

SPARSE BAYESIAN BLIND IMAGE DECONVOLUTION WITH PARAMETER ESTIMATION

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ABSTRACT

In this paper we propose a novel blind image deconvolution method developed within the Bayesian framework. A variant of the non-convex l_p -norm prior with $0 < p < 1$ is used as the image prior and a total variation (TV) based prior is utilized as the blur prior. The proposed method is derived by utilizing bounds for both the image and blur priors using the majorization-minimization principle. Maximum a posteriori Bayesian inference is performed and as a result, the unknown image, blur and model parameters are simultaneously estimated. We also show that as a special case, the developed method provides very competitive non-blind image restoration results when the blurring function is assumed to be known. Experimental results are presented to demonstrate the advantage of the proposed method compared to existing ones.

1. INTRODUCTION

The blind image deconvolution (BID) problem refers to the inverse problem in which both the image and blurring function have to be estimated from a single observation. The standard formulation of the image degradation model is given in matrix-vector form by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where the $N \times 1$ vectors \mathbf{x} , \mathbf{y} , and \mathbf{n} represent respectively the original image, the available noisy and blurred image, and the noise with independent elements of variance $\sigma_n^2 = \beta^{-1}$, and \mathbf{H} represents the blurring matrix created from the blur point spread function \mathbf{h} . The images are assumed to be of size $m \times n = N$, and they are lexicographically ordered into $N \times 1$ vectors. Given \mathbf{y} , the BID problem calls for finding estimates of \mathbf{x} and \mathbf{H} using prior knowledge on them.

A number of methods have been proposed to address BID (a recent literature review can be found in [4]). Estimating camera motion from a single photograph was the focus in [6], where the unknown image and blur were estimated in a two step process. Estimating camera motion was also investigated in [1, 9] where the unknown image and blur were estimated in a simultaneous fashion. Additionally, [1] concentrated on synthetic experiments where the performance of the algorithm was evaluated by the improvement in signal to noise ratio.

In this paper we propose a novel Bayesian algorithm for BID that utilizes a variant of the non-convex l_p -norm prior as

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the unknown image prior and the TV prior as the unknown blur prior. In previous works (e.g., [1, 9]), the unknown parameters are chosen as a sequence of numbers that yield the unknown image estimate with a good visual quality. Instead, in this paper we estimate the unknown parameters by taking the restorations of both the unknown image and unknown blur into account. Finally, we evaluate the performance of the proposed algorithm with comparisons with [1].

This paper is organized as follows. In Section 2 we provide the proposed Bayesian modeling of the BID problem. The Bayesian inference is presented in Section 3. Experimental results are provided in Section 4 and conclusions drawn in Section 5.

2. BAYESIAN MODELING

In order to model the unknown components of the BID problem within the Bayesian framework, the definition of the joint distribution $p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h}, \mathbf{y})$ is required. Assuming that \mathbf{x} and \mathbf{h} are independent, the joint distribution $p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h}, \mathbf{y})$ can be factorized in terms of the observation model $p(\mathbf{y}|\beta, \mathbf{x}, \mathbf{h})$, the prior distributions $p(\mathbf{x}|\alpha)$ and $p(\mathbf{h}|\gamma)$, and the hyperparameter distributions $p(\alpha)$, $p(\beta)$ and $p(\gamma)$, that is,

$$p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h}, \mathbf{y}) = p(\mathbf{y}|\beta, \mathbf{x}, \mathbf{h})p(\mathbf{x}|\alpha)p(\mathbf{h}|\gamma)p(\alpha)p(\beta)p(\gamma). \quad (2)$$

As already discussed in the previous section, the observation noise is modeled as a zero mean white Gaussian random vector. Therefore, the observation model is defined as

$$p(\mathbf{y}|\beta, \mathbf{x}, \mathbf{h}) \propto \beta^{N/2} \exp \left[-\frac{\beta}{2} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \right], \quad (3)$$

where β is the precision of the multivariate Gaussian distribution.

As the image prior we utilize a variant of the generalized Gaussian distribution, given by

$$p(\mathbf{x}|\alpha) = \frac{1}{Z_{GG}(\alpha)} \exp \left[-\sum_{d \in \mathcal{D}} \alpha_d \sum_i |\Delta_i^d(\mathbf{x})|^p \right], \quad (4)$$

where $Z_{GG}(\alpha)$ is the partition function, $0 < p < 1$, α denotes the set $\{\alpha_d\}$ and $d \in \mathcal{D} = \{h, v, hh, vv, hv\}$. $\Delta_i^h(\mathbf{u})$ and $\Delta_i^v(\mathbf{u})$ correspond to, respectively, the horizontal and vertical first order differences, at pixel i , that is, $\Delta_i^h(\mathbf{u}) = u_i - u_{l(i)}$ and $\Delta_i^v(\mathbf{u}) = u_i - u_{a(i)}$, where $l(i)$ and $a(i)$ denote the nearest neighbors of i , to the left and above, respectively. The operators $\Delta_i^{hh}(\mathbf{u})$, $\Delta_i^{vv}(\mathbf{u})$, $\Delta_i^{hv}(\mathbf{u})$ correspond to, respectively,

horizontal, vertical and horizontal-vertical second order differences, at pixel i .

In order to eliminate the need to estimate each α_d we assume that $\alpha_h = \alpha_v = \alpha$ and $\alpha_{hh} = \alpha_{vv} = \alpha_{hv} = \alpha/2$. Additionally, in a similar way it was proposed in [8], the partition function will be approximated as $Z_{GG}(\alpha) \propto \alpha^{-\lambda_1 N/p}$, where λ_1 is a positive real number. We then simplify (4) accordingly to obtain the following image prior

$$p(\mathbf{x}|\alpha) \propto \alpha^{\lambda_1 N/p} \exp \left[-\alpha \sum_{d \in \mathcal{D}} 2^{1-o(d)} \sum_i |\Delta_i^d(\mathbf{x})|^p \right], \quad (5)$$

where $o(d) \in \{1, 2\}$ denotes the order of the difference operator $\Delta_i^d(\mathbf{x})$.

For the blur we utilize the total-variation prior given by

$$p(\mathbf{h}|\gamma) \propto \gamma^{\lambda_2 N} \exp[-\gamma \text{TV}(\mathbf{h})], \quad (6)$$

where λ_2 is a positive real number and $\text{TV}(\mathbf{h})$ is defined as

$$\text{TV}(\mathbf{h}) = \sum_i \sqrt{(\Delta_i^h(\mathbf{h}))^2 + (\Delta_i^v(\mathbf{h}))^2}. \quad (7)$$

In this work we use flat improper hyperpriors on α, β and γ , that is, we utilize

$$p(\alpha) \propto \text{const}, \quad p(\beta) \propto \text{const}, \quad p(\gamma) \propto \text{const}. \quad (8)$$

Note that with this choice of the hyperpriors, the observed image \mathbf{y} is made solely responsible for the estimation of the image, blur and hyperparameters.

3. BAYESIAN INFERENCE

Bayesian inference on the unknown components of the blind image deconvolution problem is based on the estimation of the unknown posterior distribution $p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h} | \mathbf{y})$, given by

$$p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h} | \mathbf{y}) = \frac{p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h}, \mathbf{y})}{p(\mathbf{y})}. \quad (9)$$

In this work, we adopt the maximum *a posteriori* (MAP) approach to obtain a single point $(\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\mathbf{x}}, \bar{\mathbf{h}})$ estimate, denoted as $\bar{\Theta}$, that maximizes $p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h} | \mathbf{y})$ as follows,

$$\begin{aligned} \bar{\Theta} &= \underset{\Theta}{\text{argmax}} p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h} | \mathbf{y}) \\ &= \min_{\Theta} \left\{ \frac{\beta}{2} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 + \right. \\ &\quad \left. + \alpha \sum_{d \in \mathcal{D}} 2^{1-o(d)} \sum_i |\Delta_i^d(\mathbf{x})|^p + \gamma \text{TV}(\mathbf{h}) + \right. \\ &\quad \left. - \frac{\lambda_1 N}{p} \log \alpha - \frac{N}{2} \log \beta - \lambda_2 N \log \gamma \right\}. \end{aligned} \quad (10)$$

As can be seen from (10), obtaining the point estimate that maximizes the posterior distribution $p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h} | \mathbf{y})$ is not straightforward since it requires the minimization of a non-convex functional. Note that maximizing posterior distribution $p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h} | \mathbf{y})$ with the maximum *a posteriori* approach is equivalent to the variational Bayesian based maximization (see [2]) for the special case when all the posterior distributions are assumed to be degenerate.

In this paper, we resort to a majorization-minimization approach to bound the non-convex image prior $p(\mathbf{x}|\alpha)$ by the functional $M_1(\alpha, \mathbf{x}, \mathbf{V})$, that is

$$p(\mathbf{x}|\alpha) \geq \text{const} \cdot M_1(\alpha, \mathbf{x}, \mathbf{V}). \quad (11)$$

The majorization-minimization approach has been utilized in several approaches for image restoration [2, 3].

The functional $M_1(\alpha, \mathbf{x}, \mathbf{V})$ is derived by considering the relationship between the weighted geometric and arithmetic means, which is given by

$$z^{p/2} v^{1-p/2} \leq \frac{p}{2} z + \left(1 - \frac{p}{2}\right) v, \quad (12)$$

where z, v and p are positive real numbers. We first rewrite (12) as

$$z^{p/2} \leq \frac{p}{2} \frac{z + \frac{2-p}{p} v}{v^{1-p/2}}. \quad (13)$$

Using (13) we obtain

$$|\Delta_i^d(\mathbf{x})|^p \leq \frac{p}{2} \frac{[\Delta_i^d(\mathbf{x})]^2 + \frac{2-p}{p} v_{d,i}}{v_{d,i}^{1-p/2}}. \quad (14)$$

Therefore we have

$$\begin{aligned} p(\mathbf{x}|\alpha) &= \text{const} \cdot \alpha^{\lambda_1 N/p} \exp \left[-\alpha \sum_{d \in \mathcal{D}} 2^{1-o(d)} \sum_i |\Delta_i^d(\mathbf{x})|^p \right] \\ &\geq \text{const} \cdot \alpha^{\lambda_1 N/p} \exp \left[-\frac{\alpha p}{2} \sum_{d \in \mathcal{D}} 2^{1-o(d)} \sum_i \frac{[\Delta_i^d(\mathbf{x})]^2 + \frac{2-p}{p} v_{d,i}}{v_{d,i}^{1-p/2}} \right]. \end{aligned} \quad (15)$$

and so the $M_1(\alpha, \mathbf{x}, \mathbf{V})$ is defined as

$$\begin{aligned} M_1(\alpha, \mathbf{x}, \mathbf{V}) &= \\ &\alpha^{\lambda_1 N/p} \exp \left[-\frac{\alpha p}{2} \sum_{d \in \mathcal{D}} 2^{1-o(d)} \sum_i \frac{[\Delta_i^d(\mathbf{x})]^2 + \frac{2-p}{p} v_{d,i}}{v_{d,i}^{1-p/2}} \right], \end{aligned} \quad (16)$$

where \mathbf{V} is a matrix with elements $v_{d,i}$.

Similarly, the majorization-minimization criterion is used to bound the blur prior $p(\mathbf{h}|\gamma)$ utilizing the functional $M_2(\gamma, \mathbf{h}, \mathbf{u})$. Let us define, for γ and any N -dimensional vector $\mathbf{u} \in (\mathbb{R}^+)^N$, with components $u_i, i = 1, \dots, N$, the following functional

$$M_2(\gamma, \mathbf{h}, \mathbf{u}) = \alpha^{\lambda_2 N} \exp \left[-\frac{\gamma}{2} \sum_i \frac{(\Delta_i^h(\mathbf{h}))^2 + (\Delta_i^v(\mathbf{h}))^2 + u_i}{\sqrt{u_i}} \right]. \quad (17)$$

Using the inequality in (13) with $p = 1$, for $z \geq 0$ and $v > 0$

$$\sqrt{z} \leq \sqrt{v} + \frac{1}{2\sqrt{v}}(z - v), \quad (18)$$

we obtain

$$p(\mathbf{h}|\gamma) \geq \text{const} \cdot M_2(\gamma, \mathbf{h}, \mathbf{u}). \quad (19)$$

The lower bounds of $p(\mathbf{x}|\alpha)$ and $p(\mathbf{h}|\gamma)$ defined above lead to the following lower bound of the distribution $p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h}, \mathbf{y})$

$$p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h}, \mathbf{y}) = p(\alpha)p(\beta)p(\gamma)p(\mathbf{x}|\alpha)p(\mathbf{h}|\gamma)p(\mathbf{y}|\beta, \mathbf{x}, \mathbf{h}) \\ \geq \text{const} \cdot \mathbf{M}_1(\alpha, \mathbf{x}, \mathbf{V})\mathbf{M}_2(\gamma, \mathbf{h}, \mathbf{u})p(\mathbf{y}|\beta, \mathbf{x}, \mathbf{h}).$$

Therefore, a single point estimate that maximizes a lower bound of the posterior distribution $p(\alpha, \beta, \gamma, \mathbf{x}, \mathbf{h} | \mathbf{y})$ is found as follows

$$\bar{\Theta} = \min_{\Theta} \left\{ \frac{\beta}{2} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 + \frac{\alpha p}{2} \sum_{d \in \mathcal{D}} 2^{1-o(d)} \sum_i \frac{[\Delta_i^d(\mathbf{x})]^2 + \frac{2-p}{p} v_{d,i}}{v_{d,i}^{1-p/2}} + \frac{\gamma}{2} \sum_i \frac{(\Delta_i^h(\mathbf{h}))^2 + (\Delta_i^v(\mathbf{h}))^2 + u_i}{\sqrt{u_i}} + -\frac{\lambda_1 N}{p} \log \alpha - \frac{N}{2} \log \beta - \lambda_2 N \log \gamma \right\}. \quad (20)$$

Using (20) for all unknowns in an alternating fashion, we obtain the final algorithm as shown below.

Algorithm. Given $\alpha^1, \beta^1, \gamma^1, \mathbf{u}^1$ and \mathbf{V}^1 , where the rows of \mathbf{V}^k are denoted by $\mathbf{v}_d^k \in (\mathbb{R}^+)^N$, with $d \in \{h, v, hh, vv, hv\}$ and initial estimate of the blurring filter \mathbf{h}^1 .

For $k = 1, 2, \dots$ until a stopping criterion is met:

1. Calculate

$$\mathbf{x}^k = \left[\beta^k (\mathbf{H}^k)^t (\mathbf{H}^k) + \alpha^k p \sum_d 2^{1-o(d)} (\Delta^d)^t \mathbf{W}_d^k (\Delta^d) \right]^{-1} \\ \times \beta^k (\mathbf{H}^k)^t \mathbf{y}, \quad (21)$$

where \mathbf{W}_d^k is a diagonal matrix with entries $\mathbf{W}_d^k(i, i) = v_{d,i}^{p/2-1}$.

2. Calculate

$$\mathbf{h}^{k+1} = \left[\beta^k (\mathbf{X}^k)^t (\mathbf{X}^k) + \gamma^k \sum_{d \in \{h, v\}} (\Delta^d)^t \mathbf{U}_d^k (\Delta^d) \right]^{-1} \\ \times \beta^k (\mathbf{X}^k)^t \mathbf{y}, \quad (22)$$

3. For each $d \in \{h, v, hh, vv, hv\}$ calculate

$$v_{d,i}^{k+1} = [\Delta_i^d(\mathbf{x}^k)]^2, \quad (23)$$

4. Calculate

$$u_i^{k+1} = [\Delta_i^h(\mathbf{h}^{k+1})]^2 + [\Delta_i^v(\mathbf{h}^{k+1})]^2, \quad (24)$$

5. Calculate

$$\alpha^{k+1} = \frac{\lambda_1 N / p}{\sum_{d \in \mathcal{D}} 2^{1-o(d)} \sum_i |\Delta_i^d(\mathbf{x}^k)|^p}, \quad (25)$$

$$\beta^{k+1} = \frac{N}{\|\mathbf{y} - \mathbf{H}^{k+1} \mathbf{x}^k\|^2}, \quad (26)$$

$$\gamma^{k+1} = \frac{\lambda_2 N}{\text{TV}(\mathbf{h}^{k+1})}, \quad (27)$$

In this work we set the values of the parameters p, λ_1 and λ_2 equal to 0.8, 0.5 and 0.5, respectively. The experimental results in Section 4 validate the proposed parameter estimation procedure. The robustness of the proposed method will be tested and evaluated under various blurring and noisy conditions. Additionally, since the proposed algorithm is initialized with the unit impulse response as the initial blur estimate, it is particularly important in the first few iterations to keep parameters α and γ relatively high compared to the parameter β . This procedure prevents the proposed algorithm from converging to the undesirable blur estimate of unit impulse.

Note that if the blur \mathbf{h} and the hyperparameters α, β and γ are assumed to be known, the proposed algorithm coincides with the iteratively re-weighted least squares (IRLS) algorithm presented in [7].

4. EXPERIMENTAL RESULTS

In this section we present experimental results obtained with the use of the proposed algorithm. As the performance metric, we utilize the improvement in signal to noise ratio (ISNR), which is defined as $10 \log_{10} (\|\mathbf{x} - \mathbf{y}\|^2 / \|\mathbf{x} - \hat{\mathbf{x}}\|^2)$, where \mathbf{x}, \mathbf{y} and $\hat{\mathbf{x}}$ are the original, observed and estimated images, respectively. We evaluate the performance of the proposed algorithm, which will be denoted as ALG, on two images (Lena and Cameraman) blurred with different motion blurs, which are shown in Figure 1. Realizations of white Gaussian noise are added to obtain degraded images with blurred signal to noise (BSNR) ratios of 30 or 40dB, depending on the test configuration.

In the first set of experiments, we compare the performance of the proposed method with the state of the art non-blind deconvolution algorithms. Algorithms denoted as BMK1 and BMK2 represent the first and second methods in [2], respectively. The algorithm in [7] is denoted as IRLS and the algorithm denoted as CGMK represents the method in [5]. Finally, we denote the non-blind version of the proposed method as ALG-NB.

The comparison between the algorithms for the Cameraman and Lena images, blurred with the uniform blur of size 9x9, is shown in Table 1. For the first set of experiments, the blur support was assumed *a priori* to be 15x15. It should be pointed out that the parameters of ALG are estimated automatically as described in Section 3. On the other hand, the parameters of the IRLS method are manually selected which presents the highest performance in terms of the ISNR metric of both ALG and ALG-NB methods. As can be seen from Table 1, ALG performs very well and the ISNR values obtained are within few tenths of dB from their respective non-blind upper bounds. Additionally, ALG-NB is very competitive with the state of the art CGMK algorithm. Example restored images from Table 1 are shown in Figure 2.

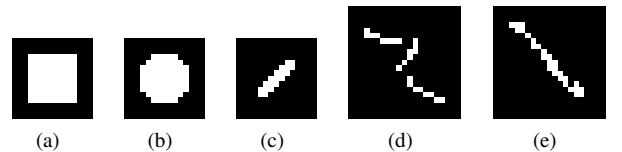


Figure 1: (a) Blur 1: Uniform 9x9, (b) Blur 2: Out-of-focus (radius=4), (c) Blur 3: Diagonal motion, (d) Blur 4: Non-parametric motion (1), (e) Blur 5: Non-parametric motion (2).

Table 1: Comparison with non-blind methods for Uniform 9x9 blur.

BSNR	Cameraman		Lena	
	Method	ISNR[dB]	Method	ISNR[dB]
40dB	<i>BMK1</i>	8.46	<i>BMK1</i>	8.41
	<i>BMK2</i>	8.25	<i>BMK2</i>	8.43
	<i>CGMK</i>	9.02	<i>CGMK</i>	8.74
	<i>IRLS</i>	8.88	<i>IRLS</i>	8.86
	<i>ALG-NB</i>	8.73	<i>ALG-NB</i>	8.79
	<i>ALG</i>	8.63	<i>ALG</i>	8.66
30dB	<i>BMK1</i>	4.47	<i>BMK1</i>	5.61
	<i>BMK2</i>	4.11	<i>BMK2</i>	5.46
	<i>CGMK</i>	6.16	<i>CGMK</i>	6.26
	<i>IRLS</i>	6.14	<i>IRLS</i>	6.29
	<i>ALG-NB</i>	5.73	<i>ALG-NB</i>	6.09
	<i>ALG</i>	4.93	<i>ALG</i>	5.72

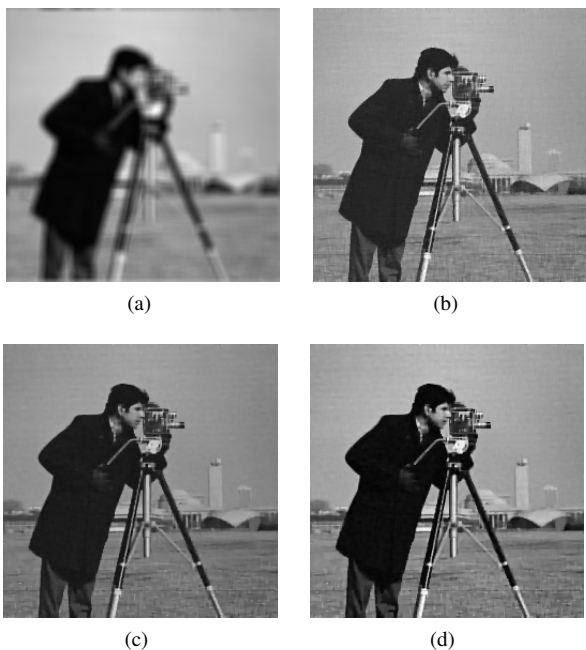


Figure 2: (a) Image degraded by the uniform size 9x9 blur (BSNR=40dB), (b) Restored image using CGMK (ISNR=9.02dB), (c) Restored image using ALG-NB (ISNR=8.73dB), (d) Restored image using ALG (ISNR=8.63dB).

In the second set of experiments, we compared the ALG algorithm with a blind image deconvolution algorithm recently proposed in [1], which will be denoted as AA. The blur support was *a priori* assumed to be 15x15. The ISNR results obtained by the algorithms AA and ALG for the first three blur PSFs are shown in Table 2. It is clear that ALG provides a very competitive restoration performance compared to AA. Example restorations from Table 2 obtained by ALG are shown in Figure 3. Note that compared to AA, no post processing of the restored image is performed while calculating ISNR.

Finally, in the third set of experiments, we evaluated the performance of the ALG algorithm in the presence of non-parametric blurs. The blur support was *a priori* assumed to

Table 2: ISNR values for the Lena image degraded by parametric motion blurs shown in Figure 1.

Image	BSNR [dB]	Blur	Method	ISNR [dB]
Lena	30	Blur 1	AA	4.27
			ALG	5.72
		Blur 2	AA	4.45
			ALG	6.33
		Blur 3	AA	5.74
			ALG	4.90

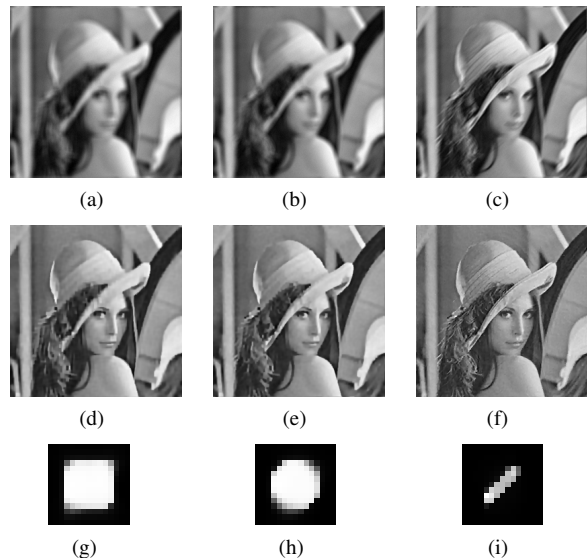


Figure 3: (a,b,c) Images degraded respectively by the uniform size 9x9 blur, circular radius 4 blur and diagonal blur (BSNR=30dB); (d,e,f) Restored images from (a,b,c), respectively; (g,h,i) Restored blurs from (a,b,c), respectively.

be 21x21. The ISNR results are shown in Table 3, and example restored images and blurs are shown in Figure 4. It is clear from the Table 3 and Figure 4 that ALG also performs very well with blindly restoring images exposed to non-parametric blurs. Note that although no ad hoc post-processing methods have been utilized during blur estimation (such as denoising, thresholding, etc), as is the case with many existing algorithms, the blur point spread functions are estimated with high accuracy.

Table 3: ISNR values for the Cameraman image degraded by non-parametric motion blurs shown in Figure 1.

Image	BSNR [dB]	Blur	Method	ISNR [dB]
Cameraman	40	Blur 4	ALG	3.34
		Blur 5	ALG	5.31
	30	Blur 4	ALG	4.68
		Blur 5	ALG	5.79

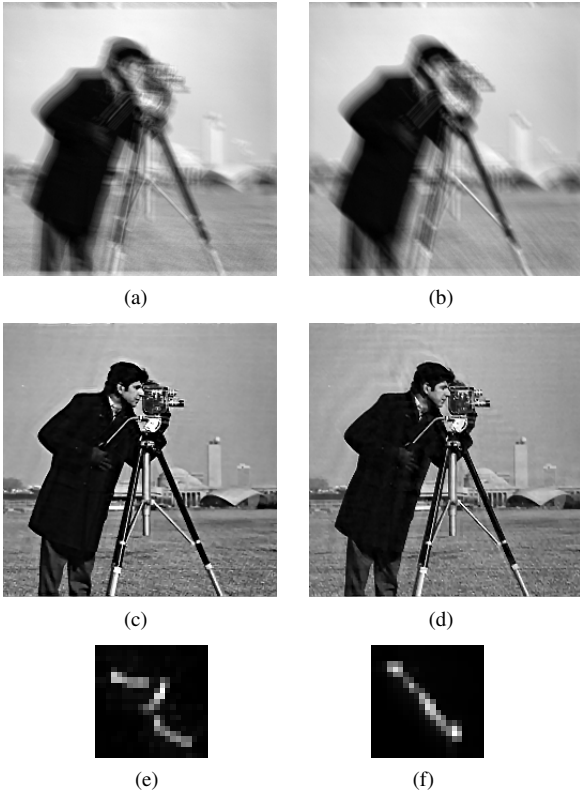


Figure 4: (a,b) Images degraded by the non-parametric blurs (BSNR=40dB); (c,d) Restored images from (a,b), respectively; (e,f) Restored blurs from (a,b), respectively.

5. CONCLUSIONS

In this paper a novel blind image deconvolution algorithm is presented. The proposed algorithm was developed within a Bayesian framework utilizing an l_p -norm based sparse prior on the image, and a total-variation prior on the unknown blur. Experimental results demonstrate that using sparse priors and the proposed parameter estimation, both the unknown image and blur can be estimated with very high accuracy. Furthermore, we have shown that, as a special case of the proposed algorithm, very competitive non-blind image restorations can be obtained if the blurring function is assumed to be known.

Finally, it was shown that the performance of the proposed algorithm is higher than existing state-of-the-art blind image deconvolution algorithms. Future work includes extending the proposed method for blind deconvolution of color images.

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