FS), who provides maintenance service for aging (and out-of-production) aircraft.

Spare parts supply chains of FS consist of three important features which make them interesting for scientific analysis. First the majority of the fleet maintained by FS consists of *out-of-production* aircraft which is associated with increasing supply and demand risk. Declining fleet size and lower utilization yield decreasing demand for after-sales services by its very nature. Despite this trend, it is still critical for the company to satisfy their customers' maintenance needs in order to maintain the company's brand and reliability.

Second, similar to other OEMs, FS is subject to significant competition from third-party service providers for its after-sales services. Those companies compete with their relatively low prices. Therefore, FS needs to provide its timely services for reasonable prices. This stands for high spare parts availability together with low inventory cost. This competition from the third-party service providers is aggravated by the existence of internet-based secondary markets, which is the third important feature of the company.

Internet-based secondary markets, e.g. ilsmart.com, fipart.com, are online trading platforms on which traders, brokers, part suppliers and even customers can trade spare parts. The effects of these markets on FS are twofold: On the positive side they constitute a potential supply source with lower prices and fast deliveries. On the negative side the availability of spare parts are not guaranteed and they aggravate the price competition that FS is faced with.

Each of these three important features are addressed in this thesis. In addition, Chapter 2 presents a detailed description of the relevant business context for this thesis.

This PhD project is a continuation of a close collaboration between Erasmus University and FS. This collaboration is a branch of a larger project, PROSELO by DINALOG, that resulted in two PhD dissertations as well as numerous master theses on service logistics before the publication date of this manuscript. By considering supply problems for spare parts of aging capital products, this PhD dissertation completes a series of previous analyses on spare parts supply chains within the PROSELO project.

1.5 Overview of the thesis

This PhD thesis consists of four chapters on supply-side problems of spare parts of aging capital products. The thesis starts with an empirical analysis on supply problems using purchase history data and specific case studies (Chapter 2). This initialization is consistent with our practice-oriented approach to scientific inquiry in the field of operations management. Chapter 3 uses the findings of the empirical study as input and justification for its modeling assumptions. Therefore, Chapters 2 and 3 consist of two complementary legs of a supply risk mitigation solution. Chapters 4 and 5 focus on secondary markets and its effects on spare parts inventory control. We summarize our results in Chapter 6 of the manuscript.

Our study on supply risk of spare parts starts with an empirical investigation into the characteristics, severity and symptoms of supply problems. In this manner, Chapter 2 is devoted to a statistical analysis of purchase history data of spare parts taken from FS. We find that lead time is a statistically significant indicator for future supply disruptions. This finding indicates that supply disruptions are coupled with random lead times for spare parts of out-of-production systems.

In Chapter 3 we analyze nonstationary random lead time and supply disruptions in a single supplier setting. We prove that the state-dependent base stock policy is optimal assuming that order crossovers are not allowed. In addition, we find that the coupled effect of random lead time and disruptions can be larger than the summation of individual effects. Therefore, it is important for OEMs to consider these two factors together in their inventory control policy.

In Chapter 4 we consider a dual sourcing problem setting. Increasingly, maintenance companies utilize secondary markets to satisfy their demand since those markets provide immediate deliveries and cheaper prices. However, secondary markets include spare parts in various conditions such as serviceable, overhauled, as-removed etc. Giving such parts to customers, who seek for new spare parts, leads to a substitution penalty, which has to be considered in the inventory policy. Analytically, we find that the optimal policy is complex and dependent on the current state of the system. Therefore, we develop heuristic methods which perform well compared to the optimal policy. In addition, we extend the heuristic approach to address nonstationary random demand.

Chapter 5 of the thesis considers secondary markets from a different perspective. As mentioned above, spare parts prices on secondary markets are lower than the prices of part suppliers. Therefore, these markets stand for potential supply sources as analyzed in Chapter 4. In addition, these cheap spare parts attract some customers of OEMs since secondary markets are accessible to all parties. Hence, OEMs' replenishment and pricing policy should take secondary markets as (limited) supply sources as well as competitors who attract demand. By assuming a linear, price-dependent demand we consider the profit maximization of an OEM for by analyzing the competition with secondary markets in a dual sourcing setting.

The research output presented in Chapters 2-5 are based on different research papers written together with several scholars. Chapter 2 is a joint work with Taoying Farenhorst-Yuan and Rommert Dekker. For this chapter, I am thankful to Erwin van der Laan for his valuable comments and guidance. In addition, I acknowledge the importance of the work by Xishu Li, who extended the study presented in Chapter 2 with a more advanced model and additional data. Chapter 3 is based on a research conducted with Erwin van der Laan and Rommert Dekker. Chapter 4 is a result of a year-long collaboration with Alan Scheller-Wolf from Tepper School of Business, whom I visited in the second year of my PhD for three months. Finally the research in Chapter 5 is conducted under the supervision of Erwin van der Laan and Rommert Dekker.

Chapter 2

Empirical Analysis for Supply Risk for Spare Parts of Out-of-Production Systems

2.1 Introduction

In todays competitive business world, all aspects of spare part management are crucial for all parties involved in maintenance/operations of capital goods. Customers seek to minimize downtime costs of their capital goods by ensuring timely high quality maintenance service with affordable prices. Original Equipment Manufacturers (OEMs), on the other hand, consider after-sale market as a key element not only for sustained customer loyalty, but also to increase their annual revenues. Bandush and Dezvane (2003) report that up to 30% of total revenues of many manufacturers come from service activities. It is a well-established phenomenon that companies, which are capable of having sustainable growth in revenue even in a stagnating economy , focus on providing high quality service to their customer base (Wise and Baumgartner, 1999; Bundschuh and Dezvane, 2003; Cohen et al., 2006). By getting closer to their downstream, they become more familiar with the customer needs which cannot be learnt by any other means (Wagner et al., 2012).

For OEMs, demand for after-sales services is not the only input that must be followed. In spare parts supply chains, these companies exist in the middle and they have to follow suppliers, as well as the customer needs and expectations. In order to follow changes in customer and supplier side, companies employ analytic tools that can extract information from the data. Studies in supply chain literature provide statistical evidence for the benefit of business analytics applications for companies (Trkman et al., 2010; de Oliveira et al., 2012; Trkman, 2010). For OEMs, monitoring spare part supply becomes more critical after the end-of-production date of capital goods since decreasing demand rate and introduction of new models might lead spare parts suppliers to stop production as explained in Chapter 1.

Spare part supply failure is defined as "losing original manufacturers or suppliers of items or raw materials" (Feldman and Sandborn, 2007). This loss may take place with or without advance notification. In some cases, suppliers notify OEMs of end-of-support decision and give them last-time buy opportunities. Nevertheless, most changes in the supply-side occur without any advance notice and needless to say, the latter case puts OEMs in a more difficult situation. On the other hand, OEMs need to satisfy their customer demand to keep the reliability of its capital products. Therefore, they develop alternative supply sources by using various methods such as searching for a new supplier, investigation into secondary markets, developing a new supplier or re-designing the spare part or the entire subsystem which the spare part is installed in. Except the first two, other procedures might be a very long and costly especially for high-tech spare parts of capital goods. Hence, predicting possible changes in supply side is crucial for OEMs since it enables them to take proactive actions for supply failure risk.

For prediction of future changes in the supply-side, the analysis of existing data with an analytic tool is crucial. The data consists of various types of information for a vast amount of part numbers. Analytic tools, which can quantify the supply risk using the data, allow OEMs to take proactive actions to mitigate the supply failure risk. This is also the reason for rapid development of business analytics applications for supply risk management in the literature (Sandborn et al., 2007; Solomon et al., 2000; Trkman et al., 2010).

In this study, we consider four supply chain characteristics, price, lead time, and order interval and order size, as supply failure indicators for prediction. The main research question is as follows: *Can changes in supply chain characteristics be associated with supply risk of spare parts?* Specifically, a spare part supplier may increase its price or postpone delivery dates due to various reasons such as maintaining the profitability or simply to motivate its customers to find another supplier. Hence, fluctuations in price or lead time of a spare part may indicate its suppliers intention to stop the production of the spare parts in future. Furthermore, supply failure risk might arise from improper ordering policies of OEMs. When a supplier receives demand orders infrequently or in very low volumes, it might consider altering its product family and announcing end-of-support which yields supply failure for an OEM.

This research is triggered by an OEM that provides maintenance service for out-of-production aircraft in the Netherlands. Company managers approached the authors with supply failure problems for many spare parts. For most of the parts, supply failures take place without advance notification and they have to deal with lack of proper part supply. They asked for possible indicators that can be used as advance warning in case they do not have a partnership agreement with the supplier.

Our literature review indicates that spare parts supply may also fail due to the factors other than suppliers' decisions such as technology maturity, decreasing demand volume, raw material availability or environmental regulations. These factors are considered in many commercial applications for approximate predictions of supply failure time. Nevertheless in our discussions with employees, we uncovered that only 30% of failures can be explained with those factors. The majority of failures, on the other hand, occur due to suppliers' decision with or without any indication due to economic reasons. Such supply problems are the most difficult and costly ones for the company. Supply chain characteristics; price, lead time, and order interval and order size, are evaluated as potential indicators of supply failure in this study.

In addition to this practical problem, our study aims to fill a research gap in spare part management literature. In their comprehensive study Rojo and Roy (2010) report that the relationship of supply failure with supply chain characteristics and market trends is a research gap that has never been studied before. Considering that the majority of supply problems stems from economic reasons (Figure 2.2), however, supply chain characteristics and market trends potentially have a significant statistical power to explain the supply risk in spare parts.

To investigate our research question, we obtain purchase history data that belongs to spare parts with failed and healthy supply. Supply chain characteristics are measured and their relationships with the failure probability are tested with logistic regression which is a suitable statistical method for dichotomous dependent variables (Agresti, 1996). In this analysis, supply failure stands for the dependent variable whereas supply chain characteristics of each part constitute independent variables. Furthermore, results of logistic regression are tested and confirmed with non-parametric hypothesis tests.

This chapter consists of six sections. Related literature is reviewed in Section 2.2 and the details of problem setting are discussed in Section 2.3. Our research methodology and analysis results are presented in Sections 2.4 to 2.6. Managerial implications of our results take place in Section 2.7.

2.2 Related Literature

Relevant literature for our research question comprises two different research streams. On one hand, there are studies on technology obsolescence, mostly published in engineering journals, which deal with spare part supply problem for high-tech capital goods. On the other hand, studies in operations management and operations research journals consider supply risk for a wide variety of items. Scholars publishing in the former group use statistical tools and qualitative arguments for technology obsolescence issue whereas studies in the latter group employ empirical (and rarely) mathematical models for different aspects of the supply risk. In this chapter, we review both research streams briefly and discuss the position of our study in the literature.

Technology obsolescence studies consist of empirical works for estimating for supply failure time. In this literature, losing a supplier of a spare part or a specific raw material is called "obsolescence" (Rojo and Roy, 2010; Singh and Sandborn, 2006; Singh et al., 2004), and estimating the obsolescence date of a part is referred as "obsolescence forecasting". It is stated that early methods in this research stream are "scorecard" and "availability factor". The former method relies on indexing technology maturity of different components of a spare part and giving specific weights to each component. The risk measure is a weighted average of scores. The latter method employs obsolescence dates of similar parts and part components to calculate obsolescence-risk-free time window for the part (Singh and Sandborn, 2006; Solomon et al., 2000). In recent studies, researchers employ sales data and a life cycle curve, which is assumed to be a Gaussian curve, to forecast obsolescence date of a part. Solomon et al. (2000) assume that an obsolescence takes place in a fixed time window, which is expressed with mean and standard deviation of the Normal distribution fitted to the sales data. This study is extended by Sandborn et al. (2007) by considering part-specific obsolescence windows depending on the manufacturer-specific characteristics. Note that sales data used in these approaches is not always available for OEMs of capital products. Even if it is, reliability and accuracy of such data would be questionable especially if these methods are employed by different parties. In a related research stream, scholars consider leading indicators for major demand changes of short-life cycle products (Meixell and Wu, 2001; Wu et al., 2006; Aytac and Wu, 2013). In this data-driven approach historical demand data is evaluated with different scenarios in a Bayesian fashion. Although the primary motives of these studies are analyzing demand scenarios, they report a nonzero time lag between shifts in demand pattern and some statistically significant indicators. These approaches are not useful for OEMs they exist in the middle of spare part supply chains and their demand is indirectly related with their supply risk. Furthermore, these methods ignore the effects of the supply risk on supply chain characteristics. Obsolescence forecasting and risk assessment studies are reviewed by Rojo and Roy (2010) who report that the relationship between market dynamics and obsolescence risk is understudied in the literature. To the best of our knowledge, apart from Li et al. (2015), there is no study focusing on supply chain indicators for obsolescence risk assessment in the literature. Our study is aimed to fill this gap.

In operations management literature, effects of supply disruption on supply chains are studied by various researchers (Craighead et al., 2007; Ellis et al., 2010; Hendricks and Singhal, 2005a,b; Kleindorfer and Saad, 2005; Zsidisin et al., 2004). Supply disruption refers to all temporary problems including machinery failures as well as a political turmoil in suppliers country. Kleindorfer and Saad (2005) provide a very insightful framework that summarizes disruption studies in operations management literature. They state that management of disruption risk comprises three main tasks: specification of the risk source (see Blackhurst et al. (2008); Chopra et al. (2004)), assessment and quantification of the disruption risk (Zsidisin et al., 2004; Hendricks and Singhal, 2005a,b; Wagner and Bode, 2006); and mitigation of the supply risk (Chopra and Sodhi, 2012; Craighead et al., 2007). In addition, Ellis et al. (2010) study the role of buyers risk perception on their decision making processes. They conclude that decisions of purchase managers are affected by their risk perceptions which are directly related to the buyers control over the risk source. The implicit assumption in all of these studies is that the material supply continues unchanged after a disruption ends. On the contrary, spare part supply failure problems are of more permanent nature and usually the entire supply chain changes when a supply failure case is solved. In that sense, these problems constitute a more problematic subclass of supply disruptions in the literature. To the best of our knowledge, our study is the first one that concentrates on the spare part supply failure problem empirically using logistic regression.

Logistic regression is reported to be appropriate for nonlinear relationship between independent variables and dichotomous dependent variable (Agresti, 1996). This model is utilized in operations management literature in various studies (Keizers et al., 2003; Lapré and Scudder, 2004). Keizers et al. (2003) conduct an empirical study, with logistic regression, in order to diagnose the production planning model being used at a Maintenance Repair Organization (MRO). They search room for improvement in the existing job scheduling system without dealing with the technical details of the underlying algorithm. In another study, Lapré and Scudder (2004) try to explain trade-off between cost and service quality in aviation industry. They transform the dependent variable (service quality) into a dichotomous variable in order to solve the problems with logistic regression. Also Gravier and Swartz (2009) consider logistic regression model for the relationship between technological attributes of a part and its obsolescence risk. In our study, on the other hand, logistic regression is used for studying the relationship between supply risk and changes in supply chain characteristics, which are measured with slopes of respective linear regression models. It is noteworthy that our analysis approach resembles to the meta-modeling applications that are common in simulation literature (Jin et al., 2001). In the following section, a detailed description of the problem setting and conditions, in which the maintenance company operates, are presented.

2.3 Aircraft Original Equipment Manufacturer

Like all capital goods, aircraft are subject to preventive and corrective maintenance during its economic life time. In those maintenance operations, dysfunctional parts need to be replaced with new (or functional) ones. In case of spare part unavailability, the aircraft stays out-of-operation (Aircraft on Ground-AOG) until the spare part is delivered to the repair shop. Therefore, spare part availability is one of the key factors that determine customer satisfaction for maintenance companies in all sectors.

On the other hand, aircraft maintenance and part manufacturing have some special features making the spare part supply chain management problem more complex. Companies in aviation industry are subject to strict regulations from international aviation authorities such as European Aviation Safety Agency (EASA) or Federal Aviation Administration (FAA). These regulatory institutions set up safety rules to airline operators and issue certificates for manufacturers and maintenance organizations. It is strictly stated that parts to be installed to an aircraft must bear a certificate issued by an EASA (or FAA)-approved manufacturing company. Also maintenance tasks can only be performed by approved MROs. These regulations, which almost all airline operators in the Western world are subject to, limit the number of manufacturers in spare part supply. Needless to say, these factors make the management of spare part supply failure problem more critical and more complex.

The company, which we contact with, is an OEM/MRO providing service for out-of-production aircraft in the Netherlands. It manages a supply chain of spare parts to satisfy its customers maintenance demand. Nevertheless, since those capital goods are not manufactured anymore, their spare parts are subject to significant supply failure risk. Due to various factors, part manufacturers tend to stop their production and allocate their capacities to their other products. When this happens, the company faces with a supply failure problem which threatens its service level due to lack of part supply for maintenance. In order to solve these problems, the company has a dedicated technical support group in its purchasing department.

Investigations into this support group reveal that the company follows a seven-step procedure for solution of spare part supply failure cases. These are: last-time buy, buying from secondary market, resourcing to other suppliers, development of Part Manufacturer Approval (PMA), and development of repair, redesign of part and redesign of the system. These procedures are executed in the order of expected cost. A detailed description of each step is presented in the following paragraphs.

Last-time buy is an opportunity of making a final purchase before a supplier stops the manufacturing. The company forecasts its total spare part demand until the expected end-of-service time of aircraft fleet in operation and places a final order to the supplier. Although last-time buys yield high inventory holding costs, this procedure is the most desired solution for the company since it is an opportunity of purchasing a certified spare part from its supplier. Yet it can only be done if the supplier issues a warning beforehand.

Buying at the secondary (surplus) market is the second step for solution of supply failure problems. Secondary market is a common name for spare part trade between different players, such as MROs, repair shops, dismantlers or airline operators in aviation industry. This trade takes place through online trading platforms such as fipart.com or ilsmart.com. Different parties log in to these websites and search for the spare parts that they need. Although secondary market is a cheaper solution for the supply failure there might be some quality problems with those parts and its not a guaranteed supply source in the long term.

Resourcing is the next solution if the last time buy and secondary markets are found to be impossible. For resourcing of a spare part, the company needs to find an alternative, EASA (or FAA) - approved manufacturer. The main purpose of this procedure is to find a new supplier for the identical spare part that customer demands. Nevertheless an alternative supplier is not always available for a spare part.

If no alternative supplier can be found, development of a Parts Manufacturer Approval is taken into account. In this procedure, the company takes the responsibility of the manufacturer and issues its own airworthiness certificate for the spare part. Therefore, a series of quality control tests is applied to manufactured parts by the company. When this long and complicated procedure is impossible, development of repair comes into play as the next procedure. Broken parts are collected from customers and diagnosis procedures are run to test for repairability. After repaired parts are subject to functionality tests, they join the serviceable spare part inventory for the next customer demand.

Redesign of the part is the second most expensive solution of supply failure cases. Components are re-designed with current raw material and manufacturing technologies and they are sent to approval to regulation authorities. Approval of a design is followed by selection of manufacturer and quality control processes for new parts. After all these procedures, new parts become available for selling to customers.

If redesigning the spare part is impossible due to mismatches between functionalities of new design and existing system, redesign of the system is considered in order to solve the problem. This alternative, however, is the most expensive and the longest one for the company. Naturally, it is always the least desired solution in supply failure cases.

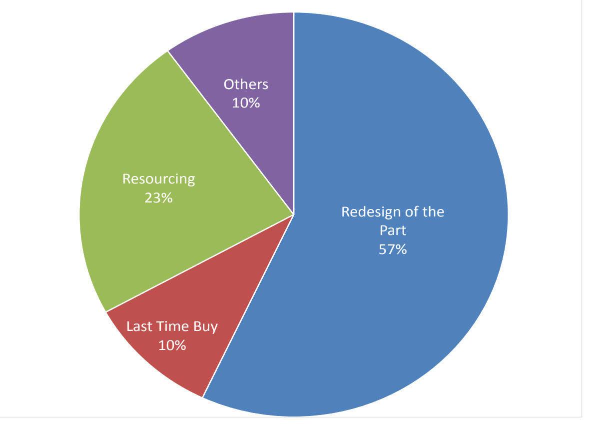

Figure 2.1: Percentages of Seven-Step Supply Failure Solutions

Statistics for solved supply failure problems indicate that part redesign is the most frequently used solution whereas the sum of cheaper solutions constitutes only a minority of all cases (See Figure 2.1). Although it takes longer (up to six months) and costs more, other alternatives are impossible in most cases. Hence, taking precautions proactively against future supply failure problems is critical for staying competitive for the company.

Proactive supply risk management requires advance indicators of failure to allow the company to start preparations for a replacement part. In this study, we focus the relationship between supply failures and variations in supply chain characteristics which are price, lead time, order interval and order size by hypothesizing that those measures can be used as advance indicators for supply problems. Our hypotheses and method of analysis are discussed in the following section.

2.4 Methodology

Failure in spare part supply is a major risk that needs to be considered by OEMs. Our investigations through different departments of the company indicate that the majority of supply problems are due to suppliers' decisions triggered by economic factors such as profitability, capacity utilization etc. In this study, we hypothesize that the effects of those factors on supply chain characteristics, price and lead time, as well as inventory control parameters (frequency and size of replenishment orders), are significant indicators for supply failures. Therefore, the relationship between these features and supply failure risk is analyzed through an empirical analysis of spare part purchases of the company.

The research steps of this empirical analysis is as follows: Firstly, we accessed the database of spare parts for which supply failure already took place. Those parts are grouped according to their functions in an aircraft. Secondly, control-group parts, which have functioning supply chains, are chosen by using the functionality criteria. Next, purchase history data for failed-supply and control group parts is analyzed and supply chain characteristics are measured for each spare part. Finally, hypotheses for the relationship between supply failure probability and supply chain characteristics are built and they are tested with logistic regression model. Results of empirical analysis are verified with the experts in the company.

Data collection phase of this study starts with our access to failed-supply part database maintained by the technical support group in the procurement department. We collect part number, the last supplier, and failure reason and solution information from that database. Analysis of this data set shows that the majority of supply failure cases happen due to suppliers production stop decision rather than technology obsolescence or raw material availability (Figure 2.2). Also 42% of all supply failure cases are found with no specific cause information in that database.

Investigations into these statistics reveal that missing cause data stems from the fact that this column didn't exist when the database was first started. It is added two years after the initialization. Hence, the supply failure entries in this period have no specific cause information. Experts in the technical support group state that this 42% can be assumed to have the same distribution with the rest of supply failure cases.

After a preliminary analysis on failed-supply database, we obtain the purchase history data for those parts and group them based on their structures and functionalities. Our classification scheme results with three main categories for failed-supply spare parts: airframe components, electronics parts and interior parts. The idea behind this classification is that the dynamics of supply chain characteristics might change for different part groups.

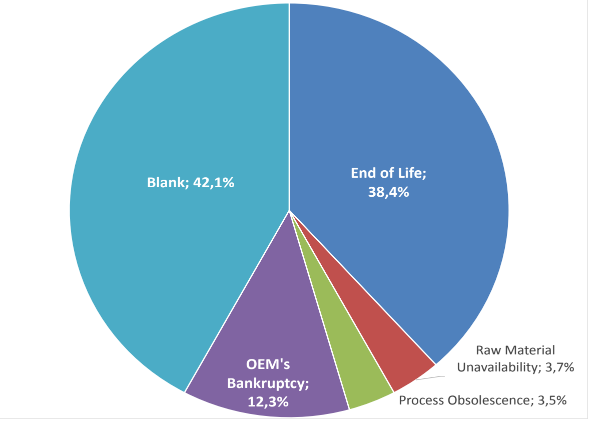

Figure 2.2: Cause Distribution of Supply Failure Cases

For instance, electronic parts have shorter life-span compared to structural parts due to rapidly changing production technologies in electronics industry. Also, external factors, such as environmental regulations or raw material availability, have different impacts on various part groups because of differences between their raw materials and production technologies.

Specifically, airframe components are structural parts in an aircraft. Their main function is providing integrity and preserving the electronic and mechanical components inside. Landing gears or doors are good examples of airframe components in an aircraft. Electronic components, on the other hand, function in the control of aircraft. Their lead times are relatively shorter compared to the other part groups. Finally, interior components have direct contact with passengers. Their supply chains are subject to changes in raw material technologies and strict environmental regulations. Seats or passenger panels are good examples for interior components in an aircraft.

The classification of failed-supply parts is followed by development of control groups in this study. In order to test the impact of supply failure risk, spare parts with a functioning supply chain are taken as control group. We select 50 spare parts for each group among recently purchased 8000 parts within the last six months.

Purchase history data of selected control parts are analyzed carefully and some parts are eliminated from the analysis since they are found suspicious due to successive switches between different suppliers. With this elimination we aim to be completely certain about the health of supply chain for control group parts. The sample size of each group is given in Table 2.1. Also, the data set for a representative spare part, Part A, is depicted in Figures 2.3 and 2.4 below.

Table 2.1: Parts Groups for Obsolete and Control Groups

Group Number	Part Group	Failed-Supply Parts	Control Group
1	Airframe Components	10	45
2	Electronic Parts	33	42
3	Interior Parts	16	48
Total		**59**	**137**

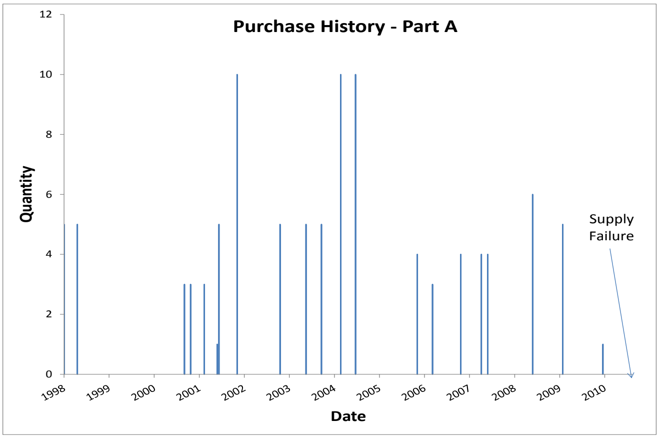

Figure 2.3: Purchase History Data Including Order Size and Order Dates

For the representative part in Figures 2.3 and 2.4, the purchase history dates back to 1998. This is an electronic component and a typical slow moving item. Replenishment orders vary between one and ten (Figure 2.3) and each replenishment order was delivered in less than three months regularly until 2007 (Figure 2.4). In that year fluctuations in supply chain characteristics started and had continued until 03-02-2011. Having received an order from its customer on that day, the company sent a Request for Quote to the part manufacturer which replied with its production stop notification on the same day. Since then the company has sourced this item from the secondary market. Like this part, we possess such historical information for 196 spare parts in our data set. Using this data we calculate each supply chain characteristics as follows:

Firstly, price stands for the amount of money paid to the manufacturer of a spare part. Since we consider historical purchase data in our analysis, we remove the effect

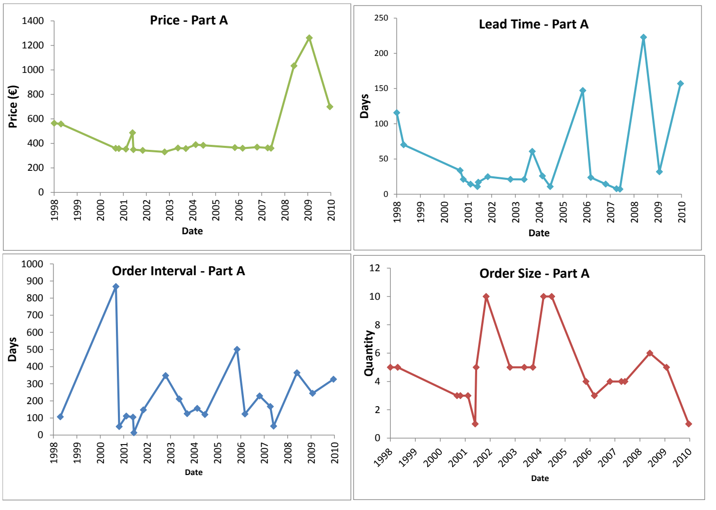

Figure 2.4: Order Interval, Order Size, Price and Lead Time

of inflation from actual price values. In order to do so, we discounted all prices with a certain inflation rate. Also, the effect of fluctuating currency rate is removed from price values of spare parts from multiple suppliers located in different countries.

Secondly, lead time is defined as the amount of time between order and delivery date for each replenishment order. For calculation, we take the time at which the order is physically delivered to the warehouse of the company. In case of partial deliveries, we take the completion date of the whole order as the delivery date. Although this approach seems to create additional variation for lead times, we consider the partial delivery as a different form of lead time increase in our analysis.

Order interval is the time period between successive replenishment orders. Starting from the first entry, order dates of successive purchase entries are subtracted from each other to calculate this measure. An important detail in this calculation is that the time period between last purchase and supply failure date is found to be significantly larger than the average order period for some parts. An exemplary case for such a dominating last time interval is given in Figure 2.5.

The purchase history for the part given in Figure 2.5 shows some irregularities at the beginning. Towards the end, the order sizes rise and the last purchase order is placed on 12-3-2008. Afterwards, no replenishment order was given to the manufacturer and on 10-08-2011 the company received a notification from the supplier stating that the

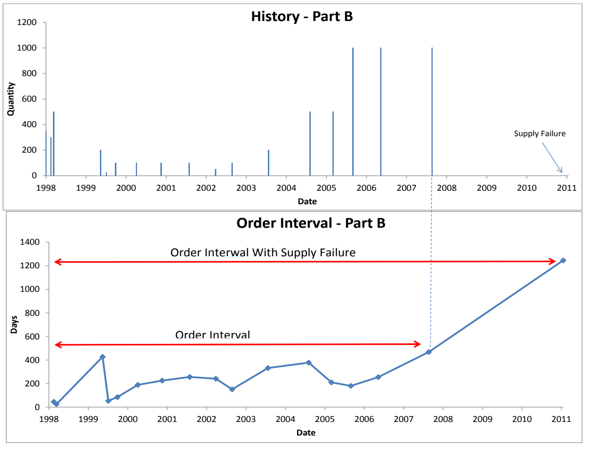

Figure 2.5: Consideration of the Time Period between the Last Purchase and Supply Failure Date

production of this part is completely stopped. In order to capture the effect of such long periods between the supply failure and the last purchase, we define another variable, *order interval with supply failure* (*OrderIntSupFail*), including this interval. At this point we should point out that this extended order interval variable will be compared with regular order intervals in control group since it is impossible to calculate the time period between supply failure and the last purchase for failed-supply parts.

Finally, order size stands for quantity of the item that is ordered to the manufacturer and this variable directly taken from purchase history data without any additional adjustment procedure. Our hypotheses capturing the relationship of these supply chain features with the supply failure probability are discussed in the following subsection.

2.4.1 Hypotheses for Suply Chain Characteristics and Supply Failure Risk

Based on the supply chain management literature, we develop hypotheses for the relationship between supply failure probability and the four supply chain characteristics, price, lead time, order size, larger order interval. In this section, each hypothesis is discussed respectively.

Price of spare part is the first relationship we analyzed. This hypothesis relies on the fact that decreasing installed base of aging capital products yield decaying spare parts demand. Therefore producing such spare parts no longer contributes to the profitability of a supplier who tends to compensate his decreasing profitability by increasing his prices. As an extreme example, spare parts of antique cars are usually more expensive than parts of modern automobiles since such parts are not suited for massive production due to extremely small installed base. Similarly, Cattani and Souza (2003) prove that spare parts suppliers need extra incentives (given that their prices are fixed) to postpone their end-of-support announcements and keep their production line open. In the absence of such (contractual) incentives, it is natural to argue that spare parts suppliers increase their prices as capital products get old. This relationship is articulated in our first hypothesis as follows:

H1: *Increasing price of a spare part is associated with increasing supply failure probability.*

Lead times of replenishment orders are expected to become longer as spare parts orders placed by OEMs of aging capital products decay. The rationale behind this hypothesis follows from the study by Duenyas and Neale (1997). They show that in a queuing system with two customer classes, if one customer has a higher expected demand rate and priority than the other, orders from the customer with low demand rate is backlogged until they reach a certain threshold. The translation of this result to our context is as follows: Spare parts suppliers produce for old and new capital products. Since new capital products yield more spare parts demand compared to the aging ones, suppliers usually give less priority to those orders and backlog them until they reach a certain threshold. This stands for increasing mean and variability of lead times for spare parts of aging capital products. This result also explains the *end-of-support* announcement by suppliers which can be denoted by the backlogging threshold being equal to infinity. Note that suppliers' attitude towards backlogging orders is referred as consolidation of orders. The relationship between supply risk and lead time of spare parts is hypothesized in the following statement:

H2:*Increasing supply failure risk is associated with increasing lead time of replacement orders.*

Note that the previous two characteristics are mainly supplier-related and they are beyond OEMs' control. Since OEMs of capital products are in the middle of those supply chains, spare parts orders received by a spare part supplier is output of the OEM's inventory control policy parameters such as order size and order interval. Specifically, order size is defined as the amount of item that is placed to a part supplier at each replenishment time. Naturally, decreasing order sizes from an OEM is expected to yield lower profitability for the supplier which motivates him to stop producing that spare part and announce end-of-support. This relationship is articulated in the following hypothesis:

H3: *Smaller replenishment order sizes yield higher supply failure probabilities for spare parts.*

Order interval, is the amount of time between successive orders. Less frequent orders to spare part suppliers might lead to a higher probability of supply risk for OEMs. Since each manufacturing equipment ties up some capital for a company, keeping production equipment available for spare parts with infrequent orders decrease return of investment for suppliers. Therefore, spare parts suppliers tend to sell or discard these equipment to increase their liquidity. This relationship between order interval of an OEM and its supply failure risk is presented in the following hypothesis:

H4: *Longer time periods between successive orders, i.e. larger order intervals, lead to a higher probability of supply failure.*

Our last hypothesis in this study is about the relationship between the time period since the last replenishment order and supply failure probability. The reason behind this argument is that a supplier might intend to cancel its manufacturing capability if it does not get any replenishment order for a long period of time (See, Figure 2.5). This might be an important factor for spare parts of out-of-production capital goods which are characteristically subject to decreasing consumption rates. Not ordering a spare part for a long period of time lowers "perceived" order rate of an OEM and this perception can motivate a supplier to announce end-of-support. The hypothesis can be articulated is as follows:

H5: *Longer time periods between supply failure date and the last purchase are associated with higher probabilities of supply failure.*

In our five hypotheses, the association between the supply failure probability and the supply chain characteristics is captured. To test these claims, purchase history data of 196 spare parts is analyzed with the logistic regression model which is presented in the following section.

2.5 Analysis for Testing Hypotheses

Logistic regression is a specific type of Generalized Linear Models. It is suitable for binary response variables and being used in various fields in empirical studies (Agresti, 1996). Since, the existence of supply failure for a spare part is represented with a binary variable in this study, logistic regression is chosen as the analysis methodology. Furthermore, another interesting property of this model is that change in the dependent variable is not the same for all values of independent variables. Specifically, dependent variable changes less near 0 and 1 against one unit increase in independent variable when it is compared to changes around 0.5 (Agresti, 1996). This property also makes the logistic regression model very suitable for the probability of supply failure using independent variables. In fact, Agresti (1996) states that the dependent variable of logistic regression constitutes cumulative distribution function of logistic distribution when there is a single independent variable with a non-negative coefficient. The functional form of the multivariate logistic regression model is given in Equation 2.1 below.

$$\Pi(x) = \frac{exp(\alpha + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n)}{1 + exp(\alpha + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n)}, \tag{2.1}$$

where, $\Pi(x)$ stands for the supply failure probability of all spare parts.

Modeling of supply failure probability with logistic regression requires a single measure, which is capable of representing "meaningful" changes in supply chain characteristics, for each independent variable in logistic regression equation. Specifically, historical data of supply chain characteristics is subject to noise due to variations in daily transactions of the company. However, all of these variations are not significant for supply failure. Our investigations indicate that changes in trends are more important than daily variations for explaining supply failure risk through supply chain characteristics. Therefore, we need to apply a transformation to remove the noise from the data at hand. To do so, we develop a two-step transformation scheme in this study.

In the transformation scheme, a linear regression model is fit to all time series of each supply chain characteristic. In these regression models, purchase dates constitute the predictors of characteristics and entire time series data of each spare part is employed. Then, slopes of the linear regression models are used to calculate the values for the independent variables (x_{ij}) as follows:

$$x_{ij} = 1, \text{ if } \alpha_{ij} > 0; \ 0, \text{otherwise}, \tag{2.2}$$

where α_{ij} is the slope of jth characteristic for part i. By employing this transformation, "necessary" amount of information is taken from data set and we obtain an "overview" of trend changes in all supply chain characteristics.

The categorical dependent variable is formed as follows:

$$y_i = 1, \text{ if the supply is failed; 0, otherwise}, \tag{2.3}$$

where i stands for the spare part index.

At this point we should acknowledge the fact that our transformation scheme has two drawbacks: Firstly, it might underestimate the impact of sudden breaks in the supply chain characteristics. Since linear regression considers the entire history, using its slope possibly undershoots the impact of the structural changes taking place after a certain point in time. This may lead to underestimation of supply failure risk of spare parts. Secondly, using slopes of linear regression without significance (or p-value) information might yield misleading transformations and inaccurate results about the relationship between supply failure and supply chain characteristics. In order to handle these issues, we replicate logistic regression analysis with truncated data sets and apply non-parametric hypothesis tests respectively.

For the first drawback, we check the existence sudden swings and evaluate their effects on our results, if any. To do so, we conduct the same transformation and logistic regression analysis with the data sets including purchase entries only after certain time points. Particularly, we build respective data sets with purchase entries after 2006, 2007 and 2008 and replicate the same analysis using each data set. We find that our results with these data sets yield no significant covariates in the logistic regression. Our interpretation of this result is that there isn't any important structural change, which might affect the results of our analysis, in the historical data of supply chain characteristics. This interpretation is also verified with visual checks of data set at hand.

For the second concern, we employ non-parametric hypothesis tests to our data to check our claims about supply chain characteristics with a different approach. Non-parametric tests are common way of testing hypotheses about sample statistics without assuming any distribution for the sample. Test statistics of these methods are based on the rank and sign transformations which follow certain properties, such as asymptotic normality or binomial distribution under certain assumptions (Maritz, 1995). In non-parametric tests, these properties of rank and sign transformations are employed in order to accept (or reject) the null hypothesis.

In our analysis, Mann-Whitney U (or Wilcoxon rank-sum test) and two-sample median tests, which are non-parametric hypothesis tests for distributions and medians of two independent samples, are employed to compare supply chain characteristics for failed-supply and control groups. The main motivation behind these tests is that we aim to verify logistic regression results and test the impact of our transformation scheme, in which we employ the slopes of linear regression, on the results with a method which is completely independent of any assumption.

For these two tests, we employ the following transformation: the first observation of each supply chain characteristic is subtracted from the last observation and the ration of this difference to the first observation is calculated. For characteristic i with n observations, the variable percent change is formulated as follows:

$$s_i = \frac{x_{i,n} - x_{i,1}}{x_{i,1}} \tag{2.4}$$

Despite abovementioned drawbacks, our analysis method has certain advantages from both theoretical and practical perspectives. Firstly, it is possible to focus on the impacts of long term trends in independent variables using this method. The slope of a regression line summarizes the trend information in the data set and removes the variations from daily transactions. Secondly, our method is generic, simple and easy-to-understand for managers. Measuring changes with a linear regression model and using its slope for further calculations allow managers to have faith in the results since they are capable of understanding the dynamics of the methodology.

Another advantage of our methodology is that it is easy to implement in companies. In our calculations, we use an Excel spread sheet for calculations of independent variables and SPSS 20 for obtaining the logistic regression results. In other words, a supplier assessment tool can easily be developed in the companies with our approach. We should stress that such an assessment tool has absolute importance for all service companies dealing with thousands of spare parts and suppliers. Having advance indicators for possible future supply failure may allow a company to start direct negotiations with the supplier or proactively apply solution steps in Section 2.3.

As noted above, most of supply failure cases are solved with re-design of spare parts in the company. Due to various reasons, such as raw material availability, changes in manufacturing technology or supplier bankrupts, the first five solution steps of the seven step procedure are usually found to be inapplicable in the company. Nevertheless, redesign procedures take a long period of time and such a reactive and time-consuming approach threatens the service level of the company. Therefore, advance indications of any future supply failure problem contribute to the risk management and customer satisfaction at the company.

Results of logistic regression model and non-parametric tests are presented and discussed in the following section.

2.6 Results

In order to analyze the effects of changes in supply chain characteristics on the supply failure, lead time, order size, order interval and order interval with supply failure variables are calculated as described in the previous section. For price variable, specifically, we remove the inflation effect using 2% and 4% discounting rates respectively before the transformation. 4% discount rate is suggested by the company experts as a common approach to inflation removal in aviation industry whereas the usage of 2% is motivated by the average inflation rate in the Netherlands. By discounting with the two inflation rates, we obtain two different data sets including five supply chain characteristics for 196 spare parts. Since results of statistical analyses are similar for the two data sets, we only present results for 2% discount rate in this section.

The main research methodology in our study relies on the comparison of four supply chain characteristics for failed-supply and control part groups. For this purpose, we employ two different transformation schemes, in Equations 2.2, 2.3 and 2.4, to obtain a single value out of time series data of each supply chain characteristic. In the first scheme (Equation 2.2 and 2.3), we calculate slopes of linear regression and transform them to binary variables. Percentage of positive-valued observations in failed-supply and control groups for each characteristic and results of two-proportion z-tests, which compare these percentages with each other using pooled standard errors, are given in Table 2.2.

Table 2.2: Percentage of Spare Parts with Positive-Slope Supply Chain Characteristics

Supply Chain Charact.	**Failed-Supply**	**Control Group**	**P Value of Hyp. Test**
Order Interval	61,7%	62,0%	0.873
Order Size	56,7%	50,8%	0.305
Price	63,3%	77,5%	0.029
Lead Time	53,3%	41,2%	0.128
Order Int. Sup.Fail	81,7%	-	0.009

Results in Table 2.2 indicate that, there is statistically significant difference between two part groups only for price variable. For control group parts, the amount of parts with increasing price values constitutes 77.5% percent of parts in this group whereas this ratio is 63.3% in failed-supply parts. Also, the second significant difference between two groups is found in comparison of extended order interval values with order intervals in control group. Since there is no supply failure in control group parts, *OrderIntSupFail* variable is compared with order interval variable of control group. The result of this test indicates that the time period between the last purchase and the supply failure does create statistically significant difference.

The second transformation employed in this study (Equation 2.4) uses the difference between the first and the last observations of each supply chain characteristic. The ratio of this difference to the first observation gives us the percent change in each variable for each spare part. Average percent changes and p-values of two-sample t-tests are presented in Table 2.3.

Table 2.3: Average Percent Change in Supply Change Characteristics

	Order Interval	Price	Order Size	Lead Time
Failed-Supply	73.83%	75.78%	245.76%	229.68%
Control Group	295.41%	379.17%	154.36%	100.60%
P-Value of Hyp. Test	0.005	0.001	0.283	0.147

Results of hypothesis tests indicate that two groups have different means for price and order interval variable whereas percent change for lead time and order size are not statistically significant from each other. For all supply chain characteristics, we observe increases up to 54 times of the initial value. It seems like these extreme values are the most dominant effect in our hypothesis tests. Thats why we employ non-parametric tests, which are presented in Table 2.4 and 2.5 below, for comparison of supply chain characteristics.

Table 2.4: Non-Parametric Test Results for Distributions of Percent Change Variables

Variable	P-value	Decision
Order interval	0.315	Retain the null hypothesis
Price	0	Reject the null hypothesis
Order size	0.173	Retain the null hypothesis
Lead time	0.39	Retain the null hypothesis

Table 2.5: Non-Parametric Test Results for Medians of Percent Change Variables

Variable	P-value	Decision
Order interval	0.532	Retain the null hypothesis
Price	0	Reject the null hypothesis
Order size	0.616	Retain the null hypothesis
Lead time	0.212	Retain the null hypothesis

For comparison of percent change values across supply failure variable, we employ Mann-Whitney U non-parametric test for distributions and two-sample test for median. These tests have sameness of distributions and equality of medians as null hypotheses respectively. Results of these tests indicate that the distribution and medians of percent values are the same for all supply chain characteristics but the price. The distribution and the median of percent changes for price variable are significantly different for two groups. Therefore, we reach the conclusion that the difference we found in Table 2.3 is meaningful for price value whereas significance of order interval difference is only stems from the extreme values yielded by the transformation scheme.

In order to analyze the relationship between supply failure probability and supply chain measures, we run the logistic regression model. Two data sets are entered to SPSS

20, and binary logistic regression is run for each of them. For logistic regression, a categorical variable, which stands for spare part groups, is added to the model to check for the differences between supply failure probabilities in each part group.

For goodness-of-fit of logistic regression model, Hosmer-Lemeshow test and likelihood ratio test is considered. Hosmer-Lemeshow test compares estimated values with observations. The null hypothesis is that the estimated values are not significantly different from observations for all groups. The test statistics approximately follow Chi-square distribution with $(g - 2)$ degrees of freedom, where g stands for the number of groups (Agresti, 1996). Likelihood ratio test, on the other hand, compares two different models through their maximum value of log-likelihood functions. Usually in this test the model including only constant term is compared with the one including all predictors. The test statistic follows Chi-square distribution with degrees of freedom which is equal to the number of predictors in the model.

Results of Hosmer-Lemeshow test (See Table 2.6) and likelihood ratio tests indicate that the logistic regression fits our data set well. Hosmer-Lemeshow test shows that estimated values are not different from observations whereas likelihood ratio test, of which the test statistics is equal to 229.63 (p value 0.0001), shows full model is significantly different from the one including only regression constant.

Table 2.6: Hosmer - Lemeshow Test

Chi-square	df	P-Value
4.429	8	0.816

Logistic regression results are presented in Table 2.7. As can be seen from these results, price change is found to be the most significant factor in the regression model. Nevertheless, counter-intuitively, the coefficient of price variable is negative in the regression equation. It indicates that price increase is more associated with the control group than failed-supply parts. This interesting result is discussed with the managers at the company in order to find some explanations and verify our intuition into the problem. Two plausible explanations are raised in these discussions.

Firstly, suppliers want to sell all spare part inventory at hand when they are about to make a production stop decision (end-of-support). Especially for out-of-production capital goods, suppliers, which keep spare part stocks, provide price discounts to their customers to eliminate the excess inventory. Secondly, demand for spare parts of out-of-production capital goods characteristically declines due to renewal and discard of old products. In order to compensate their manufacturing set-up costs and to keep their operations profitable, spare part manufacturers need to increase their prices regularly. On the contrary, when they intend to stop production or lose their interest in manufacturing a part, they do not make price increase for it.

Table 2.7: Results of Logistic Regression

Variables in the Equation					
	B	S.E.	Wald	Df	P-Value
OrderInt	-0.1	0.33	0.091	1	0.763
OrderSize	0.206	0.325	0.402	1	0.526
Price	-0.857	0.356	5.789	1	0.016
LeadTime	0.624	0.329	3.586	1	0.058
CompLevel	0.189	0.208	0.829	1	0.363
Constant	-0.952	0.585	2.65	1	0.104

Another slightly significant factor is changes in the lead time. Significance of this variable, with positive regression coefficient, is consistent with our intuition into the supply failure and confirms our second hypothesis. It indicates that spare part manufacturers tend to postpone their delivery dates as they intend to stop the production of a spare part. To sum up, regression results confirm our second hypothesis whereas other hypotheses about the association of supply failure with price, order size and order period are falsified. These findings are consistent with the statistics provided in Table 2.2 through 2.5 above.

Furthermore, as discussed in Section 2.4, it is possible to consider the time period between the last purchase and the supply failure date in the order interval (see Figure 2.5). To see the explanatory power of this extended order interval, we replace the variable *OrderInt* with *OrderIntSupFail* in the regression equation. Regression results are given in Table 2.8 and 2.9.

Table 2.8: Hosmer-Lemeshow Test for Model with Modified Order Interval

Chi-square	df	P-value
4.503	8	0.809

First of all we should note that Hosmer-Lemeshov test shows that the new regression model explains more variability in dependent variable than the previous model (See, Table 2.6). Also log-likelihood value of this model is calculated as 222.429 (p-value 0.0001). Therefore, we can state that the model with *OrderIntSupFail* has more power and provide better predictions of supply failure.

In addition, regression coefficients in Table 2.9 indicate that extended order interval and price are strongly significant covariates in the regression equation. Also, one can argue that lead times have some explanatory power in this new model depending on the

Table 2.9: Logistic Regression Results with Order Interval with Supply Failure Date

Variables in the Equation					
	B	S.E.	Wald	Df	P-Value
OrderIntSupFail	1.032	0.399	6.698	1	0.01
OrderSize	0.081	0.334	0.058	1	0.809
Price	-0.813	0.363	5.01	1	0.025
LeadTime	0.597	0.337	3.14	1	0.076
CompLevel	0.215	0.216	0.994	1	0.319
Constant	-1.775	0.652	7.4	1	0.007

significance level selection. Therefore, results of this logistic regression model confirm our fifth and second hypothesis about the supply failure.

The same analysis is run for each part type in order to see the implications of supply failure on different part types. Logistic regression results for electronic parts are given in Table 2.10 and 2.11. Also, significance level and sings of regression coefficients of logistic regression models for other part groups are depicted in Table 2.12 below.

Apart from high significance level given in Table 2.10, price is found to be a significant covariate in the logistic regression equation. Also, order interval is the second important variable in the model depending on the selection of significance level. These results show that price increase is more associated with control-group electronic parts and explanations above are valid for this part type. In addition, Table 2.11 shows that increasing order intervals are associated with healthy-supply parts. This stands for a clear evidence of decreasing demand volumes that explains negative coefficient of price covariates. Interestingly, changes in lead time are found to be insignificant for this part type. Significant covariates for other part types are given in Table 2.12.

Table 2.10: Hosmer and Lemeshow Test for Logistic Regression for Electronics Parts

Chi-square	df	P-value
1.926	6	0.926

Table 2.11: Logistic Regression Results for Electronics Parts

	B	S.E.	Wald	Df	P-Value
OrderInt	-0.924	0.536	2.976	1	0.085
OrderSize	0.279	0.506	0.304	1	0.581
Price	-1.148	0.583	3.871	1	0.049
LeadTime	0.508	0.508	1.001	1	0.317
Constant	0.854	0.681	1.573	1	0.21

Table 2.12: Summary of Regression Results for Each Part Type

	Airframe Comp.		**Electronic Parts**		**Interior Comp.**	
	Sign	P-Value	Sign	P-Value	Sign	P-Value
Order Interval	(+) / .	0.8 / .	(-) / .	0.09 / .	(+) / .	0.58 / .
OrderIntSupFail	. / (+)	. / 0.23	. / (-)	. / 1	. / (+)	. / 0.03
Order Size	(+) / (-)	0.83 / 0.81	(+) / (+)	0.58 / 0.63	(+) / (-)	0.79 / 0.97
Lead Time	(+) / (+)	0.02 / 0.03	(+) / (+)	0.32 / 0.29	(+) / (+)	0.94 / 0.58
Price	(-) / (-)	0.04 / 0.04	(-) / (-)	0.05 / 0.03	(-) / (+)	0.98 / 0.70

For the results in Table 2.12, we should note that the logistic regression models are calculated first with order interval variables. Afterwards, order intervals of failed-supply group are replaced with extended order interval variable, which includes the time period between supply failure and the last purchase entry. Results of these two models are separated with a slash in each cell in Table 2.12. Specifically, results on the left hand side of slashes belong to models with order intervals whereas values on the right-hand-side are calculated with the variable *OrderIntSupFail*.

Obviously, different factors are important for supply failure of different part types in this analysis. For instance, lead time and price variables are significant for airframe parts whereas extended order interval is the only important factor for interior component. Although these results are not powerful enough due to sample size limitations (see, Table 2.1), they show us the benefit of having respective analysis for different part groups. In addition to the general results about the whole spare part population, such information might enable procurement department employee to make replenishment orders according to part-group-specific characteristics which may result with better management of supply failure risk.

2.7 Managerial Implications

In this chapter, we study the relationship between supply chain characteristics and supply failure probability using an empirical model. Managerial implications of our analysis can be evaluated in two different levels: relevance and usage of our results in the specific company, which we contact with, and generic implications for all companies providing maintenance service for capital goods.

For the OEM, our results indicate that fluctuations in price and lead time of replenishment orders are associated with the supply failure risk of a spare part. Therefore, company managers should follow trend changes in price and lead time of spare parts in order to anticipate irregularities as early as possible. In addition, we find the effect of time period between supply failure and the last purchase significant in our analysis. This finding implies that inventory controllers should place regular replenishment orders for spare parts and those orders should be kept in small sizes for slow moving items. This prevents long time periods between successive orders which might lead to a supply failure case. Needless to say, such a strategy helps for reducing inventory holding cost as well.

Especially, for a company dealing with thousands of parts and suppliers, supply risk assessment has critical importance. By using an assessment tool based on our study, procurement managers can anticipate the risky suppliers which are likely to stop production of spare parts. They can start direct communications with them in order to confirm the findings of our model and they even start supply failure solution procedures proactively.

As discussed above, more than half of the supply failure cases are solved with redesign of the whole part, which is the fifth step in supply failure solution procedure being used in the company. In most cases, redesign procedures take a long period of time depending on the complexity level. Therefore, having advance indications about a possible supply failure also enables the company to start solution procedures before the next demand shows up. Besides, having such indications might increase the usage of less costly solution steps, including making a last-time buy, developing an alternative supplier for the spare part.

Advance indications of potential supply failure can easily be computed by using the empirical model in this study. Having built the data set including categorical dependent and independent variables with an Excel spread sheet, one can compute the supply failure probability for each part using the regression coefficients in Table 2.3 and the formulation given in Equation 2.1. Furthermore, these probabilities can be combined with criticality of spare parts in order to obtain the assessment of supply failure risk of each spare part. We should note that such a risk assessment tool requires re-calculations of regression coefficients as new failed-supply parts appear over time. Naturally, it is also possible to develop a computer program making those calculations internally and producing part (or supplier)-level risk assessment with continuous access to purchase history data and built-in criticality information.

In generic terms, we provide an easy-to-implement supply-failure risk assessment method, which can be converted to a business analytics tool. Thanks to simple structure of logistic regression equation, our analysis procedure can easily be understood by managers. We should admit that it is hard to make generic remarks about our findings since our data set is limited to only one company. Nevertheless, our analysis approach allows practitioners in different sectors to build their own models by considering sector-specific factors that might affect the supply failure probability.

Furthermore, our results constitute counter-evidence for constant price and deterministic lead time assumptions in inventory control models. We find that lead time and price of spare parts change significantly as capital products get older. Therefore, managers in service sector should approach those inventory control models with reluctance to mitigate the impacts of supply failure on their companies. Increasing lead times and fluctuating prices might cripple the service level and annual profits of a company in service sector. From academic point of view, on the other hand, these results point out the requirement of new models assuming random and non-stationary lead time and price for optimal procurement and inventory control.

2.8 Conclusion

Supply failure for spare parts of capital goods is a common problem for all parties that are involved in maintenance activities. The life cycle of spare parts is shorter than the capital goods. This mismatch becomes more problematic when the parent product becomes out-of-production and it requires time consuming solutions in most cases. On the other hand, operators of these capital products cannot tolerate high downtime costs which may lead them to phase those products out earlier than expected in case of spare part scarcity. Replacing capital products before the end of their economic life time is not only a loss for the whole economy, but also it stands for loss of business for OEMs that provide service.

In order to mitigate the effect of the mismatch between the parent product and its parts, companies should take proactive actions for supply failure of spare parts (Rojo and Roy, 2010). Possible actions for a supply failure problem consist of seven-step solution procedure starting from a last-time buy from the part supplier and ending with redesign of the entire system. Regardless of the procedure applied, the efficiency of the solution procedure heavily depends on the existence of advance signals about the future supply failure. Hence development of an analytic tool that can produce advance signals of potential supply failures is crucial for maintenance companies.

Our literature review reveals that different factors might be indicators of future supply failure for spare parts. Technology maturity, environmental regulations, or number of potential suppliers in the market might indicate a potential supply failure in future. Furthermore, supply chain characteristics, price, lead time, order interval and order size, might also be indicators for supply failure based on the suppl chain management literature. Since the former group of indicators are well studied by many researchers, we focus on the latter variables in this study.

To test these claims, purchase history data for failed-supply and healthy parts, which is the control group in this study, is obtained from the company. Changes in supply chain characteristics are measured and two different transformations are applied. In the first scheme, changes in supply chain characteristics over time are expressed by a single variable using slope of linear regression. This transformation allows us focusing on the effect of trends in spare part supply chain on supply failure risk while removing the variation from daily transactions. Calculated values constitute predictors of supply failure probability in logistic regression equation.

In our second transformation, on the other hand, the first and the last entries of the supply chain characteristics are used and percent change between these two values are calculated. Obtained values are used in non-parametric hypothesis tests in which we compare supply chain characteristics for failed-supply and control groups.

Results of our analyses indicate that three factors are significant for supply failure probability. Longer lead times and longer intervals between the last purchase and the supply failure date are associated with supply failure risk. This indicates that increasing lead times of spare parts might be a signal for future supply failure and a long period of time since the last purchase might have a boosting effect on the failure probability. Price increase, on the other hand, is found to be more related with the control group in the analysis. The interpretation of this result is that due to decreasing demand volumes, manufacturers need to increase their prices to keep their operations profitable. When they lose their interest in production of a spare part, they do not make additional increase to sell their spare part inventory.

On one hand, we should acknowledge the fact that our results are far from generality since our data set is limited to only one company. More general results require data set from different companies in different sectors. On the other hand, this weakness does not overshadow the methodology we follow in this study. We propose an easy-to-implement and generic analysis approach for supply failure risk of spare parts. Practitioners in different sectors can easily use this approach with the addition of sector-specific factors for the supply failure. Combining the results of our approach with criticality information of spare parts yields a comprehensive assessment tool for supply failure risk which is a critical issue in maintenance of (especially out-of-production) capital goods.

Furthermore, this study constitutes a first attempt for investigation of the relationship between supply chain measures and upstream risk. Our results point out new research questions for mathematicians in inventory control theory for requirement of new models with more realistic assumptions.

2.9 Epilogue

The material presented in this chapter was extended by Li et al. (2015) using the proportional hazard model and additional data for failed-supply and control group parts. The proportional hazard model is widely utilized in the maintenance literature to estimate equipment breakdowns (Özekici, 2013). Furthermore, Gallagher et al. (2010) used the same model for estimating survival probability of cancer patients. The study by Li et al. (2015) extended the usage of the proportional hazard model to supply failure risk. Also, validation of supply risk estimation was conducted with questionnaires sent to suppliers. It appeared that the results of that study are consistent with the results presented in this chapter.

At the end of the analysis, we formed an application team to build a supply risk assessment tool using the proportional hazard model. The tool is intended to work with a spare parts population consisting of more than 500,000 part numbers. Such a massive application required intensive analysis of the program and its results. Therefore, the application project was designed to consist of three different phases: *building*, *verification* and *validation*.

Building was completed by two graduate students, Joeri Admiraal and Tommy Blom, from the Department of Econometrics at Erasmus University. The tool was programmed with Access macros and SQL queries, which are the programs used in the company. The program consists of procedures for cleaning and processing raw data and calculating the covariates. When the program was complete it yielded extremely low survival probabilities for a very large number of parts. This marks the milestone for the next phase of the project: *verification*.

The *verification* phase started with manually checking all procedures to remove any bugs from the tool. Later, we focused on the parts with extremely low survival probabilities. When we had a closer look at the records, we found some unreasonably small order intervals for some parts. A deeper investigation revealed that the employees maintaining the records were paying more attention to order quantity and price information than the order delivery time. Since the delivery time of replenishment orders is the main input for lead time calculations and order interval, such an approach in record keeping creates irregular measurements for lead time covariates of the model. One good example to such improper record keeping application is the following.

Sometimes, warehouse personnel miscount the number of spare parts delivered to the company and deliveries are recorded in the system with delivery time and order number information. When someone finds the mistake, he puts another record with the same order number but the correct order size. Naturally, the second record enters with its own system time, which creates two different lead times for the same order. Since occasionally such irregularities and "data corrections" take place for many parts, an extra piece of program is required to detect and correct such glitches in the database.

Another difficulty regarding the application project was the limited number of observations for some parts. The data analysis revealed that some non-moving parts had been ordered once or twice since the starting date of the record keeping. Due to the lack of data to calculate covariates, the proportional hazard model generated extremely low survival probabilities, which may lead to a high number of false positives (type-1 error), that is calculating a low survival probability for a "healthy" spare part. To overcome this problem, we removed the parts with less than five observations. Note that false negatives (type-2 error), which represent undetected supply failures, are more dangerous for the company than false positives. However, having too many false positives may cripple the reliability of the tool and might create too much additional work, since each detected supply risk case needs to be investigated manually.

At the time of writing this manuscript, the application of the supply risk assessment tool was in the validation phase, which includes manual evaluation of the results of the risk assessment tool by contacting each supplier.

Chapter 3

Spare Parts Management Under Markov-Modulated Supply Risk

3.1 Introduction

Capital goods usually have a long life span. For instance, aircraft can last up-to 30 years. After the maturity phase of the life cycle, the number of systems in operation starts to decline, since better systems are on the market. Asset owners' have the intention to keep their existing capital goods in operation to maximize their return on investment and for this they rely on maintenance companies. The latter, however, are troubled with the risk of losing their suppliers due to changes in technology (Rojo and Roy, 2010), suppliers' financial problems and bankruptcy (Babich et al., 2007), or simply due to parts becoming less profitable for suppliers, which in our experience is the most common cause. After losing a supplier, maintenance companies will try to restart their spare parts supply process. Depending on the complexity of the manufacturing process and raw material availability this may take up-to one year, especially if the part number needs to be changed and re-certified. Aviation is one of the typical sectors where long recovery times from disruptions occur, partly due to the fact that each supplier manufactures more than one spare part and suppliers possess the technical drawings and proprietary rights.

Empirical evidence from aviation (Li et al., 2015) suggests that supply disruption risk is coupled with lead time variation. Analysis of the empirical data reveals that *increasing* lead time variability is the most important indicator for the risk of losing suppliers for spare parts.

The link between lead time variability and loss of supplier can be explained as follows. Consider the entire manufacturing processes of a supplier as a single queue with a batch processor and arrivals from two customers whose orders cannot be processed in a single batch. When Customer 1 has higher priority than Customer 2, while having a higher expected order rate, Duenyas and Neale (1997) show that orders from Customer 2 are delayed, become more variable, and even completely declined as priority difference gets larger. In our context, Customer 2 may stand for a maintenance company providing service for aircraft, whose priority gets lowered due to aircraft entering their post-maturity phases. This motivates to consider supply disruptions together with lead time variability for inventory control of spare parts.

In this study, we analyze the effects of coupling *non-stationary* random lead times and supply disruptions on inventory performance, which is unique in literature. To this end we formulate a dynamic programming model for the control of spare parts inventory that combines Markov-modulated random lead times with supply disruption risk. A state-dependent base-stock policy is proven to be optimal by showing the equivalence of our original multi-state functional equation to a single-state one which is the technical contribution of this chapter. Furthermore, we suggest a new queuing system, which generates Markov modulated random lead times without order crossovers.

We evaluate the coupled effect of random lead time and supply disruptions as well as their individual effects on total cost under different scenarios. To this end, we set-up scenarios specifying presence or absence of random lead times and supply disruption risk, the type of disruptions (i.e. long and infrequent vs. short and frequent), and the stability characteristics of the supply system. The stability characteristics refer to non-increasing supply risk ("stable supply") and increasing supply risk over time ("unstable supply'). The main results are twofold: First, we find that both random lead times and supply disruptions have substantial effects on costs and service rates. More importantly, their coupled effect can be up-to 10% of the optimal total cost even for low levels of individual risks whereas the coupled effect might be as high as 30% when lead time variability and disruption risk is high. Second, the effect of nonstationarity on the total cost can be as high as the summation of all risk factors combined. In other words, these risk factors should not be studied in isolation, but should be explicitly modeled together in inventory control of spare parts.

An important tactical issue for dealing with supply disruptions is prevention vs. treatment decision which is also recognized as *proactive* vs *reactive* approaches to supply risk mitigation. Due to limited resources, managers usually have to give more priority to either one of these. To address this tactical decision, we compared the savings from decreasing disruption probabilities with increasing recovery probabilities. Prevention of disruptions is found to be more cost-effective when the disruption risk level is low. Increasing severity of the disruptions risk, on the other hand, makes the treatment more beneficial *ceteris paribus.*

As managerial insights we find that managers should consider lead time variability together with supply disruption in their spare parts inventory management as their coupled effect is much bigger than the summation of their individual effects. As we find that nonstationarity is a very important aspect, managers should utilize certain threat signals indicating the level of risk and take proactive or reactive action accordingly.

The remainder of this chapter is structured as follows: In the next section, we position our study within the relevant extant literature. In Section 3.3, we introduce a motivational business case which puts our mathematical model into a business context. Next (Section 3.4), we present our mathematical model and the characterization of the optimal policy. Section 3.5 is devoted to our impact analysis of non-stationary supply risk factors on inventory performance. In the final section (Section 3.6), we discuss our main findings and directions for future research.

3.2 Literature

Relevant literature for our work consists of two main parts: random lead time and supply disruption studies. In the inventory management literature there is ample amount of research on both topics. Stochastic lead times have been of interest for scholars since the 1950s and various approaches have been developed for inventory control in this setting. Supply disruption studies were rather scarce in the early times of inventory research but the subject has been studied extensively during the last two decades. Since our study considers these two forms of supply risk together, we review both topics.

We divide studies on stochastic lead times into two subcategories based on the assumption on the movements of outstanding orders. In the first category, order crossovers are not allowed. This type of supply resembles sequential processors such as queueing systems working under the FIFO principle (Zipkin, 2000).It is known that inventory position constitutes sufficient information for optimal control of such systems (Kaplan, 1970; Ehrhardt, 1984; Song and Zipkin, 1996). Kaplan (1970) shows that the no crossover assumption allows the reduction of a multi-state dynamic programming formulation to a single state one which considers inventory position. The main idea is the same as the one introduced by Scarf (1960) for deterministic lead times. Since the crossover assumption implies that delivery of an order means the delivery of all prior outstanding orders, the inventory system can be controlled optimally through inventory position.

Kaplan's work is extended by Ehrhardt (1984) who utilizes the random variable A_t representing the position of the outstanding order that is delivered in period t (Nahmias, 1979). Independence of A_t and A_{t+1} provides a useful tool for optimality of base stock policies (Ehrhardt, 1984). Since the no order crossover assumption makes the lead times of successive periods dependent, calculating lead time distributions is difficult. To overcome this, Zipkin (1986) suggests the notion of "virtual customer" in a single server queueing system working under the FIFO principle as an exemplary stochastic process for stochastic lead times. In this queueing system, an order is associated with a customer arriving to the queue (Zipkin, 2000). Another significant contribution to this research stream is by Song and Zipkin (1996), who consider Markov-modulated random lead times without supply disruptions. The same lead time process in a multi-echelon setting is considered by Muharremoglu and Tsitsiklis (2008). In addition to these studies, Song (1994b, 1994a) are important contributions which deepen our understanding of stochastic lead times and their effects on base stock levels and optimal costs. These contributions are useful for the monotonicity conditions developed in this chapter.

In the second category of stochastic lead time studies, which is not directly relevant to our study, order crossovers are allowed and lead times of sequential periods are assumed to be identically distributed. This type of supply process resembles a queueing system with an infinite number of parallel servers (Zipkin, 2000). Key papers in this research stream are Robinson et al. (2001), Bradley and Robinson (2005), Hayya et al. (2008).

Supply disruption studies constitute the second main research stream relevant to our research. Disruptions are defined as *temporary* unavailability of supply due to various exogenous reasons. They are characterized by the interarrival times of "up" and "down" states (Tomlin, 2006). In other words, two features of supply disruptions are of interest from an inventory control perspective: length and frequency. For this problem, Özekici and Parlar (1999), an extension to Parlar et al. (1995), consider an exogenous Markov chain which drives supply availability as well as system parameters such as ordering cost and holding cost. Their major assumption is immediate delivery of replenishment orders. Li et al. (2004) analyze supply disruptions occurring with an alternating renewal process, in which interarrival times of successive disruptions follow a general distribution. They find that a base stock policy is optimal if disruptions follow a non-decreasing failure rate distribution. Tomlin (2006) suggests dual sourcing, inventory holding, and acceptance as potential strategies for dealing with supply risk and proves the optimal strategy for deterministic demand. In our study we focus on the coupled effect of supply *disruptions* and random lead times, which are driven by an exogenous Markov chain. Hence, another focus of our study is the nonstationarity in supply-side risk.

Markov chains for modeling dynamic environmental changes is not a new idea. Song and Zipkin (1993), Beyer and Sethi (1997), Gallego and Hu (2004), Cheng and Sethi (1999), Song and Zipkin (1996), Muharremoglu and Tsitsiklis (2008), Scheller-Wolf and Tayur (1999), Arifoğlu and Özekici (2010; 2011) consider Markov chains as a driving mechanism of exogenous factors. Apart from the latter two, all papers assume perfectly observable Markov chains as we choose to do. This modeling choice can be motivated with the empirical evidence by Li et al. (2015).

Studies considering random lead times together with supply unavailability is very scarce in the literature. To the best of our knowledge, the only example is by Mohebbi (2003) who considers both factors in a lost sales environment. In that study Mohebbi assumes an (s,Q) policy and analyzes the system performance numerically. In our study, the focus is on the *non-stationary* nature of random lead times as well as supply disruptions. We also develop a dynamic programming formulation considering order movements explicitly. In addition, Tomlin and Snyder (2006) and Song and Zipkin (1996) are the closest studies to our work. Tomlin and Snyder (2006) consider Markovian supply disruptions with "age-dependent" durations and zero lead times, whereas Song and Zipkin (1996) evaluate Markov-modulated random lead times without disruption risk. Our study extends these two by considering both factors in the same model. As important theoretical contributions, we prove that the base stock policy is optimal for this setting and we develop monotonicity conditions. As an important managerial contribution, we show that from a cost and service level perspective the *coupled* effect of supply disruptions and random lead times is much more prominent then the case where they are considered separately.

3.3 A Motivational Case Study

As a motivational example of this study, we selected a spare part from the group of parts having supply problems. This part (say Part A) is a strip made of polyurethane which for some models is applied on the aircraft's nose in order to distribute static electricity from the nose towards the body. Despite its simplicity the part is critical since accumulated static electricity may jeopardize radio communication.

Our communication with the MRO indicated that the supplier announced end of support on October 10, 2011, since the raw material polyurethane was no longer available. Receiving this notification, the MRO started an investigation with the engineering department and they realized another raw material was available which could provide the same functionality and could be used as a substitute. Since the engineering department needed to develop new technical drawings, the supplier could only re-start manufacturing as of December 9, 2011. The two-month disruption resulted in unsatisfied demand and a decreasing service rate for the company.

Analysis of purchase history data indicated that lead time fluctuations typically increase towards the disruption (as in Figures 3.1 and 3.2). Employees of the procurement department confirmed this phenomenon and explained that suppliers tend to delay the manufacturing of parts that are at end-of-life, since they give priority to other parts.

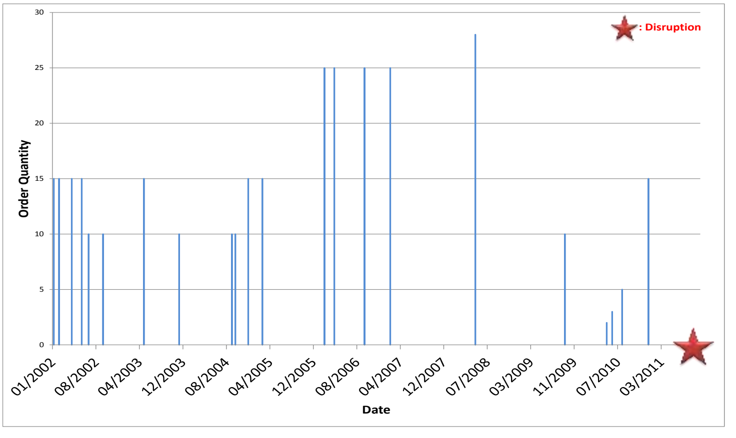

Figure 3.1: Order Quantities for Part A

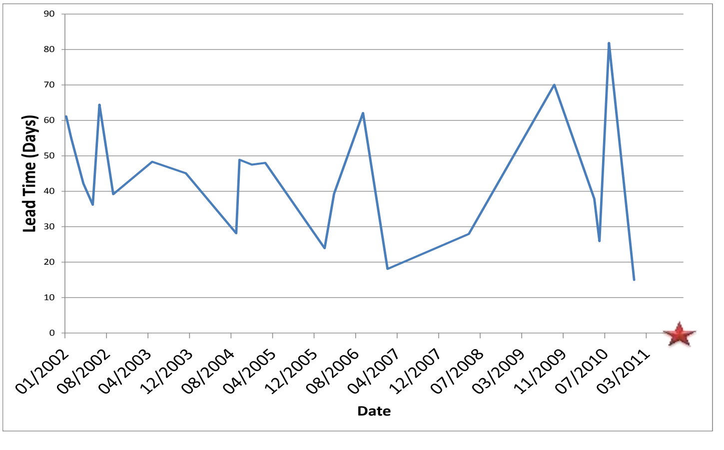

Figure 3.2: Lead Time Process for Part A

As demonstrated in our motivational example, lead time variability and supply disruptions make spare parts management a challenging task for MROs, especially if they use performance contracts. The timely availability of spare parts is essential for avoiding significant downtime costs, e.g aircraft on ground for airline companies (Wong et al., 2007). Although these two risk factors are recognized by managers, most decision tools do not consider them in their calculations due to lack of information or proper models. Personal communications with maintenance companies reveal that managers have an incomplete understanding of the effects of random lead time and supply disruption on their total costs and service levels. In addition, *nonstationarity* of these risks, for which empirical evidence is presented by (Li et al., 2015), is typical for spare part supply chains, as product components and production technologies have their own life-cycles.

In this study, we analyze the effects of coupling *non-stationary* random lead times and supply disruptions on inventory performance, which is unique in literature. To this end we formulate a dynamic programming model for the control of spare parts inventory that combines Markov-modulated random lead times with supply disruption risk. A state-dependent base-stock policy is proven to be optimal by showing the equivalence of our original multi-state functional equation to a single-state one. The main technical contribution of this chapter follows from the proof of optimal policy and monotonicity analysis of the mathematical model, as well as a new stochastic system which can be used for modeling and analysis of Markov-modulated sequential supply systems (Zipkin, 2000).

In addition, we conduct mathematical analysis of the optimal policy for supply failures, which we define as permanent loss of spare parts supply, in Appendix 3.C. As a more problematic subclass of supply disruptions, the optimal policy does not change if the supply system cannot be restarted after a disruption occurs.

Using our mathematical model and the optimal policy, we evaluate the coupled effect of random lead time and supply disruptions as well as their individual effects on the total cost under different scenarios. To this end, we set-up scenarios specifying presence or absence of random lead times and supply disruption risk, the type of disruptions (i.e. long and infrequent vs. short and frequent), and the stability characteristics of the supply system. The stability characteristics refer to non-increasing supply risk ("stable supply") and increasing supply risk over time ("unstable supply').

The main results of these analyses are twofold: First, we find that both random lead times and supply disruptions have substantial effects on both costs and service rates. More importantly, their coupled effect can be up-to 10% of the optimal total cost even for low levels of individual risks. Second, the effect of nonstationarity on the total cost can be as high as the summation of all risk factors combined. In other words, these risk factors should not be studied in isolation, but should be explicitly modeled together in inventory control of spare parts. Note the that results for supply failure case in Appendix 3.C is qualitatively the same but the deviations from the optimal policy are larger when the supply risk factors are ignored.

An important tactical issue for dealing with supply disruptions is prevention vs. treatment decision which is also recognized as *proactive* vs *reactive* approaches to supply risk mitigation. Due to limited resources, managers usually have to give more priority to either prevention or treatment of supply disruptions. To address this tactical decision making problem, we compared the savings from decreasing disruption probabilities with increasing recovery probabilities. Results of these experiments revealed that prevention creates more savings when the disruption risk level is low. Increasing severity of the disruption risk makes the treatment more beneficial *ceteris paribus* .

3.4 Model Formulation

In order to address nonstationary supply risk factors, we consider an exogenous, discrete-time Markov chain that drives the supply system. The states of the Markov chain, which we assume to be perfectly observable, consist of two groups: healthy states and disruption states. In healthy states, the inventory manager can place replenishment orders to the supplier considering the known lead time distribution and supply disruption probability of each state. The lead time distribution and disruption probabilities vary across the healthy states of the Markov chain.

When the Markov chain is in state i, two events are possible at the end of a period: either the supplier stays healthy and jumps to a healthy state with probability $q(i)$, or a supply disruption occurs and the system goes to state d^i with probability $\bar{q}(i)$. In case of disruption, the system is in state d^i, either the system comes back to the associated healthy state with probability $\xi(i)$ or it stays in the same disruption state. $\xi(i)$ can be interpreted as the probability of finding a solution to the supply problem by procurement department within one time period. Note that considering a different disruption state for each healthy state (i and d^i), which is a modeling choice rather than technical requirement, allows us to assign different recovery probabilities for each disruption state. This can be motivated by the fact that solving supply problems may become more difficult as fleet in operation gets older. One possible configuration of the Markov chain, which is consistent with this description, is given in Figure 3.3.

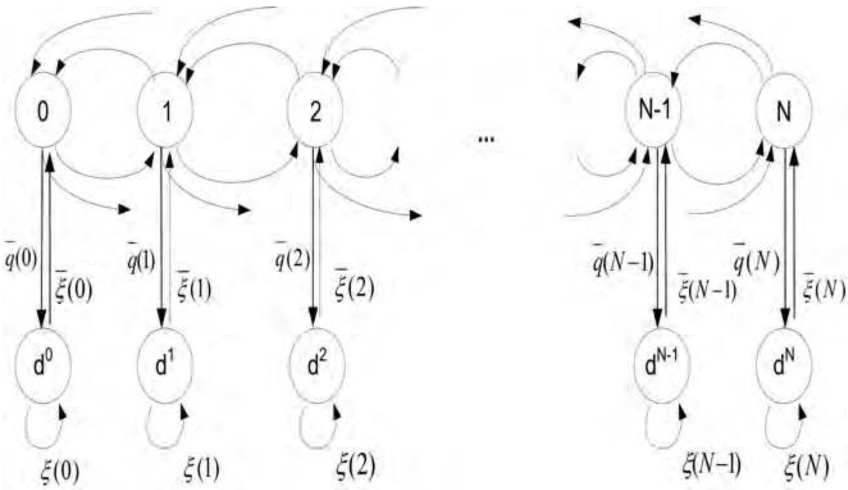

Figure 3.3: State space of the Markov chain driving the supply process.

The transition matrix $\mathcal{P}$ on state space B of the Markov chain has the following form:

$$\mathcal{P} = \begin{pmatrix} \mathcal{Q}P & (I - \mathcal{Q}) \\ \Xi & (I - \Xi) \end{pmatrix} \tag{3.1}$$

where

$$\mathcal{Q}(i,i) = q(i), \mathcal{Q}(i,j) = 0 \ \ \forall\, i \neq j \in B^h,$$

$$\Xi(i,i) = \xi(i), \Xi(i,j) = 0, \ \ \forall\, i \neq j \in B^h,$$

and

$$P = \{p_{ij} : \sum_j p_{ij} = 1, \ \ \forall i, j \in B^h\}.$$

p_{ij} is the transition probability from healthy state i to healthy state j, and B^h is the totally ordered subset of B including healthy states of the Markov chain. Note that if we assume the same ordering relationship between disruption states, i.e. $i \succeq j \Rightarrow d^i \succeq d^j, \ \forall\, i, j \in B^h$, then B becomes a lattice since the product set of two chains forms a lattice. As a modeling choice, disruption probabilities as well as the first two moments of lead time distributions are assumed to be increasing in the indices of the Markov chain states in Figure 3.3. In other words, more "problematic" states are positioned to the "right-hand side" of the Markov chain.

Replenishment orders are delivered to the inventory after a random number of periods. This randomness makes the supply system intractable due to the random movements of outstanding orders. To deal with this problem, Kaplan (1970) suggests the no order crossover assumption which guarantees that no order can be delivered after the ones placed later. We adopt this contribution to our study and assume that outstanding orders cannot cross each other in the supply system.

Assumption 1 *Outstanding orders cannot cross each other in the supply system.*

As a consequence, the position $A(i)$ of an outstanding order when the supply system is in state i is a random variable that is independent of the position of all prior outstanding orders (Nahmias, 1979).

In our context, where we have supply disruptions together with random lead times, outstanding orders after disruption require special attention. In practice, when a disruption occurs, the status of outstanding orders depends on various factors such as the size of the supplier, the commitment level between the two firms, the existence of contractual fines, etc. All possible scenarios for outstanding orders exist between two extreme cases: Firstly, all outstanding orders are preserved after supply disruption, that is, outstanding orders will still be delivered although no new order placements are possible. This scenario is consistent with make-to-stock manufacturing systems and deliveries from overseas manufacturing plants. Secondly, suppliers might cancel all outstanding orders after disruption. This situation is more consistent with make-to-order systems that manufacture slow-moving and high-value capital products and/or their components. In such a case, the company does not receive previous orders nor can it place new ones and has to continue with its existing inventory until the supply system recovers. In this study, we consider the first case, which is articulated in the following assumption. The second case can be analyzed in a similar way by setting delivery probabilities in disruption periods to zero.

Assumption 2 *Deliveries of outstanding orders continue (no new orders can be placed) during disruption periods.*

The order of events in each period is as follows: The inventory manager perfectly observes the supply system and decides that period's replenishment order. The acquisition cost is paid at the time of order placement. After random delivery and demand are realized, holding and shortage costs are incurred and the supplier's state changes. For notation we refer to Table 3.1.

The random outstanding orders in each period require a multi-state recursive equation for the total discounted cost over the planning horizon. Due to the curse of dimensionality, however, mathematical analysis is problematic. Therefore, we develop an equivalent single-state cost function (see Appendix 3.A). The main idea of the state-reduction is

Table 3.1: Notation

h	: holding cost per unit per period
p	: shortage cost per unit per period
c	: acquisition cost per unit per period
α	: discount rate per period
D	: random demand of a single period
D_l	: l-period convolution of random demand D.
$L(i)$	: random lead time of an order when the supplier is in state i
$A^l(i)$	: position of the earliest outstanding order to be delivered within $l-1$ periods.
i_+^*	: random variable indicating the next healthy state after state i.
d^i	: random variable indicating the disruption state of healthy state i.

to combine all future holding and backlog costs with the current period's acquisition costs (Kaplan 1970). In the remainder of the chapter, we continue with the reduced cost function.

We define holding and backlog cost l-periods from now, given that the current inventory level is x, as follows:

$$C^l(x) = \alpha^l \mathbf{E}\big[h \max(x - D_{l+1}, 0) + p \max(D_{l+1} - x, 0)\big]. \tag{3.2}$$

In *stationary* random lead time models, the cost function in Equation 3.2 would be weighted with lead time probabilities to obtain the expected single period costs. In *non-stationary* systems, however, lead time probabilities should be considered together with Markov transition probabilities until delivery takes place, as they are dependent. Given that the supplier is in state i and the inventory level is equal to x, the single period cost function, due to Song and Zipkin (1996), is as follows:

$$\hat{C}(i,x) = \sum_{l \geq 0} Pr\{L(i) \leq l \leq L(i_+)\} C^l(x), \tag{3.3}$$

The single period cost function for a disruption state, $\hat{C}(d^i, x)$, can be obtained by replacing i and i_+ with d^i and d^i_+ (see Theorem 2 in Appendix 3.A).

If an order is placed to the supplier when it is in state i, the probability of this order being delivered within l periods is $Pr\{L(i) \leq l\}$. The probability of the next period's order being delivered later than l-periods is $Pr\{L(i_+) > l\}$. Therefore $Pr\{L(i) \leq l \leq L(i_+)\} = Pr\{L(i) \leq l\} - Pr\{L(i_+) > l\}$ gives the probability of this period's order only covering the demand of the next l-periods. This concept is dubbed *inventory coverage* by Song and Zipkin (1996). From that perspective, the function $\hat{C}(.)$ is a mere extension of the single-period cost function in Ehrhardt (1984) and Kaplan (1970). Furthermore, the following lemma from Song and Zipkin (1996) is useful for developing insight into the probability statement $Pr\{L(i) \leq l \leq L(i_+)\}$ and cost function $\hat{C}(i,x)$.

Lemma 1 *(Song and Zipkin, 1996) If the (supply) process* **i** is stationary, then

$$Pr\{L(i) \leq l \leq L(i_+)\} = Pr\{L(i) = l\}.$$

Multi-period cost function for the problem is given in Equation 3.4 which consists of single period cost and two cost terms associated with two possibilities for a healthy state i: Cost term associated with being in another healthy state is $\alpha q(i)\mathbf{E}\tilde{f}_{n-1}(i_+^*, y-D)$, and the cost term for the possibility of jumping to the disruption in the next period $\alpha\bar{q}(i)\mathbf{E}\tilde{g}_{n-1}(d^i, y-D)$. In Equations 3.4 and 3.5, x represents the sum of outstanding orders at the beginning of a period, whereas y stands for the inventory position after the order placement. These equations can be interpreted as follows: In each period the decision maker places an order considering the supplier's state, its disruption probability, holding, shortage, and acquisition costs.

$$\tilde{f}_n(i,x) = \min_{y \geq x}\{c(y-x) + q(i)\hat{C}(i,y) + \bar{q}(i)\hat{C}(d^i,y) + \alpha q(i)\mathbf{E}\tilde{f}_{n-1}(i_+^*, y-D) \\ + \alpha\bar{q}(i)\mathbf{E}\tilde{g}_{n-1}(d^i, y-D)\},\ i, i_+^* \in B^h, \tag{3.4}$$

and

$$\tilde{g}_n(d^i,x) = \hat{C}(d^i,x) + \alpha\xi(i)\mathbf{E}\tilde{g}_{n-1}(d^i, x-D) + \bar{\xi}(i)\mathbf{E}\tilde{f}_{n-1}(i, x-D). \tag{3.5}$$

The following section gives summary of analytic characterization of the optimal base stock levels.

3.4.1 Optimal Policy

To analyze the function $\tilde{f}_n(i,x)$ and derive the optimal control policy, we utilize the following transformation first introduced by Veinott (1965): $W_n(i,x) = \tilde{f}_n(i,x) + cx$. This leads to $W_n(i,x) = \min\{G_n(i,y) : y \geq x\}$, where,

$$G_n(i,y) = cy(1-\alpha q(i)) + q(i)\hat{C}(i,y) + \bar{q}(i)\hat{C}(d^i,y) + \alpha q(i)EW_{n-1}(i_+^*, y-D) + \alpha\bar{q}(i)E\tilde{g}_{n-1}(d^i, y-D). \tag{3.6}$$

In Equation 3.6, $cy(1-\alpha q(i))$ stands for the trade-off between purchasing this period or leaving it to the next one. This trade-off includes the effect of discounting combined with the disruption risk. Also, we should note that the structure of Equation 3.6 is the same as the function $G(y)$ in Song and Zipkin (1996) if we take $q(i) = 1$. Lemma 2 establishes the convexity of single period cost functions.

Lemma 2 *Both of $\hat{C}(i, y)$ and $\hat{C}(d^i, y)$ are convex in y.*

The proof is given in Appendix 3.A. Theorem 1 states the convexity of Equations 3.5 and 3.6, and the optimal policy.

Theorem 1 *Following statements are true:*

1. *$\tilde{g}_n(d^i, x)$, $G_n(i, x)$, $W_n(i, x)$ are convex in x,*
2. *a state-dependent base stock policy is optimal.*

The proof of the theorem is presented in Appendix 3.A. The optimal policy can be characterized with $S_n(i)$, which is the optimal inventory position after the replenishment order when there are n periods ahead and the supply system is in state i. We analyzed monotonicity conditions for $S_n(i)$ and derived sufficient conditions for monotone base stock levels over Markov states. Unfortunately, these conditions are very intricate and it is hard to develop intuition from them. Therefore, we omit them in this chapter and proceed to the analysis of random lead times supply disruption (and their coupled effects) on total cost and service levels.

3.5 Impact Analysis for Nonstationary Supply Risk Factors

To investigate the combined effect of random lead time and supply disruption, we need to construct a stochastic process which generates Markovian random lead times and disruption events (Section 3.5.1). Using this stochastic process, we calculate the optimal base stock levels under different risk scenarios (that is, considering only one risk factor, or both, or none). Subsequently, the performance of these optimal policies are tested with simulation in the benchmark scenario, which includes both risk factors (Section 3.5.2). In this way, we analyze the impact of ignoring one or both of the risks in terms of costs and service level.

3.5.1 A Queueing System to Model Random Lead Time and Supply Disruptions

We need a stochastic process which a) is driven by an exogenous Markov chain, b) is capable of producing state-dependent lead time distributions, and c) precludes order crossovers. Song and Zipkin (1996) suggest three different stochastic processes that satisfy these conditions. In this study, we consider a queueing system consisting of two semi-dependent queues.

For the exogenous Markov chain (condition a), we consider a discrete-time Bernoulli queue, which is dubbed Queue #1 and depicted in Figure 3.4. The number of items in this queue determines the healthy states of the Markov chain. To include supply disruptions, we modify this queuing system with state-dependent disruption probabilities. Specifically, at the end of each period when there are i items in the system, the supply process stays healthy with probability $q(i)$ or a disruption occurs with probability $1 - q(i)$. Given that it stays healthy, an item arrives at Queue #1 with probability e, and an item leaves the queue with probability d. These probabilities allow us to directly calculate the Markov chain transition matrix in (3.1). After a supply disruption, neither arrivals nor departures are enabled until the system jumps back to the associated healthy state.

To generate state-dependent random lead times without order crossover, we consider another discrete-time queue with *partial-batch bulk* service, with batch size K, and finite queue capacity, C, where $K = C$. This queueing system is dubbed Queue #2 in Figure 3.4. A possible example of such a queueing system, besides our supply chain context, is a ferry port, in which the queueing area is equal to the capacity of a single ferry. The FIFO rule applied to this queue precludes order crossovers, whereas the partial-batch server provides completely random deliveries independent of previous orders. In our supply chain context, an example is a production manager who makes decisions for consolidating customer orders.

The effect of the Markov chain on the delivery system in Queue #2 is obtained by the process rate of the partial-batch server, which is dependent on the number of items in Queue #1. In our ferry port example, the Markovian state variable may stand for random weather conditions affecting the departure or arrival of ferries, whereas in our context it could be an exogenous factor effecting the consolidation frequency.

In each period, an item arrives at Queue #2 with probability a. This item is associated with that period's replenishment order if there is an available space in the queue (enters position 0 to random outstanding order vector). Otherwise, the arriving item is discarded and that period's order is added to the order associated with the last item in the queue

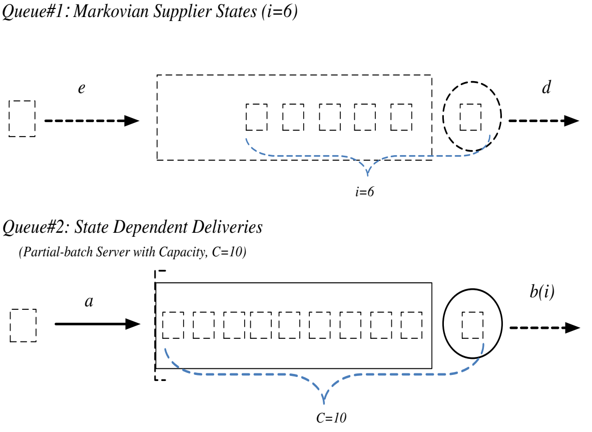

Figure 3.4: Queueing Systems for Impact Analysis

(position 1 in outstanding order vector). In each period, the server releases all items in the queue, since $K = C$, with probability $b(i)$, where i is the state of the Markov chain. Otherwise, all items wait in the queue. One possible realization of the system is depicted in Figure 3.4 when $i = 6$ and $C = K = 10$.

For a non-stationary supply system, there are two possible extreme scenarios: the system stays healthy over the entire planning horizon, or it proceeds to more risky states and eventually fails. In order to evaluate these two scenarios, we run the stochastic process with different e and d values. Due to the way that we order Markov chain states (Section 3.4), $e < d$ implies the stable supply scenario in which the supplier moves to a healthier state with higher probability than moving to a riskier state. A possible example of this situation is spare parts which are at the beginning of their life cycle. Even if exogenous changes occur, the supply system stays stable for such parts. In the other extreme, we have the unstable scenario, that is $e > d$. This situation occurs especially when capital products are at the final phase of their life cycle. Suppliers of spare parts tend to stop manufacturing over time and this tendency is reflected in increasing lead times and higher supply disruption probabilities.

Furthermore, two types of disruptions are recognized in the literature: Long and infrequent disruptions (LID) and short and frequent disruptions (SFD). In order to evaluate the effect of these two types of disruptions, we consider them in our scenario analysis in addition to the stable and unstable supply process.

In the scenario analysis, we evaluated the effect of each individual risk factor as well as their combined effect on total cost and service levels. Besides the scenarios with random lead times, we consider scenarios with deterministic lead times, while keeping expected lead times equal (Runs #1 & #3 and #2 & #4 in Table 3.2). Similarly, besides the scenarios with supply disruption risk, we run scenarios without disruption risk by setting associated probabilities to zero (Runs #1 & #5 and #3 & #7 in Table 3.2). Since each computation is conducted for unstable and stable supply scenarios together with LID and SFD respectively, we obtain sixteen different scenarios as given in Table 3.2.

Table 3.2: Specifications of Scenarios with Markovian Lead Time and Supply Disruptions

Run#	Supp. Tendency	Disrpt. Type	Rnd. LT	Disruption	Scenario
1	Unstable	LID	NO	NO	Det. LT
2	Stable	LID	NO	NO	Det. LT
3	Unstable	LID	YES	NO	Rnd. LT
4	Stable	LID	YES	NO	Rnd. LT
5	Unstable	LID	NO	YES	Det. LT & Disrpt.
6	Stable	LID	NO	YES	Det. LT & Disrpt.
7	Unstable	LID	YES	YES	Rnd. LT & Disrpt.
8	Stable	LID	YES	YES	Rnd. LT & Disrpt.
9	Unstable	SFD	NO	NO	Det. LT
10	Stable	SFD	NO	NO	Det. LT
11	Unstable	SFD	YES	NO	Rnd. LT
12	Stable	SFD	YES	NO	Rnd. LT
13	Unstable	SFD	NO	YES	Det. LT & Disrpt.
14	Stable	SFD	NO	YES	Det. LT & Disrpt.
15	Unstable	SFD	YES	YES	Rnd. LT & Disrpt.
16	Stable	SFD	YES	YES	Rnd. LT & Disrpt.

In addition to these runs, we consider a scenario with state-*independent* deterministic lead time without supply disruption. In this scenario, which is dubbed Run#0, deterministic lead time is assumed to be equal to the average of expected lead times of all states. Calculated optimal base stock levels of each scenario in Table 3.2 are used in a simulation model of the benchmark scenario to compare the effect of ignoring both supply risks on the inventory performance. The development of the scenarios and their results are presented in the following sections.

3.5.2 Setup of The Computational Study

In order to evaluate deviation of each run from the optimal policy (benchmark), we fed finite-horizon order-up-to levels into a simulation model. This procedure started with selection of parameter values for disruption and supply recovery probabilities as well as random lead time for each state.

In order to obtain realistic and applicable results, we considered a Markov chain consists of three healthy and three disruption states. Each disruption state was assumed to be associated with only one healthy state and the set of healthy states were assumed to be totally ordered within each other.

For state-dependent random lead times, we considered two different sets of parameter values for the service rate of Queue #2 ($b(i)$), given in Table 3.3. As can be seen from the table, parameter set 1 was considered to see the effect of significant lead time variations over Markov states. The parameter set 2 aimed to investigate the supply risk factors when the first two moments of lead time distributions are very close to zero. This way, we aimed to develop a better understanding for the interaction between supply disruption and random lead time.

Table 3.3: Random Lead Time Parameters for Each Markov State

Parameter Set	State 0	State 2	State 3
Set 1	0.6	0.4	0.2
Set 2	0.9	0.85	0.8

For disruption behavior of the model, we calculated disruption and recovery probabilities ($q(i)$ and $\xi(i)$ for $i = 1, 2, 3$) that make expected number of disruption periods equal to 5%, 10% and 15% of the planning horizon under four different supply scenarios: stable-LID, unstable-LID, stable-SFD and unstable-SFD. Details of these calculations and calculated parameter values are given in Appendix 3.B.

Using the parameter values in Table 3.3 and Appendix 3.B, we calculated the optimal base stock levels using the value iteration algorithm for 100 periods. The finite horizon base stock levels for the benchmark scenario (unstable-LID with both lead time and supply disruption risks) are given in Figure 3.5. As can be seen, all base stock levels converge to an infinite horizon base stock level and end-of-horizon effect appears there are 10 periods are remaining in the planning horizon. Also, there are significant differences between base stock levels of different states.

To evaluate the performance of the optimal policy, we developed a simulation model. The main reason behind this approach was that it requires to evaluate multi-state dynamic programming formulation for the total cost. This also stands for the main motivation behind Theorem 2 in Appendix 3.A.

The performance measures we track in our simulation model are total discounted cost, total discounted backlog cost, ready rate (fraction of time with positive stock on hand) and

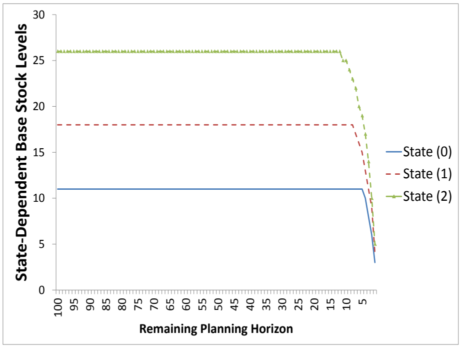

Figure 3.5: Base Stock Levels for the Benchmark Scenario

fill rate (fraction of demand that can be satisfied immediately from stock on hand (Axsäter, 2006)). Total cost and total backlog costs are common performance measures in inventory control simulations. Ready rate and fill rate are important service measures for the service sector, since most customer contracts utilize one of these (Oliva and Kallenberg, 2003). To determine the number of replications, we first conduct a pilot study consisting of 5000 replications. Results of this study are used to compute the total number of replications which is set to 50,000. To control the variance, we use common random numbers which cause dependency between replications. Therefore, paired t-tests are employed to check whether there is statistically significant difference between scenarios.

The discount rate per period is set to 0.995, which leads to a 6% annual discount rate over the entire planning horizon, since a period stands for a month for our empirical analysis presented below. Without loss of generality, we set the acquisition cost equal to 2 per item. The holding cost is equal to 0.2 and backlog cost is equal to 4 per item per period (0.1 and 2 are taken as holding and backlog cost rate multipliers). Random demand in each period is assumed to follow a Poisson distribution with mean 2.

3.5.3 Coupled Effect of Random Lead Time and Supply Disruption

In this section we present total cost and service rate values for unstable-LID scenario with 5% of the planning horizon as disruption periods. The rest of results ((un)stable-SFD with 10% and 15% disruption) are given with stacked-bar charts indicating deviations from the optimal total cost created by each supply risk factor.

Results of unstable-LID scenario indicate that ignoring random lead times creates larger total and backlog cost compared to ignoring disruption risk in the system. Calculated deviations from the benchmark (policy with random lead time and supply disruption in Figure 3.6) are 11.3% and 4.9% of the optimal cost for these two risk factors whereas ignoring both risk factors (policy with deterministic lead time) creates 27.6% deviation from the benchmark. In other words, the combined effect of the two risk factors is almost twice of the summation of their individual effects. This indicates the importance of considering both risk factors in a single model which is commonly ignored in practice as well as in literature. Finally, we consider the value of recognizing nonstationarity by comparing state-independent deterministic lead time with the benchmark. The deviation from the optimal cost is 45.52%. Therefore, we conclude that value effect of nonstationarity on total cost is as large as the summation of all other risk factors for this parameter set.

Similar results are obtained for service rates using Figure 3.7. Specifically, ignoring all risk factors yield a service level around 65% which is unacceptable in practice. On the other hand, recognizing random lead times creates service level very close to the benchmark. Note that these deviations under different scenarios explain extra surplus inventories in service sector as follows: Inventory managers implicitly aware that their models do not consider many different risk factors. Hence, they prefer carrying extra inventory to avoid stock-outs due to unexpected supply chain glitches. We can argue that employing threat-dependent control policies might lead them to obtain the same service levels with lower inventory levels.

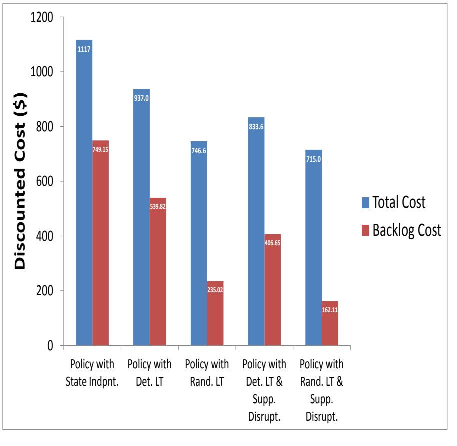

Figure 3.6: Discounted Total and Backlog Cost for Unstable-LID Scenario for Lead Time Parameter Set 1

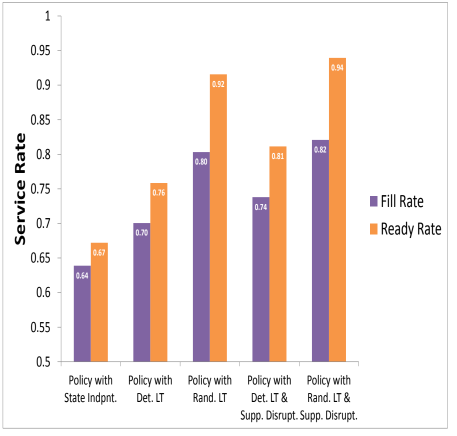

Figure 3.7: Service Rates for Unstable-LID Scenario for Lead Time Parameter Set 1

Due to space limitations, analysis results for stable-LID and (un)stable-SFD scenarios are presented through total cost deviations from the benchmark in Figures 3.8 and 3.9. In these charts, the effect of each supply risk is calculated using total cost values from the associated scenario. Specifically the formulation of the percent cost deviation due to random lead time for unstable-LID is

$$\Delta_{RLT} = \frac{TC_5 - TC_7}{TC_7},$$

where TC_i is the discounted total cost of Run$\#i$ in Table 3.2. The percent deviation due to disruption ($\Delta_{Disrupt}$) is calculated by replacing TC_5 with TC_3 in the same formula. Formulations for deviations due the coupled effect $\Delta_{coupled}$ and nonstationarity $\Delta_{nonsta.}$ are given below:

$$\Delta_{coupled} = \frac{TC_1 - TC_7}{TC_7} - \Delta_{RLT} - \Delta_{Disrupt},$$

$$\Delta_{nonsta.} = \frac{TC_0 - TC_7}{TC_7} - \Delta_{coupled}.$$

With these formulations we aim to see effects of each factors on the total cost when they are ignored by the decision maker. Naturally, percent deviations for other scenarios, stable-LID and (un)stable-SFD, can be calculated using appropriate runs from Table 3.2. Percent deviations for all scenarios are presented in Figures 3.8 and 3.9.

Results in Figures 3.8 and 3.9 indicate that increasing expected number of disruptions leads to larger deviations due to disruption as well as the coupled effect whereas it depletes

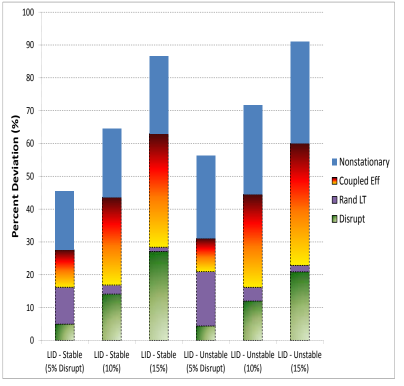

Figure 3.8: Percent Deviation from the Optimal Total Cost for LID, Lead Time Set-1

the deviation due random lead time. This is due to the fact that increasing disruption risk leads to higher inventory levels which mitigates the effect of random lead time on inventory performance. Furthermore, the effect of nonstationarity is larger for unstable supply scenarios compared to stable ones. Expectedly, when the supply health tends to get worse over time, such as in aging aircraft, ignoring supply-side risk creates larger deviation in total discounted costs.

Another important observation can be done between the two types of disruptions. Results indicate that long and infrequent disruptions have larger effect on system performance compared to short and frequent disruptions although the expected number of disruption periods are the same. This also holds for the coupled effect of random lead time and supply disruptions.

Results of the same experiments with random lead time parameter set 2 (Table 3.3) are given in Figures 3.10 and 3.11. Under this scenario, the effect of disruption is as large as 50% and the coupled effect is up-to 11% of total optimal cost. Also, we find that the effect of nonstationarity on total cost deviation is almost zero in these runs (that's why we didn't depict them in Figures 3.10 and 3.11). This indicates that state-dependent lead time distributions are more important for nonstationarity than state dependent disruption probabilities.

Furthermore, a closer look to all figures reveal that the deviation due to disruption and random lead time is larger in the stable supply scenario compared to the unstable one. The reason behind this intriguing result is that under stable supply scenario system

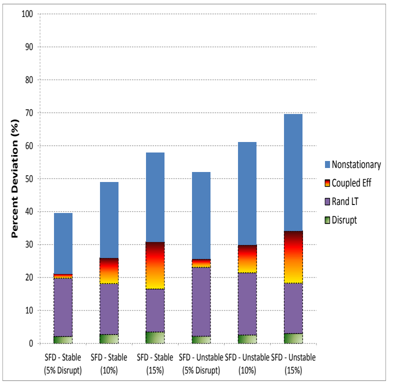

Figure 3.9: Percent Deviation from the Optimal Total Cost for SFD, Lead Time Set-1

carries less inventory which can compensate the effects of disruptions and random lead time.

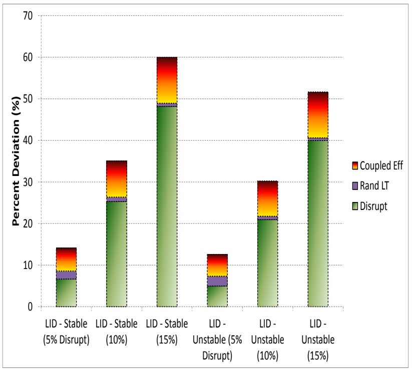

Figure 3.10: Percent Deviation from the Optimal Total Cost for LID in Lead Time Set-2

In order to see the same effects under different cost parameters, we run a sensitivity analysis in which we consider $\{0.2, 0.3, 0.4, 0.6\}$ as holding cost rates and $\{0.9, 0.95, 0.99, 0.995\}$ as service levels which are used to calculate backlog cost rates using the critical fractile.

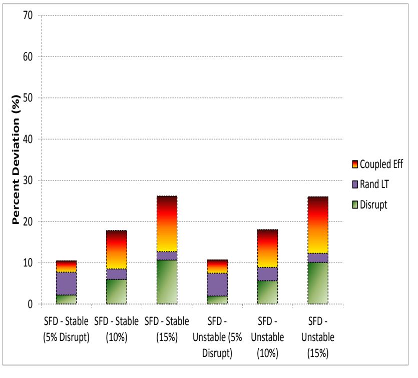

Figure 3.11: Percent Deviation from the Optimal Total Cost for SFD in Lead Time Set-2

Calculated holding and backlog cost rates are multiplied with the acquisition cost, 2 per item, to obtain cost parameters of the analysis.

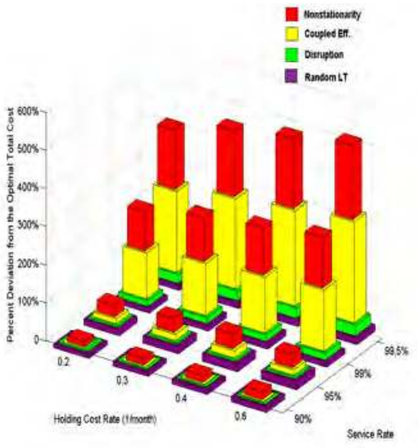

Figure 3.12: Percent Deviations from the Total Costs for LID-Unstable with 5 % Expected Disruption

Results of the sensitivity analysis are only presented for LID-unstable and SFD-stable scenarios since these two scenarios stand for upper and lower bounds for effects of supply risk on total cost and service rates. Our sensitivity analysis indicates that the coupled

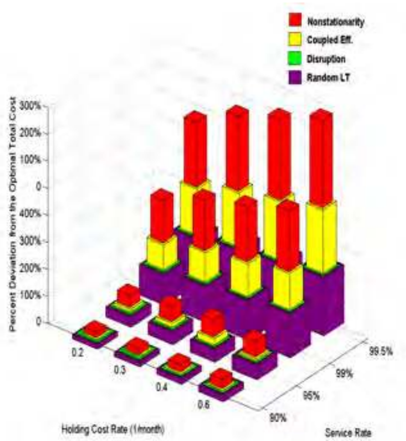

Figure 3.13: Percent Deviation from the Total Costs for SFD-Stable with 5% Expected Disruption

effect of random lead time and disruptions can be larger than 200% of the total optimal cost for high service levels under the LID-unstable scenario. When all risks are ignored the total cost deviation can rise up-to 500% of the optimal cost when the service rate is set to 99.5%. The nature of the deviation is much different for short and frequent disruptions for which the effect of random lead time is as high as the coupled effect of the two scenarios. These results indicate the importance of considering both risk factors in a single model to aim higher service rates with more reasonable inventory levels. In the next section, we discuss the comparison between two tactical level approaches to supply risk mitigation: prevention and solution.

3.5.4 Dealing with Supply Disruptions: Prevention vs. Treatment

On the tactical level, employees of procurement departments can deal with supply disruption in two different forms: prevention and/or solution. Prevention of supply disruptions can be performed by means of advance warning signals which utilize some indicators to predict future supply disruptions (Li et al. (2015), Hendricks and Singhal (2003), Hendricks and Singhal (2005a), Tomlin and Snyder (2006)). By using these advance signals, employee of procurement departments can *proactively* start a procedure to address the problem. Such procedures can be exemplified with direct communication with supplier or development of an alternative supplier by procurement departments.

All efforts for preventing supply disruptions can be represented as decreasing disruption probabilities of healthy states in our model. To analyze the value of these efforts for a company, we executed calculations with decreasing supply disruption probabilities and measure the percent decrease in total cost. Specifically, we calculated optimal base stock levels, cost and service rate figures by replacing $q(j)$ with $\tilde{q}(j)$ given below:

$$\tilde{q}(j) = q(j) - \beta, \text{ for } j \neq 0 \text{ and } \beta = 0.002 * k, \tag{3.7}$$

for $k = 1..9$. β represents the average change in supply disruptions as a result of efforts in the associate department of the company whereas k stands for the amounts of effort put into the prevention. We set the unit change in disruption probability to a very small value (0.002) in order to see the effects of smallest changes on the total cost.

Conversely, solution of disruptions stands for the efforts spent to investigate and solve the disruption problems and restart the supply process after the disruption takes place. All these efforts are recognized as *reactive* approaches to disruption mitigation. The effect of extra efforts into developing solutions *reactively* can be represented with increasing disruption recovery probabilities which can be formulated below:

$$\tilde{\xi}(j) = \xi(j) + \gamma,\ j \neq 0 \ \text{ and } \gamma = 0.002 * k,\ \text{ for } k = 1..9. \tag{3.8}$$

By replacing disruption recovery probabilities with $\tilde{\xi}(i)$, we calculated base stock levels and total cost for $k = 1, ..9$, and compared these results with the case where $k = 0$. Results of all these calculations are given in Figures 3.14 and 3.15.

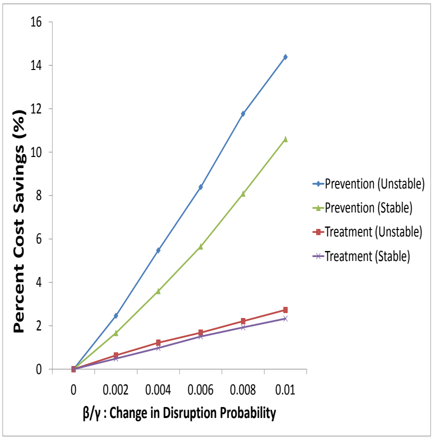

Figure 3.14: Percent Savings in Total Cost for Prevention and Solution of Disruptions (LID, 5% Disruption

Our results indicate that the prevention is much more important than the solution when the expected number of disruption periods is set to 5. Savings are especially prominent (14% for $\beta = 0.1$) for long and infrequent type of disruptions in unstable supply scenario. This indicates the value of advance warning signals for dealing with supply risk (e.g. (Li et al., 2015)) when they are applied with threat-dependent policies as in our study or (Tomlin and Snyder, 2006). As an expected result, efforts for dealing with supply risk are found to be to be more important in unstable supply than stable supply and in LID compared to SFD.

We conducted the same analysis with different expected disruption periods. Since the largest savings are obtained in LID scenarios in the previous results (Figure 3.14), we only consider LID for this sensitivity analysis of which results are given in Figure 3.17 and 3.16.

In these figures, expected periods of disruptions are given in the x-axis whereas maximum savings for each value of k are presented with iso-saving curves. Also, we give regions where prevention (solution) is more beneficial than (prevention) with different colors. Results indicate that the prevention (*proactive* approach) is more advantageous for smaller levels of disruption risk whereas the solution (*reactive* approach) becomes more important as the disruption risk level is increasing. Furthermore, the threshold levels, below which the solution is better than the prevention, depends on the level of disruption risk as well as the amount of efforts spent in these two activities. Another important observation is that the solution becomes more beneficial in higher disruption risk levels in unstable supply scenario compared to the stable one. This indicates that proactive approaches are more valuable for unstable supply case.

3.5.5 Application of the Model to Part A

To gain further understanding on the practical value of our model, we evaluated the performance of the optimum policy and infinite horizon base stock levels on the empirical data that belongs to the part A presented in Section 3.3. This analysis was conducted in four successive stages. First, we calculated historical inventory levels using the current inventory level and monthly demand and purchase data obtained from the MRO. Our calculations start in 2006 as this is the starting date of the demand data. Second, we calculated historical supply risk levels using purchase history data. Third, we calculated model parameters for optimization. Fourth, we evaluated the savings obtained from the optimal policy.

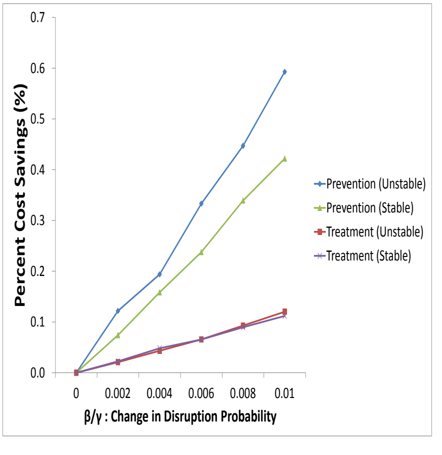

Figure 3.15: Percent Savings in Total Cost for Prevention and Solution of Disruptions (SFD, 5% Disruption)

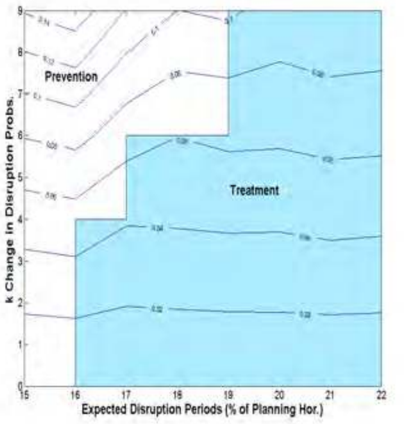

Figure 3.16: Percent Savings in Total Cost for Prevention and Solution of Disruptions (LID-Unstable)

Regarding the second stage, we used the statistical model by (Li et al., 2015) to calculate survival probabilities of the supplier of Part A for each month. To transform survival probabilities to transition probabilities p_{ij} in (3.1) of a Markov chain with two states (state 0 is healthy; state 1 is unhealthy) we chose 0.75 as a threshold level. For months when the survival probability is higher than 0.758, the supplier is assumed to

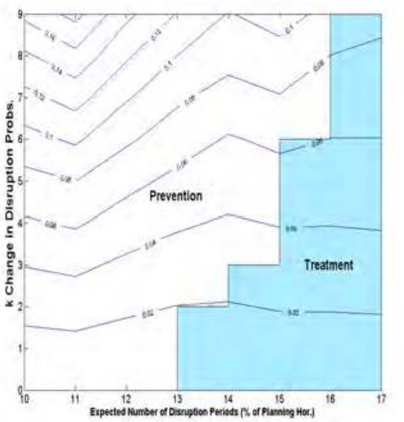

Figure 3.17: Percent Savings in Total Cost for Prevention and Solution of Disruptions (LID-Stable)

be in state 0 whereas crossing this level represents the Markov chain jumping to state 1. Using this discretization a time series data is obtained from monthly survival probabilities of the supplier.

In the third stage, we used purchase history data for similar parts from the same supplier to calculate the maximum likelihood estimator for the geometric distribution, which is the lead time distribution in our queuing system of Section 3.5.1. Calculated lead time parameters are given in Table 3.4. For the disruption probabilities of the model, $q(i)$, we used cross-validation results by Li et al. (2015). In these tests, 160 out of 186 parts had survival probability less than 0.75. Investigation into the suppliers of those parts indicated that 21 of those were already obsolete at the time of the analysis. We used this statistic as an estimator for disruption probability of state 1 ()"unhealthy" state) whereas the disruption probability of state 0 is assumed to be 0. For the disruption recovery probability, $\xi(i)$, we used the average solution time for disruption cases, which is 4.15 months. Due to lack of data, we assumed that the recovery probabilities are identical for both Markov states. All estimated parameters of the model are presented in Tables 3.4 and 3.5.

Acquisition cost of the part is 158.39 whereas the backlog cost rates are calculated using the critical ratio and assuming holding cost rate is 0.1 with service lvel being equal to 0.9 and 0.99. Calculated optimal base stock levels for two different service levels are given in Table 3.4.

Table 3.4: Parameters and the Result of the Model for Part A

Markov States	Lead Time MLE	$q(i)$	$\xi(i)$	Base Stock (0.9)	Base Stock (0.99)
Healthy	0.53	0	0.2406	**9**	**16**
Unhealthy	0.47	0.1346	0.2406	**15**	**30**

Table 3.5: Transition Probabilities (p_{ij}) for the Markov Chain of Part A.

	Good	Bad
Healthy	0.929	0.071
Unhealthy	0.032	0.968

In stage 4, we compare the optimal inventory levels with the historical inventory levels (Figure 3.18). Our policy not only provides smoother inventory levels, but also gives a better preparation for the disruption, which took place in December 2011. Discounted total costs indicate that our policy creates savings of 10.25% and 20.31% compared to "Business-As-Usual" (BAU) for service rates 0.90 and 0.99. Surprisingly, we find higher service rates leading to more cost savings which comes from the backlog costs in the year of 2006 (see Figure 3.18). Other savings come from lower inventory holding costs during the undisrupted months of the supply system. Unfortunately, demand during disruption was not captured in our demand data. Hence, we can only speculate about the savings on the backlog cost during the disruption, which took two months for this part.

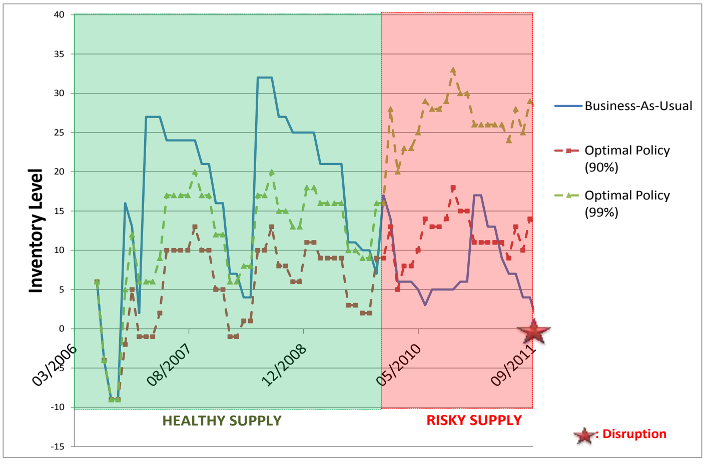

Figure 3.18: Inventory Levels for Part A

In addition, we conducted the same analysis with different holding costs and threshold levels for discretization of survival probabilities. The results of this analysis are given in Table 3.6. In general, we reached the conclusion that increasing threshold levels leads to less cost savings compared to BAU for this part since the company switches to the higher base stock level (base stock level of state 1). This result can be observed for 0.99 service levels in Table 3.6. For 0.90, on the other hand, setting the threshold level 0.65 leads to some backlog costs around 2009 which leads to less savings compared to the threshold level of 0.75 (setting the threshold level to 0.85 saves backlog costs for the same time period).

Results of this analysis indicates that for high service levels (0.99) increasing threshold level decreased the savings from the optimal policy since it indicates that the company is more risk-averse against disruptions. For moderately high service levels (0.90), we found different results since decreasing threshold level leads to some extra backlog costs which didn't appear when the threshold level is set to 0.75. These new backlog costs depleted cost savings from inventory holding.

Table 3.6: Application with Different Threshold Levels, Service Rates and Holding Costs

		Threshold Levels		
Holding Cost Rate	Service Rate	0.65	0.75	0.85
0.1	0.9	6.02%	10.25%	4.84%
0.1	0.99	23.14%	20.31%	13.18%
0.3	0.9	16.93%	15.30%	16.22%
0.3	0.99	32.98%	28.65%	21.23%

At this point we should stress that the author's personal communication with managers in the service sector indicated that 4.15 months of average solution time for disruption cases is very optimistic. It is argued that solution to disruption problems of spare parts require engineering knowledge and usually engineering departments of companies are extremely busy with new product development and research processes. Hence, disruptions get lower priority in companies and may last up-to three years. A unique feature of the MRO, whom authors have contact with, is that it has a dedicated technical group for rapid solution of disruptions. Therefore, we postulate that the relevance of our study is even higher than may be reflected in this section.

3.6 Summary and Discussion

Supply-side risks for spare parts of capital products are very important for maintenance companies. Empirical evidence suggests that towards the end-of life of capital products their spare parts suppliers eventually stop their manufacturing and/or delay deliveries to the maintenance companies. This behavior creates random lead times coupled with supply disruption risks, which are nonstationary in nature. In order to address the combined effect of these two risks, we consider a supply system driven by an exogenous Markov chain in a finite horizon setting.

Given that order crossovers are not allowed, we prove that the state-dependent base stock policy is optimal. Analysis reveals that intricate sufficient conditions are necessary for establishing the monotonicity of optimal base stock levels.

We conduct an impact analysis to address the effect of ignoring random lead times and disruption risks under stable and unstable supply scenarios. Our experiments indicate that ignoring stochastic lead times or supply disruptions leads to a significant increase in costs and a decrease in service levels, especially when the health of the supplier tends to get worse over the planning horizon. We observe that the combined effect of supply disruption and random lead times is even more than the sum of the individual effects and can create cost differences of up to 90% when the nonstationary nature of these risks is ignored.

Our analysis indicates that efforts in *prevention* of supply disruptions are more valuable than the capability of *solving* supply disruptions quicker. Hence, supply risk assessment tools and their employment in risk dependent policies are critical in the service sector.

An application of our model, which compares the optimal policy with historical inventory levels, indicates that recognizing random lead time together with supply disruption risk not only creates at least 11% savings in total discounted costs, but also makes the company more prepared for supply disruptions.

3.7 Epilogue

The research output of this chapter was presented to the OEM. The managers in the company decided to further test the implementation of the model. This phase of the study should start with replicating the application in Section 3.5.5 to a larger, random sample of spare parts. Such a large-scale application requires calculation of survival probabilities using the model by Li et al. (2015), choosing a proper threshold for transformation of survival probabilities to Markov states and estimating lead time parameters which will be used to obtain state-dependent base stock levels.

Calculation of survival probabilities should be conducted using the coefficients of the proportional hazard model (Li et al., 2015) for each month. Afterwards, survival probabilities should be transformed into Markov states using a threshold level. In our application, we used 0.75 as a threshold level below which a supplier was assumed to be in the 'risky' state. Choosing a proper threshold level is a decision that should be done by managers based on their attitude towards supply risk. Since survival probabilities calculated by the proportional hazard model are generally decreasing, choosing a high level of threshold means a risk-averse approach to supply disruptions, as the system will jump to the risky state early. On the other hand, a low threshold level indicates a risk-seeking attitude of managers, since the system will jump to the risky state late.

Once historical Makov states are obtained using the proportional hazard model, the next step should be parameter estimation for state-dependent lead time distributions using the lead time data of each part. This can be done with the maximum likelihood estimator of the geometric distribution as in Section 3.5.5. Note that it is also possible to evaluate different stochastic systems generating lead time distributions other than geometric, such as negative binomial, Poisson etc. Estimated lead time parameters should be used to calculate state-dependent base stock levels for each part.

Calculation of base stock levels was conducted with the value iteration algorithm in our application. After a proper truncation of the state space, the value iteration algorithm was run until it converged to infinite horizon solution. In MATLAB 2014a, this calculation took us approximately 20 min. for a single part. To implement this model for a large part sample, one should develop a computer program with more low-level programming languages, e.g. C++, C# etc., which would definitely shorten the calculation time. Alternatively, one can search for an efficient heuristic approach with small deviation from the optimal cost. In their paper, Song and Zipkin (1996) present a numerical experiment using a myopic cost function which generates near-optimal base stock levels for Markov-modulated random lead time model *without* supply disruption. Therefore, the myopic cost function given in this chapter might be a good starting point for a search of heuristic approach.

In order to test the performance of the state-dependent base stock policy, one should consider historical inventory levels which would be used to obtain 'Business-as-usual' (BAU) cost indicating the inventory-related costs (acquisition, holding and backlog) if this model is not implemented. Since the company does not keep monthly inventory levels in its database, one should calculate historical inventory levels using classic inventory recursion, demand and purchase history, and current inventory level of each part. The current inventory level will be the boundary condition for the inventory recursion. Naturally, the implementation of this model for all part numbers in the OEM's database would start if the inventory-related cost obtained from this model is lower than BAU cost as in Table 3.6.

In addition to the possibility of implementing this model in a stand-alone software which can communicate with the ERP system of the OEM, this model may be considered as a module in a larger supply risk mitigation system considering the existence of multiple suppliers, and secondary markets (as supply source), possibility of capacity reservation or capacity flexibility contracts with suppliers. A replenishment policy for parts, which can be supplied from a supplier and secondary markets, is addressed in the next chapter.

3.A State Reduction for Random Lead Time and Supply Disruptions

Although we use some results and notations from Song and Zipkin (1996) in our model, the addition of supply disruption to the state reduction is new. The Markov chain transition diagram is given in Figure 3.3.

We start with the notation for random variables governing the movements of outstanding orders placed in previous periods. Outstanding orders are kept in a random vector $\underline{z} = \{z_{it}\}_{i \geq 0}$ where z_0 represents the current period's order. Using outstanding orders in each position, we can define another random variable x_t^j which represents inventory level and outstanding orders together at time t as follows:

$$x_t^j = x_t + \sum_{k \geq j} z_{kt},$$

where x_t stands for inventory level at time t. In this notation, inventory position is expressed as x_t^0. Since we consider a finite horizon problem and the time index is given as subscripts of cost functions, we suppress t from this point on. We should note that, an outstanding order vector can be transformed into an inventory position vector, $\underline{x} = \{x^j\}_{j \geq 0}$.

In our analysis, the random moves of outstanding orders will be represented with the following variable.

Definition 1 *Define $N(j|i)$ as the position of an outstanding order that moves to position j within k-periods time (Song and Zipkin, 1996).*

Using the variable above, we can write state dependent random lead time, $L(i)$, as follows:

$$\{L(i) = k\} = \{N^{k+1}(\infty|i) = 0\} = \{A^{k+1}(i) = 0\},$$

where position ∞ stands for the delivery of the outstanding order and random variable $A^l(i)$ is defined in Table 3.1 in Section 3.4 and by Song and Zipkin (1996).

$$Pr\{A^{l+1}(i) > 0\} = Pr\{L(i) > l\}. \tag{3.9}$$

Definition 2 *Define equation $f_n(i, \underline{x})$ as the optimal cost function when the supplier is in healthy state i and there are n periods ahead in the planning horizon.*

Definition 3 *Define equation $g_n(d^i, \underline{x})$ as the total discounted holding and backlog costs after supply disruption takes place and the planning horizon is equal to n.*

In our model, we assume the supplier is available in state $i \in B^h$ and placing a new order becomes impossible when the supplier reaches states $d^i \in B/B^h$. Each disruption state is denoted with an index, i, which represents the state just before the supply disruption. Namely, if the supply disruption takes place when the supplier is in state 2, the disruption state is named d^2.

As stated above, the cost function $f_n(.)$ is the summation of acquisition, holding, and backlog cost for n-period dynamic programming model given that the supplier is available at the beginning of period n. When the supply system in healthy state i, it might disrupt with probability $1 - q(i)$ or it stays healthy with probability $q(i)$. After the disruption, which is represented with Markov chain's jumping to the associated disruption state d^i, we accumulate holding and backlog costs in the cost function $g_n(.)$. When the supply system is in the disruption state d^i, there are two possibilities for the next period: it stays in state d^i with probability $\xi(i)$, or it starts again with probability $\bar{\xi}(i)$. Considering the no order crossover assumption explained in Section 3.4, our dynamic programming equation can be written as follows:

$$f_n(i, \underline{x}) = \min \Big\{ c(x^0 - x^1) + \mathbf{E}C(x^{A^1(i)} - D) + \alpha q(i)\mathbf{E}f_{n-1}(i_+^*, \{x^{N(j|i)} - D\}_{j\geq 1}) \quad (3.10)$$
$$+ \alpha\bar{q}(i)\mathbf{E}g_{n-1}(d^i, \{x^{N(j|i)} - D\}_{j\geq 1}) \Big\},$$

where

$$g_n(d^i, \underline{x}) = \mathbf{E}C(x^{A^1(d^i)} - D) + \alpha\xi(i)\mathbf{E}g_{n-1}(d^i, \{x^{N(j|d^i)} - D\}_{j\geq 1}) \quad (3.11)$$
$$+ \alpha\bar{\xi}(i)\mathbf{E}f_{n-1}(i, \{x^{N(j|i)} - D\}_{j\geq 1}).$$

In Equation 3.10, i_+^* represent the next healthy state after the healthy state i. The first term of the equation is acquisition cost. The second term is one period holding and backlog cost which is defined as follows:

$$C(x) = h\max(x, 0) + b\max(-x, 0).$$

The third and fourth terms represent the two above-mentioned possibilities of the supply system given that it is in a healthy state. The terms of in Equation 3.11 arr the same with Equation 3.10 except the acquisition cost.

$f_n(i, \underline{x})$ is a multi-state recursive equation which is hard to minimize. The state reduction algorithm for this function is proved in this appendix. To this end, we need the following definition and lemma. Definition 4 is an adaptation of the random lead time definition by Song and Zipkin (1996) to disruption states.

Definition 4 *Define $L(d^i)$ as the (random) amount of periods required to receive all outstanding orders when the supply is in disruption state d^i. It can be characterized as follows:*

$$Pr\{L(d^i) > l\} = Pr\{A^{l+1}(d^i) > 1\}. \tag{3.12}$$

In Lemma 3, we present three identities which have the similar proofs. Since they are directly used in Theorem 2, we express them explicitly. Note that we define d^i_+ as the random variable indicating the next state of the supply system after the disruption state d^i.

Lemma 3 *The following entities is true for $L(d^i)$:*

1. $Pr\{L(d^i) > l-1, i_+ = d^i\} = Pr\{L(i) > l, i_+ = d^i\} + Pr\{L(i) \leq l \leq L(d^i), i_+ = d^i\}$,
2. $Pr\{L(d^i_+) > l-1, d^i_+ = d^i\} = Pr\{L(d^i) > l, d^i_+ = d^i\} + Pr\{L(d^i) = l, d^i_+ = d^i\}$,
3. $Pr\{L(d^i_+) > l-1, d^i_+ = i\} = Pr\{L(d^i) > l, d^i_+ = i\} + Pr\{L(d^i) \leq l \leq L(i), d^i_+ = i\}$

Proof For the first entity, given that $\{i_+ = d^i\}$ we can state the following:

$$\begin{aligned}
Pr\{L(i) > l\} &= Pr\{A^{l+1}(d^i) > 1\}, && (3.13)\\
&= Pr\{A^{l+2}(i) > 1, A^{l+1}(d^i) > 1\} + Pr\{A^{l+2}(i) \leq 1, A^{l+1}(d^i) > 1\},\\
&= Pr\{A^{l+2}(i) > 1\} + Pr\{A^{l+2}(i) \leq 1, A^{l+1}(d^i) > 1\},\\
&= Pr\{A^{l+2}(i) > 0\} + Pr\{A^{l+2}(i) = 0, A^{l+1}(d^i) > 1\} - Pr\{A^{l+2}(i) = 1, A^{l+1}(d^i) = 1\}, && (3.14)\\
&= Pr\{L(i) > l+1\} + Pr\{A^{l+2}(i) = 0\} - Pr\{A^{l+2}(i) \leq 1, A^{l+1}(d^i) = 1\}, && (3.15)\\
&= Pr\{L(i) > l+1\} + Pr\{A^{l+2}(i) = 0\} - Pr\{A^{l+1}(d^i) = 1\},\\
&= Pr\{L(i) > l+1\} + Pr\{L(i) \leq l+1\} - Pr\{L(d^i) \leq l\},\\
&= Pr\{L(i) > l+1\} + Pr\{L(i) \leq l+1\} - Pr\{L(i) \leq l+1, L(d^i) < l\}, && (3.16)\\
&= Pr\{L(i) > l+1\} + Pr\{L(i) \leq l+1 \leq L(d^i)\}. && (3.17)
\end{aligned}$$

In the above derivation, Equation 3.13 follows from Definition 4. Equation 3.14 comes from the fact that in disruption, no new order can be placed ($Pr\{A(d^i) = 0\} = 0$). Hence,

$$Pr\{A^{l+2}(i) = 1\} = Pr\{A^{l+2}(i) = 1, A^{l+1}(d^i) = 1\} + Pr\{A^{l+2}(i) = 1, A^{l+1}(d^i) > 1\},$$

. Similar equation for $\{A^{l+2}(i) = 0\}$ yields Equation 3.15. Also the same fact (no orders during disruption) yields

$$\{A^{l+2}(i) \leq 1, A^{l+1}(d^i) = 1\} = \{A^{l+1}(d^i) = 1\}.$$

Last expression in Equation 3.16 is given in Song and Zipkin (1996). Finally, note that $\{A^{l+2}(i) > 0\} = \{L(i) > l+1\}$. This completes the proof of the first identity.

Given that $d^i_+ = d^i$, the second identity is expressed as follows:

$$\begin{aligned}
Pr\{L(d^i_+) > l\} &= Pr\{A^{l+1}(d^i_+) > 1\}, \\
&= Pr\{A^{l+2}(d^i) > 1, A^{l+1}(d^i) > 1\} + Pr\{A^{l+2}(d^i) = 1, A^{l+1}(d^i) > 1\}, \\
&= Pr\{A^{l+2}(d^i) > 1\} + Pr\{A^{l+2}(d^i) = 1\} - Pr\{A^{l+2}(d^i) = 1, A^{l+1}(d^i) = 1\}, \\
&= Pr\{L(d^i) > l+1\} + Pr\{L(d^i) \leq l+1\} - Pr\{A^{l+1}(d^i) = 1\}, \\
&= Pr\{L(d^i) > l+1\} + Pr\{L(d^i) \leq l+1\} - Pr\{L(d^i) \leq l\}, \\
&= Pr\{L(d^i) > l+1\} + Pr\{L(d^i) = l+1\}.
\end{aligned} \tag{3.18}$$

For the third entity we can state the following under the condition of $\{d^i_+ = i\}$,

$$\begin{aligned}
Pr\{L(i) > l\} &= Pr\{A^{l+1}(i) > 0\}, \\
&= Pr\{A^{l+2}(d^i) > 1, A^{l+1}(i) > 0\} + Pr\{A^{l+2}(d^i) = 1, A^{l+1}(i) > 0\}, \\
&= Pr\{A^{l+2}(d^i) > 1\} + Pr\{A^{l+2}(d^i) = 1, A^{l+1}(i) > 0\}, \\
&= Pr\{L(d^i) > l+1\} + Pr\{A^{l+2}(d^i) = 1\} - Pr\{A^{l+2}(d^i) = 1, A^{l+1}(i) = 0\}, \\
&= Pr\{L(d^i) > l+1\} + Pr\{L(d^i) \leq l+1\} - Pr\{A^{l+2}(i) = 0\}, \\
&= Pr\{L(d^i) > l+1\} + Pr\{L(d^i) \leq l+1\} - Pr\{L(i) \leq l+1\}, \\
&= Pr\{L(d^i) > l+1\} + Pr\{L(d^i) \leq l+1 \leq L(i)\}.
\end{aligned} \tag{3.19}$$

Equations 3.17, 3.19 and 3.18 give the desired results. ■

Even though there is some technical difference in the proof, the probability expressions derived in Lemma 3 are the same with Song and Zipkin (1996). In Theorem 2 we prove state reduction transformation for the whole model in Equation 3.10. The following lemma are will be used in that theorem.

Theorem 2 *The main model in Equation 3.10 can be transformed into single-stage recursive equation as follows:*

$$f_n(i, \underline{x}) = \tilde{C}(i, \underline{x}) + \tilde{f}_n(i, x^1), \tag{3.20}$$

and

$$g_n(d^i, \underline{x}) = \tilde{C}_d(d^i, \underline{x}) + \tilde{g}_n(d^i, x^1), \tag{3.21}$$

where

$$\tilde{f}_n(i,x^1) = \min_{y\geq x}\{c(y-x^1)+\hat{C}(i,y)+\alpha q(i)\mathbf{E}\tilde{f}_{n-1}(i_+^*,y-D)+\alpha\bar{q}(i)\mathbf{E}\tilde{g}_{n-1}(d^i,y-D)\},\ i_+^*\in B^h,$$

and

$$\tilde{g}_n(d^i,x) = \hat{C}(d^i,x)+\alpha\xi(i)\mathbf{E}\tilde{g}_{n-1}(d^i,x-D)+\bar{\xi}(i)\mathbf{E}\tilde{f}_{n-1}(i,x-D).$$

The single period cost function in the recursive equation is as follows:

$$\hat{C}(i,x) = \sum_{l\geq 0} Pr\{L(i)\leq l\leq L(i_+)\}C^l(x).\ \forall\, i,i_+\in B.$$

and

$$\tilde{C}(i,\underline{x}) = \mathbf{E}\sum_{l\geq 0} 1\{L(i)>l\}C^l(x^{A^{l+1}(i)}).$$

Proof by induction. Under proper initial conditions we can state

$$\tilde{g}_N(d^i,x) = \tilde{f}_N(i,x) = 0, \forall i\in B/B^h, \forall x.$$

Hence,

$$f_N(i,\underline{x}) = \tilde{C}(i,\underline{x}),$$

and

$$g_N(d^i,\underline{x}) = \tilde{C}_d(d^i,\underline{x}).$$

This provides the initialization. Now suppose the statement it is true for $n-1$. For n,

$$\begin{aligned} f_n(i,\underline{x}) = \min_{x^0\geq x^1} \big\{ & c(x^0-x^1)+\mathbf{E}C(x^{A^1(i)}-D)+\alpha q(i)\tilde{C}(i_+^*,\{x^{N(j|i)}-D\}_{j\geq 1})+ \\ & +\alpha q(i)\mathbf{E}\tilde{f}_{n-1}(i_+^*,x^0-D)+\alpha\bar{q}(i)\tilde{C}_d(d^i,\{x^{N(j|i)}-D\}_{j\geq 1})+\alpha\bar{q}(i)\mathbf{E}\tilde{g}_{n-1}(d^i,x^0-D)\big\}, \end{aligned} \tag{3.22}$$

and

$$\begin{aligned} g_n(d^i,\underline{x}) = & \mathbf{E}C(x^{A^1(d^i)}-D)+\alpha\xi(i)\big[C(d^i,\{x^{N(j|d^i)}-D\}_{j\geq 1})+\mathbf{E}\tilde{g}_{n-1}(d^i,x^1-D)\big]+ \\ & \alpha\bar{\xi}(i)\big[\tilde{C}(i,\{x^{N(j|d^i)}-D\}_{j\geq 1}+\mathbf{E}\tilde{f}_{n-1}(i,x^1-D)\big]. \end{aligned} \tag{3.23}$$

Now we are going to analyze terms in Equations 3.22 and 3.23 respectively. For single period holding and backlog costs, in Equation 3.23, we will present the following statements, which are adapted from Song and Zipkin (1996).

$$\mathbf{E}C(x^{A^1(i)}-D) = Pr\{L(i)=0\}C^0(x^0)+\mathbf{E}1\{L(i)>0\}C^0(x^{A^1(i)}), \tag{3.24}$$

$$\mathbf{E}C(x^{A^1(d^i)} - D) = Pr\{L(d^i) = 0\}C^0(x^1) + \mathbf{E}1\{L(d^i) > 0\}C^0(x^{A^1(d^i)}). \tag{3.25}$$

Also, the third term of Equation 3.22 can be written as follows:

$$\tilde{C}(i_+^*, \{x^{N(j|i)} - D\}_{j\geq 1}) = \mathbf{E}\sum_{l\geq 0} 1\{L(i_+) > l\}C^l(x^{N(A^{l+1}(i_+)|i)} - D). \tag{3.26}$$

Note that Song and Zipkin (1996) show that $N^{l+1}(j|i) = N(N^l(j|i_+^*)|i)$ and $\{L(i_+^*) > l\} = \{L(i) \leq l+1 \leq L(i_+^*)\} \cup \{L(i) > l+1\}$. Note that i and i_+^* are two healthy states of supply process. Given that system stays in the healthy states (this condition is considered with probability $q(i)$), following statements hold:

$$\alpha C^l(x^{N(A^{l+1}(i_+^*)|i)} - D) = C^{l+1}(x^{A^{l+2}(i)}) \text{ (by definition of } C^l(.)), \tag{3.27}$$

and

$$\alpha\tilde{C}(i_+^*, \{x^{N(j|i)} - D\}_{j\geq 1}) = \mathbf{E}\sum_{l\geq 0} 1\{L(i_+^*) > l\}\alpha C^l(x^{N(A^{l+1}(i_+^*)|i)} - D), \tag{3.28}$$

$$= \sum_{l\geq 0} Pr\{L(i) \leq l+1 \leq L(i_+^*)\}C^{l+1}(x^{A^{l+2}(i)}) + \sum_{l\geq 0}\mathbf{E}1\{L(i) > l+1\}C^{l+1}(x^{A^{l+2}(i)}). \tag{3.29}$$

Since $\{L(i) \leq l+1\} \Rightarrow \{A^{l+2}(i) = 0\}$, Equation 3.29 becomes,

$$\tilde{C}(i_+^*, \{x^{N(j|i)} - D\}_{j\geq 1}) = \sum_{l\geq 0} Pr\{L(i) \leq l+1 \leq L(i_+^*)\}C^{l+1}(x^0) + \sum_{l\geq 0}\mathbf{E}1\{L(i) > l+1\}C^{l+1}(x^{A^{l+2}(i)}), \tag{3.30}$$

$$= \sum_{l\geq 1} Pr\{L(i) \leq l \leq L(i_+^*)\}C^l(x^0) + \sum_{l\geq 1}\mathbf{E}1\{L(i) > l\}C^l(x^{A^{l+1}(i)}). \tag{3.31}$$

The fifth term in Equation 3.22 is analyzed as follows:

$$\alpha\bar{q}(i)\tilde{C}_d(d^i, \{x^{N(j|i)} - D\}_{j\geq 1}) = \alpha\bar{q}(i)E\sum_{l\geq 0} 1\{L(d^i) > l\}C^l(x^{N(A^{l+1}(d^i)|i)} - D). \tag{3.32}$$

Let's make the following observations: Before a supply disruption occurs, each period's order is added to the position 0 in inventory position vector $\underline{x}$. Thanks to no order crossover assumption, delivery probabilities in each term can be expressed with the position of delivered order in the outstanding order vector: $\{A^{l+1}(i) = 0\} = \{L(i) \leq l\}$.

Recall that the outstanding order vector is transformed the inventory position vector $\underline{x}$. After the supply disruption, on the other hand, new orders are impossible. Given that the supply system is in state i and will jump to the disruption state is d^i in the next period (this condition is considered with probability $\bar{q}(i)$), the following statements hold:

$$\{N(A^{l+1}(d^i)|i)\} = \{A^{l+2}(i)\}.$$

Using the second expression of Lemma 3, we can write the following:

$$\begin{aligned}
\alpha\bar{q}(i)\tilde{C}(d^i, \{x^{N(j|i)} - D\}_{j\geq 1}) &= \bar{q}(i)\mathbf{E}\sum_{l\geq 0} 1\{L(d^i) > l\}\alpha C^l(x^{A^{l+2}(i)} - D), \\
&= \bar{q}(i)\Bigg(\sum_{l\geq 0} Pr\{L(i) \leq l+1 \leq L(d^i)\}C^{l+1}(x^0) + \mathbf{E}\sum_{l\geq 0} 1\{L(i) > l+1\}C^l(x^{A^{l+2}(i)})\Bigg), \\
&= \bar{q}(i)\Bigg(\sum_{l\geq 1} Pr\{L(i) \leq l \leq L(d^i)\}C^l(x^0) + \mathbf{E}\sum_{l\geq 1} 1\{L(i) > l\}C^l(x^{A^{l+1}(i)})\Bigg). \qquad (3.33)
\end{aligned}$$

Using Equations 3.24, 3.31, 3.33, we write Equation 3.22 as follows:

$$\begin{aligned}
&f_n(i, \underline{x}) = \\
&\quad \tilde{C}(i, \underline{x}) + \min_{x^0 \geq x^1} \big\{c(x^0 - x^1) + \hat{C}(i, x^0) + q(i)\mathbf{E}\tilde{f}_{n-1}(i_+^*, x^0 - D) + \bar{q}(i)\mathbf{E}\tilde{g}_{n-1}(d^i, x^0 - D)\big\}, \\
&\quad = \tilde{C}(i, \underline{x}) + \tilde{f}_n(i, x^1).
\end{aligned}$$

This proves the theorem for Equation 3.20. To prove Equation 3.21, we focus on the second and the fourth terms in Equation 3.23. The second term is very similar to the fifth term of Equation 3.20 except the index of random variable $N(j|.)$.

$$\alpha\xi(i)C(d^i, \{x^{N(j|d^i)} - D\}_{j\geq 1}) = \alpha\xi(i)\mathbf{E}\sum_{l\geq 0} 1\{A^{l+1}(d^i) > 1\}C^l(x^{N(A^{l+1}(d^i)|d^i)} - D).$$

The following statement is true given that the supply process is in the disruption state d^i and it will stay in the same state in the next period (This condition is expressed with probability $\xi(i)$ in the second term in Equation 3.23):

$$\{N(A^{l+1}(d^i)|d^i)\} = \{A^{l+2}(d^i)\}. \qquad (3.34)$$

Using the equality in (3.34), we can write the following:

$$\begin{aligned}
&\alpha\xi(i)C(d^i,\{x^{N(j|d^i)}-D\}_{j\geq 1}) = \\
&\qquad \xi(i)\mathbf{E}\sum_{l\geq 0}\alpha C^{l+1}(x^{A^{l+2}(d^i)})\big(1\{L(d^i)>l+1\}+1\{L(d^i)=l+1\}\big),\\
&\qquad = \xi(i)\mathbf{E}\sum_{l\geq 1}C^{l}(x^{A^{l+1}(d^i)})\big(1\{L(d^i)>l\}+1\{L(d^i)=l\}\big),\\
&\qquad = \xi(i)\mathbf{E}\sum_{l\geq 1}1\{L(d^i)>l\}C^{l}(x^{A^{l+1}(d^i)})+\xi(i)\sum_{l\geq 1}Pr\{L(d^i)=l\}C^l(x^1).
\end{aligned} \tag{3.35}$$

Similar expansion will be applied to the fourth term in Equation 3.23.

$$\alpha\bar{\xi}(i)\tilde{C}(i,\{x^{N(j|d^i)}-D\}_{j\geq 1}) = \bar{\xi}(i)\mathbf{E}\sum_{l\geq 0}1\{L(i)>l\}\alpha C^l(x^{N(A^{l+1}(i)|d^i)}-D).$$

Given that the supply process is in the disruption state d^i and it will jump back to healthy state i (this condition is expressed with $\bar{\xi}(i)$), the following statement hold:

$$\{N(A^{l+1}(i)|d^i)\} = \{A^{l+2}(d^i)\}.$$

Using this equality and the third expression of Lemma 3, we can make the following statement:

$$\begin{aligned}
&\bar{\xi}(i)\tilde{C}(i,\{x^{N(j|d^i)}-D\}_{j\geq 1}) = \sum_{l\geq 0}\mathbf{E}1\{L(i)>0\}C^{l+1}(x^{A^{l+2}(d^i)}),\\
&\quad = \sum_{l\geq 0}\mathbf{E}\bigg(1\{L(d^i)>l+1\}+1\{L(d^i)\leq l+1\leq L(i)\}\bigg)C^{l+1}(x^{A^{l+2}(d^i)}),\\
&\quad = \sum_{l\geq 1}\mathbf{E}1\{L(d^i)>l\}C^l(x^{A^{l+1}(d^i)})+\sum_{l\geq 1}Pr\{L(d^i)\leq l\leq L(i)\}C^l(x^0).
\end{aligned} \tag{3.36}$$

Using Equations 3.25, 3.35 and 3.36, we can rewrite Equation 3.23 as follows:

$$\begin{aligned}
g_n(d^i,\underline{x}) = &\sum_{l\geq 0}\mathbf{E}[1\{L(d^i)>l\}C^l(x^{A^{l+1}(d^i)})]+\xi(i)\sum_{l\geq 0}Pr\{L(d^i)=l\}C^l(x^0)\\
&+\bar{\xi}(i)\sum_{l\geq 0}Pr\{L(d^i)\leq l\leq L(i)\}C^l(x^0)+\alpha\xi(i)\mathbf{E}\tilde{g}_{n-1}(d^i,x^1-D)\\
&+\alpha\bar{\xi}(i)\mathbf{E}\tilde{f}_{n-1}(i,x^1-D),
\end{aligned}$$

which is equal to

$$\begin{aligned} g_n(d^i, \underline{x}) &= \tilde{C}_d(d^i, \underline{x}) + \hat{C}_d(d^i, x) + \alpha\xi(i)\mathbf{E}\tilde{g}_{n-1}(d^i, x^1 - D) + \alpha\bar{\xi}(i)\tilde{f}_{n-1}(i, x^1 - D), \\ &= \tilde{C}_d(d^i, \underline{x}) + \tilde{g}_n(d^i, x). \end{aligned}$$

■

Lemma 4 *$\hat{C}(i, y)$ is convex in y.*

Proof

The convexity of $\hat{C}(i, y)$ and $\hat{C}(d^i, y)$ follows from the convexity of $C^l(x)$ which can be easily shown by taking the second forward difference of $C^l(x)$. ■

Lemma 5 *Suppose B^h is an totally ordered state space of a stochastically monotone Markov chain and $h(i, u)$ is a function from $B^h \times \mathbb{R}$ to $\mathbb{R}$. For two elements $\{i, j : j \succeq i,\ j \in B^h, i \in B^h\}$ and a given u, $h(i, u) \geq h(j, u)$ implies $Eh(i_+, u) \geq Eh(j_+, u)$ where i_+ represents the next state of the Markov chain given that the present state is i.*

Proof

$$\begin{aligned} \mathbf{E}h(j_+, u) - \mathbf{E}h(i_+, u) &= \sum_{k \in B} p_{jk} h(k, u) - \sum_{k \in B} p_{ik} h(k, u), \\ &= \sum_{k \in B^h} h(k, u)(p_{jk} - p_{ik}). \end{aligned}$$

Using summation by parts,

$$\begin{aligned} \sum_{k \in B^h} \mathbf{E}h(k, u)(p_{jk} - p_{ik}) &= \sum_{k=0}^{N-1} \sum_{u=o}^{k} (p_{ju} - p_{iu})(h(k, u) - h(k+1, u)) + \sum_{u=0}^{N} (p_{ju} - p_{iu}) h(N, u), \\ &= \sum_{k=0}^{N-1} (1 - \sum_{u=k+1}^{N} p_{ju} - 1 + \sum_{u=k+1}^{N} p_{iu})(h(k, u) - h(k+1, u)), \\ &= \sum_{k=0}^{N-1} (\sum_{u=k+1}^{N} p_{iu} - \sum_{u=k+1}^{N} p_{ju})(h(k, u) - h(k+1, u)) \leq 0. \end{aligned}$$

The last inequality follows from the negativity of $(\sum_{u=k+1}^{N} p_{iu} - \sum_{u=k+1}^{N} p_{ju})$ which is implied by the monotonicity of the Markov chain. ■

3.B Parameter Values for Supply Disruption Behavior of the Model

In this section, we present our calculation scheme of parameter values to obtain desired numbers of expected disruption periods. For this purpose, we run an algorithm which calculates expected expected amount of periods spent in disruption states for an irreducible Markov chain in a finite horizon. The algorithm utilizes conditional expectations for each period in the following way: Define $\Gamma_n(i)$ as expected number of periods spent in disruption states in n-period planning horizon. Then,

$$\Gamma_n(i) = \sum_{j \in B^h} \bar{q}(i) p_{ij} \Gamma_{n-1}(j) + q(i) \Gamma_{n-1}(d^i),$$

and

$$\Gamma_n(d^i) = 1 + \xi(i)\Gamma_{n-1}(i) + \bar{\xi(i)}\Gamma_{n-1}(d^i),$$

where $\Gamma_1(d^i) = 1$ and $\Gamma_1(i) = 0$ for all $i \in B^h$. Using a Markov chain for which the state space consists of three healthy states, we run two nested for loops for $\alpha \in [0.001, 0.4]$ and $\beta \in [0.19, 0.25]$ which constitutes disruption and recovery probabilities as in Table 3.7. Resulting parameter values from this search is given in Table 3.8.

Table 3.7: Parametrized Disruption and Recovery Probabilities of the Markov Chain

	State 0	State 1	State 2
Supply Disruption ($q(i)$)	0.001	$3/2\alpha$	2α
Disruption Recovery ($\xi(i)$)	β	$\beta/2$	$\beta/3$

3.C Analysis for Supply Failure

In this appendix, we consider supply failures, which we define as permanent loss of suppliers. Note that it is not difficult to show that the Theorem 2 hold when $\xi(i) = 1$, $\forall i \in B$, therefore omitted here. In this section, we first present monotonicity conditions of base stock levels for the supply failure case. Later, we will proceed to our numerical experiments. Our results indicate that the coupled effect of random lead time and disruptions and the effect of nonstationarity are elevated versions of the disruption results presented in Section 3.5.

We will start our analysis by formulating our model for Markov-modulated supply failure and random lead time. We should start with the result of supply failure (permanent

Table 3.8: Parameter Values for Disruption Behavior of the Model

5% Disruption Periods	e	d	$q(0)$	$q(1)$	$q(2)$	$\bar{\xi}(0)$	$\bar{\xi}(1)$	$\bar{\xi}(2)$
stable-LID	0.8	0.1	0.001	0.0105	0.014	0.256	0.128	0.085
stable-SFD	0.8	0.1	0.001	0.0075	0.01	0.248	0.124	0.083
unstable-LID	0.12	0.1	0.001	0.0375	0.05	0.99	0.495	0.33
unstable-SFD	0.12	0.1	0.001	0.0255	0.034	0.95	0.475	0.317
10% Disruption Periods								
stable-LID	0.8	0.1	0.001	0.0195	0.026	0.216	0.108	0.072
stable-SFD	0.8	0.1	0.001	0.0135	0.018	0.204	0.102	0.068
unstable-LID	0.12	0.1	0.001	0.0765	0.102	0.98	0.49	0.327
unstable-SFD	0.12	0.1	0.001	0.0555	0.074	0.96	0.48	0.32
15% Disruption Periods								
stable-LID	0.8	0.1	0.001	0.0285	0.038	0.192	0.096	0.064
stable-SFD	0.8	0.1	0.001	0.0225	0.03	0.212	0.106	0.071
unstable-LID	0.12	0.1	0.001	0.1245	0.166	0.99	0.495	0.33
unstable-SFD	0.12	0.1	0.001	0.0915	0.122	0.99	0.495	0.33

loss of supplier) on the single period cost function $\hat{C}(d^i, y)$. As stated in Theorem 1 of Song and Zipkin (1996), $P\{L(i) \leq l \leq L(i_+)\} = Pr\{L(i) = l\}, \forall l \geq 0$, when $i_+ = i$. Therefore,

$$\hat{C}(d^i, x) = \sum_{l \geq 0} Pr\{L(d^i) = l\} C^l(x) = \sum_{l \geq 0} Pr\{L(d^i) = l\} C^l(x), \tag{3.37}$$

assuming the random movements of outstanding orders in a disruption state (d^i) follows the same distribution with its associated healthy state (i).

Then multi-period cost function can be expressed as follows:

$$\tilde{f}_n(i, x) = \min\{c(y - x) + q(i)\hat{C}(i, y) + \bar{q}(i)\hat{C}(d^i, y) + \alpha q(i)\mathbf{E}\tilde{f}_{n-1}(i_+, y - D) + \alpha\bar{q}(i)\mathbf{E}\tilde{g}_{n-1}(d^i, y - D) : y \geq x\}, \tag{3.38}$$

where

$$\tilde{g}_n(d^i, x) = \hat{C}(d^i, x) + \alpha\mathbf{E}\tilde{g}_{n-1}(d^i, x - D). \tag{3.39}$$

Transformation $W_n(i, x) = \tilde{f}_n(i, x) + cx$ leads to

$$W_n(i, x) = \min\{G_n(i, y) : y \geq x\}, \tag{3.40}$$

where,

$$G_n(i,y) = cy(1-\alpha q(i)) + q(i)\hat{C}(i,y) + \bar{q}(i)\hat{C}(d^i,y) + \alpha q(i)\mathbf{E}W_{n-1}(i_+, y-D) \quad (3.41)$$
$$+ \alpha\bar{q}(i)\mathbf{E}\tilde{g}_{n-1}(d^i, y-D).$$

The following theorem is presented without proof since its proof is the same with Theorem 1.

Theorem 3 *The following statements are true.*

a) Both of $\hat{C}(i,y)$ and $\hat{C}(d^i,y)$ are convex in y.

b) $\tilde{g}_n(d^i,x)$ is convex in x,

c) $G_n(i,y)$ is convex in y,

d) $W_n(i,x)$ is convex in x,

e) the state-dependent base stock policy is optimal.

The state-dependent optimal base stock policy can be characterized with $S_n(i)$, which is the optimal inventory position after the replenishment order when there are n periods ahead and the supply system is in state i. In the remainder of the section, we analyze the monotonicity conditions of time-dependent inventory policy parameters to derive further managerial insight into the problem.

We assume the inventory position to be a discrete variable, since this is more realistic in a spare parts context. Hence, define the forward difference operator $\Delta_x h(x) = h(x+1) - h(x)$. We suppress the subscript for notational simplicity throughout the appendix. Unless otherwise stated, Δ implies the forward difference of functions with respect to inventory level or inventory position variables, e.g. x or y. The first forward difference of our cost functions

$$\Delta W_n(i,x) = \begin{cases} 0, & \text{for } x < S_n(i), \\ \Delta G_n(i,x), & \text{for } x \geq S_n(i). \end{cases}$$

where

$$\Delta G_n(i,x) = c(1-\alpha q(i)) + q(i)\Delta\hat{C}(i,x) + \bar{q}(i)\Delta\hat{C}(d^i,x) + \alpha q(i)\mathbf{E}\Delta W_{n-1}(i_+, x-D)$$
$$+ \alpha\bar{q}(i)\Delta\mathbf{E}\tilde{g}_{n-1}(d^i, x-D),$$

and

$$\Delta\tilde{g}_n(d^i,x) = \Delta\hat{C}(d^i,x) + \alpha\Delta\mathbf{E}\tilde{g}_{n-1}(d^i, x-D). \quad (3.42)$$

Since costs after supply failure constitute a major component of $G_n(i,x)$, we should start our monotonicity analysis with $\tilde{g}_n(d^i,x)$. For the analysis we utilize stochastic ordering (Shaked and Shanthikumar, 2007), which is defined as follows.

Definition 5 *X and Y are two random variables. $X \geq_{st} Y$ if*

$$P\{X > l\} \geq P\{Y > l\} \;\; \forall l \in (-\infty, \infty).$$

The effect of stochastically ordered lead times on the infinite horizon base stock policy is analyzed from the average and discounted cost perspectives by Song (1994a) and Song (1994b), respectively. In the latter study Song proved that the stochastic order relationship is sufficient for $\alpha = 1$. We use similar arguments to understand this effect on the single period costs before and after supply failure in a finite-horizon setting. In the following lemma, we show the monotonicity conditions after supply failure for $\alpha < 1$ and prove that the monotonicity is preserved over the rest of the planning horizon.

Lemma 6 *For two states of the Markov chain $\{i, j : j \succeq i,\ j,\, i \in B^h\}$ if*

1. $L(i) \geq_{st} L(j)$,
2. $\frac{p}{h+p} \geq \Omega_1(i,j,\alpha,y)$,

where

$$\Omega_1(i,j,\alpha,y) = \frac{\phi(i,\alpha,y) - \phi(j,\alpha,y)}{\psi(i,\alpha) - \psi(j,\alpha)}, \tag{3.43}$$

$$\psi(i,\alpha) = \sum_{l \geq 0} \alpha^l Pr\{L(i) = l\},$$

and

$$\phi(i,\alpha,y) = \sum_{l \geq 0} \alpha^l Pr\{L(i) = l\} Pr\{D_{l+1} \leq y\},$$

then

a) $\Delta\hat{C}(d^i,x)$ is non-increasing in i,

b) $\Delta\tilde{g}_n(d^j,x) - \Delta\tilde{g}_n(d^i,x) \leq 0$.

Proof of Lemma 6:
The proof of the lemma consists of two parts. First we prove that, for given conditions, the statement a holds. Note that statement a is proved for $\alpha = 1$ by Song (1994a).

Hence, we will only focus on $\alpha < 1$ here. To establish conditions 1 and 2, we start with the following statement:

$$\begin{aligned}\Delta\hat{C}(d^i, x) &= \sum_{l\geq 0} Pr\{L(i) = l\}\Delta C^l(x), \\ &= \sum_{l\geq 0} Pr\{L(i) = l\}\alpha^l(-p + (p+h)Pr\{D_{l+1} \leq y\}).\end{aligned}$$

Let us define two generating functions:

$$\begin{aligned}\psi(i, \alpha) &= \sum_{l=0}^{\infty} \alpha^l Pr\{L(i) = l\}, \\ \phi(i, \alpha, y) &= \sum_{l\geq 0} \alpha^l Pr\{L(i) = l\}Pr\{D_{l+1} \leq y\}.\end{aligned}$$

Using these statements we write

$$\Delta\hat{C}(d^i, x) - \Delta\hat{C}(d^j, x) = -p(\psi(i, \alpha) - \psi(j, \alpha)) + (p+h)(\phi(i, \alpha, y) - \phi(j, \alpha, y)),$$

for $\alpha < 1$. The second condition of the lemma implies the statement a if $\psi(j, \alpha) \geq \psi(i, \alpha)$ and $\phi(j, \alpha, y) \geq \phi(i, \alpha, y)$ given that $L(i) \geq_{st} L(j)$. Hence we only need to show (for statement a) that $L(i) \geq_{st} L(j)$ implies $\psi(j, \alpha) \geq \psi(i, \alpha)$ and $\phi(j, \alpha, y) \geq \phi(i, \alpha, y)$. Note that $L(i) \geq_{st} L(j)$ is defined as

$$Pr\{L(i) \leq l\} \leq Pr\{L(j) \leq l\} \;\; \forall l \in [0, \infty). \tag{3.44}$$

Let us define

$$\tilde{\psi}(i, \alpha) = \sum_{l=0}^{\infty} \alpha^l Pr\{L(i) \leq l\}. \tag{3.45}$$

Equation 3.44 yields

$$\tilde{\psi}(j, \alpha) \geq \tilde{\psi}(i, \alpha).$$

By interchanging summation indices in Equation 3.45, we can state that

$$\tilde{\psi}(i, \alpha) = \frac{\psi(i, \alpha)}{1 - \alpha},$$

implying

$$\psi(j, \alpha) \geq \psi(i, \alpha).$$

To establish $\phi(j,\alpha,y) \geq \phi(i,\alpha,y)$ for $\alpha < 1$, we write

$$\phi(i,\alpha,y) - \phi(j,\alpha,y) = \sum_{l\leq 0} \alpha^l Pr\{D_{l+1} \leq y\}\big[Pr\{L(i)=l\} - Pr\{L(j)=l\}\big].$$

Using summation by parts,

$$\begin{aligned}
&\phi(i,\alpha,y) - \phi(j,\alpha,y) = \\
&\quad -\sum_{l\geq 0}\sum_{k=0}^{l} \big(Pr\{L(i)=k\} - Pr\{L(j)=k\}\big)(Pr\{D_{l+2} \leq y\}\alpha^{l+1} - Pr\{D_{l+1} \leq y\}\alpha^l), \\
&\quad = -\sum_{l\geq 0}(Pr\{L(i)\leq l\} - Pr\{L(j)\leq l\})(Pr\{D_{l+2} \leq y\}\alpha^{l+1} - Pr\{D_{l+1} \leq y\}\alpha^l).
\end{aligned}$$

The negativity of the first parenthesis comes from the stochastic order in condition 1, and the negativity of the second parenthesis comes from the convolution of identical random variables. Therefore,

$$\phi(i,\alpha,y) - \phi(j,\alpha,y) \leq 0,$$

which proves statement a. Statement b is proved by induction. For initialization,

$$\Delta\tilde{g}_1(d^j,x) - \Delta\tilde{g}_1(d^i,x) = \Delta\hat{C}(d^j,x) - \Delta\hat{C}(d^i,x) \leq 0.$$

Assume it is true for n. For $n+1$,

$$\Delta\tilde{g}_{n+1}(j,x) - \Delta\tilde{g}_{n+1}(i,x) = \Delta\hat{C}(d^j,x) - \Delta\hat{C}(d^i,x) + \alpha\mathbf{E}\Delta\tilde{g}_n(d^j,x-D) - \alpha\mathbf{E}\Delta\tilde{g}_n(d^i,x-D).$$

First two terms are non-positive due to the assumption of the lemma, the non-positivity of the last comes from the induction hypothesis. ■

Lemma 6 shows that the stochastic ordering between lead times of different states is not enough for the discounted cost case. We also need the function $\Omega_1(.)$, which captures the relationship between discount rate and lead time demand. The behavior of this function for its different parameters is presented in Figure 3.19.

Our numerical experiments indicate that $\Omega_1(.)$ is smaller than zero for the majority of the considered parameter values. This indicates that the second condition of the lemma is satisfied most of the time. Therefore, if we define an optimum inventory position after supply failure, $Z_n(d^i)$, then we can claim that it is monotonic in unhealthy states of the Markov chain. In other words, the amount of required inventory to mitigate the effect of random lead time after failure increases as the supply system gets worse. To obtain a

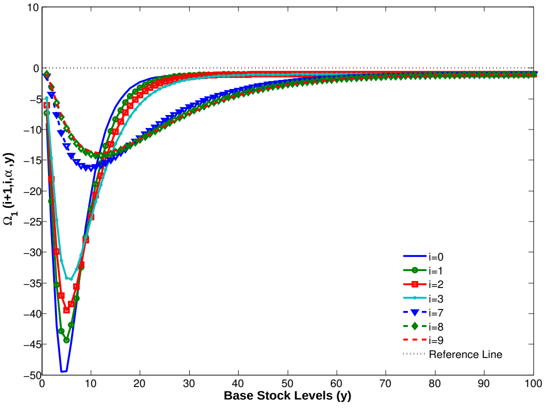

Figure 3.19: $\Omega_1(i+1, i, \alpha, y)$, $i \in [0, 9]$

similar monotonicity analysis for the whole system, we need to consider the monotonicity of the single period cost function before failure and establish its relationship with the stochastic ordering of lead time distributions.

The monotonicity conditions for the single period cost ($\hat{C}(i, x)$) for $\alpha = 1$ are given in Song and Zipkin (1996) as follows: Define $R(i, l) = Pr\{L(i) \leq l \leq L(i_+)\}$, then

$$\sum_{m=0}^{l}(R(i, m) - R(j, m)) \geq 0, \quad 0 \leq l < \infty, \tag{3.46}$$

and

$$\sum_{l=0}^{\infty} R(i, l) - \sum_{l=0}^{\infty} R(j, l) = 0. \tag{3.47}$$

For the discounted cost case ($\alpha < 1$), we need one more condition as stated in the following Lemma.

Lemma 7 *For* $\alpha < 1$, $\Delta\hat{C}(i, x) \geq \Delta\hat{C}(j, x)$ *if*

1. $\sum_{m=0}^{l}(R(i, m) - R(j, m)) \geq 0, \quad 0 \leq l < \infty,$
2. $\sum_{l=0}^{\infty} R(i, l) - \sum_{l=0}^{\infty} R(j, l) = 0,$
3. $\frac{p}{h+p} \geq \Omega_2(i, j, y),$

where

$$\Omega_2(i,j,,y) = \frac{\sum\limits_{l\geq 0} \alpha^l Pr\{D_{l+1} \leq y\}\big(R(i,l) - R(j,l)\big)}{\sum\limits_{l\geq 0} \alpha^l \big(R(i,l) - R(j,l)\big)}. \tag{3.48}$$

The proof of Lemma 7 is omitted since it is similar to the initial part of the proof of Lemma 6. $\Omega_2(.)$ in Equation 3.48 gives the functional relationship between inventory coverage, $R(i,l)$, convoluted demand, and the discount rate. The behavior of this function for different values of its parameters is given in Figure 3.20.

$\Omega_2(.)$ is difficult to interpret. As can be seen in Figure 3.20, this function maps different states of the Markov chain to a large variety of values. Since result of Lemma 7 is an input in Theorem 4, given below, we cannot say that conditions of Theorem 4 are always satisfied in our stochastic process. Also, our numerical experiments show that this function leads to negative values for smaller values of α such as 0.1. In other words, our monotonicity results are more reliable in an environment with high interest rates compared to more stable economies with lower inflation rates.

Theorem 4 states the sufficient conditions for monotonicity of optimal base stock levels given that supply failure probabilities are non-decreasing over healthy states of the Markov chain.

Theorem 4 *Suppose* $q(i) \geq q(j)$ *for two states of a Markov chain* $\{i,j : j \succeq i,\ i,j \in B^h\}$. *If*

1. $\Delta\hat{C}(j,x) \leq \Delta\hat{C}(i,x)$,

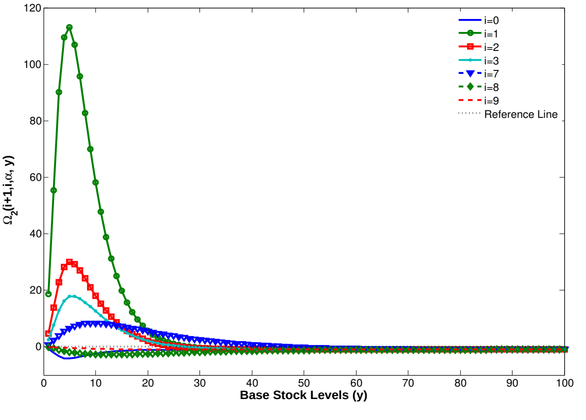

Figure 3.20: $\Omega_2(i+1,i,\alpha,y)$, $i \in [0,9]$

2. $\Delta\hat{C}(d^j, x) \leq \Delta\hat{C}(d^i, x)$,

3. $\epsilon_n^{thr} \geq \epsilon = q(i) - q(j)$ *while* $c\alpha + \Delta\tilde{g}_n(d^j, x) > 0$,

where,

$$\epsilon_n^{thr}(i, j, \alpha, x) = \bar{q}(i) \left(\frac{\Delta g_n(d^i, x) - \Delta g_n(d^j, x)}{c\alpha + \Delta g_n(d^j, x)} \right), \tag{3.49}$$

all hold, then

a) $\Delta W_{n-1}(j, x) - \Delta W_{n-1}(i, x) \leq 0$,

b) $\Delta G_n(j, x) - \Delta G_n(i, x) \leq 0$,

c) $S_n(j) \geq S_n(i)$,

Proof Take two states, $j \succeq i$ on the state space B^h of a Markov chain. For $n = 1$,

$$\Delta W_0(j, x) = \Delta W_0(i, x) = 0,$$

and

$$\Delta G_1(j, x) - \Delta G_1(i, x) = c\alpha\epsilon + q(j)\Delta\hat{C}(j, x) - q(i)\Delta\hat{C}(i, x) + \bar{q}(j)\Delta\hat{C}(d^j, x) - \bar{q}(i)\Delta\hat{C}(d^i, x).$$

We know $q(j)\Delta\hat{C}(j, x) - q(i)\Delta\hat{C}(i, x) \leq 0$ under the assumptions of the theorem. So,

$$c\alpha\epsilon + \bar{q}(j)\hat{C}(d^j, x) - \bar{q}(i)\Delta\hat{C}(d^i, x) \leq 0 \Rightarrow \Delta G_1(j, x) - \Delta G_1(i, x) \leq 0.$$

Since,

$$c\alpha\epsilon + \bar{q}(j)\Delta\hat{C}(d^j, x) - \bar{q}(i)\Delta\hat{C}(d^i, x) = \bar{q}(i) \left(\Delta\hat{C}(d^j, x) - \Delta\hat{C}(d^i, x) \right) + \epsilon \left(c\alpha + \Delta\hat{C}(d^j, x) \right),$$

$c\alpha + \Delta\hat{C}(d^j, x) \leq 0$ implies $\Delta G_1(j, x) - \Delta G_1(i, x) \leq 0$. If $c\alpha + \Delta\hat{C}(d^j, x) > 0$ then, $\epsilon \leq \epsilon_1^{thr}(x)$ implies the desired inequality since $\Delta\tilde{g}_1(d^i, x) = \Delta\hat{C}(d^i, x)$. Therefore b is true for $n = 1$ under these assumptions. b implies c.

Assume the theorem is true for n. For $n + 1$,

$$\Delta_x W_n(j, x) - \Delta_x W_n(i, x) = \begin{cases} 0, & \text{for } x \leq S_n(i) \leq S_n(j), \\ -\Delta G_n(i, x) \leq 0, & \text{for } S_n(i) \leq x \leq S_n(j), \\ \Delta G_n(j, x) - \Delta G_n(i, x) \leq 0, & \text{for } S_n(i) \leq S_n(j) \leq x. \end{cases}$$

This proves a. To show b is true for $n+1$,

$$\begin{aligned}\Delta G_{n+1}(j,x) - \Delta G_{n+1}(i,x) &= c\alpha\epsilon + q(j)\Delta\hat{C}(j,x) - q(i)\Delta\hat{C}(i,x) + \bar{q}(j)\Delta\hat{C}(d^j,x)\\ &- \bar{q}(i)\Delta\hat{C}(d^i,x) + \alpha q(j)\mathbf{E}\Delta W_n(j_+, x-D) - \alpha q(i)\mathbf{E}\Delta W_n(i_+, x-D)\\ &+ \alpha\bar{q}(j)\mathbf{E}\Delta\tilde{g}_n(d^j, x-D) - \alpha\bar{q}(i)\mathbf{E}\Delta\tilde{g}_n(d^i, x-D).\end{aligned}$$

We know $q(j)\Delta\hat{C}(j,x) - q(i)\Delta\hat{C}(i,x) \leq 0$ from assumptions of the theorem.

$$q(j)\mathbf{E}\Delta W_n(j_+, x-D) - q(i)\mathbf{E}\Delta W_n(i_+, x-D) \leq 0,$$

can be easily shown using Lemma 5 since we show that $\Delta W_n(j,x) \leq \Delta W_n(i,x)$. Therefore,

$$c\alpha\epsilon + \bar{q}(j)\Delta\hat{C}(d^j,x) - \bar{q}(i)\Delta\hat{C}(d^i,x) + \alpha\bar{q}(j)\mathbf{E}\Delta\tilde{g}_n(d^j,x-D) - \alpha\bar{q}(i)\mathbf{E}\Delta\tilde{g}_n(d^i,x-D) \leq 0, \tag{3.50}$$

implies $\Delta G_{n+1}(j,x) - \Delta G_{n+1}(i,x) \leq 0$. Let us recall that $\Delta\tilde{g}_{n+1}(d^j,x) = \Delta\hat{C}(d^j,x) + \alpha\mathbf{E}\Delta\tilde{g}_n(d^j, x-D)$. Therefore, Equation 3.50 equals to

$$\begin{aligned}&= c\alpha\epsilon + \bar{q}(j)\Delta\tilde{g}_{n+1}(d^j,x) - \bar{q}(i)\Delta\tilde{g}_{n+1}(d^i,x),\\ &= \bar{q}(i)\left(\Delta\tilde{g}_{n+1}(d^j,x) - \Delta\tilde{g}_{n+1}(d^i,x)\right) + \epsilon\left(\alpha c + \Delta\tilde{g}_{n+1}(d^j,x)\right).\end{aligned}$$

Using Lemma 3, we can state that if $\alpha c + \Delta\tilde{g}_{n+1}(d^j,x) \leq 0$, then $\Delta G_{n+1}(j,x) \leq \Delta G_{n+1}(i,x)$. Otherwise, $\epsilon \leq \epsilon_n^{thr}(x)$ implies the desired inequality (for statement b) which also implies the statement c. ■

Theorem 4 indicates that even when forward difference of single period costs before and after supply failure ($\Delta\hat{C}(i,x)$ and $\Delta\hat{C}(d^i,x)$) are ordered over states of the Markov chain, this ordering cannot be preserved without another condition on failure probabilities of different states due to the nonstationary character of the failure risk. Since the supply failure probability changes over the states of the Markov chain, the trade-off between buying or waiting becomes more complicated. Specifically, postponing the procurement to the next period yields discounted acquisition cost together with higher stock-out risk after the supply failure. This trade-off can be seen in the third condition of Theorem 4. Also the following corollary shows that the order between single-period costs over different states is preserved when $q(i) = q, \ \forall\, i \in B^h$.

Corollary 5 *If $q(i) = q, \ \forall i \in B^h$, and conditions 1 and 2 of Theorem 4 hold, then*

a) $\Delta W_{n-1}(j,x) - \Delta W_{n-1}(i,x) \leq 0,$

b) $\Delta G_n(j,x) - \Delta G_n(i,x) \leq 0,$

c) $S_n(j) \geq S_n(i)$.

The proof of Corollary 5, can easily be done if $q(i) - q(j) = \epsilon = 0$ in the proof of Theorem 4. Please note that the sufficient conditions of Theorem 4 are intricate and may fail occasionally. However, our numerical experiments indicate that the monotonicity of optimal base stock levels still holds, especially for "risky" states of the Markov chain.

3.C.1 Results of Numerical Experiments

In this subsection, we present results of our analysis on coupled effect of random lead time and supply failures. For this, we employ the queueing system discussed in Section 3.5.1. As is pointed in Section 3.5.2, the methodology we followed is based on calculating optimal order-up-to levels using value iteration algorithm and plugging them into simulation to understand their performances under different scenarios.

We used a smaller scenario setting (than the presented in Table 3.2) only including existence of disruption, random vs. deterministic lead time and supply tendency. Hence, we evaluated 8 different scenarios for this part of the study.

One major downside of considering supply failure is the possibility of complete domination of backlog costs due to early supply failures. To prevent this, we collected empirical data from a maintenance repair organization, Fokker Services, and calibrated failure behavior of our model to the following statistics:

Table 3.9: Failure Duration Statistics From Empirical and Simulated Data

	Empirical Durations	Simulated Durations
Mean	4.1548	4.1566
Std. Dev.	6.5313	11.9145
C.I.	0.3303	0.1810

Distribution of disruption length is given in the following histogram. Note that we consider a Markov chain consisting of 10 healthy and 10 disruption states in order to see the effect of large state space on our results.

Calibrated arrival (e), departure (d) and the state-dependent supply failure probabilities for Queue #1 (Figure 3.4) are given in Table 3.10 and Table 3.11. The state-dependent delivery probabilities for Queue #2, which generates random lead times, are given in Table 3.11. Due to the delivery probabilities, the first two moments of the state-dependent lead time distributions are higher for unhealthy states than those of healthy states.

Using the parameter values in Tables 3.10 and 3.11, we calculate the optimal base stock levels using the value iteration algorithm for 100 periods. The finite horizon base

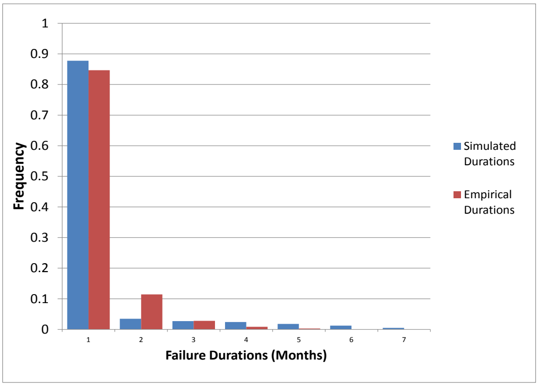

Figure 3.21: Histogram of Failure Durations for Simulated and Real Data

Table 3.10: Arrival (e) and Departure (d) Probabilities for Queue #1

Scenario	e	d
Unstable supply	0.12	0.1
Stable supply	0.08	0.1

stock levels for the benchmark scenario (with both lead time and supply failure risks) are given in Figure 3.22. Optimal base stock levels of unhealthy states (states 6-10 in Table 3.11) are depicted with bold-dashed lines whereas others are given with thin straight lines. As can be seen, even small differences in failure probabilities may lead to significant differences in optimal base stock levels. This is most apparent at the early stages of the planning horizon, whereas optimal base stock levels converge towards the end.

Unfortunately, initial supplier conditions and initial inventory levels of spare parts are unknown in our data set, e.g., the purchase history of spare parts might start with suppliers that are moderately healthy or not. To mimic this feature in our simulation, we consider 20% of 100 periods as warm-up period. During the warm-up, the supply failure events are disabled to prevent each replication from starting with failed supply. This configuration of the simulation model generates supply failures in 15% of 50,000 replications in the unstable scenario (benchmark). This ratio drops to 2% for stable supply.

The performance measures we track in our simulation model are total discounted cost, total discounted backlog cost, ready rate (fraction of time with positive stock on hand) and fill rate (fraction of demand that can be satisfied immediately from stock on hand (Axsäter,

Table 3.11: State Dependent Delivery Probabilities for Queue #2

State Index	0	1	2	3	4	5	6	7	8	9	10
Supply Failure Probabilities:	0	0	0	0	0	0	0.01	0.02	0.03	0.04	0.05
Delivery Probabilities:	0.6	0.55	0.5	0.45	0.4	0.35	0.3	0.25	0.2	0.15	0.1

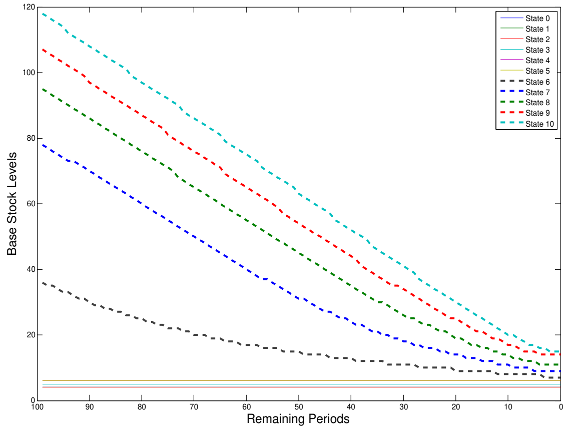

Figure 3.22: Base Stock Levels for the Benchmark Scenario

2006)). Total cost and total backlog costs are common performance measures in inventory control simulations. Ready rate and fill rate are important service measures for the service sector, since most customer contracts utilize either of these (Oliva and Kallenberg, 2003). To determine the number of replications, we first conduct a pilot study consisting of 5000 replications. Results of this study are used to compute the total number of replications which is set to 50,000. To control the variance, we use common random numbers which cause dependency between replications. Therefore, paired-t-tests are employed to check whether there is statistically significant difference between scenarios.

The discount rate per period is set to 0.995, which leads to a 6% annual discount rate over the entire planning horizon, since a period stands for a month in our calibration. Other system parameters are taken as follows: Without loss of generality, we set the acquisition cost equal to 2 per item. The holding cost is equal to 0.2 and backlog cost is equal to 4 per item per period. Random demand in each period is assumed to follow a Poisson distribution with mean 2.

Simulation results for the five different scenarios with unstable supply are given in Figures 3.23 and 3.24. Table 3.12 shows the percent difference of each scenario compared to the benchmark. These results indicate that the effect of ignoring random lead time together with supply failure can increase the total cost of up to 51% for the unstable supply scenario (run 0). Ignoring only supply failure risk creates 13.24% loss (run 3). Taking both forms of supply risk into account suppresses backlog costs (Figure 3.23) and leads to an increase of up to 17% in ready rate and up to 13% in fill rate. In terms of costs, comparing run 3 with run 5, we observe that the effect of supply failure is more prominent than that of the random lead time. Also, the effect of the nonstationarity in supply risks (run 0 versus run 1) is as important as the effect of supply failure (run 3 versus run 7).

In the stable supply scenario (Figure 3.26 and Table 3.13), the cost of ignoring supply risk factors is less in terms of all the performance measures. Our simulation results indicate that ignoring both the random lead time and the supply failure (run 2) can lead to a decrease of up to 11.2% (ready rate) and up to 7.1% (fill rate). One important observation is the decreased effect of supply failure compared to the unstable supply scenario. Obviously, considering supply-side risks, especially supply failure, has a higher priority when the supply system is unstable. Also, the effect of ignoring nonstationarity in the system seems to be less important in the stable supply case.

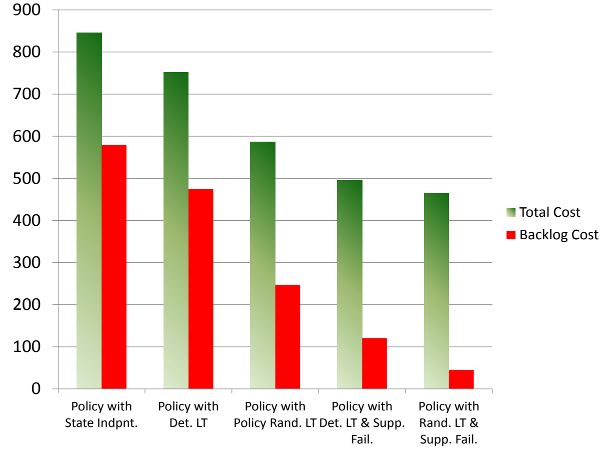

Figure 3.23: Total Costs for Unstable Supply

In the maintenance sector, operators of capital goods can incur extremely high downtime costs even if a single spare part is unavailable. For instance, consider aircraft on ground situations in the airline industry (Wong et al., 2007) or shutdowns in refineries (Trimp et al., 2004). This is one of the main motivations of performance-based contracts

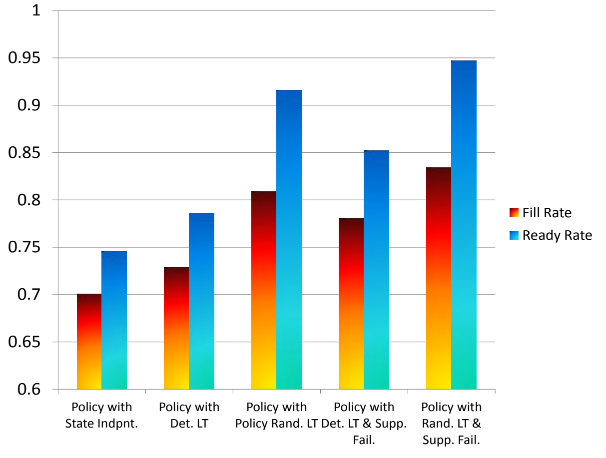

Figure 3.24: Service Measures for Unstable Supply

Table 3.12: Percentage Differences When Supply Side Risk Is Ignored (Unstable Supply Scenario)

Run#	Policy Name	Total Cost	Backlog Cost	Fill Rate	Ready Rate
0	State Indp. Det. LT	50.9 ± 1.3	14204 ± 621	16.1 ± 0.1	21.2 ± 0.8
1	State Dep. Det. LT	37.5 ± 1.2	10163 ± 502	12.8 ± 0.1	17.04 ± 0.8
3	State Dep. Rand. LT	13.2 ± 0.8	4243 ± 315	3.1 ± 0.1	3.3 ± 3.2
5	Det. LT & Supp. Fail.	6.5 ± 0.1	913 ± 13.1	6.5 ± 0.03	10.1 ± 0.5
7	Rand. LT & Supp. Fail.	0	0	0	0

Table 3.13: Percentage Differences When Supply Side Risk Is Ignored (Stable Supply Scenario)

Run#	Policy Name	Total Cost	Backlog Cost	Fill Rate	Ready Rate
0	State Indp. Det. LT	13.9 ± 0.6	2364 ± 194.8	8 ± 0.07	12.4 ± 0.1
2	State Dep. Det. LT	11.1 ± 0.5	1826 ± 162.3	7.1 ± 0.06	11.2 ± 0.06
4	State Dep. Rand. LT	2.1 ± 0.4	542.3 ± 111.3	0.56 ± 0.04	0.61 ± 0.04
6	Det. LT & Supp. Fail.	6.4 ± 0.1	690.5 ± 9.8	6 ± 0.03	9.9 ± 0.04
8	Rand. LT & Supp. Fail.	0	0	0	0

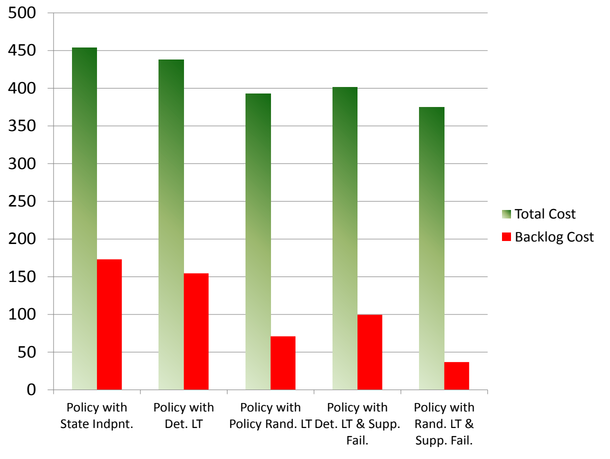

Figure 3.25: Total Costs for Stable Supply

and criticality of service rates. From a modeling perspective, this implies extremely high backlog costs. To understand the effect of supply risk for extreme backlog cost rates, we run the above analysis under various backlog multipliers, which we define as backlog cost over acquisition cost (Figures 3.27 and 3.28).

The combined effect of random lead times and supply failure increase to 2500% when the backlog multiplier is set to 100 in the unstable supply scenario (Figure 3.27). Furthermore, the effects of supply failure and random lead times seem to be very close, whereas

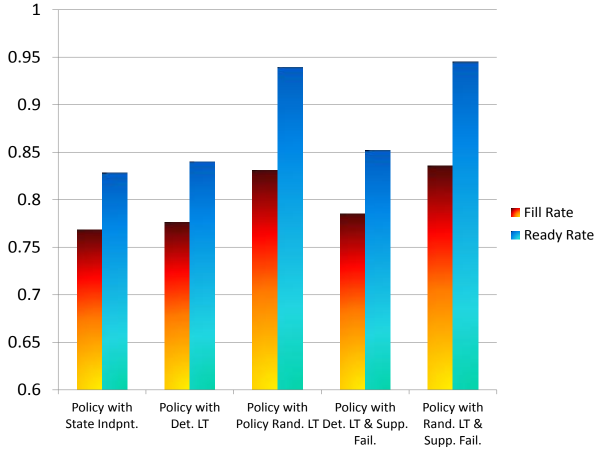

Figure 3.26: Service Measures for Stable Supply

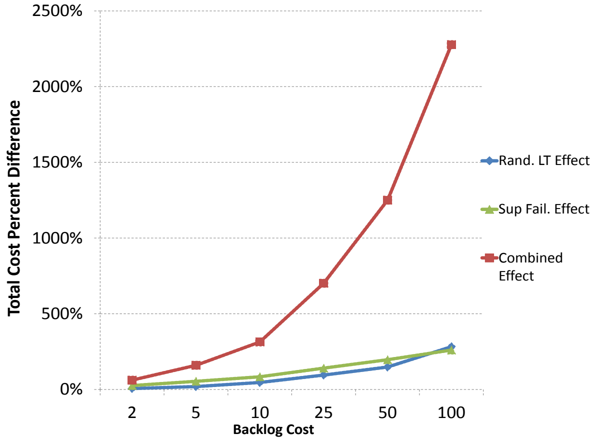

Figure 3.27: Effect of Random Lead Times and Supply Failure Under Different Backlog Costs (Unstable Supply)

their combined effect is much more than the sum of the individual effects. For the stable supply scenario (Figure 3.28), the combined effect can create cost increases of up to 800%. Here we can clearly observe the dominating effect of random lead times compared to the effect of supply failure. These results may provide an explanation for the overstocking behavior often observed in the maintenance sector (Ghobbar and Friend, 2002). One

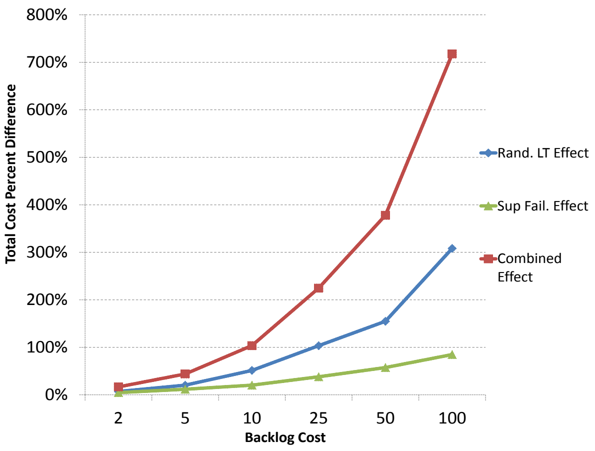

Figure 3.28: Effect of Random Lead Times and Supply Failure Under Different Backlog Costs (Stable Supply)

can argue that the managers are aware of the potential affects of supply problems on their service level. However, they tend to keep more stock than they need due to lack of quantitative models or proper decision making tools.

3.C.2 Summary and Discussion

In this part of the thesis, we present a more problematic subclass of supply disruptions: supply failures which we define to be permanent loss of supplier. We present analytical and numerical analysis from our model.

In general our results are in parallel with the ones presented in Sections 3.5.3 with disruptions. We find the coupled effect of random lead time and supply failure larger than the coupled effect with disruptions. This result is parallel to our understanding.

We can easily extend results of prevention vs treatment analysis presented in Section 3.5.4 to supply failures. Since supply failures aggravates the effect of supply risk on total cost, we expect proactive approach even more important compared to disruptions. Similarly this result implies the importance of advance warning signals and statistical indicators as in Chapter 2.

Chapter 4

Dual Sourcing with Stock-out Dependent Substitution

4.1 Introduction

In many businesses, companies prefer having multiple suppliers in order to ensure uninterrupted supply of their raw material or components of their products. Among these suppliers, the procurement department may have a preferred one due to a close relationship or strategic partnership between companies, as in 'Partnering for Success Program' by Boeing (Wilhelm, 2014). Supplier preference of a company may also stem from a quality difference between candidate suppliers. In fact, Abdolshah (2013) recognizes quality to be the most important criterion of vendor selection problems. When the supplier preference is due to a quality difference, companies may primarily purchase from a regular supplier with higher quality, and keep a back-up supplier which provides possibly lower quality items, on short notice. Hence another natural factor in supplier selection is the difference between supplier lead times.

This sort of lead time difference may stem from many factors such as geographical location, manufacturing process, supplier capacity etc. Regardless of the cause, the effect of lead time difference on procurement decisions is recognized as a very important factor affecting total cost and service level of companies (Chopra et al., 2004). In addition, having different products with similar functionality but possibly different quality levels introduces substitution into the problem setting.

In practice, substitution may arise in many different forms, depending on customer behavior and decision makers' capability of manipulating customer demand. Among different types of substitution, two of the most common are price-dependent and stock-out dependent substitution. The former considers the preference of customers when they face multiple products with similar functionality but different price and quality levels. In the latter case, customers are assumed to choose their first preference when that product exists in the inventory. In case of a stock-out they are offered another product. Hence, the probability of substitution depends on the stock levels of products in this case.

We consider a procurement problem including two suppliers: the first supplier delivers high quality items after a nonzero lead time, whereas the second supplier, potentially a spot market, has a random capacity and provides cheaper, lower quality items with immediate delivery. Hence the trade-off is among speed, quality and capacity uncertainty. In addition we consider the demand-side effects of having a price and quality difference between suppliers. Specifically, customers are assumed to prefer the higher quality product, and they are offered substitution in case of stock-out. When this happens, we assume that customers accept the substitution and the company incurs a substitution cost which may be interpreted as a discount on the product price or a penalty for the customer's dissatisfaction.

One of many possible examples for this problem setting in practice is sourcing from secondary markets and original equipment manufacturers (OEMs) for maintenance companies. In the following section, we present a business case of a maintenance repair organization (MRO), which is the primary motivation of this study.

4.1.1 Motivational Examples

The authors have contacts with a mid-size Maintenance Repair Organization (MRO) in Europe which provides maintenance service for aircraft. The fleet, operated by asset owners or airlines, needs (un)planned maintenance, which creates random spare parts demand. In addition, the company sells spare parts to other maintenance centers as well as airline operators. There are more than 500,000 spare parts (numbers) in the company's spare parts database.

For spare parts sourcing, the company utilizes original equipment manufacturers (OEM, *regular supplier*) as well as spot markets (*back-up supplier*). OEMs provide brand new parts to the company in perfect quality with positive lead times (in magnitude of months) whereas spare parts on spot markets can be in various conditions, e.g. overhauled, serviceable, as-removed etc., with virtually always immediate delivery (a couple of weeks at most). This is due to the fact that traders on spot markets do not manufacture parts directly, but instead sell their existing inventory.

In general, OEMs are assumed to have infinite capacity since annual spare parts demand is relatively small compared to the capacity of their suppliers. Sourcing from spot markets, on the other hand, is limited to the amount of spare parts available (*capacity of the back-up supplier*) at a decision epoch. This availability is random and depends on various factors such as the size of the installed base in use, surplus inventory from some airline operators, cannibalism of dismantled aircraft etc. Characteristics of both supply sources are summarized in Table 4.1.

On the demand side, customers have different attitudes towards the price and the condition of spare parts. Some customers are willing to pay extra for brand new spare parts and they demand high service rates from the MRO. These (loyal) customers have the highest priority and their demand must be satisfied as soon as possible. In case of a new part stock-out, the company utilizes spare parts from spot markets to avoid aircraft-on-ground situations, which creates costs for airline operators and may lead to large contractual fines to the MRO. However, supplying spare parts in other than new condition pays less (there is a discount on the price), and it may have an implicit cost for the company due to the fact that repetition of such cases might hurt the relationship with their loyal customers. Although the company also has price-sensitive customers, it gives lower priority to such customers and focuses on quality-sensitive customers.

In addition, the company's (spare parts) demand depends on the aircraft in operation (the installed base). An increasing number of aircraft in use stimulates the customer demand, whereas a declining number of aircraft slow the demand rate. In general, the company has access to fleet utilization information which can be used as an indicator for changing demand rate.

Another application area of our problem setting is component supply for a manufacturing company which can use new parts as well as remanufactured components from used products (Robotis et al., 2005). After collection, used products are disassembled and some of their components are refurbished for use in production of new products. In each period, the production planner first checks the (random) amount of existing remanufactured components, and then orders new ones from its suppliers, which can deliver after a nonzero lead time due to manufacturing and/or transportation. Assuming customer attitude towards the products including remanufactured components is different (in a negative way), we need to consider a nonzero substitution cost, such as a discount on selling price, together with other inventory-related cost rates (Robotis et al., 2005).

4.1.2 The Contribution of the Study

In this study, we established a single period cost function similar to Robotis et al. (2005), which we then employ in a multi-period dynamic programming model. Since mathematical analysis of the multi-period cost function reveals that it has convexity (and pseudo-convexity) only under some restrictive assumptions, we developed a heuristic myopic approach to calculate the policy parameter for the back-up supplier, whereas a simulation-based algorithm is developed for orders to the regular supplier. In our numerical experiments, we compare our policy with three other heuristic approaches from Sheopuri et al. (2010) (they show that these heuristics outperform other existing methods in the literature) as well as either optimal solutions or lower bounds.

Our results indicate that our method produces policies close to the optimum. These findings indicate that there is a significant motivation for managers to recognize the quality difference between suppliers (and the customer preference among them) which is commonly ignored in literature as well as inventory management software used in practice.

Another finding from our experiments is that the performance gap between our method and other heuristics gets larger when the capacity of the back-up supplier has positive or negative drift. This feature causes the deviation of other heuristics increases rapidly whereas our method generates more reliable results. This feature is especially important in the case of spare parts sourcing from spot markets, as availability on spot markets grows or shrinks by the installed based. Furthermore, we extend our results to Markov-modulated demand to capture nonstationarity in demand. These generated policies are tested with empirical data from a Maintenance Repair Organization who employs spot markets for sourcing spare parts.

This chapter consists of six main sections. In the next section, we present a brief review of relevant literature, placing our contribution in context. In Section 4.3 we present the single period problem and its mathematical properties; this comprises a fundamental building block of our multi-period problem. Section 4.4 is devoted to the development of multi-period formulation which is the primary focus of this study. This is followed by the presentation of our heuristic method and its performance compared to other methods in Section 4.5. Extension of our heuristic is to Markov modulated demand is provided in Section 4.6 whereas Section 4.7 includes an application of Markov-modulated demand policies to an empirical demand data. In Section 4.8 we present summary and our conclusions.

4.2 Literature Review

Related literature to our work consists of two major research streams: studies on dual-sourcing and substitution literature. We will provide a brief review of contributions to the both research streams in this section. While doing so, more attention is spent on latest contributions to the both areas of the inventory control literature.

Early contributions on dual sourcing problems are by Barankin (1961), Daniel (1963), Fukuda (1964) and Whittemore and Saunders (1977). Among these studies Fukuda (1964) proved the optimality of the dual index policy for two supply options with k and $k + 1$ periods of lead times. Whittemore and Saunders (1977) showed that the optimal policy is highly state-dependent and complex when the lead time difference between suppliers is more than one period. Similarly Feng et al. (2006) showed that the base stock policy is only optimal under restrictive conditions (contrary to the claim by Zhang (1996)) when there are three suppliers with lead times of k, $k + 1$, and $k + 2$ periods. Lawson and Porteus (2000) showed the optimality of a modified base stock policy for the multi-echelon dual sourcing problem in a serial supply chain. In his problem setting, decision maker in each echelon decides between expedited (immediate delivery), regular (1 period later) or delayed delivery option (2 periods later). Veeraraghavan and Scheller-Wolf (2008) contributed to this literature showing the separability of the two policy parameters which is exploited for calculation of optimal parameter values in a fast algorithm. Their method relies on simulation for overshoot distributions to calculate the base stock level for expedited supplier. Arts et al. (2011) contributed to this research stream by providing an approximation for the overshoot distribution using Markov chains. Another important contribution is provided by Sheopuri et al. (2010) who proved the equivalence of dual sourcing problems to the lost sales problems and use this property to develop new heuristics which outperform dual index policy when the lead time difference between two suppliers is longer than one. They also used this equivalence to provide another proof (in addition to Whittemore and Saunders (1977)) for non-optimality of order-up-to policies for dual sourcing problems with arbitrary lead time difference.

In addition to the studies on the optimal policy, scholars consider other inventory control policies for dual sourcing problems. Studies by Scheller-Wolf et al. (2007), Song and Zipkin (2009), Moinzadeh and Schmidt (1991), Allon and Van Mieghem (2010), Ju et al. (2015) can be considered in this sub-category of dual sourcing literature. Among these studies, the study by Ju et al. (2015) is the closest one to our study in the sense that they consider the quality of back-up supplier, which is expressed with a Binomial yield, in their problem setting. An extensive review of the dual sourcing literature (until 2003) can be found in Minner (2003).

Substitution studies in the inventory control literature can be categorized as firm-driven and customer-driven substitution (Hopp and Xu, 2008). In firm driven substitution, the seller makes the decision of substituting unsatisfied demand with other products. Therefore, substitution decision is considered as a part of the problem (Bassok et al., 1999; Van Mieghem and Rudi, 2002). For the customer-driven substitution, customer preference is considered in a probabilistic manner (Nagarajan and Rajagopalan, 2008; Hopp and Xu, 2008). We only review firm-driven substitution studies in this section, since our problem setting is closer to that research stream. A review of the literature on the firm-driven substitution can be found in Kök et al. (2009).

Bassok et al. (1999) consider firm-driven downward substitution for a single period model for N products with immediate deliveries. They show some characteristics of the optimal policy and provide a greedy algorithm which is shown to be optimal under a certain assumption. Van Mieghem and Rudi (2002); Van Mieghem (2004) consider multiple-storage points with multiple products in a newsvendor setting, which they call "newsvendor network". Harrison and Van Mieghem (1999) provide optimality for a single period model and show the conditions for the optimality of the myopic policy in a multi-period setting. Rao et al. (2004) consider a multi-period substitution problem with stochastic programming and suggest a heuristic procedure utilizing optimization, dynamic programming and simulation-based optimization. He claims that his approach performs reasonably well and is capable of solving industrial scale problems. Axsäter (2003a,b) recognize applicability of lateral transshipment models to firm-driven substitution problem. They develop a lateral transshipment decision rule for N warehouses following the (R,Q) policy for inventory control (Axsäter, 2003b). A recent review of lateral transshipment literature is provided by Paterson et al. (2011).

In this study, we consider quality difference between suppliers in a dual sourcing problem. Apart from the study by Ju et al. (2015), quality difference has not been addressed in the literature although it is recognized as the one of the key elements for supplier selection (Abdolshah, 2013). Note that Ju et al. consider the quality difference with Binomial random yield whereas our approach utilizes a constant cost rate for satisfying high quality product with a low quality product. Furthermore, our study contributes to substitution study by considering the lead time difference between suppliers in a multi-period setting.

Table 4.1: Supply-side Characteristics of the Problem Setting

Characteristic	Regular Supplier	Back-up Supplier
Capacity	Infinite	Markovian
Quality	High	Low
Lead Time	Positive	Immediate delivery

4.3 Single Period Model

We consider a periodic review model, denoting the low quality inventory as x and the high quality inventory as y, as in Figure 4.1. The regular, high quality supplier has unlimited capacity, whereas the capacity of the back-up supplier is random, we model this capacity as a Markov chain with a known transition matrix. (Note that the Markovian characteristic of the capacity is only relevant in the multi-period setting, presented in the next section.) Furthermore, we assume that the regular supplier delivers high quality products with a positive lead time, L^R, whereas orders to the back-up supplier are delivered in the same period.

We assume constant unit acquisition cost with the regular supplier's cost c^r greater than the back-up supplier's cost c^s. The characteristic features of the two suppliers are summarized in Table 4.1.

We assume that the company only receives demand for high quality products; incoming high quality demand is satisfied from existing high quality inventory in each decision period. In case of a stock-out, low quality products are used as substitutes (*downward substitution,* Figure 4.1). We assume that customers accept this substitution. When both high and low quality inventories are zero, demand is backlogged. These demand-side assumptions are motivated by the business case explained in Section 4.1.1.

Other assumptions are as follows: Excess inventory on hand incurs linear holding costs with cost rates $h^r \geq h^s$ for high quality and low quality items, respectively. Unsatisfied demand incurs per-period backlog cost rate b. We assume backlogging is more costly than substitution, $b \geq \psi$. All cost parameters and state variables are listed in Table 4.2.

The events of each period take place in the following order: First, previous orders from the regular supplier arrives. The decision maker reviews the inventory levels of the two products and checks the capacity of the back-up supplier. He places his orders to both channels, incurring the acquisition costs. The back-up supplier delivers the ordered low-quality products immediately and random demand arrives. If existing high quality inventory is sufficient to satisfy the demand, all customers leave the system with high quality items. If the demand is larger than the high quality inventory level then all

Table 4.2: Cost parameters and state variables of the problem setting

Variable	Explanation
t	time index in the multi-period period.
y	inventory level for high quality products.
x	inventory level for low quality products.
c^r	acquisition cost from regular supplier.
c^s	acquisition cost from back-up supplier.
h^r	holding cost of high quality items obtained from regular supplier.
h^s	holding cost of low quality items obtained from back-up supplier.
ψ	cost of satisfying high quality demand with low quality product.
b	backlog cost.
α	discounting rate.
$\phi(.)$	cumulative distribution function (cdf) of random demand.
K	capacity of back-up supplier in current period.
q_t^r	order to regular supplier at period t

high quality items are used and remaining demand is satisfied with low quality items. Unsatisfied demand after the substitution, if any, is backlogged. At the end of the period, inventory holding, substitution and backlog costs are incurred.

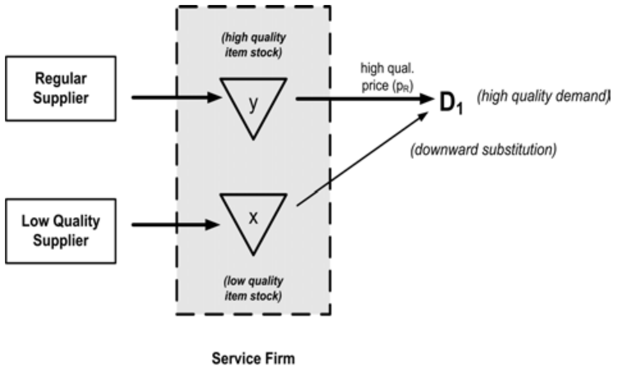

Figure 4.1: Dual Sourcing with Single Demand Class, Different Quality Levels and Demand Substitution

The single period cost function with high (y) and low (x) quality stocks is as follows:

$$L(y,x) \triangleq \int_{s=0}^{y} [h^r(y-s)+h^s x]d\phi(s)+\int_{s=y}^{y+x} [h^s(x+y-s)+\psi(s-y)]d\phi(s) \int_{s=y+x}^{\infty} [b(s-y-x)+\psi x]d\phi(s). \quad (4.1)$$

where $y \in \mathbb{R}, x \in \mathbb{R}^+$ and $\phi(.)$ is cdf of demand. Please note that our single demand class assumption implies that the low quality inventory level (x) can only take non-negative values. The first integral considers the case when incoming demand is smaller than the high quality inventory level; in this case, we incur holding costs accordingly. In the second integral, we cover the possibility of having customer demand exceed the high quality stock, but fall below the total inventory. In that case, substitution cost and holding cost of low quality items are incurred. In the last integral, we consider the possibility of demand being larger than the summation of high quality (y) and substitutable (x) products. In that case, backlog and substitution costs are incurred. The following lemma establishes the structural property of the cost function.

Lemma 8 *$L(y, x)$ in Equation 4.1 is jointly convex in y and x.*

The convexity of the single period cost function implies that inventory levels that minimizes the single period cost exist. This result provides the main building block for the multi-period problem in the following section.

4.4 Multi-Period Model

For the multi-period problem, define $V_t(K, y, x)$ as the minimum cost function of the system when the inventory levels are y and x for high and low quality items, the capacity of the back-up supplier is K and there are t periods until the end of the planning horizon. In our analytical formulation, we assume the lead time of the regular supplier is one period. The dynamic programming formulation of the multi-period problem is as follows:

$$G_t(K, v_t, w_t) = L(y, w_t) + c^s(w_t - x) + c^r(v_t - y) + \int_{s=0}^{y} \alpha E V_{t+1}(K_+, v_t - s, w_t) d\phi(s)$$
$$+ \int_{s=y}^{y+w_t} \alpha E V_{t+1}(K_+, v_t - y, w_t + y - s) d\phi(s) + \int_{s=y+w_t}^{\infty} \alpha E V_{t+1}(K_+, v_t + w_t - s, 0) d\phi(s), \quad (4.2)$$

and

$$V_t(K, y, x) = \min_{\substack{x \le w_t \le x+K, \\ y \le v_t}} \{G_t(K, v_t, w_t)\}, \quad y \in \mathbb{R}, \ x \in \mathbb{R}^+, \quad (4.3)$$

where v_t and w_t stand for order-up-to levels for the regular and the back-up suppliers respectively, and K_+ stands for the random capacity of the back-up supplier in the next period. In this recursive equation, we consider the same cases in the single period function.

Specifically, if the single period demand is lower than the existing high quality inventory level, this demand is supplied from the high quality inventory stock and the state of the system at the beginning of the next period is given in (4.2) in the first integral. If the demand is higher than the high quality inventory level but less than the sum of the inventories, substitution takes place where the amount of substituted demand can be denoted by $D - y$. In such a case, the current period's order constitutes the starting high quality inventory level at the beginning of the next period, as in the second integral. If the demand is larger than the summation of both types of stocks, then all existing low quality items are used for substitution and the rest of the demand is backlogged (the third integral).

Unfortunately, the analysis of this cost function reveals that joint convexity (and even pseudo-convexity) in v_t and w_t holds only under quite restrictive conditions. A complete convexity analysis of the multi-period function is given in Appendix 4.B. These analyses indicate that the optimal policy is complex and state-dependent which we confirmed through numerical experiments. To solve the problem, we developed a heuristic solution presented in the next section.

4.5 Heuristic Approach

Mathematical analysis of the multi-period cost function reveals that the optimal policy is state-dependent and highly complex. Therefore development of a simple and applicable heuristic approach is potentially valuable. To this end, we developed a heuristic policy which considers two order-up-to levels for high and low quality inventories respectively. The low quality order-up-to level, w, is found by using a myopic cost function, whereas calculation of the high quality order-up-to level, v, relies on a simulated distribution of the total amount of lead time demand satisfied via substitution. The methodology we used to calculate these two order-up-to levels are presented in the successive subsections.

4.5.1 Order-up-to Level for Low Quality Inventory

In order to derive the myopic policy for w, we apply the following transformation which is similar to Song and Zipkin (1993) and Veinott (1965).

$$W_t(K, y, x) = V_t(K, y, x) + c^s x + c^r y. \tag{4.4}$$

Then, our cost function becomes

$$W_t(K, y, x) = \min_{\substack{x \leq w \leq x+K, \\ y \leq v}} \left\{ \tilde{G}_t(K, y, v, w) \right\},$$

where

$$\begin{aligned} \tilde{G}_t(K, y, v, w) = H(y, v, w) + \int_{s=0}^{y} \alpha E W_{t+1}(K_+, v - s, w) d\phi(s) \\ + \int_{s=y}^{y+w} \alpha E W_{t+1}(K_+, v - y, w + y - s) d\phi(s) + \int_{s=y+w}^{\infty} \alpha E W_{t+1}(K_+, v + w - s, 0) d\phi(s), \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} H(y, v, w) =& c^r v(1 - \alpha) + w(c^s - c^r \alpha) + L(y, w) + \alpha(c^r - c^s) \Big[\int_{s=y}^{y+w} (y + w - s) d\phi(s) \\ & + w Pr\{D \leq y\} \Big] + \alpha c^r \mu. \end{aligned} \tag{4.6}$$

We define $H(y, v, w)$ as the myopic single period cost function. It captures holding, substitution and backlog costs of having y and w amounts of high and low quality items in stock given that the order-up-to level of high quality inventory is v. The role of the single period cost function is obvious in this equation and the terms in the last brackets stand for the expected low quality products left after demand realization. Therefore, the myopic cost function captures the tradeoff between buying at the current period instead of the next one and savings from high quality acquisition cost due to substitution.The mathematical structure of the myopic cost function is given in the following lemma.

Lemma 9 *$H(y, v, w)$ is jointly convex in v and w for a given y.*

Convexity of $H(y, v, w)$ is not surprising given the fact that the single period cost function is jointly convex and the function is separable in v and w. Also the convexity of the nonlinear terms in brackets (in Equation 4.6) follows from Lemma 8.

In addition to the convexity, we should note that $H(y, v, w)$ includes a linear term in v which implies that v becomes zero when we minimize the single period myopic cost function. Hence, this function can only provide potentially good base stock levels for the back-up supplier, w, for a given high quality inventory level y. The following result establishes the relationship between y and $w^*(y)$ that minimize the myopic cost function.

Lemma 10 *Suppose $w^*(y)$ minimize $H(y, v, w)$ for a given y. Then $w^*(y)$ satisfies*

$$y + w^*(y) = F^{-1}(1 - \gamma),$$

where $F(.)$ is the cdf of one-period demand and

$$\gamma = \frac{c^s(1-\alpha) + h^s}{b - \psi + \alpha(c^r - c^s) + h^s}. \tag{4.7}$$

Note that since the cost of the high quality item is assumed to be larger than the market price c^s of the low quality item ($c^r \geq c^s$) and then the backlog cost is smaller than the substitution cost, $\gamma \leq 1$. Lemma 10 indicates that the summation of the low-quality order-up-to level (w) and high quality inventory level (y) is equal to a constant at the minimum of the myopic cost function. This constant factor, γ, which is similar to the well-known critical fractile (Porteus, 2002), is a function of the difference between backlog cost and substitution cost. Increasing the difference of these two parameters motivates substitution (a similar relationship also holds for the acquisition cost difference between two suppliers). The following corollaries indicate other intuitive relationships between low quality items and cost parameters. The results are evident from Equation 4.7.

Corollary 6 *$w^*(y)$ is decreasing and linear in y.*

Corollary 7 *For a given high quality inventory level, the order-up-to level for market purchases $w^*(y)$ is a decreasing function of substitution cost and holding cost of low quality inventory.*

This concludes the methodology we use for calculating order-up-to levels for low quality inventory. The simulation-based approach for calculating high quality order-up-to level is presented in the next section.

4.5.2 Base Stock Level for High Quality Inventory

In order to calculate orders to the regular supplier at period t, denoted by q_t^r, we need to establish the relationship between high quality inventory level and the demand distribution. To this end, let us define a random variable S_t as the demand satisfied by low quality products in period t. For given low quality base stock level, w_t, and high quality inventory level at period t, y_t, S_t is given as follows:

$$S_t = \begin{cases} \min(w_t, D_t - y_t), & \text{if } D_t \geq y_t, \\ 0, & \text{otherwise.} \end{cases}$$

Note that the substitution that takes place in each period can also be considered lost sales as in Sheopuri et al. (2010) from the regular supplier's perspective. Similarly, the amount of demand supplied with high quality products is defined as $Z_t = D_t - S_t$.

We can derive recursive equations for the high quality inventory level by using S_t and Z_t given that the lead time of regular supplier is $L^R \geq 1$.

$$\begin{aligned} y_{t+1} &= y_t + q^r_{t-L^R} - (D_t - S_t), \\ &= y_t + q^r_{t-L^R} - Z_t. \end{aligned}$$

Multi-period recursive equations for high quality inventory are given as follows:

$$\begin{aligned} y_{t+2} &= y_{t+1} + q^r_{t+1-L^R} - Z_{t+1}, \\ &\dots \\ y_{t+L^R} &= y_{t+L^R-1} + q^r_{t+L^R} - Z_{t+L^R}. \end{aligned}$$

Let us define $D_t^{(m)}$ as the convoluted random demand between periods t and $t+m-1$, and $Z_t^{(m)}$ the total demand satisfied by high quality products over the same interval. Using these random variables, we can write the high quality inventory level at period t as follows:

$$y_t = IP_{t-L^R} - Z^{(L^R)}_{t+L^R}, \tag{4.8}$$

where IP_t stands for inventory position for high quality products. This equation indicates the dependence between the amount of substitution and the high quality inventory level for a given inventory position. Due to the difficulty of deriving an analytical expression of substituted demand, our heuristic approach relies on the simulation of $Z_t^{(L^R)}$. Note that from this point on we drop the subscript indicating time.

Suppose that the distribution of non-substituted demand, $Pr\{Z^{(L^R)} = k\}$ is known and Θ_v is the set including all possible realization of $Z^{(L^R)}$ for a given inventory position v. Using Equation 4.8 and considering the capacity of the back-up supplier, the myopic cost function for given high and low quality base stock levels (v and w) can be written as follows:

$$g(y, v) := H(y, v, \min(w^*(y), K)), \tag{4.9}$$

whereas the expected myopic cost function is

$$\bar{g}(v) := \sum_{u \in \Theta_v} Pr\{Z^{(L^R)} = u\} g(v - u, v), \tag{4.10}$$

by using the simulated distributions of $Z^{(L^R)}$. In our heuristic, we use $v^* = \arg\min_{v \in \mathbb{R}} \bar{g}(v)$ as the base stock level for orders to regular supplier. The following theorem characterizes v^* under certain conditions.

Theorem 8 *If $h^r - h^s + \psi \geq \alpha(c^r - c^s)$, then the following statements hold.*

1. *$g(u, v)$ is a convex function of v for each u,*
2. *$\bar{g}(v)$ is a convex function of v,*
3. *$v^* = \arg\min_{v \in \mathbb{R}} g(u, v)$ exists.*

Even when the condition of Theorem 8 does not hold, we leverage the theorem to provide inspiration to our heuristic. Specifically, Lemma 10 and Theorem 8 are utilized for the calculation of base stock levels for the regular and the back-up suppliers in our heuristic algorithm given below.

Algorithm 1 The Algorithm for Modified Myopic Approach

1: **for all** v **do**
2: Simulate $Pr\{Z^{L^R} = i\}$, $\forall\, i \in \Theta_v$;
3: Calculate $\bar{g}(v) = \sum_{i \in \Theta_v} Pr\{Z^{L^R} = i\} g(v - i, v)$;
4: **end for**
5: Calculate $v^* = \arg\min \bar{g}(v)$;

As stated above, our heuristic approach relies on the distribution of Z^{L^R} which is obtained through simulation. Since this random variable depends on the high quality base stock level v, the simulation has to be run for all values of v. In the third step of the algorithm, we calculate the base stock level for the regular supplier for a given v and realization of Z^{L^R}. At the end of the third step, the calculated expected myopic cost functions, $\bar{g}(v)$, are stored in a vector of which the minimum is found in the final step of the algorithm. Note that this heuristic approach requires a run of simulations to develop empirical distributions at the beginning of the planning horizon. In our experiments, presented below, the average required time for this process was approximately 30 seconds. Once the base stock level for high quality products are calculated, the rest of the decisions are made in milliseconds. In the following section, we present the performance of our heuristic policy.

4.5.3 Dual Sourcing Heuristics form Sheopuri et al. (2010)

In our numerical experiments, we consider three heuristic policies from Sheopuri et al. (2010): vector-based heuristic, demand allocation(U) and demand allocation(L).

Vector based heuristic is an adaptation of an heuristic policy from lost sales literature to the dual sourcing problems. It uses a ratio $c^s/(c^s+h)$ and all outstanding orders to place new orders to the regular supplier. That heuristic was first suggested by lost sales problems by Morton (1971). Orders to the back-up supplier are placed by using the base stock policy (for the expedited supplier) from Veeraraghavan and Scheller-Wolf (2008).

Demand allocation heuristics are essentially base stock policies for the regular supplier. They prescribe ordering (to regular and back-up suppliers) as much as the previous period's demand in order to keep the inventory position at the base stock level. For division of the order between suppliers, Sheopuri et al. (2010) suggested three myopic cost functions capable of considering all outstanding orders. Then they show that the order quantity minimizing the first *extended* myopic function, denoted by $q^1(D_{t-1})$, overestimates orders to the back-up supplier whereas minimizers of other two functions, denoted by $q^2(D_{t-1})$ and $q^3(D_{t-1})$, underestimate it. They call $q^1(D_{t-1})$ as Demand Allocation(U) heuristic whereas $\max(q^2(D_{t-1}), q^3(D_{t-1}))$ as Demand Allocation(L).

4.5.4 Numerical Results

In this section, we compare our heuristic method with the heuristics by Sheopuri et al. (2010). In addition, we calculated the optimal policy for a set of numerical experiments using full dynamic programming with a proper discretization. For the rest, we considered best dual index policy as the benchmark for our policy which is commonly used (Arts et al., 2011; Veeraraghavan and Scheller-Wolf, 2008; Sheopuri et al., 2010), and recognized to be the "best available heuristic in the literature" (Sheopuri et al., 2010). Although Sheopuri et al. (2010) showed their heuristics' costs lower than the best dual index policy, the optimality gap of these policies is still an unanswered question. Therefore, we chose to use the best dual index policy as a benchmark in this study.

During optimization of a multi-state, recursive equation, one has to deal with a large state space even after a proper truncation is applied. This is especially when the lead time of regular supplier is longer than one period (the size of the state space is more than 8 million when the lead time is equal to three periods). This very large number of states stems from the fact that outstanding orders, current inventory levels and capacity of the back-up supplier need to be kept as state variables.

The test bed we employed in our study consists of two major components. First, we devise our test bed with ten different factors. We treat the five most important as variable whereas the rest are constant. Second, we extend the test bed in Sheopuri et al. (2010) to evaluate different drifts (increasing and decreasing) for the evolution of the capacity of the back-up supplier. Details of each test bed and results obtained from them are presented below.

Our Test Bed

In our test bed we vary lead time of the regular supplier, planning horizon, substitution cost rate, holding cost rate for low quality items, and the rate of demand distribution. Values for these factors are given in Table 4.3. Constant factors, on the other hand, are backlog cost, high quality holding cost, acquisition costs for the regular supplier and the back-up supplier. Parameter values of constant factors are given in Table 4.4. In addition, we assumed that the Markovian capacity of the back-up supplier would take integer values between zero and four, and it would follow the transition matrix P given below.

Table 4.3: Values of Variable Factors for Our Test Bed

Regular Supplier Lead Time	**Horizon**	**Substitution Cost Rate**	**Low Quality Holding Cost**	**Demand**
1	24	50	6	Poisson(2)
3	36	100	8	Poisson(4)
		150	10	
		200		

Table 4.4: Constant Factors for Our Test Bed

Backlog Cost Rate	**Acquisition Cost Regular Supp.**	**Acquisition Cost Back-up Supp.**	**High Quality Holding Cost**
200	100	60	10

$$P = \begin{pmatrix} \alpha & 1-\alpha & 0 & 0 & 0 \\ \beta & \gamma & \beta & 0 & 0 \\ 0 & \beta & \gamma & \beta & 0 \\ 0 & 0 & \beta & \gamma & \beta \\ 0 & 0 & 0 & 1-\alpha & \alpha \end{pmatrix}, \tag{4.11}$$

where $\alpha = \gamma = 0.5$, and $\beta = 0.25$.

In this experiment set-up, a full factorial design with given values yields 48 experiments for each lead time option for the regular supplier. Results, comparing our algorithm those from Sheopuri et al. (2010), by the optimal cost are summarized in Table 4.5 whereas a detailed list of all results are provided in Appendix 4.C.

Table 4.5: Deviations from the Optimal Policy for Test Bed 1

	Vector-Based	**Demand Alloc.(U)**	**Demand Alloc.(L)**	**Mod. Myopic**
Average	11.5%	56.4%	62.7%	6.0%
Std.Dev.	8.7%	45.1%	39.4%	2.0%
Max	30.8%	136.4%	136.4%	12.2%
Min	3.2%	7.0%	7.0%	3.2%

As can be seen from Table 4.5, our Modified-Myopic policy deviates 6% from optimal, whereas the vector-based heuristic from Sheopuri et al. (2010) deviates by 11%. A closer look at the results indicates that the majority of the deviation of the vector-based heuristic stems from runs with L^R is equal to 3 (Figure 4.2). In fact, when $L^R = 1$ results of paired t-tests indicate that vector-based heuristic is better in 20 of 48 runs whereas in 4 runs the difference was statistically insignificant. This means that the modified myopic policy is better than the vector-based heuristic in 50% of runs. But for runs with $L^R = 3$, the modified myopic policy is significantly better in 36 out of 48 runs. Thus compared by the vector-based heuristic, the modified myopic policy performs much better when the lead time of the regular supplier is longer. The performance of demand allocation heuristics (U&L) is significantly worse than vector-based heuristic.

Extended Version of the Test Bed of Sheopuri et al. (2010)

For our second test bed, we extended that of Sheopuri et al. (2010), which considers geometric demand along with different lead times and cost parameters. Furthermore, Sheopuri et al. set the cost of the regular supplier to zero and try different values for the cost of the back-up supplier since they benchmark against dual index policies and it is shown that the performance of a dual index policy is mainly effected by the cost difference

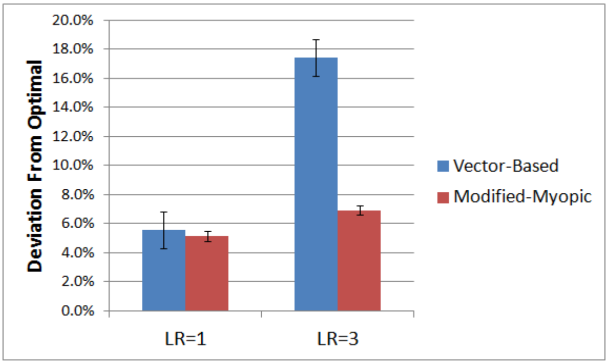

Figure 4.2: Deviation from the optimal policy for different lead times

between the suppliers (Veeraraghavan and Scheller-Wolf, 2008; Sheopuri et al., 2010). We adapt this approach to our problem setting by setting the cost parameters of the both suppliers to zero and trying nonzero values for the substitution cost. Cost parameters and lead times of the regular supplier are given in Table 4.6.

Table 4.6: Parameters of the Test Bed 2

Reg.Supp LT	High Qual. Hold.	Low Qual. Hold.	Backlog	Substitution	Demand
1	5	3	15	20	Geom.(0.4)
2		5	85/3	40	Geom.(0.5)
3			95	60	

Combining factors in Table 4.6 and removing runs with substitution costs being larger than backlog costs from the test bed, we obtain 144 runs (48 runs with each lead time value) which constituted the “core” of test bed 2. In these core runs, we consider the transition matrix P in Equation 4.11 for the capacity of the back-up supplier.

As an extension, we evaluated two alternate scenarios for the capacity of the back-up supplier: a drift towards larger or smaller values depending on its transition matrix. These scenarios could represent situations which a spot market (or gray market) had increasing (or decreasing) availability due to changes in the installed base. The considered transition matrix for these two scenarios are given by P^+ and P^- below. Also, we considered two different sets of values for β and γ for each scenario. The main motivation behind this approach was to evaluate the sensitivity of our heuristic under different back-up supplier characteristics.

$$P^+ = \begin{pmatrix} \alpha & 1-\alpha & 0 & 0 & 0 \\ \beta & \gamma & \gamma & 0 & 0 \\ 0 & \beta & \gamma & \gamma & 0 \\ 0 & 0 & \beta & \gamma & \gamma \\ 0 & 0 & 0 & 1-\alpha & \alpha \end{pmatrix}, \; P^- = \begin{pmatrix} \alpha & 1-\alpha & 0 & 0 & 0 \\ \gamma & \gamma & \beta & 0 & 0 \\ 0 & \gamma & \gamma & \beta & 0 \\ 0 & 0 & \gamma & \gamma & \beta \\ 0 & 0 & 0 & 1-\alpha & \alpha \end{pmatrix},$$

where $(\alpha, \beta, \gamma) = \{(0.5, 0.2, 0.4), (0.5, 0.163, 0.419)\}$. Expected back-up supplier capacities in these four different scenarios are listed as follows: $\Pi_1^+ = 2.811$, $\Pi_2^+ = 3$, $\Pi_1^- = 1.189$, $\Pi_2^- = 1$. By this extension, the total number runs evaluated in the second test bed is 720.

Due to large state spaces, we only calculated the optimal policy for $L^R = 1$ and the core run set (48 runs with the transition matrix P) for $L^R = 3$. In the rest of the test bed, heuristic policies are compared to the best dual index policy (which is calculated by a search over all possible values of policy parameters for each given random sample path of demand and market capacity). To see the gap between the best dual index and the optimal policy, we calculated the performance of both for the core run set with $L^R = 3$. The deviation of the best dual index policy (from the optimal) is -0.03%. Also note that for 69% of runs the best dual index policy is lower than the optimal policy due to the fact that we search for the best policy parameters for *each demand sample path*.

Statistics for deviations of all heuristics from the benchmarks are given in Table 4.7 whereas the results of all heuristics will be provided by the author upon request. Our results indicate that the vector-based policy deviates from the benchmark by an average of 48% while the modified myopic policy's deviation is only 19.1%. The demand allocation heuristics by Sheopuri et al. (2010) suffer the largest deviations among all candidates. In addition, deviations of the modified-myopic heuristic have smaller standard deviation and a much smaller maximum compared to the vector-based and demand allocation heuristics. In addition, we considered dual index policy for this problem setting. To optimize the parameter values, we use "brute-force" search over the parameter space. Results indicate that the dual index policy is slightly better than our heuristic for this test bed.

From computation time point of view, Sheopuri et al. (2010) cite that his heuristic has the same computational complexity with the dual index policy by Veeraraghavan and Scheller-Wolf (2008), who consider infinite horizon, average cost criteria. In this study, we obtain dual index parameters using brute force search with simulation, therefore computation time of dual index policy is not a fair benchmark to compare the performance of our method (brute force search takes too much time). However, Sheopuri et al. (2010)

state that their policy has the same computational complexity with the dual index policy (Veeraraghavan and Scheller-Wolf, 2008). Therefore, we compare our method with vector-base heuristic. Results indicate that our method is almost 2 times faster than the vector-based heuristic. Therefore, we conclude that although dual index policy has a marginally smaller deviation (1%), our policy is much efficient than this policy.

Table 4.7: Deviations from the Benchmark for Test Bed 2

	Vector-Based	**Dem. Alloc.(U)**	**Dem. Alloc.(L)**	**Modf. Myopic**	**Dual Ind.**
Average	48.3%	210.5%	218.6%	19.1%	17.6%
Std.Dev	53.9%	110.5%	110.5%	13.4%	9.3%
Max	304.9%	586.7%	586.7%	63.9%	46.3%
Min	0.2%	35.4%	29.1%	0.3%	0.6%

A closer look at the deviations indicates that the largest portion of deviations appear for runs with $L^R > 1$. Also we find that scenarios with positive market drift leads to lower deviations compared to the ones with negative drift. This indicates that cases with decreasing capacity of the back-up supplier are more difficult to deal with compared to the situation where the capacity of the back-up supplier improves over time. Furthermore, the breakdown of the deviations into holding, substitution and backlog costs (Table 4.8) shows that the substitution cost of the modified myopic heuristic has 0.01% average deviation from the optimal policy for $L^R = 1$ and the main deviations stem from high quality orders. This result is not surprising given the fact that we use the base stock level minimizing the myopic cost function in our solution.

Table 4.8: Breakdown of Cost Deviations

	Heuristic Name	**Holding**	**Substitution**	**Backlog**
L^R=1	Vector-Based	17.3%	2.5%	-6.3%
	Modified-Myopic	1.1%	0.01%	3.4%
	Dual Index	1.0%	-3.1 %	4.7%
L^R=2	Vector-Based	45.4%	-8.1%	20.2%
	Modified-Myopic	-6.1%	5.5%	28.8%
	Dual Index	6.1%	3.3 %	10.2 %
L^R=3	Vector-Based	51.9%	-14.1%	36.1%
	Modified-Myopic	4.7%	-1.7%	21.5%
	Dual Index	6.7 %	5.2 %	8.2 %

4.6 Markov-Modulated Demand

In practice, a demand distribution rarely follows a stationary distribution for the entire life cycle of a product. In most cases, customer demand is affected by environmental or product-related factors such as annual economic growth rate, age of the product, seasonal variations (e.g. holiday season in U.S.), etc. Such demand nonstationarity can potentially be addressed in two different ways: via an exogenous Markov chain that drives the demand process, or by using Bayesian approach for the demand distribution. In this section, we consider the former method to address the demand nonstationarity in our problem setting.

In single sourcing problems with Markov modulated demand, the optimality of a state-dependent base stock policy has been proven by many scholars (Song and Zipkin, 1993, 1996; Gallego and Hu, 2004; Muharremoglu and Yang, 2010). Assuming that the decision maker can perfectly observe the demand state, each state has an associated base stock level that is used for replenishment order of that period. Furthermore, the optimality of the same policy has been proved when the exogenous Markov chain can only be partially observed (Arifoğlu and Özekici, 2010) or completely hidden (Arifoğlu and Özekici, 2011). To consider Markov-modulated demand in our dual sourcing setting with substitution, we extended our heuristic policy to incorporate state-dependent base stock level either for the back-up supplier or the regular supplier (but not both). These two heuristics are presented in the following sections.

4.6.1 State-Dependent Base Stock Level for the Back-up Supplier

State-dependent base stock levels for the back-up supplier are denoted with $w_j^*(y_t)$ where j stands for the state of the Markov chain. For this case, it is not difficult to show that Lemma 10 holds for each demand state respectively.

To calculate the static base-stock policy for the regular supplier, we use simulation to obtain the distribution of the demand satisfied with high quality inventory; this depends on the state of the Markov chain and is denoted by Z_j^{LR}. We denote the support of the distribution of Z_j^{LR} by Θ_v^j for a given base stock level v of high quality products.

The formal description of the heuristic can easily be obtained by adding the demand state as a parameter to the functions in Equations 4.6 and 4.9. These extended functions are used to obtain $\bar{g}(j, v)$ for each demand state j and each base stock level v, similar to the procedure shown in Equation 4.10. Using the stationary distributions of the Markov chain, π_j, we take the weighted average of $\bar{g}(j, v)$ as follows:

$$\bar{g}(j,v) = \sum_{i \in \Theta_v^j} \pi_j Pr\{Z_j^{L^R} = i\} g(j, v-i, v), \tag{4.12}$$

$$\mathcal{G}(v) = \sum_{j \in \Omega} \pi_j \bar{g}(j,v), \tag{4.13}$$

where Ω is the state space of the Markov chain (driving the demand distribution). Required computation steps for the heuristic policy are summarized in Algorithm 2.

Algorithm 2 State-Dependent Base Stock Level for the Back-up Supplier

1: **for all** v **do**
2: Simulate $Pr\{Z_j^{L^R} = i\}$, $\forall\, i \in \Theta_v^j, \forall j \in \Omega$;
3: Calculate $\mathcal{G}(v) = \sum\limits_{j \in \Omega} \pi_j \sum\limits_{i \in \Theta_v^j} Pr\{Z_j^{L^R} = i\} g(j, v-i, v)$;
4: **end for**
5: Calculate $v^* = \arg\min \mathcal{G}(v)$;

In the third step of the algorithm, the function $\mathcal{G}(v)$ uses the state-dependent base stock levels for the low quality inventory $w_j^*(v-i)$ for a given Markov state, j, a realization of $Z_j^{L^R}$, i, and high quality base stock level, v (This is analogous to Equation 4.9). All calculated $\mathcal{G}(v)$ are stored in a vector at the end of the third step and these stored values are used to find the high quality base stock level in the last step of the algorithm. The computational complexity of this algorithm is not different from Algorithm 1 since we this heuristic uses only single base stock level for the regular supplier. Our second approach for Markov-modulated demand is presented in the following section.

4.6.2 State-dependent Base Stock Level for the Regular Supplier

Our second approach considers state-dependent base stock levels for the regular supplier and a single base stock level for the back-up supplier since increasing number of control parameters might decreases the practicality of the method.

To this end, we consider the weighted average of state dependent base stock levels for the back-up supplier, $w_j^*(y_t)$ as the base stock level for low quality products. Define base state dependent base stock levels for the regular supplier $\bar{v^*} = (v_1^*, v_2^*, ..., v_N^*)$ for a Markov chain with N states. Since the distribution of the demand satisfied with the high quality products depends on $\bar{v^*}$, we run simulations for $Z_{\bar{v}}^{L^R}$ for all possible values of $\bar{v^*}$, which requires nested search loops as can be seen in Algorithm 3. Using the simulated distributions, we calculate the values for base stock levels for the regular supplier. The algorithm of this approach is given below for a Markov chain consisting of N states.

Algorithm 3 State-dependent Base Stock Levels for the Regular Supplier

for all v_1 **do**
  for all v_2 **do**
    ...
    for all v_N **do**
      Simulate $Pr\{Z_{\bar{v}}^{L^R} = i\}, \forall\, i \in \Theta_{\bar{v}}^{j}$;
      for all $j \in \{1, 2, ..., N\}$ **do**
        Calculate $\bar{g}_j(v_j) = \sum_{i \in \Theta_{\bar{v}}^{j}} Pr\{Z_j^{L^R} = i\} g(j, v_j - i, v_j)$;
      end for
      Calculate $\mathcal{G}(\bar{v}) = \sum_{j \in \{1,2,..,N\}} \pi_j \bar{g}_j(v_j)$;
    end for
    ...
  end for
end for
Calculate $(\bar{v}^*) = \arg\min_{v_1, v_2, ..., v_N} \mathcal{G}(\bar{v})$;

4.7 Empirical Study

For evaluation of the performance of the modified myopic policy for Markov-modulated demand, we run an empirical analysis using data from the Maintenance Repair Organization (MRO), presented in Section 4.1.1. Our empirical study consists of three major phases: Data collection and sampling, parameter estimation, and testing the heuristic. Each phase is discussed in respective subsections below.

4.7.1 Data Collection and Sampling

In order to test our heuristic for the procurement problem of the MRO, we collected the empirical data corresponding to 14046 spare parts (due to other classifications of spare parts, which are not relevant for the results of the study, the majority of spare parts in the company's database is eliminated). We took a sample of 139 parts using a sampling procedure summarized in Appendix 4.D. The part sample, is further categorized based on parts' average annual demand and OEM prices. As a result, we obtain six part categories including 20.9 parts on average. 4 part numbers from each category are randomly selected into a test set whereas the rest of the parts are used to train model parameters. Note that one part in the test group is eliminated since there was no demand for this part for the entire planning horizon (104 months).

4.7.2 Parameter Estimation

For our next step, we sought historical availabilities for our sampled parts from a well-known spare parts trading platform (ilsmart.com), as well as fleet utilization data. The historical market availability data is used to train the Markov transition probabilities for the capacity of the back-up supplier, whereas fleet utilization data is used for training the Markov chain driving the customer demand. Twelve months of data for market availability was used.

For each part, we checked Poisson, geometric and negative binomial distributions as candidate demand distributions to be used in our heuristic policy. These distributions are recognized to be potentially relevant by Syntetos et al. (2012). Among these candidates, the geometric distribution is found to the best. Using the maximum likelihood estimators of the geometric distribution, we estimated parameters for the customer demand.

Acquisition cost from OEM was available for all parts in our sample in the database of the company. However for 13 of 23 parts in the test set, the price information was unavailable on the secondary market. Therefore, we assumed that the market price of these parts are 80% of the acquisition cost. Furthermore, we found that the available market price is higher than the price of OEM for 7 parts.

In our test group, we have 12 parts with 1 month of OEM lead time. Also we have 2 parts with 2, 4, 5 and 7 months of lead times whereas 3 parts with 3 months of lead time from OEM.

For holding cost rate, we considered {0.1,0.15,0.2,0.3} to be multiplied with the acquisition cost. For each holding cost multiplier, we considered {0.9,0.95,0.99,0.995} as services rates through which we calculated backlog cost rates using the critical ratio. Substitution cost rates are assumed to be equal to 80% of the backlog costs for the parts whose the market acquisition cost is smaller than the OEM cost.

4.7.3 Tests

Using the parameter values described as well as holding and backlog cost rates, we run our two heuristics for Markov-modulated demand and used the best dual index policy as the benchmark. The total number of parameter set (each set consists of 23 parts) is 16. The results of the two policies, the one with state-dependent parameters for the back-up supplier (formulated in Algorithm 2) and the one with state-dependent parameters for the regular supplier (formulated in 3), are presented in Table 4.9 together with dual index policy with four parameters for each state.

These results indicate that although our heuristic policies uses only three indices (instead of four), it may perform better than the dual index policy for Markov-modulated random lead time. Also note that the performance of policy3 is, naturally, better than the performance of policy2 since it regulates two indices for the high quality supplier.

Table 4.9: Deviations of the Two Heuristic Policies for Markov-Modulated Demand

L^R	**Policy2**	**Policy3**	**Dual Index**
1	22.3%	13.2%	20.9%
2	13.9%	8.5%	20.3%
3	32.4%	34.1%	17.8%
4	41.4%	29.8%	32.7%
5	56.5%	58.5%	54.7%
7	43.7%	19.0%	48.9%

4.8 Conclusion

When making sourcing decision from two suppliers, three features are usually critical: lead time, quality and cost differences between suppliers. For example, in many business settings, companies source from local and off-shore suppliers to satisfy their demand. Although it may be ignored in many cases, different suppliers rarely produce identical products; rather they are *substitutable*. If customers have different attitudes towards different quality levels, then stock-out dependent substitution should be considered.

In this study, we considered stock-out dependent substitution in dual sourcing problems. We consider two suppliers (regular and backup). In our problem setting, the regular supplier has a longer lead time, high quality, more expensive products whereas the back-up supplier provides immediate deliveries of low quality, cheaper products. Another important aspect of the back-up supplier is its random capacity, which is assumed to be Markovian. Assuming only high quality demand arrives to the system, the quality difference between products leads to stock-out dependent, *downward* substitution for high quality product. This substitution takes place in exchange for substitution cost.

The mathematical analysis of the multi-period cost function reveals that the cost function presents convexity or pseudo-convexity only under unrealistic conditions. Therefore, we proceed to a heuristic approach in order to bring a solution to the problem.

In this study, we suggest a modified version of dual index policy of which base stock levels are calculated using a myopic cost function and a simulation-based algorithm. We call this policy *modified-myopic* heuristic.

In numerical experiments we compare our policy with three different dual sourcing heuristics from the literature. These heuristics are the most recent contributions. All of these heuristics are benchmarked to the numerical optimum solution for some parts of the test bed or the best dual index policy for another. Our results indicate that, modified-myopic policy outperforms heuristics by Sheopuri et al. (2010) but is slightly worse than the dual index policy. Gaps between our policy and the others are rather close when the lead time of the regular supplier is set to 1 whereas the gap is much larger when we consider lead times larger-than one. These results also indicates existence of significant potential by recognizing the quality difference between suppliers as well as customer attitudes towards it.

Next, we extended our heuristic policy for Markov-modulated demand. Assuming the decision maker can perfectly observe the states of the Markov chain (driving the demand), we consider state-dependent base stock levels for the back-up supplier and the regular supplier in respective heuristics. These heuristics are evaluated using empirical data. Our results indicate that our policies performs well against the dual index policy which caries two base stock levels for each state of the Markov chain. Note that significant deviations of our policy from the benchmark indicate that the search for better policies for Markov-modulated demand is still open.

4.A Proofs of Theorems

Proof of Lemma 8

The components of the Hessian matrix for $L(y, x)$ is given as follows:

$$\frac{\partial}{\partial y}L(y,x) = \int\limits_{s=0}^{y} h^r d\phi(s) + \int\limits_{s=y}^{y+x} (h^s - \psi)d\phi(s) - \int\limits_{s=y+x}^{\infty} bd\phi(s), \tag{4.14}$$

$$\frac{\partial^2}{\partial y^2}L(y,x) = (h^r - h^s + \psi)\phi(y) + (b + h^s - \psi)\phi(y+x) \geq 0. \tag{4.15}$$

$$\frac{\partial}{\partial x}L(y,x) = \int\limits_{s=0}^{y} h^s d\phi(s) + \int\limits_{s=y}^{y+x} h^s d\phi(s) + \int\limits_{s=y+x}^{\infty} (-b + \psi)d\phi(s), \tag{4.16}$$

$$\frac{\partial^2}{\partial x^2}L(y,x) = (h^s - \psi + b)\phi(y+x) \geq 0, \tag{4.17}$$

$$\frac{\partial^2}{\partial x \partial y}L(y,x) = (h^s + b - \psi)\phi(y+x). \tag{4.18}$$

The positivity of second partial derivatives come from our assumptions: $h^r \geq h^s$ and $b \geq \psi$. For positive semidefiniteness of Hessian matrix we need the non-negativity of the following:

$$\frac{\partial^2 L(y,x)}{\partial x^2}\frac{\partial^2 L(y,x)}{\partial x^2} - \left[\frac{\partial^2 L(y,x)}{\partial x \partial y}\right]^2 = \phi(y+x)\phi(y)[h^r - h^s + \psi](h^s - \psi + b) \geq 0. \tag{4.19}$$

■

Proof Lemma 9

Let us write the components of Hessian matrix.

$$\frac{\partial H}{\partial v} = c^r(1-\alpha),$$
$$\frac{\partial^2 H}{\partial v^2} = 0,$$
$$\frac{\partial H}{\partial w} = c^s - c^r\alpha + L_w(y,w) + \alpha(c^r - c^s)\int\limits_{0}^{y+w} d\phi(s),$$
$$\frac{\partial^2 H}{\partial w^2} = L_{ww}(y,w) + \alpha(c^r - c^s)\phi(y+w) \geq 0,$$
$$\frac{\partial^2 H}{\partial w \partial v} = 0.$$

Hence $H(y,v,w)$ is convex in (v,w). ■

Proof of Lemma 10

As stated in Lemma 6, $w^*(y)$ can be calculated with the first order condition of $H(y, v, w)$ which is

$$c^s - c^r\alpha + L_w(y, w) + \alpha(c^r - c^s) \int_0^{y+w} d\phi(s) = 0.$$

This leads to

$$c^s - c^r\alpha + (h^s + \alpha(c^r - c^s)) \int_{s=0}^{y+w} d\phi(s) + (-b + \psi) \int_{s=y+w}^{\infty} d\phi(s) = 0.$$

$$\begin{aligned} c^s - c^r\alpha + (h^s + \alpha(c^r - c^s))F(y + w) &= (b - \psi)(1 - F(y + w)), \\ F(y + w)(b - \psi + \alpha(c^r - c^s) + h^s) &= b - \psi + \alpha c^r - c^s, \\ F(y + w) &= \frac{b - \psi + \alpha c^r - c^s}{b - \psi + \alpha(c^r - c^s) + h^s}, \\ y + w &= F^{-1}\left(1 - \frac{c^s(1 - \alpha) + h^s}{b - \psi + \alpha(c^r - c^s) + h^s}\right). \qquad (4.20) \end{aligned}$$

■

Proof of Theorem 8

Statement a implies b which leads to c. Hence we only need to show the first statement is true. Before doing so let us recall Lemma 10:

$$w^*(u - i) + u - i = F^{-1}(1 - \gamma)$$

where γ is defined in Equation 4.7. Also, we will define two variables to simplify the following derivations.

$$A = \min(F^{-1}(1 - \gamma) - u + i, K),$$

and

$$M = A + u - i.$$

Now, we can write $g(u,i)$ as follows:

$$\begin{aligned} g(u,i) =& H(u-i,u,A), \\ =& (c^R u + c^s A)(1-\alpha) + L(u-i,A) + \alpha c^r \mu + \alpha(c^s - c^R) \int\limits_{s=u-i}^{M} (s-u+i) d\phi(s) \\ & + \alpha(c^s - c^R) A Pr\{D \geq M\}. \end{aligned}$$

To get the first partial derivative of $g(u,i)$, let us first state that

$$\frac{\partial A}{\partial u} = \begin{cases} -1, & \text{if } u \geq F^{-1}(1-\gamma) + i - K, \\ 0, & \text{otherwise.} \end{cases}$$

And,

$$\frac{\partial M}{\partial u} = \begin{cases} 0, & \text{if } u \geq F^{-1}(1-\gamma) + i - K, \\ 1, & \text{otherwise.} \end{cases}$$

The first partial derivative of $g(u,i)$ w.r.t u,

$$\begin{aligned} \frac{\partial g(u,i)}{\partial u} =& (c^R + c^s \frac{\partial A}{\partial u})(1-\alpha) + L_y(u-i,A) + L_x(u-i,A)\frac{\partial A}{\partial u} \\ & + \alpha(c^s - c^R)\frac{\partial}{\partial u}\Big[\int\limits_{s=u-i}^{M} (s-u+i) d\phi(s) + A Pr\{D \geq M\}\Big]. \end{aligned} \tag{4.21}$$

Let's derive the first partial derivative of the last expression in brackets:

$$\frac{\partial}{\partial u}\Big[\int\limits_{s=u-i}^{M} (s-u+i) d\phi(s) + A Pr\{D \geq M\}\Big] \tag{4.22}$$

$$= \int\limits_{s=u-i}^{M} -d\phi(s) + \frac{\partial M}{\partial u}(M-u+i)\phi(M) - 0 + \frac{\partial A}{\partial u} Pr\{D \geq M\} - A\phi(M)\frac{\partial M}{\partial u}, \tag{4.23}$$

$$= \int\limits_{s=u-i}^{M} -d\phi(s) + \frac{\partial A}{\partial u} Pr\{D \geq M\}. \tag{4.24}$$

Hence,

$$\frac{\partial}{\partial u}g(u,i) =(c^R + c^s\frac{\partial A}{\partial u})(1-\alpha) + L_y(u-i,A) + L_x(u-i,A)\frac{\partial A}{\partial u}$$
$$+ \alpha(c^s - c^R)\Big[\int_{s=u-i}^{M} -d\phi(s) + \frac{\partial A}{\partial u}Pr\{D \geq M\}\Big].$$

For the second partial derivative, let us first state that $\frac{\partial^2 M}{\partial u^2} = \frac{\partial^2 A}{\partial u^2} = 0$. Hence,

$$\frac{\partial^2}{\partial u^2}g(u,i) =L_{yy}(u-i,A) + 2L_{yx}(u-i,A)\frac{\partial A}{\partial u} + L_{xx}(u-i,A)\left[\frac{\partial A}{\partial u}\right]^2 \tag{4.25}$$
$$+ \frac{\partial}{\partial u}\alpha(c^s - c^R)\Big[\int_{s=u-i}^{M} -d\phi(s) + \frac{\partial A}{\partial u}Pr\{D \geq M\}\Big]. \tag{4.26}$$

Let's again work on the partial derivative of the last expression.

$$\frac{\partial}{\partial u}\int_{s=u-i}^{M} -d\phi(s) + \frac{\partial^2 A}{\partial u^2}Pr\{D \geq M\} = -\phi(M)\frac{\partial M}{\partial u}(\frac{\partial A}{\partial u}+1) + \phi(u-i) + \frac{\partial^2 A}{\partial u^2}Pr\{D \geq M\}.$$

Therefore, using Equations 4.15-4.17 we can write

$$\frac{\partial^2 g(u,i)}{\partial u^2} =L_{yy}(u-i,A) + 2L_{yx}(u-i,A)\frac{\partial A}{\partial u} + L_{xx}(u-i,A)\left[\frac{\partial A}{\partial u}\right]^2$$
$$+ \alpha(c^s - c^R)\Big[-\phi(M)\frac{\partial M}{\partial u}(\frac{\partial A}{\partial u}+1) + \phi(u-i)\Big],$$

$$\frac{\partial^2 g(u,i)}{\partial u^2} =(h^r - h^s + \psi)\phi(u-i) + (h^s - \psi + b)\phi(M)\left(1 + 2\frac{\partial A}{\partial u} + \left[\frac{\partial A}{\partial u}\right]^2\right)$$
$$+ \alpha(c^R - c^s)\Big[\phi(M)\frac{\partial M}{\partial u}(\frac{\partial A}{\partial u}+1) - \phi(u-i)\Big],$$
$$=(h^r - h^s + \psi - \alpha(c^R - c^s))\phi(u-i) + (h^s - \psi + b)\phi(M)\left(1 + \frac{\partial A}{\partial u}\right)^2$$
$$+ \alpha(c^R - c^s)\Big[\phi(M)\frac{\partial M}{\partial u}\Big]^2 \geq 0.$$

The last equation comes from $\frac{\partial M}{\partial u} = \frac{\partial A}{\partial u} + 1$. The proof is complete. ■

4.B Convexity Analysis of Multi-Period Problem

The convexity of multi-period model will utilize Lemma 8 in Section 4.3. The convexity condition of the multi-period model is established in the following theorem.

Theorem 9 *If*

$$V_{t_x}(K, v-y, 0) - V_{t_y}(K, v-y, 0) \geq 0, \;\; \forall v \in \mathbb{R}, \;\; y \in \mathbb{R}, \;\; 1 \leq t \leq T, \;\; \forall K, \tag{4.27}$$

then

a) $G_t(K, v, w)$ *is jointly convex in* v *and* w*,*

b) $V_t(K, y, x)$ *is jointly convex in* y *and* x*.*

Proof Let's check the components of Hessian matrix for function $G_t(K, v, w)$:

$$\frac{\partial G_t}{\partial v} = c^R + \int_{s=0}^{y} \alpha E V_{t+1_y}(K_+, v-s, w) d\phi(s) + \int_{s=y}^{y+w} \alpha E V_{t+1_y}(K_+, v-y, w+y-s) d\phi(s)$$
$$+ \int_{s=y+w}^{\infty} \alpha E V_{t+1_y}(K_+, v+w-s, 0) d\phi(s), \tag{4.28}$$

$$\frac{\partial^2 G_t}{\partial v^2} = \int_{s=0}^{y} \alpha E V_{t+1_{yy}}(K_+, v-s, w) d\phi(s) + \int_{s=y}^{y+w} \alpha E V_{t+1_{yy}}(K_+, v-y, w+y-s) d\phi(s)$$

$$+ \int_{s=y+w}^{\infty} \alpha E V_{t+1_{yy}}(K_+, v+w-s, 0) d\phi(s), \tag{4.29}$$

$$\frac{\partial G_t}{\partial w} = \frac{\partial L}{\partial w} + c^s + \int_{s=0}^{y} \alpha E V_{t+1_x}(K_+, v-s, w) d\phi(s) + \int_{s=y}^{y+w} \alpha E V_{t+1_x}(K_+, v-y, w+y-s) d\phi(s)$$
$$+ \int_{s=y+w}^{\infty} \alpha E V_{t+1_y}(K_+, v+w-s, 0) d\phi(s), \tag{4.30}$$

$$\frac{\partial^2 G_t}{\partial w^2} = \frac{\partial^2 L}{\partial w^2} + \int_{s=0}^{y} \alpha EV_{t+1_{xx}}(K_+, v-s, w)d\phi(s) + \int_{s=y}^{y+w} \alpha EV_{t+1_{xx}}(K_+, v-y, w+y-s)d\phi(s)$$
$$+ \int_{s=y+w}^{\infty} \alpha EV_{t+1_{yy}}(K_+, v+w-s, 0)d\phi(s)$$
$$+ \alpha\phi(y+w)E[V_{t+1_x}(K_+, v-y, 0) - V_{t+1_y}(K_+, v-y, 0)], \tag{4.31}$$
$$\frac{\partial^2 G_t}{\partial w \partial v} = \int_{s=0}^{y} \alpha EV_{t+1_{xy}}(K_+, v-s, w)d\phi(s) + \int_{s=y}^{y+w} \alpha EV_{t+1_{xy}}(K_+, v-y, w+y-s)d\phi(s)$$
$$+ \int_{s=y+w}^{\infty} \alpha EV_{t+1_{yy}}(K_+, v+w-s, 0)d\phi(s). \tag{4.32}$$

Let's start with stating that the statement is true for $t = T$ since

$$V_T(K, y, x) = \min_{\substack{x \le w \le x+K, \\ y \le v}} L(v, w).$$

Suppose it is true for $t+1$. For t, considering induction hypothesis for Equation 4.29 and 4.31 together with the condition of the theorem brings the non-negativity of second partial derivatives of $G_t(K, v, w)$ w.r.t v and w. The positive semidefiniteness of Hessian matrix can be shown as follows:

Let us use symbolic letters for integrals given in Equations 4.29 - 4.32:

$$\frac{\partial^2 G}{\partial v^2} = (A_1 + B_1 + C_1), \tag{4.33}$$
$$\frac{\partial^2 G}{\partial w^2} = (A_2 + B_2 + C_1 + D), \tag{4.34}$$
$$\frac{\partial^2 G}{\partial w \partial v} = (A_3 + B_3 + C_1). \tag{4.35}$$

The desired property comes if

$$(A_1 + B_1 + C_1)(A_2 + B_2 + C_1 + D) - (A_3 + B_3 + C_1)^2 =$$
$$(A_1 + B_1 + C_1)(A_2 + B_2 + C_1) - \{A_3^2 + B_3^2 + C_1^2 + 2A_3B_3 + 2A_3C_1 + 2B_3C_1\} \ge 0.$$

Followings are true thanks to induction hypothesis and the condition of the lemma

$$(A_1A_2 - A_3^2) + (B_1B_2 - B_3^2) \ge 0,$$

and

$$(A_1 + B_1 + C_1)D \geq 0.$$

So we need to show the followings are true

$$A_1B_2 + B_1A_2 - 2A_3B_3 \geq 0,$$

and

$$C_1(A_2 + B_2 + A_1 + B_1 - 2A_3 - 2B_3) \geq 0.$$

From induction hypothesis we know $A_1A_2 - A_3^2 \geq 0$ and $B_1B_2 - B_3^2 \geq 0$. So,

$$A_1B_2 + B_1A_2 - 2A_3B_3 \geq \frac{A_3^2B_2}{A_2} + \frac{B_3^2A_2}{B_2} - 2A_3B_3 = \frac{(B_3A_2 - A_3B_2)^2}{B_2A_2} \geq 0$$

The non-negativity of the other condition can be shown in a similar way. ■

Theorem 9 establishes the fact that the convexity of the multi-period model requires that the first partial derivative of the minimum multi-period cost, $V_t(K, y, x)$, with respect to low quality inventory level x should be larger than the first partial derivative with respect to high quality inventory y. Unfortunately, we couldn't prove any sufficient conditions that satisfy this. However intuitively, we argue that when the holding cost rates of high quality and low quality inventories are equal to each other, the desired result can be obtained. This intuition is expressed in the following conjecture.

Conjecture 1 *If $h^r = h^s$, then*

$$V_t(K, y + \Delta, x) \leq V_t(K, y, x + \Delta),\ \forall y \in \mathbb{R},\ x \in \mathbb{R}^+,\ \Delta \in \mathbb{R}^+.$$

Before testing our intuition with numerical experiments, we also investigated the pseudo-convexity of the multi-period cost function which can provide us a single minimizing point for order-up-to levels of the two inventories. To this end, we adapt the definition of pseudo-convexity from Bazaraa and Shetty (1979).

Definition 6 *(Bazaraa and Shetty, 1979) Let E_n be a n-dimensional Euclidean space, S be a nonempty open set in E_n, and left $f : S \to E_1$ be differentiable on S. The function f is pseudo-convex if for each $x_1, x_2 \in S$ with $\nabla f(x_1)^T(x_2 - x_1) \geq 0$ we have $f(x_2) \geq f(x_1)$.*

Note the similarity between the first order convexity and pseudo-convexity conditions. During our investigation into pseudo-convexity, we need the following extension on the support of the minimum cost function.

$$V_t(K, y, u) = V_t(K, y + u, 0), \forall\, u \in \mathbb{R}^-. \tag{4.36}$$

This extension does not have any effect on the structure of the optimal policy since it is never optimal to have an order-up=-to level smaller than zero. This reasoning directly follows from the fact that the system only backlogs high quality demand. Now, we are ready to state the theorem for pseudo-convexity:

Theorem 10 *For period t, suppose v_t^*, w_t^* are two values such that $\frac{\partial}{\partial v} G_t(K, v^*, w^*) = \frac{\partial}{\partial w} G_t(K, v_t^*, w_t^*) = 0$ for a given K, $\dot{y}$ and $\dot{x}$. If*

$$\frac{\partial}{\partial y} V_t(K, v_t^* - \dot{y}, w_t^*) - \frac{\partial}{\partial x} V_t(K, v_t^* - \dot{y}, w_t^*) \geq 0,$$

a) $G_t(K, v, w)$ is pseudo-convex in v and w $\forall$ $y \in \mathbb{R}$ and $x \in \mathbb{R}^+$,

b) (v^*, w^*) is unique, global minimizer of $G_t(K, v, w)$ for a given K,

c) $V_t(K, y, x)$ is jointly convex in y and x.

Proof

For $t = N$, $V_N(K, y, x) = 0$. So,

$$G_N(K, v, w) = c^s(w - x) + c^r(v - y) + L(y, w).$$

Since $L(y, w)$ is jointly convex in y and w, $G_N(K, v, w)$ is jointly convex in v and w which proves the statements a, b and c. Assume a-c are true for $t+1$. For t, we need to show the following condition for pseudo-convexity of $G_t(K, v, w)$: For two different points (v_1, w_1) and (v_2, w_2)

$$\begin{bmatrix} \frac{\partial}{\partial v} G_t(K, v_1, w_1) \\ \frac{\partial}{\partial w} G_t(K, v_1, w_1) \end{bmatrix}^T \begin{bmatrix} v_2 - v_1 \\ w_2 - w_1 \end{bmatrix} \geq 0 \Rightarrow G_t(K, v_2, w_2) \geq G_t(K, v_1, w_1).$$

Let us start with the right-hand-side inequality for a given $\dot{y}$.

$$\begin{bmatrix} \frac{\partial}{\partial v} G_t(K,v_1,w_1) \\ \frac{\partial}{\partial w} G_t(K,v_1,w_1) \end{bmatrix}^T \begin{bmatrix} v_2-v_1 \\ w_2-w_1 \end{bmatrix} = (v_2 - v_1)\Bigg[c^R + \int_{s=0}^{\dot{y}} \alpha EV_{t+1_y}(K_+, v_1 - s, w_1) d\phi(s) \tag{4.37}$$
$$+ \int_{s=\dot{y}}^{\dot{y}+w_1} \alpha EV_{t+1_y}(K_+, v_1 - \dot{y}, w_1 + \dot{y} - s) d\phi(s) + \int_{s=\dot{y}+w_1}^{\infty} \alpha EV_{t+1_y}(K_+, v_1 + w_1 - s, 0) d\phi(s)\Bigg]$$
$$+ (w_2 - w_1)\Bigg[\frac{\partial}{\partial w} L(\dot{y}, w_1) + c^s + \int_{s=0}^{\dot{y}} \alpha EV_{t+1_x}(K_+, v_1 - s, w_1) d\phi(s)$$
$$+ \int_{s=\dot{y}}^{\dot{y}+w_1} \alpha EV_{t+1_x}(K_+, v_1 - \dot{y}, w_1 + \dot{y} - s) d\phi(s) + \int_{s=\dot{y}+w_1}^{\infty} \alpha EV_{t+1_y}(K_+, v_1 + w_1 - s, 0) d\phi(s)\Bigg].$$

We handle the terms in Equation 4.37 in four different groups: In the first group we will evaluate the terms of a single period:

$$\begin{aligned} (v_2 - v_1)c^R + (w_2 - w_1)(c^s + \frac{\partial}{\partial w} L(\dot{y}, w_1)) &= \nabla(c^r v_1 + c^s w_1 + L(\dot{y}, w_1))^t \begin{bmatrix} v_2-v_1 \\ w_2-w_1 \end{bmatrix}, \\ &\leq (c^r v_2 + c^s w_2 + L(\dot{y}, w_2)) - (c^r v_1 + c^s w_1 + L(\dot{y}, w_1)). \end{aligned} \tag{4.38}$$

The inequality follows from the convexity of $L(\dot{y}, w)$. In the second group, we consider the first integral term.

$$\begin{aligned} &\alpha \int_{s=0}^{\dot{y}} \Big[(v_2 - v_1) EV_{t+1_y}(K_+, v_1 - s, w_1) - (w_2 - w_1) EV_{t+1_x}(K_+, v_1 - s, w_1)\Big] d\phi(s) \\ &\quad = \alpha \int_{s=0}^{\dot{y}} \nabla EV_{t+1}(K_+, v_1 - s, w_1)^T \begin{bmatrix} v_2-v_1 \\ w_2-w_1 \end{bmatrix} d\phi(s) \\ &\quad \leq \alpha \int_{s=0}^{\dot{y}} \Big[EV_{t+1}(K_+, v_2 - s, w_2) - EV_{t+1}(K_+, v_1 - s, w_1)\Big] d\phi(s) \\ &\quad = \alpha \int_{s=0}^{\dot{y}} EV_{t+1}(K_+, v_2 - s, w_2) d\phi(s) - \alpha \int_{s=0}^{\dot{y}} EV_{t+1}(K_+, v_1 - s, w_1) d\phi(s). \end{aligned} \tag{4.39}$$

The inequality in Equation 4.39 follows from the first order condition of the convexity of $V_{t+1}(.)$ implied by the induction hypothesis. The third group of terms include the second integral in Equation 4.37.

$$
\begin{aligned}
&\alpha \int_{s=\dot{y}}^{\dot{y}+w_1} E\Big[(v_2 - v_1)V_{t+1_y}(K_+, v_1 - \dot{y}, w_1 + \dot{y} - s) + (w_2 - w_1)V_{t+1_x}(K_+, v_1 - \dot{y}, w_1 + \dot{y} - s)d\phi(s)\Big] \\
&= \alpha \int_{s=\dot{y}}^{\dot{y}+w_1} \nabla EV_{t+1}(K_+, v_1 - \dot{y}, w_1 + \dot{y} - s)^T \left[\begin{smallmatrix} v_2 - v_1 \\ w_2 - w_1 \end{smallmatrix}\right] d\phi(s) \\
&\leq \alpha \int_{s=\dot{y}}^{\dot{y}+w_1} \Big[EV_{t+1}(K_+, v_2 - \dot{y}, w_2 + \dot{y} - s)d\phi(s) - EV_{t+1}(K_+, v_1 - \dot{y}, w_1 + \dot{y} - s)d\phi(s)\Big] \\
&= \int_{s=\dot{y}}^{\dot{y}+w_1} \alpha EV_{t+1}(K_+, v_2 - \dot{y}, w_2 + \dot{y} - s)d\phi(s) - \int_{s=\dot{y}}^{\dot{y}+w_1} \alpha EV_{t+1}(K_+, v_1 - \dot{y}, w_1 + \dot{y} - s)d\phi(s)
\end{aligned}
\tag{4.40}
$$

The last group of terms include the last integral:

$$
\begin{aligned}
&\alpha \int_{s=\dot{y}+w_1}^{\infty} (v_2 + w_2 - v_1 - w_1)EV_{t+1_y}(K_+, v_1 + w_1 - s, 0)d\phi(s) = \\
&\alpha \int_{s=\dot{y}+w_1}^{\infty} \nabla EV_{t+1}(K_+, v_1 + w_1 - s)^T \left[\begin{smallmatrix} v_2 + w_2 - v_1 - w_1 \\ 0 \end{smallmatrix}\right], \\
&\leq \alpha \int_{s=\dot{y}+w_1}^{\infty} EV_{t+1}(K_+, v_2 + w_2 - s, 0) - EV_{t+1}(K_+, v_1 + w_1 - s, 0)d\phi(s) \\
&= \int_{s=\dot{y}+w_1}^{\infty} \alpha EV_{t+1}(K_+, v_2 + w_2 - s, 0)d\phi(s) - \int_{s=\dot{y}+w_1}^{\infty} \alpha EV_{t+1}(K_+, v_1 + w_1 - s, 0)d\phi(s).
\end{aligned}
\tag{4.41}
$$

The summation of terms in Equations 4.39 - 4.41 leads to the following:

$$\nabla G_t(K, v_1, w_1)^t \left[\begin{smallmatrix} v_2 - v_1 \\ w_2 - w_1 \end{smallmatrix}\right] \leq c^r v_2 + c^s w_2 + L(\dot{y}, w_2)$$
$$+ \alpha \Bigg[\int_{s=0}^{\dot{y}} EV_{t+1}(K_+, v_2 - s, w_2) d\phi(s) + \int_{s=\dot{y}}^{\dot{y}+w_1} EV_{t+1}(K_+, v_2 - \dot{y}, w_2 + \dot{y} - s) d\phi(s) +$$
$$\int_{s=\dot{y}+w_1}^{\infty} EV_{t+1}(K_+, v_2 + w_2 - s, 0) d\phi(s) \Bigg] - G_t(K, v_1, w_1). \tag{4.42}$$

Note that w_1 in the integral limits in Equation 4.42 prevents us from writing

$$\nabla G_t(K, v_1, w_1)^t \left[\begin{smallmatrix} v_2 - v_1 \\ w_2 - w_1 \end{smallmatrix}\right] \leq G_t(K, v_2, w_2) - G_t(K, v_1, w_1).$$

So, we need to analyze those integral limits under $w_1 \leq w_2$ and $w_1 \geq w_2$ conditions.

<u>If $w_1 \leq w_2$</u>,

$$\int_{s=\dot{y}}^{\dot{y}+w_1} EV_{t+1}(K_+, v_2 - \dot{y}, w_2 + \dot{y} - s) d\phi(s) + \int_{s=\dot{y}+w_1}^{\infty} EV_{t+1}(K_+, v_2 + w_2 - s, 0) d\phi(s)$$
$$= \int_{s=\dot{y}}^{\dot{y}+w_2} EV_{t+1}(K_+, v_2 - \dot{y}, w_2 + \dot{y} - s) d\phi(s) + \int_{s=\dot{y}+w_2}^{\infty} EV_{t+1}(K_+, v_2 + w_2 - s, 0) d\phi(s) +$$
$$\int_{s=\dot{y}+w_1}^{\dot{y}+w_2} [EV_{t+1}(K_+, v_2 + w_2 - s, 0) - EV_{t+1}(K_+, v_2 - y, w_2 + \dot{y} - s)] d\phi(s). \tag{4.43}$$

Using Conjecture 1 we can state that $V_{t+1}(K, v_2 + w_2 - s, 0) - V_{t+1}(K, v_2 - y, w_2 + \dot{y} - s) \leq 0 \ \forall s \in E_1$ which implies the pseudo-convexity of $G_t(.)$ (statement a).

<u>If $w_1 > w_2$</u>,

$$\int_{s=\dot{y}}^{y+w_1} EV_{t+1}(K_+, v_2 - \dot{y}, w_2 + \dot{y} - s)d\phi(s) + \int_{s=\dot{y}+w_1}^{\infty} EV_{t+1}(K_+, v_2 + w_2 - s, 0)d\phi(s)$$
$$= \int_{s=\dot{y}}^{y+w_2} EV_{t+1}(K_+, v_2 - \dot{y}, w_2 + \dot{y} - s)d\phi(s) + \int_{s=\dot{y}+w_2}^{\infty} EV_{t+1}(K_+, v_2 + w_2 - s, 0)d\phi(s) +$$
$$\int_{s=\dot{y}+w_2}^{y+w_1} [EV_{t+1}(K_+, v_2 - \dot{y}, w_2 + \dot{y} - s) - EV_{t+1}(K_+, v_2 + w_2 - s, 0)]d\phi(s). \tag{4.44}$$

Our assumption in Equation 4.36 implies that $\int_{s=y+w_2}^{y+w_1} [EV_{t+1}(K_+, v_2 - y, w_2 - s, 0) - EV_{t+1}(K_+, v_2 + w_2 - s)]d\phi(s) = 0$. This completes the proof of the statement a. The statement b follows from statement a. For the convexity of $V_t(K, y, x)$ in y and x, $\nabla G_t(K, v^*, w^*) = 0$ implies that

$$\frac{\partial G}{\partial v} = c^R + \int_{s=0}^{\dot{y}} \alpha EV_{t+1_y}(K_+, v^* - s, w^*)d\phi(s) + \int_{s=y}^{\dot{y}+w^*} \alpha EV_{t+1_y}(K_+, v^* - \dot{y}, w^* + \dot{y} - s)d\phi(s)$$
$$+ \int_{s=\dot{y}+w^*}^{\infty} \alpha EV_{t+1_y}(K_+, v^* + w^* - s, 0)d\phi(s) = 0, \tag{4.45}$$

$$\frac{\partial G}{\partial w} = \frac{\partial L}{\partial w} + c^s + \int_{s=0}^{\dot{y}} \alpha EV_{t+1_x}(K_+, v^* - s, w^*)d\phi(s) + \int_{s=\dot{y}}^{y+w} \alpha EV_{t+1_x}(K_+, v^* - \dot{y}, w^* + \dot{y} - s)d\phi(s)$$
$$+ \int_{s=\dot{y}+w}^{\infty} \alpha EV_{t+1_y}(K_+, v^* + w^* - s, 0)d\phi(s) = 0. \tag{4.46}$$

Equations 4.45 and 4.46 leads to the following:

$$\int_{s=\dot{y}}^{\dot{y}+w} \alpha E \left(V_{t+1_x}(K_+, v^* - \dot{y}, w^* + y - s) - V_{t+1_y}(K_+, v^* - \dot{y}, w^* + \dot{y} - s) \right) d\phi(s) \tag{4.47}$$
$$= c^R - c^s - \frac{\partial L}{\partial w} + \int_{s=0}^{\dot{y}} \left(\alpha EV_{t+1_y}(K_+, v^* - s, w^*) - \alpha EV_{t+1_x}(K_+, v^* - s, w^*) \right) d\phi(s). \tag{4.48}$$

Let us state that

$$V_t(K, y, x) = G_t(K, v*, w*).$$

Hence,

$$\frac{\partial V_t}{\partial y} = \frac{\partial G_t^*}{\partial y} = \frac{\partial L}{\partial y} - c^R + \int_{s=\dot{y}}^{\dot{y}+w} \alpha EV_{t+1_x}(K_+, v^* - \dot{y}, w^* + \dot{y} - s) \qquad (4.49)$$

$$- \int_{s=\dot{y}}^{\dot{y}+w} \alpha EV_{t+1_y}(K_+, v^* - \dot{y}, w^* + \dot{y} - s) d\phi(s),$$

$$\frac{\partial V_t}{\partial y} = \frac{\partial L}{\partial y} - c^s - \frac{\partial L}{\partial w} + \int_{s=0}^{\dot{y}} \left[\alpha EV_{t+1_y}(K_+, v^* - s, w^*) - \alpha EV_{t+1_x}(K_+, v^* - s, w^*)\right] d\phi(s).$$

$$\frac{\partial^2 G_t^*}{\partial y^2} = \frac{\partial^2 L}{\partial y^2} - \frac{\partial^2 L}{\partial w^2} + \left[\alpha EV_{t+1_y}(K_+, v^* - \dot{y}, w^*) - \alpha EV_{t+1_x}(K_+, v^* - \dot{y}, w^*)\right] \phi(\dot{y}) \geq 0, \qquad (4.50)$$

$$\frac{\partial V_t}{\partial x} = \frac{\partial G_t^*}{\partial x} = -c^s, \qquad (4.51)$$

$$\frac{\partial^2 G_t^*}{\partial x^2} = 0, \qquad (4.52)$$

$$\frac{\partial^2 G_t^*}{\partial x \partial y} = 0. \qquad (4.53)$$

Equation 4.15 and 4.17 implies that $\frac{\partial^2 L}{\partial y^2} - \frac{\partial^2 L}{\partial w^2} = (h^r - h^s + \psi)\phi(y) \geq 0$. Therefore, the condition of the theorem implies that $\frac{\partial^2 G_t}{\partial y^2} \geq 0$ and Hessian matrix is positive semidefinite with respect to y and x. ■

Conditions of Theorem 9 and 10 are very similar to each other since both of them rely on the order between partial derivatives of the minimum cost function which is expressed in Conjecture 1. The validity of our intuition in Conjecture 1 can be seen from Figures 4.3 and 4.4 below.

The only difference between the two experiments is the holding cost rate of the low quality products, which is set to 10 in Figure 4.3 and 8 in Figure 4.4.

Note that although we generate some evidence for the sufficient conditions of Theorem 9, the equality of holding cost rates is not a realistic assumption since their acquisition costs are assumed to be different. This concludes our investigation into convexity condi-

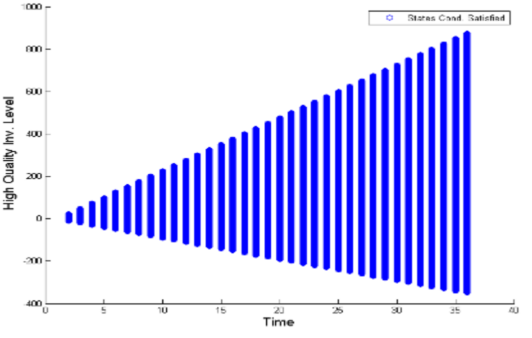

Figure 4.3: Partial Derivatives of the Minimum Cost Function. Horizon=36, Maximum Market Capacity=4, Supplier Capacity=25, Backlog Cost Rate=200, Substitution Cost Rate=50, **Holding Costs= (10,10)**, Expected Demand=2, Discount Rate=0.995

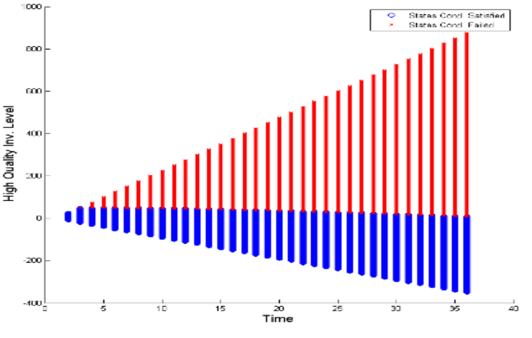

Figure 4.4: Partial Derivatives of the Minimum Cost Function. Horizon=36, Maximum Market Capacity=4, Supplier Capacity=25, Backlog Cost Rate=200, Substitution Cost Rate=50, **Holding Costs= (10,8)**, Expected Demand=2, Discount Rate=0.995

tions of multi-period cost function in Section 4.4. In the next section of this appendix, we present numerical results of our experiments with heuristic policies.

4.C Results of Numerical Experiments

Table 4.10: Test Bed 1

Run#	Horizon	L^R	Supp Cap	K	b	ψ	c^r	c^s	h^r	h^s	D	Market Sce.
Run#1	36	{1,3}	25	4.0	200	50	100	60	10	6	Pois.(2)	Stable
Run#2	24	{1,3}	25	4.0	200	50	100	60	10	6	Pois.(2)	Stable
Run#3	36	{1,3}	25	4.0	200	100	100	60	10	6	Pois.(2)	Stable
Run#4	24	{1,3}	25	4.0	200	100	100	60	10	6	Pois.(2)	Stable
Run#5	36	{1,3}	25	4.0	200	150	100	60	10	6	Pois.(2)	Stable
Run#6	24	{1,3}	25	4.0	200	150	100	60	10	6	Pois.(2)	Stable
Run#7	36	{1,3}	25	4.0	200	200	100	60	10	6	Pois.(2)	Stable
Run#8	24	{1,3}	25	4	200	200	100	60	10	6	Pois.(2)	Stable
Run#9	36	{1,3}	25	4	200	50	100	60	10	8	Pois.(2)	Stable
Run#10	24	{1,3}	25	4	200	50	100	60	10	8	Pois.(2)	Stable
Run#11	36	{1,3}	25	4	200	100	100	60	10	8	Pois.(2)	Stable
Run#12	24	{1,3}	25	4	200	100	100	60	10	8	Pois.(2)	Stable
Run#13	36	{1,3}	25	4	200	150	100	60	10	8	Pois.(2)	Stable
Run#14	24	{1,3}	25	4	200	150	100	60	10	8	Pois.(2)	Stable
Run#15	36	{1,3}	25	4	200	200	100	60	10	8	Pois.(2)	Stable
Run#16	24	{1,3}	25	4	200	200	100	60	10	8	Pois.(2)	Stable
Run#17	36	{1,3}	25	4	200	50	100	60	10	10	Pois.(2)	Stable
Run#18	24	{1,3}	25	4	200	50	100	60	10	10	Pois.(2)	Stable
Run#19	36	{1,3}	25	4	200	100	100	60	10	10	Pois.(2)	Stable
Run#20	24	{1,3}	25	4	200	100	100	60	10	10	Pois.(2)	Stable
Run#21	36	{1,3}	25	4	200	150	100	60	10	10	Pois.(2)	Stable
Run#22	24	{1,3}	25	4	200	150	100	60	10	10	Pois.(2)	Stable
Run#23	36	{1,3}	25	4	200	200	100	60	10	10	Pois.(2)	Stable
Run#24	24	{1,3}	25	4	200	200	100	60	10	10	Pois.(2)	Stable
Run#25	36	{1,3}	25	4	200	50	100	60	10	6	Pois.(4)	Stable
Run#26	24	{1,3}	25	4	200	50	100	60	10	6	Pois.(4)	Stable
Run#27	36	{1,3}	25	4	200	100	100	60	10	6	Pois.(4)	Stable
Run#28	24	{1,3}	25	4	200	100	100	60	10	6	Pois.(4)	Stable
Run#29	36	{1,3}	25	4	200	150	100	60	10	6	Pois.(4)	Stable
Run#30	24	{1,3}	25	4	200	150	100	60	10	6	Pois.(4)	Stable
Run#31	36	{1,3}	25	4	200	200	100	60	10	6	Pois.(4)	Stable
Run#32	24	{1,3}	25	4	200	200	100	60	10	6	Pois.(4)	Stable
Run#33	36	{1,3}	25	4	200	50	100	60	10	8	Pois.(4)	Stable
Run#34	24	{1,3}	25	4	200	50	100	60	10	8	Pois.(4)	Stable
Run#35	36	{1,3}	25	4	200	100	100	60	10	8	Pois.(4)	Stable
Run#36	24	{1,3}	25	4	200	100	100	60	10	8	Pois.(4)	Stable
Run#37	36	{1,3}	25	4	200	150	100	60	10	8	Pois.(4)	Stable
Run#38	24	{1,3}	25	4	200	150	100	60	10	8	Pois.(4)	Stable
Run#39	36	{1,3}	25	4	200	200	100	60	10	8	Pois.(4)	Stable
Run#40	24	{1,3}	25	4	200	200	100	60	10	8	Pois.(4)	Stable
Run#41	36	{1,3}	25	4	200	50	100	60	10	10	Pois.(4)	Stable
Run#42	24	{1,3}	25	4	200	50	100	60	10	10	Pois.(4)	Stable
Run#43	36	{1,3}	25	4	200	100	100	60	10	10	Pois.(4)	Stable
Run#44	24	{1,3}	25	4	200	100	100	60	10	10	Pois.(4)	Stable
Run#45	36	{1,3}	25	4	200	150	100	60	10	10	Pois.(4)	Stable
Run#46	24	{1,3}	25	4	200	150	100	60	10	10	Pois.(4)	Stable
Run#47	36	{1,3}	25	4	200	200	100	60	10	10	Pois.(4)	Stable
Run#48	24	{1,3}	25	4	200	200	100	60	10	10	Pois.(4)	Stable

Table 4.11: Results of Test Bed 1 for $L^R = 1$

	Optimal Cost						Vector Based Heuristic (Sheopuri et al, 2010)						Substitution Hueristic						Demand Allocation(U)						Demand Allocation(L)					
Run#	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M
R#1	8113	816	881	258	5025	1133	8845	1251	159	212	7006	217	8565	666	773	638	5463	1024	13901	79	2370	6595	1965	2865	13901	79	2370	6595	1965	2865
R#2	5674	548	675	206	3363	881	6325	851	145	156	4967	206	6029	473	563	396	3824	773	9459	56	1717	4365	1224	2080	9459	56	1717	4365	1224	2080
R#3	8594	1015	578	335	6274	392	9009	1250	319	213	7009	218	8938	1114	584	197	6600	442	16307	78	4738	6625	1984	2856	16307	78	4738	6625	1984	2856
R#4	6058	683	480	276	4286	333	6473	852	292	160	4963	206	6411	787	463	129	4657	374	11183	55	3423	4379	1242	2068	11183	55	3423	4379	1242	2068
R#5	8804	1062	437	512	6596	196	9173	1250	480	213	7013	217	9164	1237	471	253	6966	237	18603	63	6175	7243	2619	2477	18603	63	6175	7243	2619	2477
R#6	6238	717	387	416	4541	176	6620	851	437	159	4966	206	6611	867	408	176	4943	217	12824	43	4436	4826	1719	1784	12824	43	4436	4826	1719	1784
R#7	8912	1255	229	552	6804	73	9327	1249	638	211	7012	217	9313	1240	623	248	6967	236	20257	39	5318	9191	4091	1593	20257	39	5318	9191	4091	1593
R#8	6349	839	219	498	4722	70	6769	851	583	158	4971	206	6753	867	545	177	4947	218	14059	27	3841	6248	2776	1153	14059	27	3841	6248	2776	1153
R#9	8273	887	696	382	5409	899	8798	1274	251	142	6769	362	8630	980	480	364	6137	670	13956	102	2366	6622	1978	2860	13956	102	2366	6622	1978	2860
R#10	5783	598	551	294	3618	722	6285	875	216	100	4767	326	6130	696	361	231	4314	529	9486	74	1717	4372	1224	2081	9486	74	1717	4372	1224	2081
R#11	8651	1048	459	435	6402	307	9051	1296	319	211	7007	217	9062	1305	312	249	6961	236	16322	102	4728	6624	1991	2850	16322	102	4728	6624	1991	2850
R#12	6111	713	407	339	4375	278	6517	887	292	161	4971	207	6533	917	272	179	4947	218	11229	73	3416	4404	1257	2064	11229	73	3416	4404	1257	2064
R#13	8827	1063	367	591	6645	160	9213	1296	478	212	7009	217	9223	1306	469	249	6964	236	18603	77	6117	7274	2654	2454	18603	77	6117	7274	2654	2454
R#14	6255	723	342	467	4572	151	6652	889	437	157	4964	206	6657	917	408	175	4940	217	12825	54	4406	4841	1737	1772	12825	54	4406	4841	1737	1772
R#15	8922	1261	212	571	6812	67	9375	1295	638	213	7011	217	9251	1134	436	476	7058	147	20259	43	5279	9221	4110	1582	20259	43	5279	9221	4110	1582
R#16	6352	843	208	508	4726	66	6804	887	583	159	4969	206	6693	792	395	350	5018	138	14054	30	3826	6254	2782	1148	14054	30	3826	6254	2782	1148
R#17	8371	1041	511	367	5785	668	8875	1363	250	140	6760	362	8871	1351	290	195	6595	440	13931	128	2369	6581	1962	2864	13931	128	2369	6581	1962	2864
R#18	5869	705	436	287	3864	578	6356	941	217	102	4769	327	6358	958	231	129	4665	374	9522	92	1715	4387	1232	2079	9522	92	1715	4387	1232	2079
R#19	8706	1053	480	477	6381	316	9134	1363	502	144	6762	363	9133	1373	313	252	6959	236	16347	126	4723	6629	1995	2847	16347	126	4723	6629	1995	2847
R#20	6144	736	410	352	4369	277	6568	942	433	101	4765	327	6575	966	272	175	4943	218	11224	90	3414	4388	1254	2063	11224	90	3414	4388	1254	2063
R#21	8608	1095	120	582	6654	156	8886	1361	251	142	6769	362	8882	1349	291	197	6604	441	13980	127	2367	6621	1976	2861	13980	127	2367	6621	1976	2861
R#22	6051	745	112	466	4581	148	6351	942	217	101	4765	327	6353	959	231	128	4661	374	9496	92	1717	4367	1223	2081	9496	92	1717	4367	1223	2081
R#23	8809	1268	105	566	6804	66	9128	1363	500	142	6761	362	9127	1375	311	248	6957	236	16318	127	4733	6598	1982	2853	16318	127	4733	6598	1982	2853
R#24	6252	848	104	511	4723	65	6569	942	433	101	4766	326	6577	967	272	178	4943	217	11232	90	3409	4399	1259	2060	11232	90	3409	4399	1259	2060
R#25	8846	1096	361	584	6648	156	9257	1342	479	213	7006	218	9290	1373	468	251	6961	236	18612	92	6042	7327	2702	2423	18612	92	6042	7327	2702	2423
R#26	6283	744	337	469	4586	148	6699	924	438	160	4971	207	6716	965	409	177	4948	218	12866	65	4391	4874	1755	1765	12866	65	4391	4874	1755	1765
R#27	8914	1268	211	566	6803	66	9408	1342	637	210	7002	217	9265	1162	435	472	7049	147	20227	47	5238	9228	4122	1569	20227	47	5238	9228	4122	1569
R#28	6355	849	208	511	4722	66	6838	925	582	159	4966	206	6711	813	395	351	5015	138	14044	32	3789	6276	2798	1137	14044	32	3789	6276	2798	1137
R#29	15584	1143	1132	540	11316	1452	16157	944	1015	849	12014	1336	16260	1073	797	820	12485	1085	29613	70	3490	15318	6496	4204	29613	70	3490	15318	6496	4204
R#30	10923	764	870	465	7694	1131	11453	651	751	609	8421	1021	11538	745	600	580	8763	849	20164	51	2568	10189	4236	3098	20164	51	2568	10189	4236	3098
R#31	16181	1467	695	581	12974	463	16723	1269	796	677	13448	534	16771	1496	795	427	13459	594	33104	68	6951	15334	6537	4179	33104	68	6951	15334	6537	4179
R#32	11409	988	606	496	8910	409	11959	872	653	508	9476	449	12016	1028	645	342	9496	505	22748	49	5100	10223	4285	3070	22748	49	5100	10223	4285	3070
R#33	16469	1597	686	646	13247	292	17124	1268	1195	677	13450	534	17077	1619	798	459	13817	386	36512	52	9628	15862	7079	3854	36512	52	9628	15862	7079	3854
R#34	11670	1081	645	548	9118	278	12289	872	980	510	9477	450	12297	1117	698	365	9769	346	25228	37	7060	10591	4685	2831	25228	37	7060	10591	4685	2831
R#35	16671	1612	614	843	13412	190	17525	1268	1595	677	13449	535	17361	1843	790	404	14050	274	39414	29	9999	17844	8512	2996	39414	29	9999	17844	8512	2996
R#36	11857	1089	601	722	9257	188	12611	872	1306	507	9477	449	12564	1280	745	319	9957	263	27361	20	7340	12046	5734	2201	27361	20	7340	12046	5734	2201
R#37	15783	1312	868	617	11867	1120	16303	1076	1016	853	12022	1337	16404	1408	564	583	13047	802	29677	90	3488	15351	6509	4203	29677	90	3488	15351	6509	4203
R#38	11060	879	687	518	8076	899	11553	743	752	610	8425	1023	11675	971	439	434	9179	652	20201	67	2572	10202	4235	3103	20201	67	2572	10202	4235	3103
R#39	16254	1530	630	632	13047	415	16791	1334	797	680	13446	535	16883	1691	532	461	13813	386	33138	88	6943	15353	6544	4174	33138	88	6943	15353	6544	4174
R#40	11450	1028	566	530	8950	377	11993	918	651	505	9471	448	12100	1166	464	362	9762	345	22719	65	5112	10178	4265	3078	22719	65	5112	10178	4265	3078
R#41	16505	1614	650	696	13273	273	17192	1334	1196	679	13447	535	17151	1689	801	461	13814	387	36521	66	9604	15874	7094	3845	36521	66	9604	15874	7094	3845
R#42	11696	1094	628	576	9130	268	12335	917	981	509	9478	450	12345	1165	698	365	9770	346	25237	47	7048	10599	4694	2825	25237	47	7048	10599	4694	2825
R#43	16698	1617	615	856	13420	189	17607	1332	1598	683	13459	536	17419	1886	793	407	14059	274	39467	33	9969	17905	8538	2987	39467	33	9969	17905	8538	2987
R#44	11870	1099	603	725	9255	188	12660	918	1308	511	9473	450	12601	1314	747	322	9954	264	27351	23	7320	12050	5742	2196	27351	23	7320	12050	5742	2196
R#45	15909	1449	670	649	12272	870	16427	1210	1014	848	12020	1335	16553	1310	515	893	13138	698	29680	112	3489	15337	6503	4203	29680	112	3489	15337	6503	4203
R#46	11143	982	537	539	8377	708	11638	838	751	609	8420	1021	11760	912	404	641	9235	566	20192	83	2570	10186	4231	3100	20192	83	2570	10186	4231	3100
R#47	16289	1630	514	636	13177	333	17428	1211	2025	844	12015	1333	16951	1762	532	458	13813	386	33139	109	6943	15334	6543	4174	33139	109	6943	15334	6543	4174
R#48	11483	1104	488	533	9038	319	12380	838	1499	605	8420	1019	12149	1214	465	363	9761	346	22747	79	5089	10205	4288	3064	22747	79	5089	10205	4288	3064

Table 4.12: Results of Test Bed 1 for $L^R = 3$

	Optimal Cost						Vector Based Heuristic (Sheopuri et al, 2010)						Substitution Hueristic						Demand Allocation(U)						Demand Allocation(L)					
Run#	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M	Total	Hold.	Subst.	Backl.	Acq.R	Acq.M
1	8515	1054	1541	304	3607	2009	9126	1015	780	652	5660	1020	9186	1478	538	163	6203	805	9153	592	1422	1209	4073	1857	10491	443	760	2820	5523	945
2	5894	668	1317	230	1935	1744	6642	621	614	665	3939	803	6606	1047	448	114	4295	702	6346	418	1031	737	2769	1391	7277	311	589	1859	3777	741
3	9328	1419	797	409	6241	462	10092	998	1260	997	6038	799	9890	1721	893	112	6469	695	10540	513	2260	1606	4708	1453	11813	422	1238	3581	5814	759
4	6553	943	753	274	4104	479	7202	667	1043	690	4131	671	7202	1207	781	84	4498	632	7368	362	1682	1010	3205	1109	8264	294	986	2391	3985	608
5	9867	1526	968	591	6433	348	10731	993	1899	1000	6035	804	10343	1806	1122	150	6709	556	11749	461	2676	2252	5236	1124	13154	396	1407	4658	6123	570
6	7087	1004	921	492	4323	346	7732	666	1571	692	4129	674	7619	1262	1032	117	4685	524	8268	324	2051	1448	3569	877	9305	274	1134	3214	4222	461
7	10469	1528	1267	841	6572	261	11353	997	2525	999	6032	801	10826	1947	1425	134	6785	534	12674	447	3221	2584	5416	1007	14217	377	1335	5718	6384	403
8	7520	1020	1154	634	4425	287	8246	666	2087	689	4132	672	8052	1356	1324	106	4757	508	8984	311	2494	1686	3702	790	10123	258	1104	4004	4422	334
9	8675	1205	1186	347	4405	1532	9237	1076	812	683	5606	1060	9359	1659	537	159	6199	805	10507	471	757	2813	5524	942	9283	724	1414	1212	4088	1845
10	6016	773	1124	256	2381	1482	6492	732	653	452	3785	871	6738	1171	450	116	4295	705	6455	512	1030	747	2777	1389	7314	331	590	1868	3781	743
11	9446	1482	714	521	6309	421	10382	902	1497	1317	5724	942	10089	1933	745	149	6706	555	10633	600	2257	1615	4711	1451	11851	435	1231	3611	5820	754
12	6673	964	694	395	4188	431	7365	613	1199	879	3908	766	7367	1347	689	118	4689	524	7443	423	1689	1011	3207	1113	8304	303	987	2414	3993	608
13	10004	1532	932	730	6501	309	10799	1052	1898	1004	6041	803	10474	1924	1126	153	6714	557	11809	507	2674	2261	5243	1123	13172	399	1403	4670	6131	568
14	7110	1028	913	500	4328	340	7763	711	1562	689	4130	671	7704	1352	1027	117	4685	522	8288	360	2044	1440	3571	873	9284	279	1130	3195	4221	459
15	10327	1540	1058	896	6572	260	11402	1058	2514	998	6035	798	10883	1990	1273	272	6907	441	12735	474	3093	2724	5479	965	14203	380	1333	5706	6382	402
16	7539	1007	1021	789	4462	260	8295	712	2088	695	4128	673	8113	1379	1220	232	4854	427	9048	334	2422	1788	3740	765	10130	261	1109	4007	4418	335
17	8810	1293	1013	397	4827	1280	9350	1189	810	681	5613	1057	9541	1839	537	161	6199	805	9427	779	1289	1325	4362	1671	10848	470	684	3170	5677	847
18	6109	846	971	284	2748	1260	6561	813	651	444	3784	869	6857	1298	450	112	4293	704	6543	554	950	812	2960	1268	7552	331	535	2123	3894	669
19	9530	1526	688	565	6355	396	10163	1187	1622	683	5612	1059	10216	2053	747	151	6710	555	10737	648	2128	1742	4859	1361	11964	438	1180	3747	5876	723
20	6737	997	667	443	4228	403	7215	812	1298	451	3787	866	7447	1438	687	115	4684	523	7512	461	1599	1102	3304	1045	8399	307	933	2544	4039	576
21	10107	1523	965	811	6529	278	11021	1119	2262	950	5721	969	10579	2058	1121	149	6697	555	11806	558	2653	2242	5238	1115	13113	410	1397	4624	6116	565
22	7217	1026	886	616	4384	305	7992	660	1794	867	3907	764	7793	1439	1029	115	4686	523	8339	396	2047	1450	3571	874	9308	284	1135	3207	4220	461
23	10345	1542	1059	906	6580	258	11470	1113	2531	994	6030	803	10949	2054	1275	269	6910	441	12777	507	3103	2719	5480	968	14295	379	1283	5835	6410	387
24	7550	1017	1021	787	4466	259	8338	756	2088	691	4131	672	8166	1430	1221	231	4855	428	9068	360	2422	1779	3741	765	10176	262	1075	4080	4435	325
25	16830	1870	737	1011	12235	977	21191	683	1757	6017	10571	2163	17905	2005	754	1106	12955	1086	18346	809	1791	3071	10357	2318	20493	640	999	5542	12084	1228
26	11858	1219	693	924	8079	944	14375	488	1411	3783	6932	1762	13026	1381	657	1031	8991	966	13015	576	1324	2216	7135	1763	14427	456	816	3851	8295	1010
27	17971	2035	1110	1363	12828	635	22104	791	2954	5288	11257	1814	18848	2324	1277	992	13320	934	20174	735	3031	3510	10959	1938	22144	615	1725	6388	12363	1052
28	12774	1327	1143	1083	8541	681	15601	531	2340	3657	7626	1446	13842	1593	1169	948	9272	859	14365	524	2295	2503	7543	1501	15673	437	1447	4416	8488	886
29	18794	2135	1388	1686	13116	469	23590	816	4387	5242	11351	1795	19549	2446	1743	1012	13540	809	21826	679	3837	4210	11488	1613	24220	560	1839	8206	12874	742
30	13629	1405	1339	1526	8889	471	16598	548	3481	3610	7525	1435	14487	1690	1641	956	9441	759	15583	483	2960	2965	7908	1266	17265	392	1579	5772	8883	639
31	19366	2149	1725	1891	13184	417	25036	818	5848	5228	11346	1795	20051	2358	2283	1098	13553	759	23217	643	4391	4942	11871	1370	25566	544	1904	9405	13139	574
32	14216	1413	1682	1731	8973	417	17795	545	4640	3648	7527	1435	15070	1728	2124	991	9524	703	16702	454	3449	3524	8186	1089	18341	377	1706	6662	9081	514
33	17002	1921	696	1106	12391	888	21917	696	2016	6711	10013	2481	18076	2158	755	1111	12966	1087	18534	936	1779	3119	10400	2300	20570	654	996	5591	12104	1225
34	11973	1261	659	980	8203	870	14986	480	1532	4366	6704	1905	13190	1542	628	995	9098	926	13115	665	1318	2222	7156	1754	14453	468	816	3858	8302	1009
35	18177	2079	988	1592	12960	558	23076	747	3237	6186	10924	1982	18875	2315	1258	1078	13346	877	20312	818	2989	3582	11013	1909	22361	617	1666	6632	12432	1014
36	12954	1351	1020	1305	8674	604	15960	541	2637	3884	7260	1638	13850	1592	1157	999	9290	810	14444	579	2270	2533	7580	1482	15827	439	1410	4582	8535	861
37	18889	2142	1362	1782	13159	445	23673	841	4432	5320	11266	1814	19651	2541	1744	1018	13539	809	21888	735	3835	4215	11491	1612	24394	560	1763	8434	12925	711
38	13672	1408	1334	1557	8906	465	16847	566	3515	3694	7625	1448	14557	1749	1643	962	9444	760	15653	522	2965	2988	7910	1268	17408	393	1532	5946	8919	619
39	19370	2149	1725	1889	13190	417	25078	867	5844	5223	11352	1793	20290	2683	2154	1023	13717	713	23256	674	4365	4967	11888	1361	25661	543	1840	9551	13174	554
40	14206	1413	1680	1719	8977	417	17829	580	4645	3639	7529	1436	15028	1681	2161	1008	9460	718	16739	475	3432	3550	8199	1083	18423	378	1663	6775	9105	501
41	17175	1947	637	1268	12524	799	21938	776	2008	6661	10020	2473	18163	2215	696	1171	13116	965	18666	1040	1723	3169	10510	2224	20567	676	996	5577	12094	1224
42	12035	1295	630	1020	8269	821	15045	536	1526	4377	6709	1897	13173	1524	619	1054	9106	870	13204	735	1281	2258	7230	1699	14451	484	813	3852	8297	1006
43	18251	2107	963	1649	12995	536	23360	747	3360	6452	10744	2057	18979	2423	1261	1076	13341	879	20386	907	2992	3566	11010	1910	22374	626	1668	6636	12428	1015
44	13091	1371	942	1474	8759	545	16205	510	2615	4355	7114	1611	13907	1663	1154	999	9282	809	14485	642	2261	2529	7576	1476	15802	448	1405	4563	8527	858
45	18895	2147	1355	1793	13159	440	24721	798	4855	6165	10921	1982	19728	2639	1744	1006	13531	809	21917	792	3825	4203	11489	1608	24605	556	1624	8761	13009	654
46	13684	1414	1331	1566	8911	462	17430	547	3786	4174	7367	1556	14614	1810	1643	961	9440	760	15684	560	2954	2994	7912	1263	17600	388	1412	6237	8993	570
47	19461	2139	1670	2031	13235	387	25164	893	5905	5291	11263	1813	20336	2746	2155	1011	13711	713	23271	709	4351	4964	11889	1357	25787	544	1746	9758	13213	526
48	14201	1418	1679	1717	8970	417	17995	605	4670	3648	7628	1443	15134	1814	2120	979	9518	702	16732	500	3422	3533	8196	1080	18481	380	1599	6893	9128	481

4.D Sampling Procedure for Empirical Tests

In order to take a sample to train our model's transition matrix of Markovian market capaicty and find Markov-modulated distributions, we take price and annual demand rates for 14,046 part numbers. Since this large part population includes extreme values, we take logarithms of cost and annual demand rates and plot them into a scatter diagram (Figure 4.5), which is used for segmentation of part numbers.

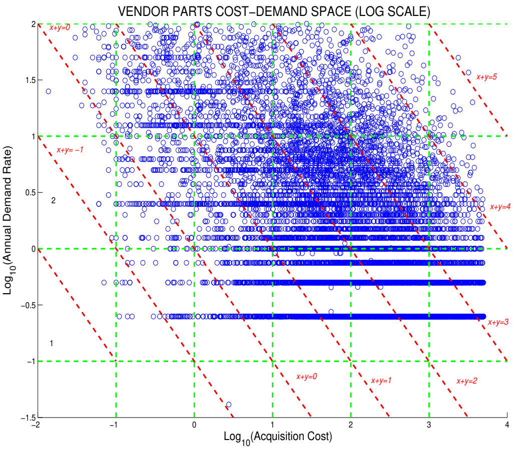

Figure 4.5: Sampling Scheme for Empirical Tests

In the scatter plot, log-cost (x axis) are distributed between -2 and 4 whereas log-demand-rates (y-axis) are between -1 and 2. We divide the scatter plots into 18 segmentss which are indicated with dashed-green lines in Figure 4.5. In order to make evenly distributed sampling within each segment we utilized diagonal lines (dashed, red lines in Figure 4.5). Note that each diagonal line represent a curve on which the multiplication of demand and cost of parts are equal to each other. In our sampling scheme, we took 4 parts around diagonal lines ($\pm 20\%$) and 4 parts from corner regions of each segment. Note that due to limited number parts in three segments on the left-hand-side of the scatter plot, our part sample consists of 139 part numbers in total.

Chapter 5

Pricing and Inventory Management Against Secondary Markets

5.1 Introduction

Producers of capital products, which are referred to as Original Equipment Manufacturers (OEMs), are increasingly paying more attention to after-sale services to their customers. Bundschuh and Dezvane (2003) report that after-sale business volumes reached 25% of total sales in the machine and plant construction industry. Similarly, Cohen et al. (2006) addresses the importance of after-sale markets by stating that performance of after-sale services is an indicator for stock prices of companies.

In a regular supply chain of an OEM, parts flow from suppliers to customers who own and/or operate capital products. During the economic life time of capital products, they need (un)planned maintenance services and spare parts to stay in operation and avoid downtime costs. For many products, OEMs are not the only spare parts provider for capital products. Third-party service providers are usually highly competitive for after-sale services and OEMs lose a significant volume of after-sale business as soon as the warranty period of a capital product ends (Cohen et al., 2006). The competition even gets stronger in existence of secondary markets.

Secondary markets for spare parts are online trading platforms, e.g. ilsmart.com or fipart.com, on which different agents, such as suppliers, traders, operators or OEMs, can trade their spare parts. Since those markets are accessible by customers, they can play two different roles: Secondary markets can be used by OEMs as a source of spare parts, or they can 'steal' some of OEMs' demand.

As a supply source, secondary markets have some specific features which should be considered explicitly. In each decision epoch, there are a certain number of available parts on the secondary markets. *Surplus inventory* and cannibalization of older products are the main sources of these spare parts. Surplus inventory represents spare parts being sold on markets since they are no longer needed by the selling agent. For instance, an airline operator who has an aircraft holds some spare parts inventory in its warehouse. When the operator sells the aircraft, those parts become surplus and usually are sold on secondary markets. Similarly if a maintenance shop purchase more spare parts than he needs from a supplier, he can sell surplus parts on secondary markets. Cannibalization is defined as removal of useful parts from an inoperative machine to use them for maintenance of another machine (Fisher, 1990). In some cases, maintenance organizations remove spare parts from capital products and sell them on secondary markets. Parts on those markets are usually cheaper than the regular supplier's price of parts and may be in different conditions, such as new, serviceable, or as-removed. For some parts, such as repairable parts of aircraft (brakes, landing gears), parts from the secondary market may be accepted as perfect substitutes of new parts from the regular supplier. For other parts, such as non-repairables, customers may be quality-sensitive and prefer new parts over parts from secondary markets. In this study we will focus on the former case, whereas the latter is left to future research.

As a source of competition, secondary markets create a challenge for OEMs' after-sale services with their cheaper prices and fast delivery times. Secondary markets consist of online trading platforms which can be directly accessed by asset owners or operators. Hence, high selling prices charged by an OEM may push some customers to purchase spare parts from secondary markets. We refer to this division of customer demand between the OEM and secondary markets as the *primary effect* of the OEM's pricing policy.

Part prices charged by an OEM also has an effect on the total spare parts demand, shared between secondary markets and the OEM. Specifically, high spare parts prices increase maintenance costs of asset owners or operators, which might trigger replacement of old capital products with new models. If there is only weak competition in the market of capital products, replacement might be a beneficial for an OEM since customers would buy his new models. In existence of a strong competitor, on the other hand, the OEM's interest might be in extending the economic lifetime of old capital products. Hence, the pricing policy should consider the effect of spare parts prices on asset owners' decisions, which we refer to as the *secondary effect* of the OEM's pricing policy.

In this study, we focus on the optimal replenishment and pricing policy for spare parts of OEMs in existence of secondary markets which have a limited number of spare parts at any given time. These available parts are a potential supply source for OEMs as well as customers. Therefore, those markets possess an advantage of cheaper parts, but the also stand for a source of competition to OEMs. To understand the optimal policy, we consider a single period model with immediate deliveries from part suppliers and secondary markets.

Demand is modeled in two stages. First, we assume that the OEM's price has a direct effect on the fraction of total demand received by the OEM and the secondary markets (primary effect). Namely, as the OEM charges higher prices for their spare parts, more customers prefer buying from secondary markets directly, which means a demand loss for the OEM. Second, we assume that the total demand (shared between the OEM and secondary markets) is a decreasing function of the OEM's spare part prices (secondary effect). The structure of the supply chain we considered in this study is given in Figure 5.1. In addition to these two effects, we consider a finite availability of parts on the secondary markets and assume that the OEM purchases from those secondary markets before demand arrives. In other words, the total demand received by the OEM is a function of his own purchases from the secondary markets.

Through our analysis we show that 1) the OEM should consider the secondary markets as their primary supply source, while the regular supplier is used only if backlogged demand cannot be satisfied through the secondary markets; this is intuitive, since purchasing from secondary markets increases the potential demand, 2) the pricing policy is either a list price or it moves between two list prices as a non-increasing function of the inventory level.

The remainder of this chapter is organized as follows. In the next section, we position our study with respect to the relevant literature. Section 5.3 is devoted to a description of a business case taken from a Western European OEM in aviation. In Section 5.4, we present the model and its mathematical properties. Numerical results are presented in Section 5.5. The last section is devoted to our conclusions and suggestions for future research.

5.2 Related Literature

The literature related with our study consists of two major parts: dynamic pricing and dual sourcing. Since the latter stream of research is reviewed in the previous chapter in detail, we focus on the former literature in this chapter. In our review, we focus on studies which we find the most important for our problem. A more comprehensive review of pricing studies is given by Chen and Simchi-Levi (2010) and Elmaghraby and Keskinocak (2003).

Studies in pricing literature can be categorized based on the possibility of replenishment during a selling season (Elmaghraby and Keskinocak, 2003). Although we consider a single period model (which precludes possibility of replenishment during the selling season by definition), we acknowledge that dynamic models with periodic replenishments (in a finite horizon) are also related with our research setting (Section 5.3). Therefore, we will review both research streams in this section.

To the best of our knowledge, the first study considering a single period model in a newsvendor setting is by Zabel (1970). He considers a single period problem with a multiplicative demand model with constant backlog and holding cost rates. He shows the optimal pricing and replenishment policies and analyzes monotonicity conditions of policy parameters over cost rates and inventory level. Polatoğlu (1991) assumes that the random demand has a general distribution between lower and upper bounds while the expected demand is a decreasing function of the price in a single period model. He finds that the optimal solution is base-stock type with a pricing level depending on the order quantity. Unimodality of the profit function plays a crucial role in his analysis. Rajan et al. (1992) considers pricing of perishable products within an inventory cycle. They derive some monotonicity properties of the optimal price over unit purchase cost and the length of inventory cycle. Gallego and Van Ryzin (1994) consider optimal dynamic pricing for a company who has to sell a fixed amount of inventory over a single period. They show monotonicity properties for the optimal price over inventory level and selling horizon. Furthermore, they prove asymptotic optimality of the constant price policy which can be used as a heuristic approach for the problem. Our study can be considered as an extension to Gallego and Van Ryzin (1994) since we assume a limited control (due to finite availability of secondary markets) over the amount of products to be sold within a selling season.

Early studies on multi-period pricing policy are (Karlin and Carr, 1962; Zabel, 1972). Karlin and Carr (1962) analyze joint pricing and inventory policy for some restrictive assumptions, such as zero holding cost and constant pricing policy over the entire planning horizon. An extension of this study is given by Zabel (1972) who considers the problem setting with additive and multiplicative demand functions. He finds that the optimal policy depends on the form of the demand function and its random variable, assuming that the demand is a convex function of the price. Thowsen (1975) considers a convex decreasing function for expected price-dependent demand in a multi-period model and shows the sufficient conditions for the optimality of list price. Polatoğlu and Sahin (2000) extend the work by Polatoğlu (1991) to a multi-period, finite horizon problem setting with lost sales. They find that the optimal replenishment policy is similar to order-up-to policy under the assumption that the profit function is unimodal. Li (1988) considers Poisson processes with price-dependent intensities for the demand process of a company. Federgruen and Heching (1999) extend that work by considering a general distribution with price-dependent distribution function for the demand. They show some characteristics of the optimal pricing and replenishment policies and present a value iteration algorithm to calculate policy parameters. The same problem setting is extended with fixed ordering cost by Chen and Simchi-Levi (2004), and with lost sales by Chen et al. (2006). Furthermore, Gong et al. (2014) consider supply risk in a dynamic pricing and inventory management problem. They consider an additive demand model and show the monotonicity of optimal pricing policy as well as the optimality of reorder point policy for replenishment of products from suppliers. They also show that both customer and retailer benefit from low supply risk. Ceryan et al. (2013) focus on capacity flexibility and dynamic pricing in a problem setting where two products can be substituted for each other's demand. They find that the pricing policy depends on production capacities for two products.

In our study we consider an additive linear model for the expected total demand and assume a split of this total demand between OEM and secondary markets. Our demand function resembles Zabel (1972) and Thowsen (1975) since it is a convex, quadratic and decreasing function of price. Unlike those studies, we explicitly model the demand function by formulating primary and secondary effects of price. Furthermore, our study is the first one considering secondary markets both as a source of supply and a competitor. These two properties are recognized by managers from the aviation sector. In the next section, we present our research setting which stands for the main motivation of this study.

5.3 Research Setting

This study is motivated by an OEM providing parts and maintenance services for out-of-production aircraft in Western Europe. Since the company does not manufacture new aircraft, the extension of economic lifetimes of existing aircraft, by providing responsive maintenance service for reasonable prices, is critical for the financial stability of the company.

The company has more than 500,000 part numbers in its database. Depending on the customer demand, it sells new parts (from a regular supplier) as well as second-hand parts in different conditions, such as overhauled or serviceable. Inventory analysts in the company state that as the fleet gets older, the customer base of the company turns from quality-sensitive into price-sensitive, i.e. customers are willing to accept parts in different conditions from secondary markets for cheaper prices. This motivates the company to consider secondary markets in their sourcing and pricing policies.

Secondary markets in aviation largely consist of internet-based online trading platforms, such as ilsmart.com ans fipart.com. Suppliers, brokers, OEMs and customers register on these platforms by paying an annual membership fee and run queries for the part numbers that they want to trade. In general, spare parts from secondary markets are cheaper than prices of regular suppliers and their conditions are worse than brand new (the condition of parts from regular supplier). Secondary markets are a beneficial supply source for the OEM given that customers are willing to accept spare parts from secondary markets as substitutes of brand new parts.

Another important aspect of secondary markets is their finite availability at any given time. Spare parts owned by different traders are registered to these markets and buyers quote the prices of these parts when they need them. Due to finite availability, one potential strategy of the OEM is to collect all the spare parts on the market to receive as much demand as possible if the marginal cost of purchasing from secondary markets does not exceed the marginal profit of selling them. Otherwise the OEM can leave the market as it is. For simplification, we assume that the OEM makes his purchase decision from markets before the customer demand arrives. Such a leader role (as in a Stackelberg game) is appropriate for the OEM due to its technical knowledge and natural advantage over other traders.

The OEM categorizes its customers as *loyal* and *price-sensitive*. Loyal customers prefer the OEM due to its reliable, high part availability and quality whereas *price-sensitive* customers prefer secondary markets. Since customers in the both categories have direct access to secondary markets, loyal customers can become price-sensitive if they decide that the OEM's spare parts are too high. Similarly, decreasing prices of the OEM might motivate some price-sensitive customers to prefer the OEM instead of secondary markets. In other words, the total amount of spare parts demand is split up between the OEM and secondary markets and the fraction of total demand received by the OEM is a decreasing function of its spare parts prices (*primary effect of pricing*).

Another important aspect of the problem setting is the tendency of asset owners to replace their fleet with new models. Increasing average age of fleet comes with increased maintenance and downtime costs which eventually leads asset owners to sell their existing aircraft and buy a new model. Pricing of spare parts has such a *secondary effect* on expected total demand (the summation of demand received by the OEM and secondary markets) due to the asset owners' attitude towards replacing their fleet in operation. The expected total demand is therefore a decreasing function of OEMs spare part prices.

In the next section, we consider the optimal pricing and replenishment policy for the OEM who has secondary markets as a supply source and a competitor. Primary and secondary effects of pricing on the company demand are considered in a single period model and the optimal pricing and replenishment policies are analyzed.

5.4 Model

We consider an OEM who is using secondary markets as a supply source and facing competition from secondary markets due to their lower prices. The OEM purchases both from regular supplier and secondary markets and sets his selling price by considering existing inventory at the beginning of the period (y), market availability (K), primary and secondary effects of pricing on demand. We define p as the selling price of spare parts whereas q^r and q^m are the amount of parts purchased from a regular supplier and secondary markets respectively. We assume that orders to both channels are delivered immediately and acquisition costs are denoted with c_r and c_m. Secondary markets have a finite number of available parts at any given time, $K \geq q^m$, whereas the regular supplier has infinite capacity.

The primary and secondary effects of the OEM's selling price (p) on the expected demand are modeled in two stages: First we assume a linear decreasing function for the total expected demand $D(p)$, which stands for the *secondary effect of pricing*. The formulation of the total expected demand is given as follows:

$$D(p) = D_0 - (p - p_{min})\frac{D_0 - \underline{D}}{p_{max} - p_{min}}, \tag{5.1}$$

where D_0 and $\underline{D}$ are maximum and minimum possible demand values when the part price is set to p_{min} and p_{max} which are maximum and minimum feasible values for the OEM's selling price.

Second, the total demand comes from price-sensitive and loyal customers which prefer to go to secondary markets and the OEM respectively. The split of the total demand between these two customer classes constitutes the primary effect of pricing since increasing the OEM's selling price leads loyal customers to become price-sensitive. We denote the demand from the former class of customers as 'price-sensitive demand' and the demand of the latter class as 'loyal demand'. We consider the following linear decreasing function to model the fraction of *loyal* demand which comes directly to the OEM:

$$\xi(p) = 1 - \frac{p - p_{min}}{p_{max} - p_{min}}.$$

Without loss of generality, we set $\underline{D} = p_{min} = 0$. For notational brevity, we define $\gamma = \frac{D_0}{p_{max}}$ and $\beta = \frac{1}{p_{max}}$, which are slopes of total demand and *loyal* demand respectively, both decreasing in p.

Since the available spare parts on the market is limited, some of price-sensitive customers might be unsatisfied after realization of demand. We assume that these customers come to the OEM to satisfy their spare parts needs by paying the OEM's selling price. Therefore, *total received demand* of the OEM is a summation of total loyal demand and unsatisfied (left-over) price-sensitive demand as formulated below:

$$\begin{aligned} D_{tot} &= D(p)\xi(p) + [D(p)(1 - \xi(p)) - K + q^m]^+ + \epsilon, \\ &= (D_0 - \gamma p)(1 - \beta p) + [(D_0 - \gamma p)\beta p - K + q^m]^+ + \epsilon, \end{aligned} \tag{5.2}$$

where $D(p)$ is given in Equation 5.1 . The first term of the equality stands for loyal demand of the OEM, whereas the second term stands for left-over, price-sensitive demand which comes to the OEM since it is not satisfied from secondary markets due to the limited availability. The last term in Equation 5.2, ϵ, is an additive random variable, with zero mean and positive support on demand. Note that in the case of large K or low q^m, $[(D_0 - \gamma p)\beta p - K + q^m]^+$ becomes zero and the company receives only the loyal demand, $D(p)\xi(p)$. A schematic representation of the model is depicted in Figure 5.1.

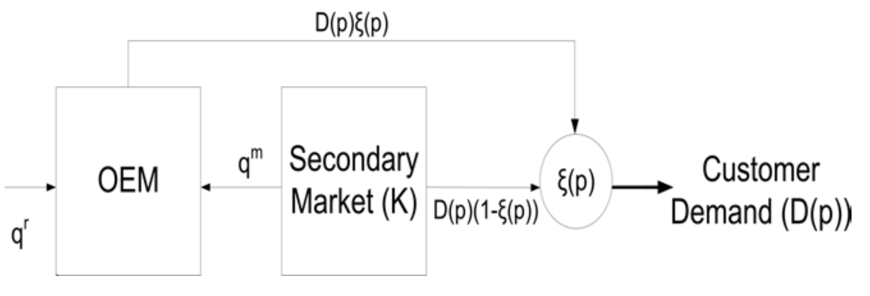

Figure 5.1: Supply Chain of an OEM with a Secondary Market

In our model we assumed the following event sequence: At the beginning of the period, the OEM decides orders to regular supplier and secondary markets (q^r and q^m), and its selling price (p) by looking at the existing inventory (y) and the market availability (K). Deliveries occur immediately. Afterwards, the total demand and its split between secondary markets and the OEM materializes. Unsatisfied price-sensitive customers come to the OEM. The total received demand is satisfied from inventory $(y + q^r + q^m)$. Finally, the holding cost (backlog cost) for excess inventory (for unsatisfied demand) is incurred. Note that we assume that the backlog cost rates for both parts of total received demand (loyal and price-sensitive) are equal to each other. Also we assume that the salvage value of excess inventory is zero and the OEM is indifferent between loyal and price-sensitive customers in its pricing policy. Furthermore customers are assumed to accept parts from markets and regular suppliers as substitutes. The last assumption can be justified for repairable items, which can be purchased from secondary markets and overhauled in a repair shop. When a part is overhauled, the cost of buying from the secondary markets is equal to the sum of the overhauling cost and the acquisition cost from the secondary market. All notations of the model are presented in Table 5.1.

Table 5.1: Notation of the model

K :	number of parts available on the market at the beginning of a period.
c_m:	cost of buying one part from the market for the OEM.
c_r:	cost of buying an part from the regular supplier.
h:	holding cost per period per part.
b:	cost of backlogged demand.
p_{min} :	minimum selling price.
p_{max} :	maximum selling price.
D_0:	maximum demand rate when the selling price is set to p_{min}.
$\underline{D}$	minimum demand rate when the selling price is set to p_{max}.
$D(p)$:	total customer demand arriving to the system.
$\xi(p)$:	fraction of loyal demand choosing the OEM.
D_{tot}:	total amount of demand received by the OEM.
q^r:	order to regular supplier.
q^m:	number of item purchased from secondary market.
p :	selling price to the customer.
$z = K - q^m$:	the amount of spare parts left on the market by the OEM.

Using these notations and assumptions, the single period profit function is formulated as follows:

$$H(K, y, p, q^m, q^r) = pD_{tot} - c_m q^m - c_r q^r - h[y + q^m + q^r - D_{tot}]^+ - b[D_{tot} - (y + q^m + q^r)]^+, \tag{5.3}$$

where D_{tot} is given in Equation 5.2. The first term of Equation 5.3 is the revenue from total received demand. The rest of the terms are acquisition cost from secondary markets and the regular supplier, holding and backlog costs.

The single-period profit maximization problem can be formulated as follows:

$$\mathbf{P1}: \quad \max_{\substack{0\le q^r,\\ 0\le q_m\le K,\\ 0\le p\le p_{max}}} E[H(K,p,y,q^m,q^r)]. \tag{5.4}$$

The constraints of **P1** consist of non-negativity of replenishment orders and selling prices, and the limit on total purchase from secondary markets. The main difficulty of this maximization problem stems from D_{tot} since it may take two different forms depending on K and the amount of left-over, price-sensitive demand. Denoting the amount of parts left on the market by the OEM with z, total received demand (D_{tot}, Equation 5.2) can be written as follows:

$$D_{tot} = \begin{cases} D(p) - z + \epsilon, & \text{if } (D_0 - \gamma p)\beta p \ge z, \\ D(p)\xi(p) + \epsilon, & \text{otherwise.} \end{cases} \tag{5.5}$$

This indicates that the objective function of **P1** changes when price-sensitive demand is larger than the amount of spare parts left on market by the OEM. This brings the necessity of analyzing **P1** on two different feasible sets: $\{(p,z) : (D_0 - \gamma p)\beta p \ge z\}$ and its complement in the feasible set of **P1**. We refer to these two subproblems as **P1.1** and **P1.2** which are analyzed in the following subsections respectively.

5.4.1 Analysis for Problem 1.1

Recall that the OEM purchases parts from the secondary markets before the demand arrives. Therefore, the amount of spare parts left on the market for price-sensitive demand, denoted by z, is a result of his replenishment policy. When the price-sensitive demand is larger than the spare parts left on the secondary market by the OEM ($D(p)\beta p \ge z$), then the company receives the difference between the total expected demand and the amount of parts left on the market ($D(p)-z+\epsilon$). A transformation $v = K+y+q^r-D(p)$ simplifies the analysis of the profit function significantly. The variable v can be interpreted as the amount of expected excess inventory after the demand is satisfied. Using this variable, the objective function and the maximization problem (**P1.1**) are written as follows:

$$\begin{aligned} H^1(K,y,p,z,v) &= p(D(p) - z + \epsilon) - c_m(K - z) - c_r(v - K - y) - h[v - \epsilon]^+ - b[\epsilon - v]^+, \\ &= (p - c_r)D(p) + p\epsilon + (c_m - p)z - c_r v - h[v - \epsilon]^+ - b[\epsilon - v]^+ + c_r y + K(c^r - c^m), \\ &= (p - c_r)D(p) + p\epsilon + (c_m - p)z - (c_r - b)v - (h + b)[v - \epsilon]^+ + c_r y + K(c_r - c_m), \end{aligned} \tag{5.6}$$

and

$$\mathbf{P1.1}: \quad \max_{(p,z,v)\in {}^1\mathcal{F}} E[H^1(K,y,p,z,v)], \tag{5.7}$$

where

$${}^1\mathcal{F} = \{(p,z,v) \in \mathbb{R}^3 : 0 \le z \le K; 0 \le p \le p_{max}; (D_0 - \gamma p)\beta p \ge z; K + y - D_0 + \gamma p \le v\}. \tag{5.8}$$

The feasible set ${}^1\mathcal{F}$ consists of four different constraints. The first one is the market capacity (the second constraint in **P1**), whereas the second one specifies possible values of price (the third constraint of **P1**). The third constraint defines the portion of the feasible set where $D_{tot} = D(p) - z$ and the last constraint is a transformed version of the non-negativity of orders to regular supplier (the first constraint in **P1**). The following lemma establishes the convexity of ${}^1\mathcal{F}$.

Lemma 11 *The following statements hold.*

(a) $(D_0 - \gamma p)\beta p - z$ *is jointly concave in* p *and* z*,*

(b) ${}^1\mathcal{F}$ *is a convex set of* (p, z, v)*.*

The proof of Lemma 11 is given in Appendix 5.A. For analyzing the objective function we use separability of the profit function $H^1(K,y,p,z,v)$ in (p,z) and v, which can be expressed as follows:

$$H^1(K,y,p,z,v) = R^1(p,z) - \mathcal{G}_1(v)$$

The first term in the profit function, $R^1(p,z) = (p-c_r)D(p) + (c^m - p)z + p\epsilon$, is interpreted as revenue function. The first term of the revenue function stands for the revenue minus to acquisition cost of the regular supplier, whereas the second term represents the loss of profit due to the amount of parts left on the market.

The second term of the profit function, $\mathcal{G}_1(v) = (c^r - b)v + (h+b)[v-\epsilon]^+ - c_r y - K(c_r - c_m)$, is the cost function when the expected amount of excess inventory after demand is v. The first two terms of this function represent acquisition, holding and backlog costs, whereas the third and fourth terms stand for the cost saving due to existing inventory and buying from secondary market instead of the regular supplier. The following lemma states the strict concavity of the profit functions.

Lemma 12 *The following statements hold.*

(a) $E[R_1(p,z)]$ *is a strictly concave function of* (p,z)*,*

(b) $\mathcal{G}_1(v)$ *is a convex function of* v,

(c) The expected profit function $E[H^1(K, y, p, z, v)]$ *in Equation 5.6 is a strictly concave function of* (p, z, v).

The proof of Lemma 12 is given in Appendix 5.A. Lemmas 11 and 12 imply that the subproblem **P1.1** is a maximization of a strictly concave function on a convex set. Hence, the optimal value can be found analytically using Karush-Kuhn-Tucker conditions. The optimal solution of the problem **P1.1** is given in the following theorem.

Theorem 11 *The optimal solution of the maximization problem (**P1.1**) is characterized as follows:*

$$(p^*, z^*, v^*) = \begin{cases} \left(\frac{p_{max}+c_r}{2}, 0, F^{-1}(\phi)\right) & \text{if } F^{-1}(\phi) \geq +K + y - D(p^*), p^* \geq c_m \\ \left(f_2^{-1}(0), 0, K + y - D(p^*)\right) & \text{if } F^{-1}(\phi) < K + y - D(p^*), p^* \geq c_m, \\ \left(f_3^{-1}(0), D(p^*)\beta p^*, K + y - D(p^*)\right) & \text{if } F^{-1}(\phi) < K + y - D(p^*),\ p^* < c_m, D(p^*) < K, \\ \left(f_4^{-1}(0), K, K + y - D(p^*)\right) & \text{if } F^{-1}(\phi) < K + y - D(p^*),\ p^* < c_m, D(p^*) > K, \end{cases} \tag{5.9}$$

where $\phi = \frac{b-c_r}{h+b}$, $F(.)$ *is the cdf of the random variable* ϵ *and the functions* $f_2(p)$, $f_3(p)$ *and* $f_4(p)$ *are given below:*

$$f_2(p) = D_0 - 2\gamma p + \gamma(b - (h+b)F(K + y - D(p))) \tag{5.10}$$

$$f_3(p) = f_2(p) - D(p)\beta p + \gamma(p - c_m)(-1 + 2\beta p), \tag{5.11}$$

$$f_4(p) = f_2(p) - K. \tag{5.12}$$

The proof of the theorem is presented in the appendix of this chapter. The optimal pricing policy is calculated with three different functions mainly depending on the market availability (K) and the inventory level y. Among three functions, $f_2(p)$ is the fundamental one since it takes place in other price functions. Therefore, all of these functions have some common properties which are useful for deriving insight into the solution. First, all functions are nonincreasing function of μ, which is the sum of market availability and the inventory level at the beginning of the period. This implies the relationship between the optimal pricing policy and the existing inventory level. Second, the effect of inventory and market availability appears through the random component of the demand. This implies that for larger values of μ, the last term of $f_2(p)$ becomes one ($\lim_{\mu\to\infty} F(\mu - D(p)) = 1$) and the optimal pricing policy becomes the list price. The list price policy stands for a constant selling price which is independent of the inventory level of a company. A constant mark-up policy used in practice is a good example of list price policy. Since our policy includes a transition between two different list prices depending on μ, we refer to this policy as *modified-list* price due to the effect of randomness on the price.

When we transform decision variables from (p, z, v) back to (p, q^r, q^m), results presented in Theorem 11 are as follows:

$$(p^*, q^{m^*}, q^{r^*}) = \begin{cases} \left(\frac{p_{max}+c_r}{2}, K, F^{-1}(\phi) - \mu + D(p^*)\right) & \text{if } F^{-1}(\phi) \geq \mu - D(p^*),\ p^* \geq c_m, \\ \left(f_2^{-1}(0), K, 0\right) & \text{if } F^{-1}(\phi) < \mu - D(p^*),\ p^* \geq c_m, \\ \left(f_3^{-1}(0), K - D(p^*)\beta p^*, 0\right) & \text{if } F^{-1}(\phi) < \mu - D(p^*),\ p^* < c_m,\ D(p^*) < K, \\ \left(f_4^{-1}(0), 0, 0\right) & \text{if } F^{-1}(\phi) < \mu - D(p^*),\ p^* < c_m,\ D(p^*) > K, \end{cases} \tag{5.13}$$

where $\phi = \frac{b-c_r}{h+b}$ and $\mu = K + y$.

Although conditions and optimal price values in Equation 5.13 are intricate, we make some important observations about the optimal solution for **P1.1**. First, secondary markets are used as the primary source of supply in the solution. Orders are issued to the regular supplier only if the summation of market availability, existing inventory minus expected demand is smaller than a certain threshold $(F^{-1}(\phi) \geq \mu - D(p^*))$. Otherwise, no order is placed to the regular supplier. This policy is called zero inventory ordering principle (Chen and Simchi-Levi, 2010). Second, increasing amount of existing inventory changes the optimal pricing from $\frac{p_{max}+c_r}{2}$ to $f_2^{-1}(0)$. The latter solution is a non-decreasing function of $K + y$ and it is always smaller than the former. In other words, increasing inventory level (or market availability) leads to a lower optimal price level. Third, the third solution in Equation 5.13 is on the constraint $D(p)\beta p \geq z$ of the feasible set ${}^1\mathcal{F}$. This might imply that when this solution is the optimal for **P1.1**, the profit of **P1.2** might be higher than that of **P1.1**.

Since it is not possible to get closed form conditions for the last three solutions in Equation 5.13 (all of them are functions of p^*), we cannot derive further insights analytically. Therefore, we employ numerical experiments to understand these conditions better in Section 5.5. In the next section, the analysis for **P1.2** is presented.

5.4.2 Analysis for Problem 1.2

In the second subprobem, we consider the possibility of demand going to secondary markets being smaller than the amount of parts left on the market, $D(p)\beta p < z$. In this case, the OEM only receives the demand from loyal customers who choose to come directly to the OEM instead of secondary markets. To make an analysis for $H^2(K, y, p, q^m, q^r)$, which is the profit function when $D(p)\beta p < z$, let us define the another decision variable

$w = \mu - z + q^r - D(p)\xi(p)$ and recall that $\mu = K + y$. Then,

$$\begin{aligned} &H^2(K, y, p, z, w) = \\ &\quad p(D(p)\xi(p) + \epsilon) - c_m(K - z) - c_r[w - \mu + z + D(p)\xi(p)] - h[w - \epsilon]^+ - b[\epsilon - w]^+, \\ &\quad = (p - c_r)D(p)\xi(p) + p\epsilon - z(c_r - c_m) - c_r w - h[w - \epsilon]^+ - b[\epsilon - w]^+ + c_r\mu - Kc_m. \end{aligned} \tag{5.14}$$

Using $H^2(K, y, p, z, w)$ as the objective function, we can define the maximization problem **P1.2** as follows:

$$\textbf{P1.2}: \quad \max_{(p,z,w)\in {}^2\mathcal{F}} E[H^2(K, y, p, z, w)], \tag{5.15}$$

where

$$\begin{aligned} {}^2\mathcal{F} = &\{(p, z, w) \in \mathbb{R}^3 : 0 \le z \le K; 0 \le p \le p_{max}\} \\ &\cap \{(p, z, w) \in \mathbb{R}^3 : -w - z + \gamma p - \gamma\beta p^2 + \mu - D_0 + \gamma\beta \le 0; (D_0 - \gamma p)\beta p < z\}. \end{aligned}$$

The feasible set ${}^2\mathcal{F}$ consists of two different subsets. The first set includes linear constraints of market availability and bounds of price. Thanks to linearity of constraints the first subset is convex. The second subset is constrained with two inequalities. The first one follows from the non-negativity of the orders to regular supplier ($q^r \ge 0$) and the transformation ($w = y + K - z + q^r - D(p)\xi(p)$). The second inequality is based on the function that separates the feasible sets of the two subproblems from each other. Both inequalities form non-convex sets since they are sub-level sets of concave functions.

The objective function of **P1.2** in Equation 5.14 can be written as $H^2(K, y, p, z, w) = R_2(p) - \mathcal{G}_2(z, w)$ where $R_2(p) = (p - c_r)(D_0 - \gamma p)(1 - \beta p)$ and $\mathcal{G}_2(z, w) = z(c_r - c_m) + c_r w + h[w - \epsilon]^+ + b[\epsilon - w]^+ - c_r\mu + Kc_m$. Therefore it is separable in p and (z, w). $R_2(p)$ stands for the profit obtained from sales of spare parts replenished by the regular supplier. The function $\mathcal{G}_2(z, w)$ represents the costs to be deducted from the profit. The first term represents the amount of extra cost that stems from choosing the regular supplier instead of the secondary market. Recall that z is defined as the amount of items left on the market. The interpretation of the other terms are the same as for $\mathcal{G}_1(z, v)$. The following lemma completes the analysis for the objective function of **P1.2**.

Lemma 13 *The following statements hold.*

a) $E[R_2(p)]$ *is a strictly concave function in* p *on* $[0, \Lambda)$ *and it is convex decreasing in* p *on* $[\Lambda, p_{max})$*, where*

$$\Lambda = \frac{c_r + 2p_{max}}{3}.$$

b) $\mathcal{G}_2(z, w)$ *is convex in* (z, w)*,*

c) $H^2(K, y, p, z, w)$ *is strictly concave on* ${}^2\mathcal{F} \cap \{p : p \in [0, \Lambda)\}$*.*

The proof of Lemma 13 is given in Appendix 5.A. $R_2(p)$ is a cubic function of p and it is strictly concave only in $[0, \Lambda]$. Lemma 13 implies that **P1.2** is a maximization of a strictly concave function on a nonconvex set. To the best of our knowledge, there is no analytical method for **P1.2**. Therefore, after developing some more insight into the structure of **P1.2**, we proceed to the numerical analysis of the problem.

In Lemma 13 we showed that the maximum value of $R_2(p)$ is in $[0, \Lambda)$. Recall that $R_2(p)$ is the positive part of the profit function $(H^2(K, y, p, z, w))$. Therefore, it is intuitive to state that the solution of **P1.2** can be found in ${}^2\mathcal{F} \cap \{p : p \in [0, \Lambda)\}$. This result is shown for the maximization problem *without* non-convex constraints in the following lemma.

Lemma 14 *The following equality holds.*

$$\max_{\substack{p \in [p_{min}, p_{max}],\\ 0 \le z \le K,\\ w \in \mathbb{R}^+.}} H^2(K, y, p, z, w) = \max_{\substack{p \in [p_{min}, \Lambda],\\ z \ge 0,\\ w \in \mathbb{R}^+.}} H^2(K, y, p, z, w)$$

The proof of Lemma 14, given in the appendix of this chapter, relies on statement a of Lemma 13 and the separability of the profit function $H^2(K, y, p, z, w)$ in p and (z, w). Unfortunately, Lemma 14 cannot be shown for **P1.2** due to the nonconvex constraints of ${}^2\mathcal{F}$. We only conjecture the result in Lemma 14 and check its validity numerically in Section 5.5.

Conjecture 2

$$\max_{\substack{(p,z,w) \in {}^2\mathcal{F},\\ p \in [0,\Lambda)}} E[H^2(K, y, p, z, w)] = \max_{(p,z,w) \in {}^2\mathcal{F}} E[H^2(K, y, p, z, w)].$$

Finally, we present the solutions of the *unconstrained* **P1.2**, which will be useful to understand the directions of the gradient vector and the solution of the overall problem.

Lemma 15 *The following statements hold:*

a) $(\frac{p_{max}+2c_r}{3}, 0, F^{-1}(\frac{b-c_r}{h+b}))$ *is the optimum solution of the problem*

$$\max_{\substack{0\le z\le K,\\ 0\le p\le \Lambda}} E[H^2(K,y,p,z,w)].$$

b) The optimum solution of the unconstrained problem does not satisfy $(D_0-\gamma p)\beta p - z < 0$. *Hence, they are not in* $^2\mathcal{F}$.

Lemma 15, of which the proof is given in Appendix 5.A, establishes that the gradient vector of the objective function is towards the constraint separating **P1.1** from **P1.2**. This indicates that the solution of **P1.2** will be on $(D_0-\gamma p)\beta p = z$ for many instances. Since the feasible set of **P1.1** includes this constraint, we expect that **P1.1** will be sufficient for solving the problem **P1** in many instances of the problem. As stated above, we do not have any analytical method to prove these properties and bring a final and definite answer to the problem. Therefore, we moved to numerical experiments to confirm our insights into the solution. Note that given the status of our analysis, we can only say that the solution presented in Theorem 11 can be used as a heuristic approach to the maximization problem **P1**. In the following section, we test the performance of this heuristic solution in an extensive numerical study.

5.5 Numerical Experiments

This section consists of three major parts. First we will present results of numerical optimization for the first subproblem (**P1.1**) to analyze the relationship between different solutions presented in Theorem 11. To this end, we employ extensive numerical experiments and find regions of the state space where each solution becomes optimal. Second, we will present some results on the validity of Conjecture 2 in Section 5.4.2. Recall that due to the nonconvex constraints of $^2\mathcal{F}$ we cannot extend the result of Lemma 14 for the whole problem although we find it to be true for most instances of our test bed. Third, we present results of our experiments for the relationship between **P1.1** and **P1.2**. Note that all of experiments are conducted in MATLAB 2014a.

In order to evaluate solutions of the problem we use a test bed consisting of a factorial design of parameters presented in Table 5.2. For inventory level, y, market capacity, K, maximum possible demand, D_0, upper bound of price, p_{max}, and acquisition cost of the regular supplier, c_r; we use actual values presented in the table. For acquisition cost from secondary markets, backlog and holding cost rates, c_m, b and h, we multiply l^{cm}, l^b, l^h with c_r. The motivation behind this approach is avoiding the violation of the condition $b > c_r \geq c_m > h$. The first inequality is classic in inventory control theory and its violation leads the decision maker to backlog the demand instead of replenishing the inventory. The second inequality follows from our assumption of the regular supplier being more expensive than secondary markets. With the last inequality we aim to avoid unrealistic scenarios since holding cost rate cannot be larger than the acquisition cost of a spare part.

Table 5.2: Experiment Factors Consisting of the Test Bed

y	K	D_0	P_{max}	c_r	l^{cm}	l^b	l^h
-2	1	5	10	5	0.1	1.1	0.05
-1	5	10	25	10	0.25	1.25	0.1
0	10	20	50	20	0.5	1.5	0.25
1	20	50	100	50	0.75	2	0.5
2					1		0.75
3							
4							
5							

In addition, some instances of the test bed, where $p_{max} < c_r$, represent unrealistic scenarios since the maximum possible price cannot be smaller than the cost of a regular supplier. Such a situation would set the OEM's profit to zero. We eliminated those instances from the test bed. As a result we obtain a test bed consisting of 112640 different instances. In our calculations we assumed that $\epsilon \sim Unif(-1, 1)$.

5.5.1 Solutions of Problem 1.1

Conditions of Theorem 11 mostly consist of functions of the optimal price value, which can only be expressed analytically for some specific distributions for ϵ. Therefore, to understand the conditions of the theorem and the relationship between solutions, we employ enumeration. To this end, we coded the problem in MATLAB and used its built-in function *fmincon* which is suitable for convex optimization problems.

Our results indicate that the first two solutions in Theorem 11 are optimal in 96% percent of cases (S1 and S2 in Figure 5.2) whereas the third solution is found to be the optimal in 3.9% of them (S3 in Figure 5.2). This distribution stems from the fact that the third solution appears when the summation of existing inventory and market capacity is larger than the expected demand. Such instances can occur in our test when D_0 is smaller than $K + y$. The fourth solution (S4) is only optimal for small values of K and large values of y, i.e. when the secondary markets do not bring competition and the inventory level is sufficient to satisfy the demand. These relationships are more obvious in Figures 5.3 and 5.4.

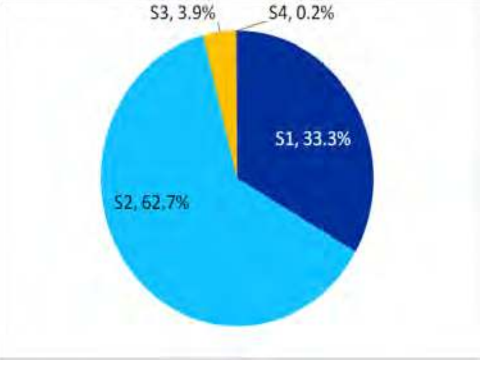

Figure 5.2: Solutions of Problem 1.1

In Figures 5.3 and 5.4, we present results of two experiments, which are different from each other by the cost of regular supplier. Decreasing the value of this parameter increases the region of S1 in which the OEM orders to regular supplier and collects all parts on the market. This counter-intuitive result stems from the increasing gap between p_{max} and c_r. Specifically, when c_r is smaller while everything else is the same, the OEM has larger potential profit margin to manipulate the demand. This motivates him to increase more inventory (by obtaining from both channels) to obtain larger profit.

Another interesting managerial question is the relationship between the optimal policy and the market cost which is given in Figure 5.5. Recall that the first two solutions prescribe to collect all the market while the first one prescribes ordering to regular supplier while the second one does not. This is the reason why solution 1 is the optimal for negative inventory levels. As the gap between costs of market and the supplier closes, collecting all the market loses its appeal and the third solution becomes the optimal. Recall that the third solution is defined with the quadratic function $D(p)\beta p = z$ which separates **P1.1** from **P1.2**. In other words, when the market cost is close to the cost of the regular supplier, the **P1.2** might be more interesting for **P1**.

5.5.2 Analysis for Conjecture 2

For analyzing **P1.2** and Conjecture 2, we enumerate the objective function as well as the constraints of the problem in MATLAB using an appropriate discretization. In these computations, some instances, including large K, D_0 and p_{max} values, are eliminated from

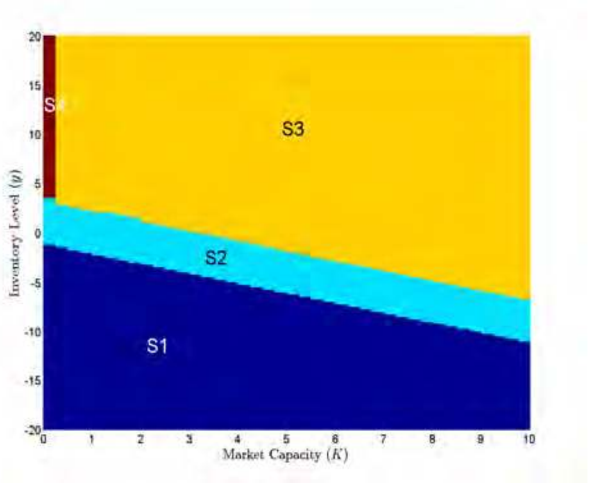

Figure 5.3: Solutions of Problem 1.1 for different Market Capacity and Inventory Level for $D_0 = 5$, p_{max}=50, c_r=40, $c_m = 20$, $b = 44$, $h = 4$

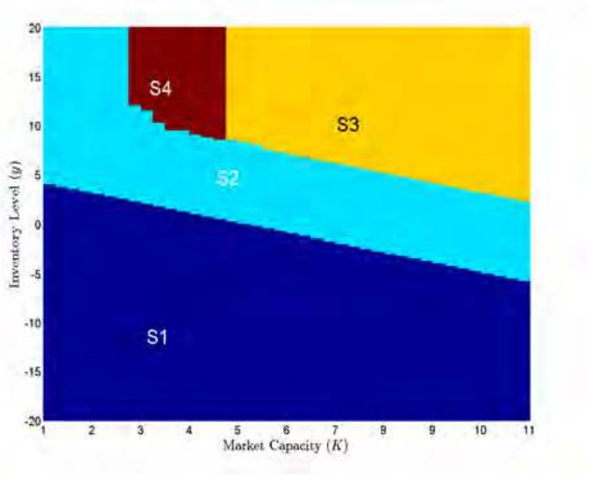

Figure 5.4: Solutions of Problem 1.1 for different Market Capacity and Inventory Level for $D_0 = 5$, p_{max}=50, c_r=25, $c_m = 20$, $b = 44$, $h = 4$

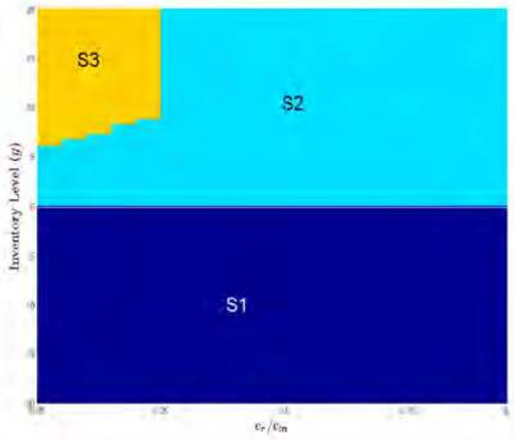

Figure 5.5: Solutions of Problem 1.1 for different Inventory Levels (y) and Market Cost (c_m) for $D_0 = 5$, p_{max}=50, $K = 5$, c_r=25, $b = 44$, $h = 4$

the test bed due to memory constraints of MATLAB The total size of the test bed we evaluated in this part of the numerical experiment is 84667.

An analysis on the results of numerical experiments reveals that in 74% of the test bed Conjecture 2 was correct, i.e. the optimum price is smaller than Λ from Lemma 13. A closer look at the instances, where Conjecture 2 fails, indicated that in 83% of cases K equals to one (Figure 5.6) whereas in 16% of them K is equal to 5. This indicates that Conjecture 2 is more reliable for larger values of market availability. Note that the conjecture holds for all instances of K equal to 20. Reasons behind this empirical evidence are twofold: For small values of market capacity, **P1.2** becomes trivial due to the fact that the constraint $D(p)\beta p < z < K$ covers a very small area of the feasible set. In terms of managerial insight, we can translate this observation as increasing values of market capacity brings more fierce competition to the OEM so, it has to adjust its pricing policy accordingly.

5.5.3 Comparison Between Problems 1.1 and 1.2

Recall from Section 5.4 that the profit maximization problem **P1** consists of two separate subproblems of which the feasible sets are mutually exclusive. The solution of the problem **P1** is equal to the maximum of **P1.1** and **P1.2** which are analyzed in Section 5.4.1 and 5.4.2 respectively. Although an analytical formulation of the solution is derived for **P1.1**,

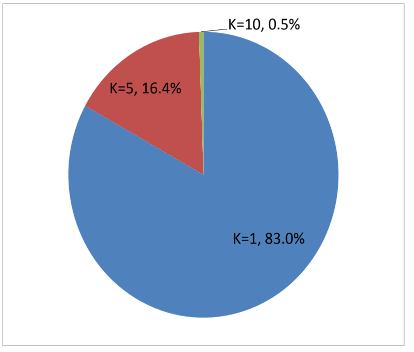

Figure 5.6: Distribution of Values of K Cases in the Market Availability Where Conjecture 1 Fails

the non-convex feasible set of **P1.2** prevents us to derive any analytical solution to be used in comparison between the two subproblem. Therefore, we circumvent this complication by comparing the two subproblems numerically on an extensive test bed which includes 84667 instances, as described in the previous section.

Recall that we expect a domination of **P1.1** over **P1.2** since the unconstrained solution of the latter problem exists in the feasible set of the former, as stated in Conjecture 2. Results of our experiments indicate that in 85% of the cases this expectation is true.

Properties of the test bed, on which **P1.2** is larger than **P1.2**, are important to understand the reliability of analytical solutions derived in Section 5.4.1. A closer look at that portion of the test bed reveals that the majority of these test instances consists of small values for D_0 and p_{max}. This can be explained by the fact that smaller values of these parameters yield larger feasible sets for **P1.2** compared to its complementary subproblem. Furthermore, numerical values of the decision variables (p, z, w) indicate that secondary markets are still in use as the primary supply source, whereas orders are placed to regular supplier according to *zero inventory ordering principle* (Chen and Simchi-Levi, 2010). The optimal prices of **P1.2** seem to be case-dependent, which means it is difficult to derive a general rule from them.

In order to understand the magnitudes of the profits, we consider the maximum profit over p_{max} ratio. This statistic is used to analyze calculated profits from the two subproblems and understand the solution for **P1**. Distributions of test instances over this ration are given in Figures 5.8 and 5.9.

In Figure 5.8, we provide two histograms: Blue bars represent the profit histogram for instances in which **P1.2** dominates. Red histogram depicts the profit distribution of **P1.2** for all instances. The former distribution has a large probability mass on the negative side and a long left-tail while the right-tail of the latter is higher and longer. This implies that **P1.2** prevents the maximization problem to have extremely small profits and complements the subproblem **P1.1**.

The picture is reversed for the portion of the test bed where **P1.1** dominates (Figure 5.9). Compared to all cases, **P1.1** has a higher right tail. Therefore, we conclude that **P1.1** is more associated with positive profits of the problem while **P1.2** stands for minimizing the loss due to inconvenient circumstances. From a managerial point of view this result indicates that using analytical solutions derived in Section 5.4.1 yields high expected profits with high standard deviation. Decision makers should pay extra attention to the cases with spare parts whose maximum possible demand (D_0) and maximum possible price (p_{max}) are low to avoid significant losses.

5.6 Conclusion

It is known that OEMs are subject to fierce competition by third-party service providers for their installed bases especially after the end of warranty period. This fact is aggravated by introduction of internet-based trading platforms on which different agents can exchange surplus or overhauled spare parts with each other. Those agents usually charge cheaper price to their customers and this constitutes a source of competition for OEMs' after-sale services.

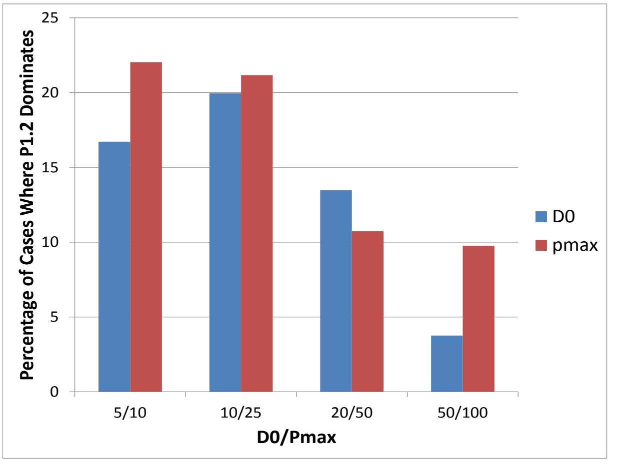

Figure 5.7: Distribution of Cases, Where **P1.2** Dominates, over D_0 and p_{max}

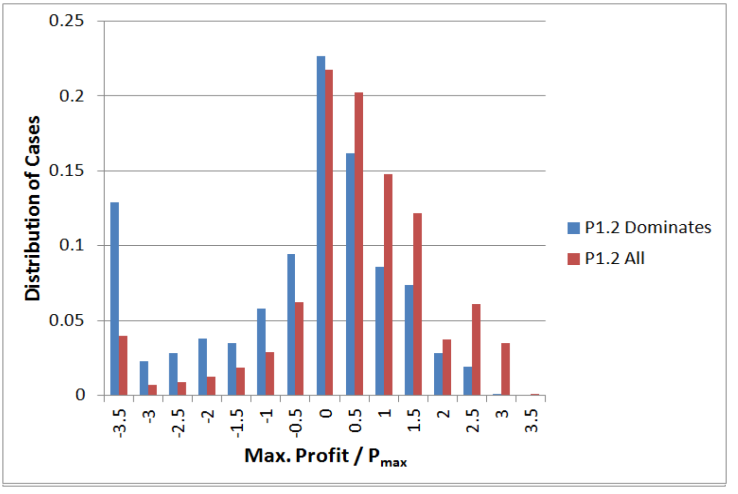

Figure 5.8: Distribution of Profit Over p_{max} Ratio

In this study, we consider a business case of an OEM who provides service to out-of-production aircraft. They are faced with a constant competition from third party service providers (maintenance and repair shops) as well as agents on internet-based secondary markets, such as ilsmart.com. These secondary markets are not only a source of competition, but also potential supply sources for spare parts that can be used by the

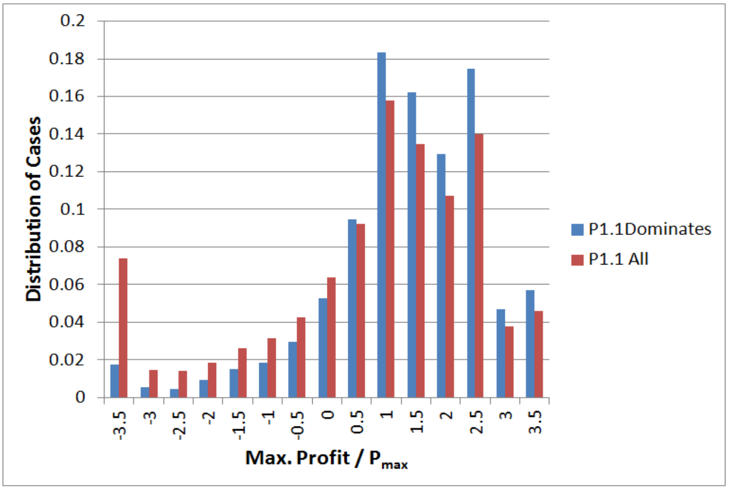

Figure 5.9: Distribution of Profit Over p_{max} Ratio

OEM. Therefore, dual sourcing and price competition should be considered for the pricing and replenishment policies of the OEM.

In order to address this problem, we consider a single-period inventory control model in which the customer demand can be manipulated with pricing. For the effect of price on spare parts demand, we recognize two separate effects: The primary effect of pricing appears through division of demand between secondary markets and the OEM, whereas the secondary effect is on the total amount of spare parts demand.

Mathematical analysis of the model indicates that the problem should be considered by dividing it into two subproblems since the objective function changes on different portions of the feasible set. We derive an analytical solution for one of the subproblems. Analytical results show that the pricing policy is a modified list price in which the modification is due to the random component of the demand. The replenishment policy, on the other hand, utilizes both channels. Secondary markets are found to be the primary supply source, while the regular supplier is used in a complementary nature with zero inventory ordering principle. This result stems from the fact that purchasing from secondary markets decreases spare parts availability on those markets and leads to increasing demand for the OEM since there are only a finite amount of spare parts available at any given time.

The analysis for the second problem indicates that the feasible set of the maximization problem consists of a non-convex constraint. Therefore the second problem is analyzed numerically. Results indicate that the replenishment principles found in the first subproblem hold for the second subproblem as well. However, the pricing policy is more state-dependent when the second subproblem dominates the first one. Furthermore, the first subproblem leads to higher, but more variable expected profit compared to the profit values of the second subproblem.

A natural extension of our study is considering the multi-period optimal policy for this problem. The policy found in this study can be used as a heuristic for the multi-period problem. The performance of this heuristic over a finite and infinite planning horizon is another research question left to future research. Also, we are planning to consider quality differences (and substitution) between the regular supplier and secondary markets as well as different demand models, such as exponential, logit etc,. for the relationship between demand and price.

5.7 Epilogue

In this section, joint optimization of pricing and inventory control policies is considered for a problem setting taken from an OEM in aviation, Fokker Services. The results of this chapter were presented to managers and engineers in the company and we encountered good acceptance of the idea and the company asked what is needed for a large-scale field test. In this section, we address some issues related to such a field test.

First, optimization of pricing and replenishment policies relies on the availability of a price-dependent expected demand function. In other words, we assume that for a given price value, we possess knowledge of expected spare parts demand the OEM will receive. However, the price elasticity of demand has not been investigated in the company since the OEM applies constant mark-up policy for pricing for a long period of time. Since changing spare parts prices in order to measure the reaction of customers is not a viable option, price elasticities of spare parts should be determined using expert judgments.

Second, we only consider an additive random variable for total expected demand. The additive demand model assumes a constant variance of demand, independent of the magnitude of the expectation. In the literature, a multiplicative demand model, in which the random component is multiplied with the expected demand, is suggested to overcome the weaknesses of the additive model. Unfortunately, the multiplicative model increases the mathematical complexity of the profit function and leads to intricate solutions for pricing and replenishment policies of the OEM. Intuitively, both models have some merit in spare parts context. We can speculate that the additive model might be useful for slow-moving parts, whereas the multiplicative model might be more appropriate for fast-movers. Therefore, one should consider both models and derive optimal formulations in order to have a comprehensive pricing application. Furthermore, other demand models, e.g. logit model by Aydin and Porteus (2008), may be addressed in future research and in the application.

Third, our model assumes that the amount of demand from price-sensitive customers is known and it can be formulated with a decreasing linear split function $\xi(p)$. One could enumerate the split function by using the installed base information possessed by the OEM. Specifically, the OEM keeps track of the number of aircraft owned/used by each operator and it has the monthly utilization data for the entire fleet. Using fleet utilizations, technical information and the number of aircraft in operation, one could make an estimate for the size of the customer demand that goes to the secondary markets, which is as such already interesting business information.

5.A Proofs of Theorems

Proof of Lemma 11: Statement a follows from the semidefiniteness of the Hessian matrix for $(D_0-\gamma p)\beta p-z$ which is easy to show. Also the same statement can be shown with the separability of the function $(D_0 - \gamma p)\beta p - z$ in p and z, and the first quadratic function is a concave function of p.

Statement b follows from the intersection of first, second and the fourth constraints of ${}^1\mathcal{F}$ and the set defined by the third consraint. Convexity of that set is implied by the fact that the third consraint is a sub-level set of the function $-(D_0-\gamma p)\beta p+z$ for zero. ■

Proof of Lemma 12:

For statement a, partial derivatives of the revenue function are as follows:

$$\frac{\partial^2 R_1(p,z)}{\partial p^2} = -\gamma, \tag{5.16}$$

$$\frac{\partial^2 R_1(p,z)}{\partial z^2} = 0, \tag{5.17}$$

$$\frac{\partial^2 R_1(p,z)}{\partial z \partial p} = -1 \tag{5.18}$$

Statement a follows from the negative semi-definiteness of the Hessian matrix. The function $\mathcal{G}_1(v)$ is a summation of acquisition cost, holding and backlog cost (Equation 5.6). It is known that it is a convex function which proves statement b. Statement c follows from the strict concavity of $R_1(p,z)$, given in a, convexity of $\mathcal{G}_1(v)$. ■

Proof of Lemma 13: Strict concavity and convexity of $R_2(p)$ follow from the second derivative of the function.

$$\frac{\partial^2 R_2(p)}{\partial p^2} = -2(\bar{D}\beta+\gamma) + 6\gamma\beta p - 2c_r\gamma\beta.$$

When $\Lambda > p$, $\frac{\partial^2 R^2(p)}{\partial p^2}$ becomes negative and positive otherwise. This proves the convex and concavity statements. Now we need to show $R^2(p)$ is decreasing function of p in $[\Lambda, p_{max})$. To this end, write the first derivative of $R^2(p)$ using $\gamma = \frac{D_0}{p_{max}}$ and $\beta = \frac{1}{p_{max}}$.

$$\begin{aligned}
\frac{\partial R^2(p)}{\partial p} &= (D_0 + \frac{2D_0c_r}{p_{max}}) + 2p(-\frac{2D_0}{p_{max}} - \frac{D_0c_r}{p_{max}^2}) + 3p^2\frac{D_0}{p_{max}^2}, && (5.19)\\
&= \frac{D_0p}{p_{max}^2}(3p - 2c_r) - \frac{D_0}{p_{max}}(4p - p_{max} - 2c_r),\\
&= \frac{D_0p}{p_{max}^2}(3p - 2c_r) - \frac{D_0}{p_{max}}(3p - 2c_r) - \frac{D_0}{p_{max}}(p - p_{max}),\\
&= \frac{D_0}{p_{max}}(3p - 2c_r)\left(\frac{p}{p_{max}} - 1\right) - \frac{D_0p}{p_{max}} - D_0,\\
&= D_0\left(\frac{p}{p_{max}} - 1\right)\left(\frac{3p - 2c_r}{p_{max}} - 1\right). && (5.20)
\end{aligned}$$

In Equation 5.20, the first term is always positive whereas second terms is negative. We will show that the last term is positive in $[\Lambda, p_{max})$ which guarantees that $R^2(p)$ is decreasing in that interval. To this end, note that $\Lambda = \frac{c_r+2p_{max}}{3}$. Take a ν s.t. $\frac{2p_{max}-c_r}{3} \geq \nu > 0$. If $p = \frac{c_r+2p_{max}}{3} + \nu$, then

$$\frac{3p - 2c_r}{p_{max}} - 1 = \frac{p_{max} - c_r}{p_{max}} \geq 0 \Rightarrow \frac{\partial R^2(p)}{\partial p} \leq 0,$$

which holds since $p_{max} \geq c_r$. Proofs of statements b and c are the same with Lemma 12, omitted here. ■

Proof of Theorem 11: The maximization problem **P1.1** can be written as follows:

$$\begin{aligned}
\max\ H^1(K, y, p, z, v) & && (5.21)\\
s.t. \qquad -z &\leq 0, && (5.22)\\
z - K &\leq 0, && (5.23)\\
-p &\leq 0, && (5.24)\\
p - p_{max} &\leq 0, && (5.25)\\
K + y - D_0 + \gamma p - v &\leq 0, && (5.26)\\
-(D_0 - \gamma p)\beta p + z &\leq 0. && (5.27)
\end{aligned}$$

For the concave profit function $(H^1(K, y, p, z, v))$ defined on the convex set ${}^1\mathcal{F}$ in Equation 5.8, the Lagrangian function is given below:

$$\begin{aligned}
\mathcal{L}(\lambda_1, ..., \lambda_6, p, z, v) &= R^1(p, z) - c^r v - h[v - \epsilon]^+ - b[-v + \epsilon]^+ + c_r(y - D_0) - \lambda_1 z \quad (5.28)\\
&+ \lambda_2(z - K) - \lambda_3 p + \lambda_4(p - p_{max}) - \lambda_5(v - \gamma p - K - y + D_0) - \lambda_6((D_0 - \gamma p)\beta p - z),
\end{aligned}$$

Taking the first partial derivatives, we write KKT conditions as follows:

$$D_0 - 2\gamma p - z + c_r\gamma - \lambda_3 + \lambda_4 + \lambda_5\gamma + \lambda_6(-D_0\beta + 2\gamma\beta p) = 0, \tag{5.29}$$

$$c_m - p - \lambda_1 + \lambda_2 + \lambda_6 = 0, \tag{5.30}$$

$$-c_r - h\int_{-\infty}^{v} \mathrm{d}\Phi(s) + b\int_{v}^{\infty} \mathrm{d}\Phi(s) - \lambda_5 = 0, \tag{5.31}$$

Primal Feasibility:

$$0 \le z \le K, \tag{5.32}$$

$$0 \le p \le p_{max}, \tag{5.33}$$

$$\gamma p - v \le -K - y + D_0, \tag{5.34}$$

$$-(D_0 - \gamma p)\beta p + z \le 0, \tag{5.35}$$

Complementary Slackness:

$$\lambda_1 z = 0, \tag{5.36}$$

$$\lambda_2(z - K) = 0, \tag{5.37}$$

$$\lambda_3 p = 0, \tag{5.38}$$

$$\lambda_4(p - p_{max}) = 0, \tag{5.39}$$

$$\lambda_5(\gamma p - v + K + y - D_0) = 0, \tag{5.40}$$

$$\lambda_6(-(D_0 - \gamma p)\beta p + z) = 0, \tag{5.41}$$

Dual Feasibility:

$$\lambda_{1,\ldots,6} \le 0. \tag{5.42}$$

To analyze the negativity of Lagrange multipliers, we consider three possibilities ($\lambda_1 < 0,\ \lambda_2 = 0$),($\lambda_1 = 0, \lambda_2 < 0$), and ($\lambda_1 = \lambda_2 = 0$) as three different regions which are denoted with R1 and R2 and R3. Each region is analyzed respectively.

R1: Note that in this region $z = 0$ due to 5.36. It is easy to show that we have only feasible solutions with the following conditions:

1. $\lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, \lambda_6 = 0,$

2. $\lambda_3 = 0, \lambda_4 = 0, \lambda_5 < 0, \lambda_6 = 0,$

When $\underline{\lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, \lambda_6 = 0}$,

$$\begin{aligned}
D_0 - 2\gamma p + c_r\gamma &= 0, \\
c_m - p - \lambda_1 &= 0, \\
-c_r + b - (h+b)F(v) &= 0.
\end{aligned}$$

These equations imply that $v^* = F^{-1}\left(\frac{b-c_r}{h+b}\right)$, $z^* = 0$, $p^* = (p_{max}+c_r)/2$, $\lambda_1 = c_m - p$ where primal feasibility requires $F^{-1}\left(\frac{b-c_r}{h+b}\right) \geq K + y - D((p_{max}+c_r)/2)$ while dual feasibility forces $(p_{max}+c_r)/2 > c_m$. Dual feasibility condition is satisfied due to our assumption in the problem setting. Recall that $D(p)$ is given in Equation 5.1.

For $\underline{\lambda_3 = 0, \lambda_4 = 0, \lambda_5 < 0, \lambda_6 = 0}$,

$$\begin{aligned}
D_0 - 2\gamma p + c_r\gamma + \lambda_5\gamma &= 0, \\
c_m - p - \lambda_1 &= 0, \\
-c_r + b - (h+b)F(v) - \lambda_5 &= 0, \\
-v + K + y - D_0 + \gamma p &= 0.
\end{aligned}$$

Solution of these equation results $p^* = f_1^{-1}(0)$, $z^* = 0$, $v^* = K + y - D(p^*)$, $\lambda_1 = c_m - p$, $\lambda_5 = -c_r + b - (h+b)F(K + y - D(p^*))$, where $f_1(p)$ is given in Equation 5.10. Primal feasibility requires $0 \leq f_1^{-1}(0) \leq p_{max}$ whereas dual feasibility implies $F^{-1}\left(\frac{b-c_r}{h+b}\right) < K + y - D(p^*)$.

R2: In this region $z = K$ due to Equation 5.37. Among 16 different possible solutions we find that only the following two are feasible:

1. $\lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, \lambda_6 = 0$,
2. $\lambda_3 = 0, \lambda_4 = 0, \lambda_5 < 0, \lambda_6 = 0$,

When $\underline{\lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, \lambda_6 = 0}$: KKT conditions can be written as follows:

$$\begin{aligned}
D_0 - 2\gamma p - K + c_r\gamma &= 0, \\
c_m - p + \lambda_2 &= 0, \\
-c_r + b - (h+b)F(v) &= 0,
\end{aligned}$$

These equations imply that $p^* = (p_{max}+c_r)/2 - K/(2\gamma)$, $z^* = K$, $v^* = F^{-1}\left(\frac{b-c_r}{h+b}\right)$, $\lambda_2 = (p_{max}+c_r)/2 - K/(2\gamma) - c_m$. Primal feasibility implies that $F^{-1}\left(\frac{b-c_r}{h+b}\right) \geq K + y - D((p_{max}+$

$c_r)/2)$ while dual feasibility forces $(p_{max} + c_r)/2 - K/(2\gamma) < c_m$. Note that the dual feasibility condition might hold if $K/D_0 > 1$ and $c_r < 2c_m$.

For $\underline{\lambda_3 = 0, \lambda_4 = 0, \lambda_5 < 0, \lambda_6 = 0}$:

$$\begin{aligned}
D_0 - 2\gamma p - K + c_r\gamma + \lambda_5\gamma &= 0, \\
c_m - p + \lambda_2 &= 0, \\
-c_r + b - (h+b)F(v) - \lambda_5 &= 0, \\
-v + K + y - D_0 + \gamma p &= 0.
\end{aligned}$$

These equations result with $p^* = f_4^{-1}(0)$, $z = K, v = K + y - D(p^*)$, $\lambda_2 = p - c_m$, $\lambda_5 = b - c_r - (h+b)F(v)$ where $f_4(p)$ is given in Equation 5.12. Primal feasibility requires $0 \leq f_4^{-1}(0) \leq p_{max}$, $K \leq (D_0\gamma p)\beta p$ whereas dual feasibility implies $f_4^{-1}(0) < c_m$, $F^{-1}\left(\frac{b-c_r}{h+b}\right) < K + y - D(p^*)$.

R3: In this region, the only feasible solution is $\underline{\lambda_3 = 0, \lambda_4 = 0, \lambda_5 < 0, \lambda_6 < 0}$. KKT conditions can be written as

$$\begin{aligned}
D_0 - 2\gamma p - z + c_r\gamma + \lambda_5\gamma + \lambda_6(-\gamma + 2\gamma\beta p) &= 0, \\
c_m - p + \lambda_6 &= 0, \\
-c_r + b - (h+b)F(v) - \lambda_5 &= 0, \\
-v + K + y - D_0 + \gamma p &= 0, \\
-(D_0 - \gamma p)\beta p + z &= 0.
\end{aligned}$$

These equations result with $p^* = f_3^{-1}(0)$, $z^* = (D_0 - \gamma p)\beta p$, $v^* = K + y - D(p^*)$, $\lambda_5 = b - c_r - (h+b)F(v)$, $\lambda_6 = p - c_m$, where $f_3(p)$ is given in Equation 5.11. Primal feasibility enforces $0 \leq f_3^{-1}(0) \leq p_{max}$, $0 \leq (D_0\gamma p)\beta p \leq K$ whereas dual feasibility requires $f_3^{-1}(0) < c_m$, $F^{-1}\left(\frac{b-c_r}{h+b}\right) < K + y - D(p^*)$.

Finally we will state that the solution $(p^*, z^*, v^*) = ((p_{max} + c_r)/2, 0, = F^{-1}\left(\frac{b-c_r}{h+b}\right))$ yield larger profit than teh solution $(p^*, z^*, v^*) = ((p_{max} + c_r)/2 - K/(2\gamma), K, F^{-1}\left(\frac{b-c_r}{h+b}\right))$. Also the primal feasbility conditions of the former implies the primal feasibility of the latter whereas the dual feability of the former always hold in our problem setting. Therefore the latter solution is eliminated which completes the proof. ■

Proof of Lemma 14: In order to prove the desired result we will use separability of the profit function $H_2(K, y, p, z, w)$ in p and (z, w). Thanks to this property and the strict concavity from Lemma 13, showing that $R_2(p)$ has a maximizer (using the first derivative of the function) in $[0, \Lambda)$ yield the desired result. To this end,

$$\frac{\partial R^2(p)}{\partial p} = (D_0 + \frac{2D_0 c_r}{p_{max}}) + 2p(-\frac{2D_0}{p_{max}} - \frac{D_0 c_r}{p_{max}^2}) + 3p^2 \frac{D_0}{p_{max}^2} = 0.$$

$\frac{\partial R^2(p)}{\partial p}$ is a quadratic function. To find its roots we will use the discriminant.

$$\Delta = 4\gamma^2(1 - \beta c_r)^2,$$

which yields the following roots: $p^{(1)} = p_{max}$ and $p^{(2)} = (p_{max} + 2c_r)/3$. The first root is larger than Λ whereas the second one is smaller than Λ. Therefore $R_2(p^{(2)}) > R_2(p^{(1)})$. ■

Proof of Lemma 15: For Statement a, the optimal value of p is given in the proof of Lemma 14. Since

$$\mathcal{G}_2(z, w) = z(c_r - c_m) + c_r w + h[w - \epsilon]^+ + b[\epsilon - w]^+ - c_r y - K(c_r - c_m),$$

is linear in z with a positive first derivative $z^* = 0$. The proof of $w^* F^{-1}\left(\frac{b-c_r}{h+b}\right)$ follows from the first partial derivative of $\mathcal{G}_2(z, w)$ w.r.t w. Statement b follows from the fact that $D(p^{(2)}) > 0$, where $p^{(2)} = (p_{max} + 2c_r)/3$. ■

Chapter 6

Summary and Conclusion

The economic lifetime of capital products is much longer than its production phase. Keeping capital products in operation requires a stable spare parts supply. When capital products are in-production it is easy to obtain spare parts, since Original Equipment Manufacturers (OEMs) can increase their order sizes to ship spare parts to their customers. After the end-of-production announcement of the OEM, obtaining spare parts becomes more difficult due to since suppliers' profitability and capacity utilization becomes an important factor for the flow of spare parts. As a consequence of this dependence this spare part flow is subject to increasing supply risk towards the end of the economic lifetime.

From the suppliers' perspective, producing spare parts of out-of-production systems may yield a steady revenue stream or it may be a source of efficiency loss due to an increasing number of production setups. To prevent excessive set-ups, suppliers might choose to maintain some inventory which then, however, would increases their holding cost and lower their financial performance. Naturally all of these decisions depend on the demand rate of spare parts. On the one hand, for capital products with large installed bases, keeping a spare part stock might be beneficial for the supplier. On the other hand, when the installed base of a capital product is small, holding the inventory increases the amount of non-moving parts, whereas producing them might cripple the average utilization of the supplier. Therefore, to consolidate current orders with future ones, suppliers choose to delay their production for incoming orders and eventually stop their spare part support. The theoretical explanation of this phenomenon is illustrated by a queuing system with two customer classes and a batch server in Chapter 3. From the OEMs' perspective, this situation implies an increasing supply risk for spare parts of aging capital products. End-of-support announcements from suppliers stand for their main source of supply risk.

Another factor that aggravates the supply risk is that spare parts suppliers may be too busy to notify OEMs of their end-of-support decisions. Dealing with large supplier bases, it is impossible for OEMs to check their suppliers continuously for an unannounced supply loss. Therefore, they can only discover a supplier's end-of-support decision when they need to buy a part. If there is no alternative supplier (and no inventory), OEMs fail to satisfy their customers' spare parts demand. Failing to satisfy the demand not only means a revenue loss, it also makes the reliability of OEMs' after-sale services questionable and might motivate asset owners to replace their capital products. Therefore, early detection, and mitigation of supply risk of spare parts is an important item in OEMs agendas. To overcome the supply risk, OEMs take proactive and reactive actions such as maintaining spare parts stock, cultivating a back-up supplier and utilization of secondary markets, if any. Since each action incurs a cost, it is important to evaluate inventory and replenishment policies of spare parts through mathematical models that explicitly take into account supply risk.

Among these means of supply risk mitigation, the utilization of secondary markets requires more attention then others. Next to being an alternative supply source, secondary markets might be a source of competition for OEMs' after-sales services. It is known that OEMs are subject to fierce competition with third party service providers for after-sale services of their capital products, especially after the end of their warranty period (Cohen et al., 2006). The existence of secondary markets make this competition even more fierce since different traders (or even customers) can trade their surplus spare parts. This situation forces OEMs to consider secondary markets in their tactical and operational decisions.

In this thesis, we consider four different spare parts management problems. Each is motivated by a business an OEM, Fokker Services, that provides maintenance services for out-of-production aircraft in the Netherlands. From a broad perspective, our study started with an empirical analysis on supply risk using purchase history and demand data from the company. After generating empirical evidence for supply risk and insight for its underlying reasons (Chapter 2), we develop mathematical models for optimal inventory control and replenishment policies in Chapters 3 and 4. In Chapter 5, we address pricing and replenishment policy for spare parts when secondary markets are used as a supply source by OEMs as well as customers.

An empirical analysis on supply risk for spare parts of aging capital products is presented in Chapter 2. At the beginning of this study, managers in the company had an incomplete understanding of supply risk for their spare parts. The company was experiencing supplier loss due to various reasons and each supply problem case triggered a solution procedure, depicted in Chapter 2, which could take a year. At the end of that procedure, the company would find a new supply source and restart its supply chain. Since this reactive approach threatened the service rate and profitability, it was necessary to develop indicators for future supply problems. The main motivation was having a risk measure which can trigger advance actions to mitigate the effects of supply disruptions on the company. To this end, we hypothesize relationships between disruptions and various supply chain features, e.g. lead time, price, order frequency, order size etc., and test them using purchase histories of spare parts whose suppliers failed before the date of analysis. Results indicate that supply risks are closely related with changes in lead times as well as the time period since the last purchase. This result not only led to another empirical study by Li et al. (2015), who considered a more advanced model for the same problem, but also it indicated the necessity of considering random lead times and supply disruptions in a single inventory control model, which is the focus of Chapter 3.

Random lead times coupled with supply disruptions are considered in Chapter 3. Our review of the queuing literature indicates that coupled lead times and supply disruptions are a natural consequence of suppliers' optimal manufacturing schedules. To address this problem, we develop a mathematical model considering Markov-modulated random lead times with supply disruptions. After proving structure of the optimal policy, we run extensive numerical experiments and evaluate different scenarios. The main output of this study is the importance of the coupled effect of random lead times and supply disruptions. Our analysis indicates that the coupled effect may give rise to extreme cost increases, especially for high service levels. Furthermore, we find that proactive actions using advance indicators are most important for mitigating the effect of supply risk.

Replenishment of spare parts from secondary markets and a regular supplier is considered in Chapter 4. Although secondary markets are useful for mitigating the supply risk, they have particular qualities which must be considered explicitly. First, secondary markets have limited and random availability of spare parts. In each decision epoch there are only a limited number of parts and the spare part availability varies over time. Second, secondary markets are cheaper and faster than regular suppliers in most cases. Third, spare parts on secondary markets may be in different conditions which brings *substitution* into the equation. We consider all of these factors in a dual sourcing setting (a regular supplier and secondary markets) and develop efficient heuristics using a myopic cost function. Later we extend our heuristic with nonstationary demand and evaluate both policies in extensive numerical experiments. Our results indicate that our policy is fast and efficient while producing near-optimal solutions when the lead time of the regular supplier is equal to one period. For larger lead times, our method deviates 17% from the optimal policy on average. Despite the significant deviation from the optimal cost, the best available alternative policy is only marginally better (about 1%), while our method is much more efficient in terms of computation time.

In Chapter 5, secondary markets are considered to be competitors for OEMs' after-sale services. Secondary markets are accessible for OEMs as well as customers. Due to competitive prices, some customers choose to buy spare parts from secondary markets. For OEMs this means demand and revenue loss. This implies the necessity of considering price competition from secondary markets in the replenishment and pricing policies. We consider a single-period model for this problem setting. Our analysis indicates that for the replenishment policy, secondary markets should be considered as the main source, whereas regular suppliers should be used as a complementary one. For pricing, a modified list price policy is optimal in which the sum of existing inventory and the market availability is the main factor for the optimal price value.

References

Abdolshah, M. 2013. A review of quality criteria supporting supplier selection. *Journal of Quality and Reliability Engineering* **2013**.

Agresti, Alan. 1996. *An introduction to categorical data analysis*, vol. 135. Wiley New York.

Allon, G., J.A. Van Mieghem. 2010. Global dual sourcing: Tailored base-surge allocation to near-and offshore production. *Management Science* **56**(1) 110–124.

Arifoğlu, K., S. Özekici. 2010. Optimal policies for inventory systems with finite capacity and partially observed markov-modulated demand and supply processes. *European Journal of Operational Research* **204**(3) 421–438.

Arifoğlu, K., S. Özekici. 2011. Inventory management with random supply and imperfect information: A hidden markov model. *International Journal of Production Economics* **134**(1) 123–137.

Arts, J., M. van Vuuren, G. Kiesmüller. 2011. Efficient optimization of the dual-index policy using markov chains. *Iie Transactions* **43**(8) 604–620.

Axsäter, S. 2003a. Evaluation of unidirectional lateral transshipments and substitutions in inventory systems. *European Journal of Operational Research* **149**(2) 438–447.

Axsäter, S. 2003b. A new decision rule for lateral transshipments in inventory systems. *Management Science* **49**(9) 1168–1179.

Axsäter, S. 2006. *Inventory Control*. 2nd ed. Springer.

Aydin, Goker, Evan L Porteus. 2008. Joint inventory and pricing decisions for an assortment. *Operations Research* **56**(5) 1247–1255.

Aytac, Berrin, S David Wu. 2013. Characterization of demand for short life-cycle technology products. *Annals of Operations Research* **203**(1) 255–277.

Babich, V., A.N. Burnetas, P.H. Ritchken. 2007. Competition and diversification effects in supply chains with supplier default risk. *Manufacturing & Service Operations Management* **9**(2) 123–146.

Barankin, E.W. 1961. A delivery-lag inventory model with an emergency provision (the single-period case). *Naval Research Logistics Quarterly* **8**(3) 285–311.

Bassok, Y., R. Anupindi, R. Akella. 1999. Single-period multiproduct inventory models with substitution. *Operations Research* **47**(4) 632–642.

Bazaraa, Mokhtar S, Chitharanjan M Shetty. 1979. *Nonlinear programming: theory and algorithms*. John Wiley & Sons.

Beyer, D., S.P. Sethi. 1997. Average cost optimality in inventory models with markovian demands. *Journal of Optimization Theory and Applications* **92**(3) 497–526.

Blackhurst, Jennifer V, Kevin P Scheibe, Danny J Johnson. 2008. Supplier risk assessment and monitoring for the automotive industry. *International Journal of Physical Distribution & Logistics Management* **38**(2) 143–165.

Bradley, J.R., L.W. Robinson. 2005. Improved base-stock approximations for independent stochastic lead times with order crossover. *Manufacturing & Service Operations Management* **7**(4) 319–329.

Bundschuh, Russell G, Theodore M Dezvane. 2003. How to make after-sales services pay off. *McKinsey Quarterly* **4** 116–127.

Cattani, Kyle D, Gilvan C Souza. 2003. Good buy? delaying end-of-life purchases. *European Journal of Operational Research* **146**(1) 216–228.

Ceryan, Oben, Özge Sahin, Izak Duenyas. 2013. Dynamic pricing of substitutable products in the presence of capacity flexibility. *Manufacturing & Service Operations Management* **15**(1) 86–101.

Chen, Xin, David Simchi-Levi. 2004. Coordinating inventory control and pricing strategies with random demand and fixed ordering cost: The finite horizon case. *Operations Research* **52**(6) 887–896.

Chen, Xin, David Simchi-Levi. 2010. Pricing and inventory management. Robert Philips, Özalp. Özer, eds., *Handbook of Pricing*. Oxford University Press, 784–822.

Chen, Youhua Frank, Saibal Ray, Yuyue Song. 2006. Optimal pricing and inventory control policy in periodic-review systems with fixed ordering cost and lost sales. *Naval Research Logistics (NRL)* **53**(2) 117–136.

Cheng, F., S.P. Sethi. 1999. Optimality of state-dependent (s, s) policies in inventory models with markov-modulated demand and lost sales. *Production and Operations Management* **8**(2) 183–192.

Chopra, S., G. Reinhardt, M. Dada. 2004. The effect of lead time uncertainty on safety stocks. *Decision Sciences* **35**(1) 1–24.

Chopra, Sunil, MS Sodhi. 2012. Managing risk to avoid supply-chain breakdown. *MIT Sloan Management Review (Fall 2004)* .

Cohen, Morris A, Narendra Agrawal, Vipul Agrawal. 2006. Winning in the aftermarket. *Harvard business review* **84**(5) 129.

Craighead, Christopher W, Jennifer Blackhurst, M Johnny Rungtusanatham, Robert B Handfield. 2007. The severity of supply chain disruptions: design characteristics and mitigation capabilities. *Decision Sciences* **38**(1) 131–156.

Daniel, K.H. 1963. A delivery-lag inventory model with emergency. *Multistage Inventory Models and Techniques* 32–46.

de Oliveira, Marcos Paulo Valadares, Kevin McCormack, Peter Trkman. 2012. Business analytics in supply chains–the contingent effect of business process maturity. *Expert Systems with Applications* **39**(5) 5488–5498.

Dipaola, A., Y. Okada. 2013. Saudi aramco plans massive spending to extend field life. http://www.bloomberg.com/news/articles/2013-10-14/saudi-aramco-seeks-to-extend-oilfield-life-with-expansion-plans.

Duenyas, Izak, John J Neale. 1997. Stochastic scheduling of a batch processing machine with incompatible job families. *Annals of Operations Research* **70** 191–220.

Ehrhardt, R. 1984. (s,S) policies for a dynamic inventory model with stochastic lead times. *Operations Research* **32**(1) 121–132.

Ellis, Scott C, Raymond M Henry, Jeff Shockley. 2010. Buyer perceptions of supply disruption risk: a behavioral view and empirical assessment. *Journal of Operations Management* **28**(1) 34–46.

Elmaghraby, Wedad, Pınar Keskinocak. 2003. Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science* **49**(10) 1287–1309.

Federgruen, Awi, Aliza Heching. 1999. Combined pricing and inventory control under uncertainty. *Operations research* **47**(3) 454–475.

Feldman, Kiri, Peter Sandborn. 2007. Integrating technology obsolescence considerations into product design planning. *ASME 2007 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. American Society of Mechanical Engineers, 981–988.

Feng, Q., S.P. Sethi, H. Yan, H. Zhang. 2006. Are base-stock policies optimal in inventory problems with multiple delivery modes? *Operations Research* **54**(4) 801–807.

Fisher, Warren W. 1990. Markov process modelling of a maintenance system with spares, repair, cannibalization and manpower constraints. *Mathematical and Computer Modelling* **13**(7) 119–125.

Fukuda, Y. 1964. Optimal policies for the inventory problem with negotiable leadtime. *Management Science* **10**(4) 690–708.

Gallagher, Martin P, Patrick J Kelly, Meg Jardine, Vlado Perkovic, Alan Cass, Jonathan C Craig, Josette Eris, Angela C Webster. 2010. Long-term cancer risk of immunosuppressive regimens after kidney transplantation. *Journal of the American Society of Nephrology* **21**(5) 852–858.

Gallego, G., H. Hu. 2004. Optimal policies for production/inventory systems with finite capacity and markov-modulated demand and supply processes. *Annals of Operations Research* **126**(1) 21–41.

Gallego, Guillermo, Garrett Van Ryzin. 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Management science* **40**(8) 999–1020.

Ghobbar, A.A., C.H. Friend. 2002. Sources of intermittent demand for aircraft spare parts within airline operations. *Journal of Air Transport Management* **8**(4) 221–231.

Gong, Xiting, Xiuli Chao, Shaohui Zheng. 2014. Dynamic pricing and inventory management with dual suppliers of different lead times and disruption risks. *Production and Operations Management* **23**(12) 2058–2074.

Gravier, Michael J, Stephen M Swartz. 2009. The dark side of innovation: Exploring obsolescence and supply chain evolution for sustainment-dominated systems. *The Journal of High Technology Management Research* **20**(2) 87–102.

Harrison, J.M., J.A. Van Mieghem. 1999. Multi-resource investment strategies: Operational hedging under demand uncertainty. *European Journal of Operational Research* **113**(1) 17–29.

Hayya, J.C., U. Bagchi, J.G. Kim, D. Sun. 2008. On static stochastic order crossover. *International Journal of Production Economics* **114**(1) 404–413.

Hendricks, Kevin B, Vinod R Singhal. 2003. The effect of supply chain glitches on shareholder wealth. *Journal of Operations Management* **21**(5) 501–522.

Hendricks, Kevin B, Vinod R Singhal. 2005a. Association between supply chain glitches and operating performance. *Management science* **51**(5) 695–711.

Hendricks, Kevin B, Vinod R Singhal. 2005b. An empirical analysis of the effect of supply chain disruptions on long-run stock price performance and equity risk of the firm. *Production and Operations management* **14**(1) 35–52.

Hopp, W.J., X. Xu. 2008. A static approximation for dynamic demand substitution with applications in a competitive market. *Operations Research* **56**(3) 630–645.

Jin, Ruichen, Wei Chen, Timothy W Simpson. 2001. Comparative studies of metamodelling techniques under multiple modelling criteria. *Structural and Multidisciplinary Optimization* **23**(1) 1–13.

Ju, Wanrong, Adriana F Gabor, JCW Van Ommeren. 2015. An approximate policy for a dual-sourcing inventory model with positive lead times and binomial yield. *European Journal of Operational Research* **244**(2) 490–497.

Kaplan, R.S. 1970. A dynamic inventory model with stochastic lead times. *Management Science* **16**(7) 491–507.

Karlin, Samuel, Charles R Carr. 1962. Prices and optimal inventory policy. *Studies in applied probability and management science* 159–172.

Keizers, Joris M, J Bertrand, Jaap Wessels. 2003. Diagnosing order planning performance at a navy maintenance and repair organization, using logistic regression. *Production and Operations Management* **12**(4) 445–463.

Kleindorfer, P.R., G.H. Saad. 2005. Managing disruption risks in supply chains. *Production and operations management* **14**(1) 53–68.

Kök, A. G., M.L. Fisher, R. Vaidyanathan. 2009. Assortment planning: Review of literature and industry practice. *Retail Supply Chain Management*. Springer, 99–153.

Lapré, Michael A, Gary D Scudder. 2004. Performance improvement paths in the us airline industry: linking trade-offs to asset frontiers. *Production and Operations Management* **13**(2) 123–134.

Lawson, D.G., E.L. Porteus. 2000. Multistage inventory management with expediting. *Operations Research* **48**(6) 878–893.

Levitt, Theodore. 1965. Exploit the product life cycle. *HARVARD BUSINESS REVIEW* .

Li, Lode. 1988. A stochastic theory of the firm. *Mathematics of operations research* **13**(3) 447–466.

Li, X., C. Heij, M. Hekimoğlu, R. Dekker. 2015. Predicting end-of-supply risk of spare parts using the proportional hazard model. *Decision Sciences* **Accepted**.

Li, Z., S.H. Xu, J. Hayya. 2004. A periodic-review inventory system with supply interruptions. *Probability in the Engineering and Informational Sciences* **18**(1) 33–53.

Maritz, Johannes S. 1995. *Distribution-free statistical methods*, vol. 17. CRC Press.

Meixell, Mary J, S David Wu. 2001. Scenario analysis of demand in a technology market using leading indicators. *Semiconductor Manufacturing, IEEE Transactions on* **14**(1) 65–75.

Minner, S. 2003. Multiple-supplier inventory models in supply chain management: A review. *International Journal of Production Economics* **81** 265–279.

Mohebbi, E. 2003. Supply interruptions in a lost-sales inventory system with random lead time. *Computers & Operations Research* **30**(3) 411–426.

Moinzadeh, K., C.P. Schmidt. 1991. An (s- 1, s) inventory system with emergency orders. *Operations Research* **39**(2) 308–321.

Morton, T.E. 1971. The near-myopic nature of the lagged-proportional-cost inventory problem with lost sales. *Operations Research* **19**(7) 1708–1716.

Muharremoglu, A., J.N. Tsitsiklis. 2008. A single-unit decomposition approach to multi-echelon inventory systems. *Operations Research* **56**(5) 1089–1103.

Muharremoglu, A., N. Yang. 2010. Inventory management with an exogenous supply process. *Operations research* **58**(1) 111–129.

Nagarajan, M., S. Rajagopalan. 2008. Inventory models for substitutable products: optimal policies and heuristics. *Management Science* **54**(8) 1453–1466.

Nahmias, S. 1979. Simple approximations for a variety of dynamic leadtime lost-sales inventory models. *Operations Research* **27**(5) 904–924.

Oliva, R., R. Kallenberg. 2003. Managing the transition from products to services. *International Journal of Service Industry Management* **14**(2) 160–172.

Özekici, S., M. Parlar. 1999. Inventory models with unreliable suppliers in a random environment. *Annals of Operations Research* **91** 123–136.

Özekici, Süleyman. 2013. *Reliability and maintenance of complex systems*, vol. 154. Springer Science & Business Media.

Parlar, M., Y. Wang, Y. Gerchak. 1995. A periodic review inventory model with markovian supply availability. *International Journal of Production Economics* **42**(2) 131–136.

Paterson, C., G. Kiesmüller, R. Teunter, K. Glazebrook. 2011. Inventory models with lateral transshipments: A review. *European Journal of Operational Research* **210**(2) 125–136.

Polatoğlu, Hakan, Izzet Sahin. 2000. Optimal procurement policies under price-dependent demand. *International Journal of Production Economics* **65**(2) 141–171.

Polatoğlu, L Hakan. 1991. Optimal order quantity and pricing decisions in single-period inventory systems. *International Journal of Production Economics* **23**(1) 175–185.

Porteus, Evan L. 2002. *Foundations of stochastic inventory theory*. Stanford University Press.

Rajan, Arvind, Rakesh, Richard Steinberg. 1992. Dynamic pricing and ordering decisions by a monopolist. *Management Science* **38**(2) 240–262.

Rao, Uday S, Jayashankar M Swaminathan, Jun Zhang. 2004. Multi-product inventory planning with downward substitution, stochastic demand and setup costs. *IIE Transactions* **36**(1) 59–71.

Robinson, L.W., J.R. Bradley, L.J. Thomas. 2001. Consequences of order crossover under order-up-to inventory policies. *Manufacturing & Service Operations Management* **3**(3) 175–188.

Robotis, A., S. Bhattacharya, L.N. Van Wassenhove. 2005. The effect of remanufacturing on procurement decisions for resellers in secondary markets. *European Journal of Operational Research* **163**(3) 688–705.

Rojo, F.J., R.S.E. Roy. 2010. Obsolescence management for long-life contracts: state of the art and future trends. *The International Journal of Advanced Manufacturing Technology* **49**(9-12) 1235–1250.

Sandborn, Peter, Frank Mauro, Ron Knox, et al. 2007. A data mining based approach to electronic part obsolescence forecasting. *Components and Packaging Technologies, IEEE Transactions on* **30**(3) 397–401.

Scarf, H. 1960. The optimality of (S,s) policies in the dynamic inventory problem. K. Arrow, S. Karlin, H. Scarf, eds., *Mathematical Methods in the Social Sciences*. Stanford University Press, Stanfor, 196–202.

Scheller-Wolf, A., S. Tayur. 1999. Managing supply chains in emerging markets. R. Ganeshan S. Tayur, M. Magazine, eds., *Quantitative Models for Supply Chain Management*. Springer, 703–736.

Scheller-Wolf, A., S. Veeraraghavan, G. van Houtum. 2007. Effective dual sourcing with a single index policy. *Working paper, Carnegie Mellon University, Pittsburgh* .

Serel, Dogan A, Maqbool Dada, Herbert Moskowitz. 2001. Sourcing decisions with capacity reservation contracts. *European Journal of Operational Research* **131**(3) 635–648.

Shaked, M., J.G. Shanthikumar. 2007. *Stochastic orders and their applications*. Springer.

Sheopuri, A., G. Janakiraman, S. Seshadri. 2010. New policies for the stochastic inventory control problem with two supply sources. *Operations Research* **58**(3) 734–745.

Singh, Pameet, Peter Sandborn. 2006. Obsolescence driven design refresh planning for sustainment-dominated systems. *The Engineering Economist* **51**(2) 115–139.

Singh, Pameet, Peter Sandborn, Todd Geiser, David Lorenson. 2004. Electronic part obsolescence driven product redesign planning. *International Journal of Agile Manufacturing* **7**(1) 23–32.

Solomon, Rajeev, Peter Sandborn, Michael G Pecht, et al. 2000. Electronic part life cycle concepts and obsolescence forecasting. *Components and Packaging Technologies, IEEE Transactions on* **23**(4) 707–717.

Song, J.S. 1994a. The effect of leadtime uncertainty in a simple stochastic inventory model. *Management Science* **40**(5) 603–613.

Song, J.S. 1994b. Understanding the lead-time effects in stochastic inventory systems with discounted costs. *Operations Research Letters* **15**(2) 85–93.

Song, J.S., P.H. Zipkin. 1993. Inventory control in a fluctuating demand environment. *Operations Research* **41**(2) 351–370.

Song, J.S., P.H. Zipkin. 1996. Inventory control with information about supply conditions. *Management Science* **42**(10) 1409–1419.

Song, J.S., P.H. Zipkin. 2009. Inventories with multiple supply sources and networks of queues with overflow bypasses. *Management Science* **55**(3) 362–372.

Syntetos, A.A., M.Z. Babai, N. Altay. 2012. On the demand distributions of spare parts. *International Journal of Production Research* **50**(8) 2101–2117.

Thowsen, Gunnar T. 1975. A dynamic, nonstationary inventory problem for a price/quantity setting firm. *Naval Research Logistics Quarterly* **22**(3) 461–476.

Tomlin, B. 2006. On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science* **52**(5) 639–657.

Tomlin, B., L.V. Snyder. 2006. On the value of a threat advisory system for managing supply chain disruptions. *Working Paper, Kenan-Flagler Business School, University of North Carolina-Chapel Hill, USA* .

Trimp, M.E., S.M. Sinnema, R. Dekker, R.H. Teunter. 2004. Optimise initial spare parts inventories: an analysis and improvement of an electronic decision tool. Tech. rep., Econometric Institute Research Papers.

Trkman, Peter. 2010. The critical success factors of business process management. *International Journal of Information Management* **30**(2) 125–134.

Trkman, Peter, Kevin McCormack, Marcos Paulo Valadares De Oliveira, Marcelo Bronzo Ladeira. 2010. The impact of business analytics on supply chain performance. *Decision Support Systems* **49**(3) 318–327.

Tsay, Andy A. 1999. The quantity flexibility contract and supplier-customer incentives. *Management science* **45**(10) 1339–1358.

Van Mieghem, J.A. 2004. Note-commonality strategies: Value drivers and equivalence with flexible capacity and inventory substitution. *Management Science* **50**(3) 419–424.

Van Mieghem, J.A., N. Rudi. 2002. Newsvendor networks: Inventory management and capacity investment with discretionary activities. *Manufacturing & Service Operations Management* **4**(4) 313–335.

Veeraraghavan, S., A. Scheller-Wolf. 2008. Now or later: A simple policy for effective dual sourcing in capacitated systems. *Operations Research* **56**(4) 850–864.

Veinott, A.F. 1965. Optimal policy for a multi-product, dynamic, nonstationary inventory problem. *Management Science* **12**(3) 206–222.

Wagner, S.M., R. Jönke, A.B. Eisingerich. 2012. Strategic framework for spare parts logistics. *California Management Review* **54**(4) 69–92.

Wagner, Stephan M, Christoph Bode. 2006. An empirical investigation into supply chain vulnerability. *Journal of purchasing and supply management* **12**(6) 301–312.

Whittemore, A.S., S.C. Saunders. 1977. Optimal inventory under stochastic demand with two supply options. *SIAM Journal on Applied Mathematics* **32**(2) 293–305.

Wilhelm, S. 2014. 500 suppliers now partnering with boeing to cut costs, execs say. http://www.bizjournals.com/seattle/blog/2014/05/500-suppliers-now-partnering-with-boeing-to-cut.html?page=all.

Wise, Richard, Peter Baumgartner. 1999. Go downstream: the new profit imperative in manufacturing. *Harvard business review* **77**(5) 133–141.

Wong, H., D. van Oudheusden, D. Cattrysse. 2007. Cost allocation in spare parts inventory pooling. *Transportation Research Part E: Logistics and Transportation Review* **43**(4) 370–386.

Wu, S David, Berrin Aytac, Rosemary T Berger, Chris A Armbruster. 2006. Managing short life-cycle technology products for agere systems. *Interfaces* **36**(3) 234–247.

Zabel, Edward. 1970. Monopoly and uncertainty. *The Review of Economic Studies* 205–219.

Zabel, Edward. 1972. Multiperiod monopoly under uncertainty. *Journal of Economic Theory* **5**(3) 524–536.

Zhang, V.L. 1996. Ordering policies for an inventory system with three supply modes. *Naval Research Logistics (NRL)* **43**(5) 691–708.

Zipkin, P.H. 1986. Stochastic leadtimes in continuous-time inventory models. *Naval Research Logistics Quarterly* **33**(4) 763–774.

Zipkin, P.H. 2000. *Foundations of inventory management*. McGraw-Hill New York.

Zsidisin, George A, Lisa M Ellram, Joseph R Carter, Joseph L Cavinato. 2004. An analysis of supply risk assessment techniques. *International Journal of Physical Distribution & Logistics Management* **34**(5) 397–413.

Nederlandse Samenvatting

In dit onderzoek analyseren we productieketens van zogeheten Original Equipment Manufacturers (OEMs) met betrekking tot reservedelen. Specifiek kijken we naar OEMs die klantenservice bieden voor kapitaalgoederen die uit productie zijn genomen. Deze productieketens zijn onderhevig aan stijgende leveringsrisico's, welke veroorzaakt worden door afnemend gebruik van corresponderende kapitaalgoederen en dalende vraag naar reservedelen.

Om deze effecten te verminderen ontwikkelen we eerst een empirisch model dat leveringsproblemen kan voorspellen. Oplossingen voor leveringsproblemen kunnen langdurige processen vereisen. Zo moeten bijvoorbeeld reservedelen worden herontworpen, of samenwerkingen met nieuwe leveranciers worden aangegaan. Vroegtijdige detectie van leveringsproblemen stelt OEMs in staat tot proactieve maatregelen en kostenbesparingen. Een uitbreiding van deze studie is toegepast bij een OEM die onderhoudsdiensten biedt voor vliegtuigen uit productie.

Een van de belangrijkste inzichten uit de empirische studie is de het belang van veranderingen in levertijden voordat verstoringen in de productieketen ontstaan. We presenteren een mathematisch model waarin een Markov model de levertijden en leveringsrisico's bepaalt en bijbehorende fundamentele eigenschappen en de optimaliteit van toestandsafhankelijke base stock policy om dit inzicht te verwerken in voorraadbeheerstrategieën voor reservedelen. Onze analyse laat zien dat het gecombineerde effect van leveringsproblemen en onzekere levertijden even groot kan zijn als som van de individuele effecten. Bovendien kan niet-stationariteit in deze risico's een kostenverhoging met zich meebrengen die groter is dan de som van de individuele effecten en hun gezamenlijke effect. Dit geldt voornamelijk wanneer de OEM streeft naar hoge servicedoelstellingen.

Ook bekijken we de effectsen van de secundaire markt op de reservedelen productieketen. Op deze markten kunnen OEMs reservelen kopen die variëren in conditie, zoals functionerend of repareerbaar. Reservedelen van secundaire markten zijn doorgaans van lagere kwaliteit, terwijl gloednieuwe reservedelen beschouwd kunnen worden als hoog kwalitatief. In dit proefschrift presenteren we een methode voor het oplossen van een

voorraadbeheerprobleem met twee leveranciers die verschillen in termen van de kwaliteit van de aangeboden reservedelen. Door dit kwaliteitsverschil expliciet te modelleren kunnen we betere dual sourcing strategien gebruiken dan wat in de literatuur staat. Ook breiden we deze methode uit om rekening te kunnen houden met niet-stationaire vraag.

Een andere eigenschap van secundaire markten is dat ze concurreren met OEMs op het gebied van klantenservice na verkoop. Goedkopere onderdelenprijzen op de secundaire markten kunnen klanten aantrekken, waardoor de vraag naar onderhoudsdiensten van OEMs afneemt. In deze dissertatie beschouwen we de prijscompetitie tussen een OEM en de secundaire markten. Het innovatieve van deze studie is de interactie tussen de voorraadstrategie en het prijsstrategie van de OEM. Wij tonen aan dat deze interactie belangrijk is, omdat beide strategieën een effect hebben op de secundaire markt. Onze analyse laat zien dat secundaire markten als voornaamste toeleverancier door OEMs gebruikt zouden moeten worden om hun winst te maximaliseren. Ook vinden we een voorraad- en prijsstrategie die resulteert in hoge maar onzekere verwachte winst.

About the Author

Mustafa Hekimoğlu holds a BSc. degree in industrial engineering from Dokuz Eylul University, İzmir and he received a MSc. degree in the same field from and Boğazici University, Istanbul. Later, he graduated from ERIM Master of Philosophy in Business Research and started his PhD in 2011. His fields of interest are supply chain management, inventory control and system dynamics modelling. He presented his scientific output in international conferences such as POMS, INFORMS, ISIR, MSOM. He contributed research papers published in *Decision Sciences* and *Energy Policy*. After the graduation, he will start working as an assistant professor at Işık University, Istanbul.

ERASMUS RESEARCH INSTITUTE OF MANAGEMENT (ERIM)

ERIM PH.D. SERIES RESEARCH IN MANAGEMENT

The ERIM PhD Series contains PhD dissertations in the field of Research in Management defended at Erasmus University Rotterdam and supervised by senior researchers affiliated to the Erasmus Research Institute of Management (ERIM). All dissertations in the ERIM PhD Series are available in full text through the ERIM Electronic Series Portal: http://repub.eur.nl/pub. ERIM is the joint research institute of the Rotterdam School of Management (RSM) and the Erasmus School of Economics at the Erasmus University Rotterdam (EUR).

DISSERTATIONS LAST FIVE YEARS

Abbink, E.J., *Crew Management in Passenger Rail Transport*, Promotor(s): Prof.dr. L.G. Kroon & Prof.dr. A.P.M. Wagelmans, EPS-2014-325-LIS, http://repub.eur.nl/pub/76927

Acar, O.A., *Crowdsourcing for Innovation: Unpacking Motivational, Knowledge and Relational Mechanisms of Innovative Behavior in Crowdsourcing Platforms*, Promotor(s): Prof.dr.ir. J.C.M. van den Ende, EPS-2014-321-LIS, http://repub.eur.nl/pub/76076

Acciaro, M., *Bundling Strategies in Global Supply Chains*, Promotor(s): Prof.dr. H.E. Haralambides, EPS-2010-197-LIS, http://repub.eur.nl/pub/19742

Akin Ates, M., *Purchasing and Supply Management at the Purchase Category Level: strategy, structure and performance*, Promotor(s): Prof.dr. J.Y.F. Wynstra & Dr. E.M. van Raaij, EPS-2014-300-LIS, http://repub.eur.nl/pub/50283

Akpinar, E., *Consumer Information Sharing*, Promotor(s): Prof.dr.ir. A. Smidts, EPS-2013-297-MKT, http://repub.eur.nl/pub/50140

Alexander, L., *People, Politics, and Innovation: A Process Perspective*, Promotor(s): Prof.dr. H.G. Barkema & Prof.dr. D.L. van Knippenberg, EPS-2014-331-S&E, http://repub.eur.nl/pub/77209

Alexiev, A.S., *Exploratory Innovation: The Role of Organizational and Top Management Team Social Capital*, Promotor(s): Prof.dr.ing. F.A.J. van den Bosch & Prof.dr. H.W. Volberda, EPS-2010-208-STR, http://repub.eur.nl/pub/20632

Almeida e Santos Nogueira, R.J. de, *Conditional Density Models Integrating Fuzzy and Probabilistic Representations of Uncertainty*, Promotor(s): Prof.dr.ir. U. Kaymak & Prof.dr. J.M.C. Sousa, EPS-2014-310-LIS, http://repub.eur.nl/pub/51560

Bannouh, K., *Measuring and Forecasting Financial Market Volatility using High-frequency Data*, Promotor(s): Prof.dr. D.J.C. van Dijk, EPS-2013-273-F&A, http://repub.eur.nl/pub/38240

Ben-Menahem, S.M., *Strategic Timing and Proactiveness of Organizations*, Promotor(s): Prof.dr. H.W. Volberda & Prof.dr.ing. F.A.J. van den Bosch, EPS-2013-278-S&E, http://repub.eur.nl/pub/39128

Benning, T.M., *A Consumer Perspective on Flexibility in Health Care: Priority Access Pricing and Customized Care*, Promotor(s): Prof.dr.ir. B.G.C. Dellaert, EPS-2011-241-MKT, http://repub.eur.nl/pub/23670

Berg, W.E. van den, *Understanding Salesforce Behavior using Genetic Association Studies*, Promotor(s): Prof.dr. W.J.M.I. Verbeke, EPS-2014-311-MKT, http://repub.eur.nl/pub/51440

Betancourt, N.E., *Typical Atypicality: Formal and Informal Institutional Conformity, Deviance, and Dynamics*, Promotor(s): Prof.dr. B. Krug, EPS-2012-262-ORG, http://repub.eur.nl/pub/32345

Bezemer, P.J., *Diffusion of Corporate Governance Beliefs: Board independence and the emergence of a shareholder value orientation in the Netherlands*, Promotor(s): Prof.dr.ing. F.A.J. van den Bosch & Prof.dr. H.W. Volberda, EPS-2010-192-STR, http://repub.eur.nl/pub/18458

Binken, J.L.G., *System markets: Indirect network effects in action, or inaction?*, Promotor(s): Prof.dr. S. Stremersch, EPS-2010-213-MKT, http://repub.eur.nl/pub/21186

Bliek, R. de, *Empirical Studies on the Economic Impact of Trust*, Promotor(s): Prof.dr. J. Veenman & Prof.dr. Ph.H.B.F. Franses, EPS-2015-324-ORG, http://repub.eur.nl/pub/78159

Blitz, D.C., *Benchmarking Benchmarks*, Promotor(s): Prof.dr. A.G.Z. Kemna & Prof.dr. W.F.C. Verschoor, EPS-2011-225-F&A, http://repub.eur.nl/pub/22624

Boons, M., *Working Together Alone in the Online Crowd: The Effects of Social Motivationsand Individual Knowledge Backgrounds on the Participation and Performance of Members of Online Crowdsourcing Platforms*, Promotor(s): Prof.dr. H.G. Barkema & Dr. D.A. Stam, EPS-2014-306-S&E, http://repub.eur.nl/pub/50711

Borst, W.A.M., *Understanding Crowdsourcing: Effects of motivation and rewards on participation and performance in voluntary online activities*, Promotor(s): Prof.dr.ir. J.C.M. van den Ende & Prof.dr.ir. H.W.G.M. van Heck, EPS-2010-221-LIS, http://repub.eur.nl/pub/21914

Brazys, J., *Aggregated Marcoeconomic News and Price Discovery*, Promotor(s): Prof.dr. W.F.C. Verschoor, EPS-2015-351-F&A, http://repub.eur.nl/pub/78243

Budiono, D.P., *The Analysis of Mutual Fund Performance: Evidence from U.S. Equity Mutual Funds*, Promotor(s): Prof.dr. M.J.C.M. Verbeek & Dr.ir. M.P.E. Martens, EPS-2010-185-F&A, http://repub.eur.nl/pub/18126

Burger, M.J., *Structure and Cooptition in Urban Networks*, Promotor(s): Prof.dr. G.A. van der Knaap & Prof.dr. H.R. Commandeur, EPS-2011-243-ORG, http://repub.eur.nl/pub/26178

Byington, E., *Exploring Coworker Relationships: Antecedents and Dimensions of Interpersonal Fit,Coworker Satisfaction, and Relational Models*, Promotor(s): Prof.dr. D.L. van Knippenberg, EPS-2013-292-ORG, http://repub.eur.nl/pub/41508

Camacho, N.M., *Health and Marketing: Essays on Physician and Patient Decision-Making*, Promotor(s): Prof.dr. S. Stremersch, EPS-2011-237-MKT, http://repub.eur.nl/pub/23604

Cancurtaran, P., *Essays on Accelerated Product Development*, Promotor(s): Prof.dr. F. Langerak & Prof.dr.ir. G.H. van Bruggen, EPS-2014-317-MKT, http://repub.eur.nl/pub/76074

Caron, E.A.M., *Explanation of Exceptional Values in Multi-dimensional Business Databases*, Promotor(s): Prof.dr.ir. H.A.M. Daniels & Prof.dr. G.W.J. Hendrikse, EPS-2013-296-LIS, http://repub.eur.nl/pub/50005

Carvalho, L. de, *Knowledge Locations in Cities: Emergence and Development Dynamics*, Promotor(s): Prof.dr. L. Berg, EPS-2013-274-S&E, http://repub.eur.nl/pub/38449

Carvalho de Mesquita Ferreira, L., *Attention Mosaics: Studies of Organizational Attention*, Promotor(s): Prof.dr. P.P.M.A.R. Heugens & Prof.dr. J. van Oosterhout, EPS-2010-205-ORG, http://repub.eur.nl/pub/19882

Cox, R.H.G.M., *To Own, To Finance, and To Insure - Residential Real Estate Revealed*, Promotor(s): Prof.dr. D. Brounen, EPS-2013-290-F&A, http://repub.eur.nl/pub/40964

Defilippi Angeldonis, E.F., *Access Regulation for Naturally Monopolistic Port Terminals: Lessons from Regulated Network Industries*, Promotor(s): Prof.dr. H.E. Haralambides, EPS-2010-204-LIS, http://repub.eur.nl/pub/19881

Deichmann, D., *Idea Management: Perspectives from Leadership, Learning, and Network Theory*, Promotor(s): Prof.dr.ir. J.C.M. van den Ende, EPS-2012-255-ORG, http://repub.eur.nl/pub/31174

Deng, W., *Social Capital and Diversification of Cooperatives*, Promotor(s): Prof.dr. G.W.J. Hendrikse, EPS-2015-341-ORG, http://repub.eur.nl/pub/77449

Desmet, P.T.M., *In Money we Trust? Trust Repair and the Psychology of Financial*

Compensations, Promotor(s): Prof.dr. D. de Cremer, EPS-2011-232-ORG, http://repub.eur.nl/pub/23268

Dietvorst, R.C., *Neural Mechanisms Underlying Social Intelligence and Their Relationship with the Performance of Sales Managers*, Promotor(s): Prof.dr. W.J.M.I. Verbeke, EPS-2010-215-MKT, http://repub.eur.nl/pub/21188

Dollevoet, T.A.B., *Delay Management and Dispatching in Railways*, Promotor(s): Prof.dr. A.P.M. Wagelmans, EPS-2013-272-LIS, http://repub.eur.nl/pub/38241

Doorn, S. van, *Managing Entrepreneurial Orientation*, Promotor(s): Prof.dr. J.J.P. Jansen, Prof.dr.ing. F.A.J. van den Bosch, & Prof.dr. H.W. Volberda, EPS-2012-258-STR, http://repub.eur.nl/pub/32166

Douwens-Zonneveld, M.G., *Animal Spirits and Extreme Confidence: No Guts, No Glory?*, Promotor(s): Prof.dr. W.F.C. Verschoor, EPS-2012-257-F&A, http://repub.eur.nl/pub/31914

Duca, E., *The Impact of Investor Demand on Security Offerings*, Promotor(s): Prof.dr. A. de Jong, EPS-2011-240-F&A, http://repub.eur.nl/pub/26041

Duursema, H., *Strategic Leadership: Moving Beyond the Leader-Follower Dyad*, Promotor(s): Prof.dr. R.J.M. van Tulder, EPS-2013-279-ORG, http://repub.eur.nl/pub/39129

Eck, N.J. van, *Methodological Advances in Bibliometric Mapping of Science*, Promotor(s): Prof.dr.ir. R. Dekker, EPS-2011-247-LIS, http://repub.eur.nl/pub/26509

Ellen, S. ter, *Measurement, Dynamics, and Implications of Heterogeneous Beliefs in Financial Markets*, Promotor(s): Prof.dr. W.F.C. Verschoor, EPS-2015-343-F&A, http://repub.eur.nl/pub/78191

Eskenazi, P.I., *The Accountable Animal*, Promotor(s): Prof.dr. F.G.H. Hartmann, EPS-2015-355-F&A, http://repub.eur.nl/pub/78300

Essen, M. van, *An Institution-Based View of Ownership*, Promotor(s): Prof.dr. J. van Oosterhout & Prof.dr. G.M.H. Mertens, EPS-2011-226-ORG, http://repub.eur.nl/pub/22643

Evangelidis, I., *Preference Construction under Prominence*, Promotor(s): Prof.dr. S.M.J. van Osselaer, EPS-2015-340-MKT, http://repub.eur.nl/pub/78202

Feng, L., *Motivation, Coordination and Cognition in Cooperatives*, Promotor(s): Prof.dr. G.W.J. Hendrikse, EPS-2010-220-ORG, http://repub.eur.nl/pub/21680

Faber, N., *Structuring Warehouse Management*, Promotor(s): Prof.dr. MB.M. de Koster, Prof.dr. Ale Smidts, EPS-2015-336-LIS, http://repub.eur.nl/pub/78603

Fourne, S.P., *Managing Organizational Tensions: A Multi-Level Perspective on Exploration, Exploitation and Ambidexterity*, Promotor(s): Prof.dr. J.J.P. Jansen & Prof.dr. S.J. Magala, EPS-2014-318-S&E, http://repub.eur.nl/pub/76075

Gharehgozli, A.H., *Developing New Methods for Efficient Container Stacking Operations*, Promotor(s): Prof.dr.ir. M.B.M. de Koster, EPS-2012-269-LIS, http://repub.eur.nl/pub/37779

Gils, S. van, *Morality in Interactions: On the Display of Moral Behavior by Leaders and Employees*, Promotor(s): Prof.dr. D.L. van Knippenberg, EPS-2012-270-ORG, http://repub.eur.nl/pub/38027

Ginkel-Bieshaar, M.N.G. van, *The Impact of Abstract versus Concrete Product Communications on Consumer Decision-making Processes*, Promotor(s): Prof.dr.ir. B.G.C. Dellaert, EPS-2012-256-MKT, http://repub.eur.nl/pub/31913

Gkougkousi, X., *Empirical Studies in Financial Accounting*, Promotor(s): Prof.dr. G.M.H. Mertens & Prof.dr. E. Peek, EPS-2012-264-F&A, http://repub.eur.nl/pub/37170

Glorie, K.M., *Clearing Barter Exchange Markets: Kidney Exchange and Beyond*, Promotor(s): Prof.dr. A.P.M. Wagelmans & Prof.dr. J.J. van de Klundert, EPS-2014-329-LIS, http://repub.eur.nl/pub/77183

Hakimi, N.A., *Leader Empowering Behaviour: The Leader's Perspective*, Promotor(s): Prof.dr. D.L. van Knippenberg, EPS-2010-184-ORG, http://repub.eur.nl/pub/17701

Hensmans, M., *A Republican Settlement Theory of the Firm: Applied to Retail Banks in England and the Netherlands (1830-2007)*, Promotor(s): Prof.dr. A. Jolink & Prof.dr. S.J. Magala, EPS-2010-193-ORG, http://repub.eur.nl/pub/19494

Hernandez-Mireles, C., *Marketing Modeling for New Products*, Promotor(s): Prof.dr. Ph.H.B.F. Franses, EPS-2010-202-MKT, http://repub.eur.nl/pub/19878

Heij, C.V., Innovating beyond Technology. Studies on how management innovation, co-creation and business model innovation contribute to firm's (innovation) performance, Promotor(s): Prof.dr.ing. F.A.J. van den Bosch & Prof.dr. H.W. Volberda, EPS-2012-370-STR, http://repub.eur.nl/pub/78651

Heyde Fernandes, D. von der, *The Functions and Dysfunctions of Reminders*, Promotor(s): Prof.dr. S.M.J. van Osselaer, EPS-2013-295-MKT, http://repub.eur.nl/pub/41514

Heyden, M.L.M., *Essays on Upper Echelons & Strategic Renewal: A Multilevel Contingency Approach*, Promotor(s): Prof.dr.ing. F.A.J. van den Bosch & Prof.dr. H.W. Volberda, EPS-2012-259-STR, http://repub.eur.nl/pub/32167

Hoever, I.J., *Diversity and Creativity*, Promotor(s): Prof.dr. D.L. van Knippenberg,

EPS-2012-267-ORG, http://repub.eur.nl/pub/37392

Hogenboom, F.P., *Automated Detection of Financial Events in News Text*, Promotor(s): Prof.dr.ir. U. Kaymak & Prof.dr. F.M.G. de Jong, EPS-2014-326-LIS, http://repub.eur.nl/pub/77237

Hoogendoorn, B., *Social Entrepreneurship in the Modern Economy: Warm Glow, Cold Feet*, Promotor(s): Prof.dr. H.P.G. Pennings & Prof.dr. A.R. Thurik, EPS-2011-246-STR, http://repub.eur.nl/pub/26447

Hoogervorst, N., *On The Psychology of Displaying Ethical Leadership: A Behavioral Ethics Approach*, Promotor(s): Prof.dr. D. de Cremer & Dr. M. van Dijke, EPS-2011-244-ORG, http://repub.eur.nl/pub/26228

Hout, D.H. van, *Measuring Meaningful Differences: Sensory Testing Based Decision Making in an Industrial Context; Applications of Signal Detection Theory and Thurstonian Modelling*, Promotor(s): Prof.dr. P.J.F. Groenen & Prof.dr. G.B. Dijksterhuis, EPS-2014-304-MKT, http://repub.eur.nl/pub/50387

Houwelingen, G.G. van, *Something To Rely On*, Promotor(s): Prof.dr. D. de Cremer & Prof.dr. M.H. van Dijke, EPS-2014-335-ORG, http://repub.eur.nl/pub/77320

Huang, X., *An Analysis of Occupational Pension Provision: From Evaluation to Redesign*, Promotor(s): Prof.dr. M.J.C.M. Verbeek, EPS-2010-196-F&A, http://repub.eur.nl/pub/19674

Hurk, E. van der, *Passengers, Information, and Disruptions*, Promotor(s): Prof.dr. L.G. Kroon & Prof.mr.dr. P.H.M. Vervest, EPS-2015-345-LIS, http://repub.eur.nl/pub/78275

Hytonen, K.A., *Context Effects in Valuation, Judgment and Choice: A Neuroscientific Approach*, Promotor(s): Prof.dr.ir. A. Smidts, EPS-2011-252-MKT, http://repub.eur.nl/pub/30668

Iseger, P. den, *Fourier and Laplace Transform Inversion with Applications in Finance*, Promotor(s): Prof.dr.ir. R. Dekker, EPS-2014-322-LIS, http://repub.eur.nl/pub/76954

Jaarsveld, W.L. van, *Maintenance Centered Service Parts Inventory Control*, Promotor(s): Prof.dr.ir. R. Dekker, EPS-2013-288-LIS, http://repub.eur.nl/pub/39933

Jalil, M.N., *Customer Information Driven After Sales Service Management: Lessons from Spare Parts Logistics*, Promotor(s): Prof.dr. L.G. Kroon, EPS-2011-222-LIS, http://repub.eur.nl/pub/22156

Kagie, M., *Advances in Online Shopping Interfaces: Product Catalog Maps and Recommender Systems*, Promotor(s): Prof.dr. P.J.F. Groenen, EPS-2010-195-MKT, http://repub.eur.nl/pub/19532

Kappe, E.R., *The Effectiveness of Pharmaceutical Marketing*, Promotor(s): Prof.dr. S. Stremersch, EPS-2011-239-MKT, http://repub.eur.nl/pub/23610

Karreman, B., *Financial Services and Emerging Markets*, Promotor(s): Prof.dr. G.A. van der Knaap & Prof.dr. H.P.G. Pennings, EPS-2011-223-ORG, http://repub.eur.nl/pub/22280

Khanagha, S., *Dynamic Capabilities for Managing Emerging Technologies*, Promotor(s): Prof.dr. H.W. Volberda, EPS-2014-339-S&E, http://repub.eur.nl/pub/77319

Kil, J., *Acquisitions Through a Behavioral and Real Options Lens*, Promotor(s): Prof.dr. H.T.J. Smit, EPS-2013-298-F&A, http://repub.eur.nl/pub/50142

Klooster, E. van 't, *Travel to Learn: the Influence of Cultural Distance on Competence Development in Educational Travel*, Promotor(s): Prof.dr. F.M. Go & Prof.dr. P.J. van Baalen, EPS-2014-312-MKT, http://repub.eur.nl/pub/51462

Koendjbiharie, S.R., *The Information-Based View on Business Network Performance: Revealing the Performance of Interorganizational Networks*, Promotor(s): Prof.dr.ir. H.W.G.M. van Heck & Prof.mr.dr. P.H.M. Vervest, EPS-2014-315-LIS, http://repub.eur.nl/pub/51751

Koning, M., *The Financial Reporting Environment: The Role of the Media, Regulators and Auditors*, Promotor(s): Prof.dr. G.M.H. Mertens & Prof.dr. P.G.J. Roosenboom, EPS-2014-330-F&A, http://repub.eur.nl/pub/77154

Konter, D.J., *Crossing Borders with HRM: An Inquiry of the Influence of Contextual Differences in the Adoption and Effectiveness of HRM*, Promotor(s): Prof.dr. J. Paauwe & Dr. L.H. Hoeksema, EPS-2014-305-ORG, http://repub.eur.nl/pub/50388

Korkmaz, E., *Bridging Models and Business: Understanding Heterogeneity in Hidden Drivers of Customer Purchase Behavior*, Promotor(s): Prof.dr. S.L. van de Velde & Prof.dr. D. Fok, EPS-2014-316-LIS, http://repub.eur.nl/pub/76008

Kroezen, J.J., *The Renewal of Mature Industries: An Examination of the Revival of the Dutch Beer Brewing Industry*, Promotor(s): Prof.dr. P.P.M.A.R. Heugens, EPS-2014-333-S&E, http://repub.eur.nl/pub/77042

Kysucky, V., *Access to Finance in a Cros-Country Context*, Promotor(s): Prof.dr. L. Norden, EPS-2015-350-F&A, http://repub.eur.nl/pub/78225

Lam, K.Y., *Reliability and Rankings*, Promotor(s): Prof.dr. Ph.H.B.F. Franses, EPS-2011-230-MKT, http://repub.eur.nl/pub/22977

Lander, M.W., *Profits or Professionalism? On Designing Professional Service Firms*, Promotor(s): Prof.dr. J. van Oosterhout & Prof.dr. P.P.M.A.R. Heugens, EPS-2012-253-ORG, http://repub.eur.nl/pub/30682

Langhe, B. de, *Contingencies: Learning Numerical and Emotional Associations in an Uncertain World*, Promotor(s): Prof.dr.ir. B. Wierenga & Prof.dr. S.M.J. van Osselaer, EPS-2011-236-MKT, http://repub.eur.nl/pub/23504

Larco Martinelli, J.A., *Incorporating Worker-Specific Factors in Operations Management Models*, Promotor(s): Prof.dr.ir. J. Dul & Prof.dr.ir. M.B.M. de Koster, EPS-2010-217-LIS, http://repub.eur.nl/pub/21527

Legault-Tremblay, P.O., Corporate Governance During Market Transition: Heterogeneous responses to Institution Tensions in China, Promotor: Prof.dr. B. Krug, EPS-2015-362-ORG, http://repub.eur.nl/pub/78649

Leunissen, J.M., *All Apologies: On the Willingness of Perpetrators to Apologize*, Promotor(s): Prof.dr. D. de Cremer & Dr. M. van Dijke, EPS-2014-301-ORG, http://repub.eur.nl/pub/50318

Li, D., *Supply Chain Contracting for After-sales Service and Product Support*, Promotor(s): Prof.dr.ir. M.B.M. de Koster, EPS-2015-347-LIS, http://repub.eur.nl/pub/78526

Li, Z., *Irrationality: What, Why and How*, Promotor(s): Prof.dr. H. Bleichrodt, Prof.dr. P.P. Wakker, & Prof.dr. K.I.M. Rohde, EPS-2014-338-MKT, http://repub.eur.nl/pub/77205

Liang, Q.X., *Governance, CEO Identity, and Quality Provision of Farmer Cooperatives*, Promotor(s): Prof.dr. G.W.J. Hendrikse, EPS-2013-281-ORG, http://repub.eur.nl/pub/39253

Liket, K., *Why 'Doing Good' is not Good Enough: Essays on Social Impact Measurement*, Promotor(s): Prof.dr. H.R. Commandeur & Dr. K.E.H. Maas, EPS-2014-307-STR, http://repub.eur.nl/pub/51130

Loos, M.J.H.M. van der, *Molecular Genetics and Hormones: New Frontiers in Entrepreneurship Research*, Promotor(s): Prof.dr. A.R. Thurik, Prof.dr. P.J.F. Groenen, & Prof.dr. A. Hofman, EPS-2013-287-S&E, http://repub.eur.nl/pub/40081

Lovric, M., *Behavioral Finance and Agent-Based Artificial Markets*, Promotor(s): Prof.dr. J. Spronk & Prof.dr.ir. U. Kaymak, EPS-2011-229-F&A, http://repub.eur.nl/pub/22814

Lu, Y., *Data-Driven Decision Making in Auction Markets*, Promotor(s): Prof.dr.ir. H.W.G.M. van Heck & Prof.dr. W. Ketter, EPS-2014-314-LIS, http://repub.eur.nl/pub/51543

Manders, B., *Implementation and Impact of ISO 9001*, Promotor(s): Prof.dr. K. Blind, EPS-2014-337-LIS, http://repub.eur.nl/pub/77412

Markwat, T.D., *Extreme Dependence in Asset Markets Around the Globe*, Promotor(s): Prof.dr. D.J.C. van Dijk, EPS-2011-227-F&A, http://repub.eur.nl/pub/22744

Mees, H., *Changing Fortunes: How China's Boom Caused the Financial Crisis*, Promotor(s): Prof.dr. Ph.H.B.F. Franses, EPS-2012-266-MKT, http://repub.eur.nl/pub/34930

Meuer, J., *Configurations of Inter-firm Relations in Management Innovation: A Study in China's Biopharmaceutical Industry*, Promotor(s): Prof.dr. B. Krug, EPS-2011-228-ORG, http://repub.eur.nl/pub/22745

Micheli, M.R., *Business Model Innovation: A Journey across Managers' Attention and Inter-Organizational Networks*, Promotor(s): Prof.dr. J.J.P. Jansen, EPS-2015-344-S&E, http://repub.eur.nl/pub/78241

Mihalache, O.R., *Stimulating Firm Innovativeness: Probing the Interrelations between Managerial and Organizational Determinants*, Promotor(s): Prof.dr. J.J.P. Jansen, Prof.dr.ing. F.A.J. van den Bosch, & Prof.dr. H.W. Volberda, EPS-2012-260-S&E, http://repub.eur.nl/pub/32343

Milea, V., *News Analytics for Financial Decision Support*, Promotor(s): Prof.dr.ir. U. Kaymak, EPS-2013-275-LIS, http://repub.eur.nl/pub/38673

Naumovska, I., *Socially Situated Financial Markets: A Neo-Behavioral Perspective on Firms, Investors and Practices*, Promotor(s): Prof.dr. P.P.M.A.R. Heugens & Prof.dr. A. de Jong, EPS-2014-319-S&E, http://repub.eur.nl/pub/76084

Nielsen, L.K., *Rolling Stock Rescheduling in Passenger Railways: Applications in short term planning and in disruption management*, Promotor(s): Prof.dr. L.G. Kroon, EPS-2011-224-LIS, http://repub.eur.nl/pub/22444

Nijdam, M.H., *Leader Firms: The value of companies for the competitiveness of the Rotterdam seaport cluster*, Promotor(s): Prof.dr. R.J.M. van Tulder, EPS-2010-216-ORG, http://repub.eur.nl/pub/21405

Noordegraaf-Eelens, L.H.J., *Contested Communication; A Critical Analysis of Central Bank Speech*, Promotor(s): Prof.dr. Ph.H.B.F. Franses, Prof.dr. J. de Mul, & Prof.dr. D.J.C. van Dijk, EPS-2010-209-MKT, http://repub.eur.nl/pub/21061

Nuijten, A.L.P., *Deaf Effect for Risk Warnings: A Causal Examination applied to Information Systems Projects*, Promotor(s): Prof.dr. G.J. van der Pijl, Prof.dr. H.R. Commandeur & Prof.dr. M. Keil, EPS-2012-263-S&E, http://repub.eur.nl/pub/34928

Oosterhout, M. van, *Business Agility and Information Technology in Service Organizations*, Promotor(s): Prof.dr.ir. H.W.G.M. van Heck, EPS-2010-198-LIS, http://repub.eur.nl/pub/19805

Osadchiy, S.E., *The Dynamics of Formal Organization: Essays on bureaucracy and formal rules*, Promotor(s): Prof.dr. P.P.M.A.R. Heugens, EPS-2011-231-ORG, http://repub.eur.nl/pub/23250

Otgaar, A.H.J., *Industrial Tourism: Where the Public Meets the Private*, Promotor(s): Prof.dr. L. Berg, EPS-2010-219-ORG, http://repub.eur.nl/pub/21585

Ozdemir, M.N., *Project-level Governance, Monetary Incentives, and Performance in Strategic R&D Alliances*, Promotor(s): Prof.dr.ir. J.C.M. van den Ende, EPS-2011-235-LIS, http://repub.eur.nl/pub/23550

Peers, Y., *Econometric Advances in Diffusion Models*, Promotor(s): Prof.dr. Ph.H.B.F. Franses, EPS-2011-251-MKT, http://repub.eur.nl/pub/30586

Peters, M., *Machine Learning Algorithms for Smart Electricity Markets*, Promotor(s): Prof.dr. W. Ketter, EPS-2014-332-LIS, http://repub.eur.nl/pub/77413

Pince, C., *Advances in Inventory Management: Dynamic Models*, Promotor(s): Prof.dr.ir. R. Dekker, EPS-2010-199-LIS, http://repub.eur.nl/pub/19867

Porck, J., *No Team is an Island: An Integrative View of Strategic Consensus between Groups*, Promotor(s): Prof.dr. P.J.F. Groenen & Prof.dr. D.L. van Knippenberg, EPS-2013-299-ORG, http://repub.eur.nl/pub/50141

Porras Prado, M., *The Long and Short Side of Real Estate, Real Estate Stocks, and Equity*, Promotor(s): Prof.dr. M.J.C.M. Verbeek, EPS-2012-254-F&A, http://repub.eur.nl/pub/30848

Poruthiyil, P.V., *Steering Through: How organizations negotiate permanent uncertainty and unresolvable choices*, Promotor(s): Prof.dr. P.P.M.A.R. Heugens & Prof.dr. S.J. Magala, EPS-2011-245-ORG, http://repub.eur.nl/pub/26392

Potthoff, D., *Railway Crew Rescheduling: Novel approaches and extensions*, Promotor(s): Prof.dr. A.P.M. Wagelmans & Prof.dr. L.G. Kroon, EPS-2010-210-LIS, http://repub.eur.nl/pub/21084

Pourakbar, M., *End-of-Life Inventory Decisions of Service Parts*, Promotor(s): Prof.dr.ir. R. Dekker, EPS-2011-249-LIS, http://repub.eur.nl/pub/30584

Pronker, E.S., *Innovation Paradox in Vaccine Target Selection*, Promotor(s): Prof.dr. H.J.H.M. Claassen & Prof.dr. H.R. Commandeur, EPS-2013-282-S&E, http://repub.eur.nl/pub/39654

Pruijssers, J.K., *An Organizational Perspective on Auditor Conduct*, Promotor(s): Prof.dr. J. van Oosterhout & Prof.dr. P.P.M.A.R. Heugens, EPS-2015-342-S&E, http://repub.eur.nl/pub/78192

Retel Helmrich, M.J., *Green Lot-Sizing*, Promotor(s): Prof.dr. A.P.M. Wagelmans, EPS-2013-291-LIS, http://repub.eur.nl/pub/41330

Rietveld, N., *Essays on the Intersection of Economics and Biology*, Promotor(s): Prof.dr. A.R. Thurik, Prof.dr. Ph.D. Koellinger, Prof.dr. P.J.F. Groenen, & Prof.dr. A. Hofman, EPS-2014-320-S&E, http://repub.eur.nl/pub/76907

Rijsenbilt, J.A., *CEO Narcissism: Measurement and Impact*, Promotor(s): Prof.dr. A.G.Z. Kemna & Prof.dr. H.R. Commandeur, EPS-2011-238-STR, http://repub.eur.nl/pub/23554

Roelofsen, E.M., *The Role of Analyst Conference Calls in Capital Markets*, Promotor(s): Prof.dr. G.M.H. Mertens & Prof.dr. L.G. van der Tas, EPS-2010-190-F&A, http://repub.eur.nl/pub/18013

Roza-van Vuren, M.W., *The Relationship between Offshoring Strategies and Firm Performance: Impact of innovation, absorptive capacity and firm size*, Promotor(s): Prof.dr. H.W. Volberda & Prof.dr.ing. F.A.J. van den Bosch, EPS-2011-214-STR, http://repub.eur.nl/pub/22155

Rubbaniy, G., *Investment Behaviour of Institutional Investors*, Promotor(s): Prof.dr. W.F.C. Verschoor, EPS-2013-284-F&A, http://repub.eur.nl/pub/40068

Schellekens, G.A.C., *Language Abstraction in Word of Mouth*, Promotor(s): Prof.dr.ir. A. Smidts, EPS-2010-218-MKT, http://repub.eur.nl/pub/21580

Shahzad, K., *Credit Rating Agencies, Financial Regulations and the Capital Markets*, Promotor(s): Prof.dr. G.M.H. Mertens, EPS-2013-283-F&A, http://repub.eur.nl/pub/39655

Sotgiu, F., *Not All Promotions are Made Equal: From the Effects of a Price War to Crosschain Cannibalization*, Promotor(s): Prof.dr. M.G. Dekimpe & Prof.dr.ir. B. Wierenga, EPS-2010-203-MKT, http://repub.eur.nl/pub/19714

Sousa, M.J.C. de, *Servant Leadership to the Test: New Perspectives and Insights*, Promotor(s): Prof.dr. D.L. van Knippenberg & Dr. D. van Dierendonck, EPS-2014-313-ORG, http://repub.eur.nl/pub/51537

Spliet, R., *Vehicle Routing with Uncertain Demand*, Promotor(s): Prof.dr.ir. R. Dekker, EPS-2013-293-LIS, http://repub.eur.nl/pub/41513

Srour, F.J., *Dissecting Drayage: An Examination of Structure, Information, and Control in Drayage Operations*, Promotor(s): Prof.dr. S.L. van de Velde, EPS-2010-186-LIS, http://repub.eur.nl/pub/18231

Staadt, J.L., *Leading Public Housing Organisation in a Problematic Situation: A Critical Soft Systems Methodology Approach*, Promotor(s): Prof.dr. S.J. Magala, EPS-2014-308-ORG, http://repub.eur.nl/pub/50712

Stallen, M., *Social Context Effects on Decision-Making: A Neurobiological Approach*, Promotor(s): Prof.dr.ir. A. Smidts, EPS-2013-285-MKT, http://repub.eur.nl/pub/39931

Tarakci, M., *Behavioral Strategy: Strategic Consensus, Power and Networks*, Promotor(s): Prof.dr. D.L. van Knippenberg & Prof.dr. P.J.F. Groenen, EPS-2013-280-ORG, http://repub.eur.nl/pub/39130

Teixeira de Vasconcelos, M., *Agency Costs, Firm Value, and Corporate Investment*, Promotor(s): Prof.dr. P.G.J. Roosenboom, EPS-2012-265-F&A, http://repub.eur.nl/pub/37265

Tempelaar, M.P., *Organizing for Ambidexterity: Studies on the pursuit of exploration and exploitation through differentiation, integration, contextual and individual attributes*, Promotor(s): Prof.dr.ing. F.A.J. van den Bosch & Prof.dr. H.W. Volberda, EPS-2010-191-STR, http://repub.eur.nl/pub/18457

Tiwari, V., *Transition Process and Performance in IT Outsourcing: Evidence from a Field Study and Laboratory Experiments*, Promotor(s): Prof.dr.ir. H.W.G.M. van Heck & Prof.mr.dr. P.H.M. Vervest, EPS-2010-201-LIS, http://repub.eur.nl/pub/19868

Troster, C., *Nationality Heterogeneity and Interpersonal Relationships at Work*, Promotor(s): Prof.dr. D.L. van Knippenberg, EPS-2011-233-ORG, http://repub.eur.nl/pub/23298

Tsekouras, D., *No Pain No Gain: The Beneficial Role of Consumer Effort in Decision-Making*, Promotor(s): Prof.dr.ir. B.G.C. Dellaert, EPS-2012-268-MKT, http://repub.eur.nl/pub/37542

Tuijl, E. van, *Upgrading across Organisational and Geographical Configurations*, Promotor(s): Prof.dr. L. van den Berg, EPS-2015-349-S&E, http://repub.eur.nl/pub/78224

Tuncdogan, A., *Decision Making and Behavioral Strategy: The Role of Regulatory Focus in Corporate Innovation Processes*, Promotor(s): Prof.dr.ing. F.A.J. van den Bosch, Prof.dr. H.W. Volberda, & Prof.dr. T.J.M. Mom, EPS-2014-334-S&E, http://repub.eur.nl/pub/76978

Tzioti, S., *Let Me Give You a Piece of Advice: Empirical Papers about Advice Taking in Marketing*, Promotor(s): Prof.dr. S.M.J. van Osselaer & Prof.dr.ir. B. Wierenga, EPS-2010-211-MKT, http://repub.eur.nl/pub/21149

Uijl, S. den, *The Emergence of De-facto Standards*, Promotor(s): Prof.dr. K. Blind, EPS-2014-328-LIS, http://repub.eur.nl/pub/77382

Vaccaro, I.G., *Management Innovation: Studies on the Role of Internal Change Agents*, Promotor(s): Prof.dr.ing. F.A.J. van den Bosch, Prof.dr. H.W. Volberda, & Prof.dr. J.J.P. Jansen, EPS-2010-212-STR, http://repub.eur.nl/pub/21150

Vagias, D., *Liquidity, Investors and International Capital Markets*, Promotor(s): Prof.dr. M.A. van Dijk, EPS-2013-294-F&A, http://repub.eur.nl/pub/41511

Veelenturf, L.P., *Disruption Management in Passenger Railways: Models for Timetable, Rolling Stock and Crew Rescheduling*, Promotor(s): Prof.dr. L.G. Kroon, EPS-2014-327-LIS, http://repub.eur.nl/pub/77155

Venus, M., *Demystifying Visionary Leadership: In search of the essence of effective vision communication*, Promotor(s): Prof.dr. D.L. van Knippenberg, EPS-2013-289-ORG, http://repub.eur.nl/pub/40079

Verheijen, H.J.J., *Vendor-Buyer Coordination in Supply Chains*, Promotor(s): Prof.dr.ir. J.A.E.E. van Nunen, EPS-2010-194-LIS, http://repub.eur.nl/pub/19594

Visser, V.A., *Leader Affect and Leadership Effectiveness:How leader affective displays influence follower outcomes*, Promotor(s): Prof.dr. D.L. van Knippenberg, EPS-2013-286-ORG, http://repub.eur.nl/pub/40076

Vlam, A.J., *Customer First? The Relationship between Advisors and Consumers of Financial Products*, Promotor(s): Prof.dr. Ph.H.B.F. Franses, EPS-2011-250-MKT, http://repub.eur.nl/pub/30585

Waard, E.J. de, *Engaging Environmental Turbulence: Organizational Determinants for Repetitive Quick and Adequate Responses*, Promotor(s): Prof.dr. H.W. Volberda & Prof.dr. J. Soeters, EPS-2010-189-STR, http://repub.eur.nl/pub/18012

Waltman, L., *Computational and Game-Theoretic Approaches for Modeling Bounded Rationality*, Promotor(s): Prof.dr.ir. R. Dekker & Prof.dr.ir. U. Kaymak, EPS-2011-248-LIS, http://repub.eur.nl/pub/26564

Wang, T., *Essays in Banking and Corporate Finance*, Promotor(s): Prof.dr. L. Norden & Prof.dr. P.G.J. Roosenboom, EPS-2015-352-F&A, http://repub.eur.nl/pub/78301

Wang, Y., *Information Content of Mutual Fund Portfolio Disclosure*, Promotor(s): Prof.dr. M.J.C.M. Verbeek, EPS-2011-242-F&A, http://repub.eur.nl/pub/26066

Wang, Y., *Corporate Reputation Management: Reaching Out to Financial Stakeholders*, Promotor(s): Prof.dr. C.B.M. van Riel, EPS-2013-271-ORG, http://repub.eur.nl/pub/38675

Weenen, T.C., *On the Origin and Development of the Medical Nutrition Industry*, Promotor(s): Prof.dr. H.R. Commandeur & Prof.dr. H.J.H.M. Claassen, EPS-2014-309-S&E, http://repub.eur.nl/pub/51134

Wolfswinkel, M., *Corporate Governance, Firm Risk and Shareholder Value*, Promotor(s): Prof.dr. A. de Jong, EPS-2013-277-F&A, http://repub.eur.nl/pub/39127

Wubben, M.J.J., *Social Functions of Emotions in Social Dilemmas*, Promotor(s): Prof.dr. D. de Cremer & Prof.dr. E. van Dijk, EPS-2010-187-ORG, http://repub.eur.nl/pub/18228

Xu, Y., *Empirical Essays on the Stock Returns, Risk Management, and Liquidity Creation of Banks*, Promotor(s): Prof.dr. M.J.C.M. Verbeek, EPS-2010-188-F&A, http://repub.eur.nl/pub/18125

Yang, S., *Information Aggregation Efficiency of Prediction Markets*, Promotor(s): Prof.dr.ir. H.W.G.M. van Heck, EPS-2014-323-LIS, http://repub.eur.nl/pub/77184

Zaerpour, N., *Efficient Management of Compact Storage Systems*, Promotor(s): Prof.dr.ir. M.B.M. de Koster, EPS-2013-276-LIS, http://repub.eur.nl/pub/38766

Zhang, D., *Essays in Executive Compensation*, Promotor(s): Prof.dr. I. Dittmann, EPS-2012-261-F&A, http://repub.eur.nl/pub/32344

Zhang, X., *Scheduling with Time Lags*, Promotor(s): Prof.dr. S.L. van de Velde, EPS-2010-206-LIS, http://repub.eur.nl/pub/19928

Zhou, H., *Knowledge, Entrepreneurship and Performance: Evidence from country-level and firm-level studies*, Promotor(s): Prof.dr. A.R. Thurik & Prof.dr. L.M. Uhlaner, EPS-2010-207-ORG, http://repub.eur.nl/pub/20634

Zwan, P.W. van der, *The Entrepreneurial Process: An International Analysis of Entry and Exit*, Promotor(s): Prof.dr. A.R. Thurik & Prof.dr. P.J.F. Groenen, EPS-2011-234-ORG, http://repub.eur.nl/pub/23422

Erasmus Research Institute of Management - ERIM

SPARE PARTS MANAGEMENT OF AGING CAPITAL PRODUCTS

In this thesis, spare parts supply chains of Original Equipment Manufacturers (OEMs) providing after-sales services to out-of-production capital products are analyzed. These supply chains are subject to non-decreasing supply risks which are byproducts of decreasing numbers of capital products and vanishing spare parts demand.

To mitigate the effect of supply risk, we develop an empirical model which can detect supply problems in advance. Since solution of supply problems may include some long procedures, advance detection of those problems allows OEMs to take proactive actions and save costs. An extended version of the study presented in this dissertation is applied in an OEM providing maintenance service for its out-of-production aircraft.

Our empirical study indicates the significance of lead time changes before supply disruptions occur. To address changing lead times in control policies for spare parts inventory, we present a mathematical model and its fundamental properties. Our analysis reveal that combined effect of supply disruptions and random lead time may be as large as the summation of individual effects of the two risk factors. This is especially true when the OEM aims to achieve high service levels.

In addition, we consider the effects of secondary markets on spare parts supply chains. Those markets include spare parts in different conditions (such as serviceable and as-removed) and these parts can be purchased by OEMs to satisfy their spare parts demand. In the last two chapters of the thesis, secondary markets are considered as a supply source and price competitor for OEMs. Our results indicate that those markets are important factors that alter the optimum policy of spare parts inventory control.

Design & layout: B&T Ontwerp en advies (www.b-en-t.nl) Print: Haveka (www.haveka.nl)

ERIM

The Erasmus Research Institute of Management (ERIM) is the Research School (Onderzoekschool) in the field of management of the Erasmus University Rotterdam. The founding participants of ERIM are the Rotterdam School of Management (RSM), and the Erasmus School of Economics (ESE). ERIM was founded in 1999 and is officially accredited by the Royal Netherlands Academy of Arts and Sciences (KNAW). The research undertaken by ERIM is focused on the management of the firm in its environment, its intra- and interfirm relations, and its business processes in their interdependent connections.

The objective of ERIM is to carry out first rate research in management, and to offer an advanced doctoral programme in Research in Management. Within ERIM, over three hundred senior researchers and PhD candidates are active in the different research programmes. From a variety of academic backgrounds and expertises, the ERIM community is united in striving for excellence and working at the forefront of creating new business knowledge.

ERIM PhD Series

Research in Management

Erasmus Research Institute of Management - ERIM
Rotterdam School of Management (RSM)
Erasmus School of Economics (ESE)
Erasmus University Rotterdam (EUR)
P.O. Box 1738, 3000 DR Rotterdam,
The Netherlands

Tel. +31 10 408 11 82
Fax +31 10 408 96 40
E-mail info@erim.eur.nl
Internet www.erim.eur.nl