IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Digital Object Identifier

Sparsity Based Approaches for Distribution Grid State Estimation - A Comparative Study

SHWETA DAHALE, (Student Member, IEEE), HAZHAR SUFI KARIMI, (Student Member, IEEE), Kexing Lai, (Member, IEEE) and Balasubramaniam Natarajan, (Senior Member, IEEE)
Electrical and Computer Engineering Department, Kansas State University

Corresponding author: Shweta Dahale (e-mail: sddahale@ksu.edu).

This material is based upon work supported by the Department of Energy, Office of Energy Efficiency and Renewable Energy (EERE), Solar Energy Technologies Office, under Award Number DE-EE0008767.

ABSTRACT The power distribution grid is typically unobservable due to a lack of measurements. While deploying more sensors can alleviate this issue, it also presents new challenges related to data aggregation and the underlying communication infrastructure. Therefore, developing state estimation methods that enhance situational awareness at the grid edge with compressed measurements is critical. For this purpose, a suite of sparsity-based approaches that exploit the correlation among states/measurements in spatial as well as temporal domains have been proposed recently. This paper presents a systematic comparison and evaluation of these approaches. Specifically, the performance and complexity of spatial methods (1-D compressive sensing and matrix completion) and spatio-temporal methods (2-D compressive sensing and tensor completion) are compared using the IEEE 37 and IEEE 123 bus test systems. Additionally, new robust formulations of these sparsity-based methods are derived and shown to be robust to bad data and network parameter uncertainties. Among the sparsity-based approaches, compressive sensing methods tend to outperform matrix completion and tensor completion methods in terms of error performance.

INDEX TERMS Bad Data, Compressive Sensing, Matrix Completion, Power Distribution, State Estimation

I. INTRODUCTION

THE goal of distribution system state estimation (DSSE) is to infer the system states based on available measurements and network information typically stored in the distribution management system (DMS) database. Conventional DSSE methods encounter significant challenges with increasing system uncertainties due to deployments of distributed energy resources (DERs) and electric vehicles (EVs) [1]. The poor observability of distribution grids [2] due to insufficient measurements further impairs the accuracy of DSSE. Alternatively, an increase in the number of grid edge sensors and smart meters lead to congestion issues in the underlying communication network [3]. Recently, there has been a plethora of efforts to develop sparsity-based DSSE techniques that can simultaneously deal with the issues of unobservability and minimal data access [4], [5]. This paper not only presents a comprehensive comparison of these potential DSSE solutions but also introduces new robust formulations to deal with bad data and parameter uncertainties.

A. RELATED WORK

Weighted Least Squares (WLS) has been the traditional approach for DSSE. In order to guarantee full observability for the WLS-based state estimation, historical data based pseudo-measurements have been used to artificially compensate for insufficient data [6]. However, the WLS + pseudo-measurement state estimation paradigm suffers from huge data requirements [6] and poor estimation performance [7].

Recently the challenges posed by unobservability and limited measurement availability at the grid edge have been addressed by sparsity-based DSSE methods. All these methods exploit the underlying smoothness or sparsity of the raw or linearly transformed measurements/system states. These methods exploit the network structure to perform state estimation at current levels of data availability and observability. Thus, the requirement of creating pseudo-measurements for ensuring observability is eliminated. Compressive Sensing (CS) based DSSE was the first class of solutions proposed where the sparsity of measured data in a linear transformation

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3035378, IEEE Access

basis was exploited to compress measurements. The spatio-temporal correlation between loads and distributed generation in a single-phase distribution system is exploited in the estimation strategies proposed in [4]. A noiseless CS approach is applied to reconstruct the real and reactive power measurements across the grid and estimate the voltage magnitude and angle. [8] extends this DSSE approach to three-phase distribution system. A dynamic compression scheme is employed in [9] on three publicly available datasets for spatial and spatio-temporal compression. Recursive dynamic state estimation is implemented in [10] by exploiting the sparsity in distribution grid data. An efficient dynamic solution for online smart grid topology identification using CS is presented in [11]. Recently, [12] and [5] introduced the idea of implementing DSSE based on matrix and tensor completion algorithms, respectively. Both these methods utilize the sparsity or smoothness of raw measurements. Sparsity in CS indicates the signal of interest is sparse in a specific transform domain. However, in matrix/tensor completion, it indicates that the singular vector of the original matrix/tensor is sparse [13]. Matrix/tensor completion methods impute missing elements in a matrix/tensor by obtaining a suitable low rank approximation of the incomplete matrix/tensor. The matrix completion algorithm exploits the underlying spatial correlation in the data. Recently in [12], matrix completion along with noise-resilient power flow constraints was employed to estimate states in a distribution grid. A matrix completion formulation that is robust to bad data is presented in [14]. An algorithm for dealing with data loss in PMU-based power systems using matrix completion and compressive sensing is proposed in [15] and [16] respectively. Apart from exploiting spatial correlation, the existence of inherent spatio-temporal correlation in states and measurements can be leveraged using tensor completion algorithms. Tensor completion based approaches for DSSE estimation is proposed in [5] and is demonstrated to provide accurate state estimation in low-observable systems. Using low-rank canonical polyadic decomposition, [17] presents a model-free state estimation and energy forecasting framework for distribution systems.

Given that sparsity-based DSSE methods work with limited data, it is important to understand the impact of bad data within this limited set. Conventionally, statistical tests, such as the χ^2-test and the largest normalized residual test are employed for bad data detection and identification, respectively [18]. Both these tests are post-SE processing techniques which rely on least-squares estimated residuals. Least absolute value (LAV) [19] [20], least median of squares (LMS) and least trimmed squares (LTS) [2] estimators have also been used to detect the bad measurements. However, these methods have huge computational cost and high measurement redundancy requirements, which limit their applications in distribution grids with low observability. Therefore, there is a need to integrate and understand the effect of bad data in sparsity-based DSSE methods.

It is important to note that DSSE assumes perfect knowledge of the network parameters. However, the underlying network parameters may be erroneous because of inaccurate manufacturing data, human data entry error, unreported device upgrade or ambient/operating condition variations [21]. Several approaches for parameter error identification have been developed, which are mainly based on residual sensitivity analysis [22] and state vector augmentation [23]. However, these methods are either post-processing methods or run into the observability problem with the increasing scale of the system. The impact of network parameter uncertainties on sparsity-based DSSE methods is currently unknown.

B. CONTRIBUTIONS

The major contributions of this paper can be summarized as follows -

1) The DSSE performance using four methods, i.e., 1-D CS, matrix completion, 2-D CS and tensor completion are compared in a systematic manner. These methods do not require measurements from all the buses at all the times. In particular, two scenarios based on the type of available measurements are considered:
   - Both power and voltage measurements are available
   - Only power measurements are available

   Based on the various standard evaluation metrics, 1-D CS approach estimates the states with higher fidelity relative to matrix completion for both the cases mentioned above. 2-D CS performs better than tensor completion for low observability situations.
2) New robust formulations of the four sparsity-based DSSE approaches are derived. This is the first work that aims to address bad data and system parameter uncertainties in sparsity-based DSSE. Numerical studies demonstrate the capability of the proposed formulations. Relative to the classical WLS method, the error performance of sparsity-based DSSE methods offer nearly 94% improvement with 10% bad data. Gains of 99% relative to WLS can be seen with 25% network parameter errors.
3) Extensive simulations are conducted on IEEE 37 and IEEE 123 bus test systems. The computational complexity associated with the four sparsity-based DSSE methods are compared and tested. It is shown that matrix completion has a higher computational complexity than 1-D CS, while 2-D CS is computationally more complex than tensor completion.

II. SPARSITY-BASED DSSE APPROACHES

DSSE refers to the procedure of obtaining the correct voltage phasors at all nodes based on real-time measurements and the power network model. The measurement set includes power injections at each bus, power flow, and current over each distribution line, along with bus voltages which are related to the states (voltage magnitude and voltage angle) by the non-linear equation (1),

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \boldsymbol{\epsilon} \tag{1}$$

where, $\mathbf{z} \in \mathbb{C}^m$ represents the vector of measurements, $\mathbf{x} \in \mathbb{C}^n$ denotes the vector of states, and the function $\mathbf{h}(\mathbf{x})$ capture the relationship between states and measurements. $\boldsymbol{\epsilon}$ represents the measurement noise vector. The state estimation task is to estimate $\mathbf{x}$ based on $\mathbf{z}$ and knowledge of $\mathbf{h}(.)$. This non-linear function $\mathbf{h}(.)$ relating the voltage states $\mathbf{x}$ and measurements $\mathbf{z}$ can be linearized around an operating point as in [24] resulting in,

$$\mathbf{x} = \mathbf{M}\mathbf{z} + \mathbf{w} \tag{2}$$

where $\mathbf{z} = ((\boldsymbol{P})^T, (\boldsymbol{Q})^T)^T$ represents the measurement vector of active and reactive power injections.

$$\mathbf{M} = \left(\mathbf{Y_{LL}^{-1}}\text{diag}(\bar{\mathbf{w}})^{-1}, -j\mathbf{Y_{LL}^{-1}}\text{diag}(\bar{\mathbf{w}})^{-1}\right) \text{ and,}$$

$$\mathbf{w} = -\mathbf{Y_{LL}^{-1}}\mathbf{Y_{L0}}\mathbf{v_0} \text{ is the zero-load voltage}$$

where, $\mathbf{v_0}$ denote the complex vectors collecting the three phase nodal voltage at the slack bus. Here, $\mathbf{Y_{LL}} \in \mathbb{C}^{3m\times 3m}$ and $\mathbf{Y_{L0}} \in \mathbb{C}^{3m\times 3}$ with m number of buses are the submatrices of the three-phase admittance matrix,

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y_{00}} & \mathbf{Y_{0L}} \\ \mathbf{Y_{L0}} & \mathbf{Y_{LL}} \end{bmatrix} \in \mathbb{C}^{3(m+1)\times 3(m+1)} \tag{3}$$

In order to solve (1), the conventional state estimation method uses a WLS approach. An invertible Jacobian matrix is required to implement WLS. However, this condition cannot be satisfied due to insufficient measurements at the grid edge. In other words, the low observability in distribution grids hinders the applicability of the conventional WLS method for DSSE. To address the low observability issue in DSSE, the correlations among states and measurements are exploited. Specifically, a spatial correlation exists among DERs based on their geographical proximity [4]. Moreover, power injection/consumption patterns of nodes may exhibit temporal correlation. The presence of spatial and/or temporal correlation results in the sparsity of states/measurements in a linear transformation basis. Thus, very few measurements from network buses are required for state estimation. The sparsity characteristics can be exploited by the following DSSE methods.

A. SPATIAL METHODS

Spatial methods such as 1-D CS and matrix completion aim to estimate the voltage states of all nodes at a given time instant.

1) 1-D Compressive Sensing

This technique exploits the sparsity of the signal of interest in a linear transformation basis. It is used to efficiently reconstruct a signal by finding sparse solutions to an underdetermined linear system. Let $\mathbf{x} \in \mathbb{R}^N$ be the original state, compressible in a linear transformation basis such that,

$$\mathbf{x} = \boldsymbol{\Psi}\mathbf{a} \tag{4}$$

where $\mathbf{a}$ has at most $K \ll N$ significant coefficients i.e., $\mathbf{x}$ is K-sparse in sparsifying basis $\boldsymbol{\Psi}$. If the sensing mechanism is such that,

$$\mathbf{y} = \boldsymbol{\Phi}\mathbf{x}; \mathbf{y} \in \mathbb{R}^M, \boldsymbol{\Phi} \in \mathbb{R}^{M\times N}, \tag{5}$$

where, $\boldsymbol{\Phi}$ is a random measurement/projection matrix (e.g., matrix elements distributed as i.i.d. Gaussian random variable with mean 0 and variance $1/M$ or Bernoulli random variables), then the original state $\mathbf{x}$ can be reconstructed by solving the following l_1 minimization problem

$$\hat{\mathbf{a}} = \min_{\mathbf{s}} \|\mathbf{s}\|_1 \quad \text{subject to } \mathbf{y} = \boldsymbol{\Phi\Psi}\mathbf{s} \tag{6}$$

$$\boldsymbol{\Psi}\mathbf{s} = \mathbf{M}\mathbf{z} + \mathbf{w} \tag{7}$$

Here, (7) represents the linearized power-flow constraints. This formulation can be modified to include noisy measurements as shown in [25]. The reconstructed state is

$$\hat{\mathbf{x}} = \boldsymbol{\Psi}\hat{\mathbf{a}} \tag{8}$$

The result of the optimization problem in (6) provides an exact reconstruction with overwhelming probability [25] if there exists a $\delta \in (0, 1)$ such that,

$$(1-\delta)\|\mathbf{s}\|_2^2 \le \|\boldsymbol{\Phi\Psi}\mathbf{s}\|_2^2 \le (1+\delta)\|\mathbf{s}\|_2^2, \tag{9}$$

holds for all K-sparse signal $\mathbf{s}$. This is called the Restricted Isometry property (RIP) of order K. The ratio M/N is termed as Compressed Measurement Ratio (CMR). More details regarding 1-D CS based DSSE can be found in [4].

2) Matrix Completion

To utilize the matrix completion approach for DSSE, a structured matrix $\mathbf{X}$ whose rows correspond to measurement locations and columns correspond to measurement types (e.g. power or voltage) is constructed for a given time. For example, each row represents a bus and each column represents a measurement associated with the bus [12]. Therefore, for every bus b $\in B$, the corresponding row in the matrix $\mathbf{X} \in \mathbb{R}^{n_1\times n_2}$ contains:

$$[\Re(v_b), \Im(v_b), |v_b|, \Re(s_b), \Im(s_b)] \tag{10}$$

where $n_1 = |B|$ and n_2 = 5 quantities per row. Since the distribution grid is unobservable, $\mathbf{X}$ will be an incomplete matrix. The smoothness in spatial variation of the physical power system quantities (voltage, power, etc) translates into low rank property for this matrix $\mathbf{X}$. This low rank property can be exploited to impute the unknown entries of $\mathbf{X}$. Specifically, matrix completion aims to determine the unknown elements in the matrix by minimizing the rank of the matrix. The convex relaxation for rank of a matrix is the nuclear norm. The sampling operator $P_\Omega(\mathbf{X})$ represents the observation matrix as,

$$[P_\Omega(\mathbf{X})]_{ij} = \begin{cases} \mathbf{X}_{ij}, & (ij) \in \Omega \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3035378, IEEE Access

Therefore, the optimization formulation (12) recovers the complete low rank matrix, as

$$\hat{\mathbf{L}} = \min_{\mathbf{L}\in\mathbb{R}^{n_1\times n_2}} \|\mathbf{L}\|_* \quad \text{subject to} \quad P_\Omega(\mathbf{X}) = P_\Omega(\mathbf{L}) \tag{12}$$

$$\mathbf{x} = \mathbf{M}\mathbf{z} + \mathbf{w} \tag{13}$$

where, (13) captures the linearized power-flow constraint relating voltage states $\mathbf{x}$ to power measurements $\mathbf{z}$. More information on matrix completion based DSSE can be found in [12].

B. SPATIO-TEMPORAL METHODS

In addition to the correlation in the spatial dimension, temporal correlation is exploited in 2-D CS and tensor completion methods for DSSE.

1) 2-D Compressive Sensing

2-D CS aims at reconstructing the signal with insufficient data using the underlying sparsity across both space and time. Let $\mathbf{X} \in \mathbb{R}^{N_{space}\times N_{time}}$ be the spatio-temporal state over N_{space} nodes and for N_{time} number of observations. It has been shown that $\mathbf{X}$ is sparse in sparsifying basis $\mathbf{\Psi}_{N_{space}}$ and $\mathbf{\Psi}_{N_{time}}$ such that [4],

$$\mathbf{X} = \mathbf{\Psi}_{N_{space}}\mathbf{A}\mathbf{\Psi}^T_{N_{time}} \tag{14}$$

The spatio-temporal compressed sensing of $\mathbf{X}$ corresponds to,

$$\mathbf{Y} = \mathbf{\Phi}_{space}\mathbf{X}\mathbf{\Phi}^T_{time} \tag{15}$$

where, $\mathbf{\Phi}_{space} \in \mathbb{R}^{m_{space}\times N_{space}}$ and $\mathbf{\Phi}_{time} \in \mathbb{R}^{m_{time}\times N_{time}}$ with entries that are i.i.d. Gaussian random variables with zero mean and respective variance of $1/m^2_{space}$ and $1/m^2_{time}$ with $m_{space} \ll N_{space}$ and $m_{time} \ll N_{time}$. The spatio-temporal data is recovered by solving the following l_1 minimization problem

$$\begin{aligned} \hat{\mathbf{a}} = &\min_{\mathbf{s}} \ \|\mathbf{s}\|_1 \\ &\text{subject to} \\ &vec(\mathbf{Y}) = (\mathbf{\Phi}_{space}\otimes\mathbf{\Phi}_{time})(\mathbf{\Psi}_{N_{space}}\otimes\mathbf{\Psi}_{N_{time}})\mathbf{s} \\ &\mathbf{x} = \mathbf{M}\mathbf{z} + \mathbf{w} \end{aligned} \tag{16}$$

Here, $vec(.)$ represents the vectorized version of a matrix and $\otimes$ represents the kronecker product.The reconstructed state is therefore,

$$vec(\hat{\mathbf{X}}) = (\mathbf{\Psi}_{N_{space}}\otimes\mathbf{\Psi}_{N_{time}})\hat{\mathbf{a}} \tag{17}$$

More details on 2-D CS based DSSE can be found in [4].

2) Tensor Completion

Similar to matrix completion, the starting point for tensor completion approach is the construction of a tensor corresponding to system states and measurements. In essence, the matrix corresponding to the state/measurement at one time instant can be extended to a sequence of matrices each corresponding to one time instant. The resulting tensor $\mathcal{T}$ will be of dimension $\mathbb{R}^{m\times 5\times N_t}$ with m buses, N_t time instants and 5 physical variables as stated in (10). $\mathcal{T}$ is a 3-way tensor indexed by two spatial variables and one temporal variable.

The goal of tensor completion is to fill in the missing entries of $\mathcal{T}$ by exploiting the sparsity in the data. Tensor completion utilizes tensor trace norm minimization formulations with linearized power-flow constraints. The tensor trace norm can also be expressed as the convex combination of trace norms of all matrices obtained by unfolding the tensor along all its modes [26]. Additional matrices $\mathcal{M}_1$, $\mathcal{M}_2$,..., $\mathcal{M}_n$ are introduced to eliminate the interdependency in the elements across the different unfoldings. In [5], a simple low rank tensor completion (SiLRTC) approach is considered that uses block coordinate descent to obtain the n matrices $\mathcal{M}_i$ and tensor $\mathcal{X}$. The suffix (i) denotes the unfolding operation applied on the tensor along the mode i. The optimal M_i can be obtained by applying a shrinkage operator $D_\mathcal{T}(\mathcal{X}_{(i)})$, where $D_\mathcal{T}(\mathcal{X}_{(i)}) = U\Sigma_\mathcal{T}V^T$ and $\Sigma_\mathcal{T} = diag(max(\sigma_j - \tau, 0))$. $\mathcal{X}$ is obtained by solving the optimization problem for certain positive values of $\beta_i s$,

$$\begin{aligned} \min_{\mathcal{X}} \quad & \sum_{i=1}^{n}\frac{\beta_i}{2}\|\mathcal{M}_i - \mathcal{X}_{(i)}\|_F^2 \\ \text{subject to} \quad & \mathcal{X}_\Omega = \mathcal{T}_\Omega \\ & \mathbf{x} = \mathbf{M}\mathbf{z} + \mathbf{w} \end{aligned} \tag{18}$$

The $\mathcal{M}_i's$ and $\mathcal{X}$ are alternatively updated until $\mathcal{X}$ converges. More details on tensor completion based DSSE can be found in [5].

III. ROBUST ESTIMATION

The basic sparsity-based DSSE methods were introduced in the previous section. In this section, formulations to systematically ensure robustness are derived.

A. ROBUSTNESS TO BAD DATA

"Bad data" refers to the data measurements that significantly deviate from the underlying actual behavior. They occur due to instrument failures, impulsive communication noises, infrequent instrument calibration, or jammed sensors. In the smart grid context, bad data can also refer to some form of malicious data injections [27], [28], [29], [30]. The sparsity of bad data reflects the fact that cyber-attacks, impulsive noise or faulty sensors are infrequent as compared to the total number of measurements. In practise, prior research efforts [31], [32] have also affirmed that at most 10% of total data ends up being bad. The presence of bad measurement data can degrade state estimation accuracy [33]. This section discusses the robust formulations of sparsity-based approaches in the presence of bad data.

1) Robust CS

Considering bad data and measurement noise, the sensing matrix $\mathbf{y}$ in (5) can be defined as,

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} + \mathbf{o} + \mathbf{e} \tag{19}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3035378, IEEE Access

where, $\mathbf{o} \in \mathbb{R}^M$ and $\mathbf{e} \in \mathbb{R}^M$ represent outlier noise and measurement noise respectively. $\mathbf{o}$ is an unknown vector with its entry $\mathbf{o}(i)$ being non-zero only if $\mathbf{y}(i)$ is a bad datum. The joint reconstruction of both $\mathbf{x}$ and $\mathbf{o}$ essentially reveals the state and identifies the faulty measurements. It is reasonable to assume that the presence of bad data in measurement dataset is sparse. Hence, by exploiting the sparsity of $\mathbf{o}$, we can recover both the states and bad data. The robust optimization formulation corresponds to,

$$\begin{aligned} \min_{\mathbf{a},\mathbf{o}} \quad & \|\mathbf{a}\|_1 + \lambda_1\|\mathbf{o}\|_1 + \lambda_2\|\mathbf{y} - \mathbf{\Phi\Psi a} - \mathbf{o}\|_2 \\ \text{subject to} \quad & \mathbf{\Psi a} = \mathbf{Mz} + \mathbf{w} \end{aligned} \tag{20}$$

where λ_1 and λ_2 are the tuning parameters. The reconstructed signal is $\hat{\mathbf{x}} = \mathbf{\Psi}\hat{\mathbf{a}}$. A similar robust formulation is obtained for 2-D CS by incorporating the l_1 minimization of bad data vector in (16).

2) Robust Matrix Completion

As mentioned earlier, the partially observed matrix $\mathbf{X}$ serves as the starting point for matrix completion. However, entry-wise measurement noise and outlier noise could exist. This in turn may degrade the performance of matrix completion based state estimation. A robust formulation is proposed to withstand these errors. The key idea behind the robust formulation is to recover the low rank matrix as well as outlier matrix from the observed matrix $\mathbf{X}$ at different levels of system observability. The partially observed matrix $\mathbf{X}$ is,

$$\mathbf{X} = \mathbf{L} + \mathbf{S} + \mathbf{Z} \tag{21}$$

where $\mathbf{L}$ – low rank matrix, $\mathbf{S}$ – sparse bad data matrix and $\mathbf{Z}$ – measurement noise matrix

Accordingly, the optimization problem (12) is reformulated to incorporate bad data and measurement noise as,

$$\begin{aligned} \min_{\mathbf{L},\mathbf{S}} \quad & \|\mathbf{L}\|_* + \lambda_1\|vec(\mathbf{S})\|_1 + \lambda_2\|\mathbf{X} - \mathbf{L} - \mathbf{S}\|_F \\ \text{subject to} \quad & \mathbf{x} = \mathbf{Mz} + \mathbf{w} \end{aligned} \tag{22}$$

where λ_1 and λ_2 are the tuning parameters. By solving (22), the low rank matrix ($\mathbf{L}$) and corrupted matrix ($\mathbf{S}$) is jointly recovered [34].

3) Robust Tensor Completion

In the tensor completion formulation (18), we need to remove outliers and recover the low rank tensor based on the global structure of the tensor [35]. Therefore, similar to the matrix completion case, the robust tensor completion formulation of tensor $\mathcal{L}$ corresponds to,

$$\begin{aligned} (\hat{\mathcal{L}}, \hat{\mathcal{S}}) = \min_{\mathcal{L},\mathcal{S}} \quad & \sum_{i=1}^{N} \|\mathcal{L}_{(i)}\|_* + \lambda\|vec(\mathcal{S})\|_1 \\ \text{subject to} \quad & \mathcal{X} = \mathcal{L} + \mathcal{S} \\ & \mathbf{x} = \mathbf{Mz} + \mathbf{w} \end{aligned} \tag{23}$$

where λ is the tuning parameter. $\mathcal{L}$, $\mathcal{S}$ are low rank and bad data tensor respectively.

B. ROBUSTNESS TO NETWORK PARAMETER UNCERTAINTY

In all the DSSE methods discussed above, accurate knowledge of the power network is assumed. The network model parameters include series resistances, reactances, tap values and suseptances. Parameter errors may create serious bias in the state estimate solutions that tend to last for a long time [36]. However, it is reasonable to assume that such errors in network parameters does not exist universally. That is, only a subset of these values may be erroneous i.e., these parameter errors occur sparsely [36]. Consider a general norm minimization problem with objective $\|\mathbf{Ax} - \mathbf{b}\|$ and variable $\mathbf{x}$ along with uncertainty for set of possible values of $\mathbf{A} \in \mathcal{A} \subseteq \mathbb{R}^{m\times n}$. The worst case robust approximation formulation [37] can be applied here. i.e., the goal is to minimize the worst-case error e_{wc},

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & e_{wc}(\mathbf{x}) \\ \text{where} \quad & e_{wc}(\mathbf{x}) = \sup\ \{\|\mathbf{Ax} - \mathbf{b}\| \mid \mathbf{A} \in \mathcal{A}\}. \end{aligned} \tag{24}$$

(24) can be cast as a LP with variables $\mathbf{x}$ and t when $\mathcal{A}$ is the singleton $\mathcal{A} = \{\mathbf{A}\}$,

$$\begin{aligned} \underset{\mathbf{x},t}{\text{minimize}} \quad & t \\ \text{subject to} \quad & -t\mathbf{1} \preceq \mathbf{Ax} - \mathbf{b} \preceq t\mathbf{1} \end{aligned} \tag{25}$$

For DSSE, uncertainties in the network parameter values results in uncertainties in submatrices of admittance matrix $\mathbf{Y_{LL}} \in \mathbb{C}^{3m\times 3m}$ where m is the number of buses. The robust formulation of the power-flow constraints with voltage $\tilde{\mathbf{x}}$ as the optimization variable corresponds to,

$$\begin{aligned} \underset{\tilde{\mathbf{x}},t}{\text{minimize}} \quad & t \\ \text{subject to} \quad & -t\mathbf{1} \preceq \Re\{\mathbf{Y_{LL}x} - (\mathbf{M_{YL}z} - \mathbf{Y_{L0}v_0})\} \preceq t\mathbf{1} \\ & -t\mathbf{1} \preceq \Im\{\mathbf{Y_{LL}x} - (\mathbf{M_{YL}z} - \mathbf{Y_{L0}v_0})\} \preceq t\mathbf{1} \end{aligned} \tag{26}$$

where

$$\mathbf{M_{YL}} := \left(\text{diag}(\bar{\mathbf{w}})^{-1}, -j\text{diag}(\bar{\mathbf{w}})^{-1}\right)$$

Thus, by minimizing the maximum residual error of the powerflow constraint, we can get robust solutions for $\tilde{\mathbf{x}}$. The complete robust formulation of CS with network parameter uncertainty corresponds to,

$$\begin{aligned} \underset{\mathbf{s},t}{\text{minimize}} \quad & \|\mathbf{s}\|_1 + \lambda t \\ \text{subject to} \quad & \mathbf{y} = \mathbf{\Phi\Psi s} \\ & -t\mathbf{1} \preceq \Re\{\mathbf{Y_{LL}}(\mathbf{\Psi s}) - (\mathbf{M_{YL}z} - \mathbf{Y_{L0}v_0})\} \preceq t\mathbf{1} \\ & -t\mathbf{1} \preceq \Im\{\mathbf{Y_{LL}}(\mathbf{\Psi s}) - (\mathbf{M_{YL}z} - \mathbf{Y_{L0}v_0})\} \preceq t\mathbf{1} \end{aligned} \tag{27}$$

where λ is the tuning parameter. Similar robust formulation can be derived for matrix completion and tensor completion approaches. Due to space constraints, those formulations have been omitted.

IV. COMPUTATIONAL COMPLEXITY

1-D CS based method involves an l_1 minimization which is solved by linear programming. Let m be the number of variables and n the number of constraints. The computational complexity of solving one newton step is $\mathcal{O}(mn.min(m,n))$ [38]. 2-D CS involves Kronecker product [39] of its sparsifying basis and measurement matrics for each of its d-sections. It solves a single higher dimensional optimization problem of complexity $\mathcal{O}(\prod_{d=1}^{D}(mn.min(m,n))^d)$.

In matrix completion, matrix of dimension $\mathbb{R}^{m\times 5}$ is recovered by minimizing the nuclear norm. The semi-definite optimization problem solved by interior point methods [40] has computational complexity of computing step direction $\mathcal{O}(m^3N_i)$ for N_i iterations. The linearized power-flow constraints involves solving of $5m$ equations whose complexity is $\mathcal{O}(m^3N_i)$.

For tensor completion, consider a state measurement tensor $\mathcal{X}$ of size $\mathbb{R}^{m\times 5\times N_t}$. This tensor can grow along its first and third mode. Thus, the resulting tensor will have $\mathcal{O}(mN_t)$ elements. SiLRTC requires $\mathcal{O}(mN_tN_i\ max(m,N_t))$ computations where N_i is the number of iterations for each frame of $\mathcal{X}$. The linearized power flow constraints require $\mathcal{O}(m^3N_tN_i)$ computations. Based on these complexity computations, 1-D CS has lower computational complexity than matrix completion and 2-D CS approach has higher computational complexity than tensor completion. The exact time complexity associated with these sparsity-based DSSE methods are discussed in the next section.

V. SIMULATION RESULTS AND DISCUSSION

In this section, the four DSSE approaches introduced earlier are evaluated and compared on IEEE 37 and 123 bus test systems. These test systems are standard three-phase unbalanced distribution grids [41]. IEEE 37 test system is a highly unbalanced delta connected system with an operating voltage of 4.8 kV and spot loads. IEEE 123 test system is a Wye connected network operating at nominal voltage of 4.16 kV and spot loads. The CVX solver [42] is utilized to solve the involved optimization formulations. For quantifying the performance of sparsity-based DSSE methods, MAPE (Mean Absolute Percentage Error) and MIAE (Mean Integrated Absolute Error) metrics are used for voltage magnitude and voltage angle, respectively. They are defined as,

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_i-\hat{x}_i}{x_i}\right| \tag{28}$$

$$MIAE = \sum_{i=1}^{N}\frac{|\theta_i-\hat{\theta}_i|}{N} \tag{29}$$

where x_i and θ_i represent the true magnitude and angle at bus i respectively. $\hat{x_i}$ and $\hat{\theta}_i$ are the estimated magnitude and angle, respectively. Different case studies are presented for evaluating the performance of these methods.

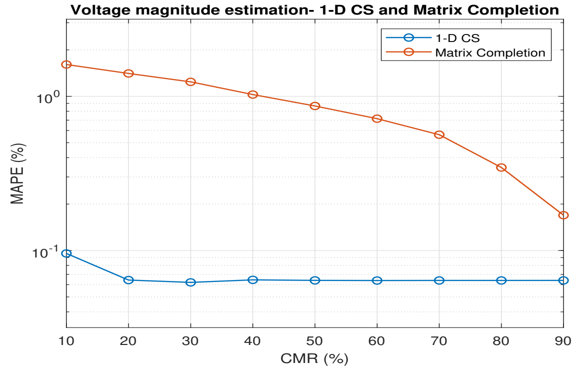

FIGURE 1. Case I: Voltage magnitude performance of spatial approaches

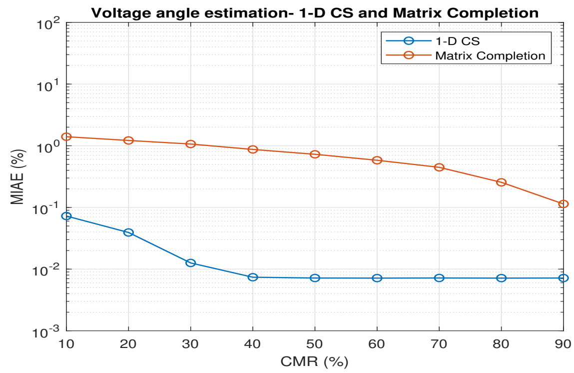

FIGURE 2. Case I: Voltage angle performance of spatial approaches

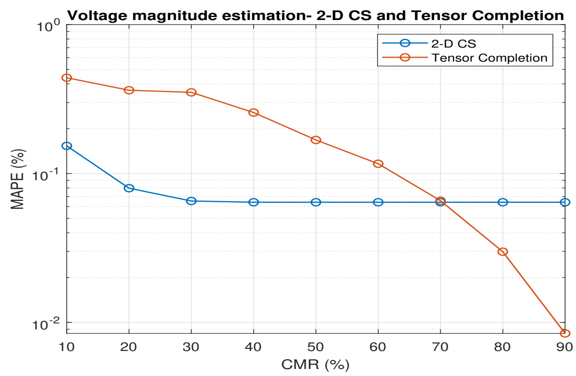

FIGURE 3. Case I: Voltage magnitude performance of spatio-temporal approaches

A. CASE I- POWER AND VOLTAGE MEASUREMENTS

Fig. 1 and Fig. 2 show the comparative performance of spatial approaches with a subset of both power and voltage measurements in the IEEE 37 bus test system. The results illustrate that 1-D CS based approach outperforms matrix completion when we find a proper transformation basis (here, DCT basis is used). This is because, using the transformation basis allows 1-D CS to better exploit the sparsity of states, which in turn aids in effective state recovery from a smaller number of measurements. Similarly, from Fig. 3 and Fig. 4, it can

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3035378, IEEE Access

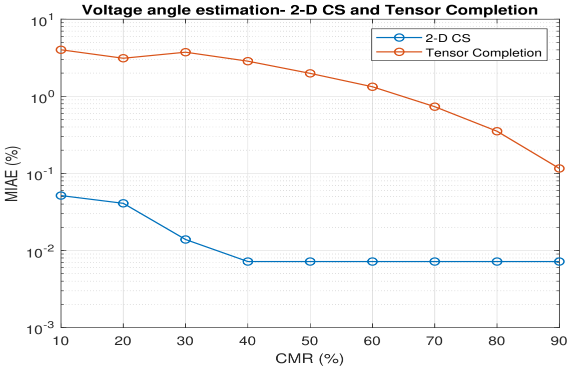

FIGURE 4. Case I: Voltage angle performance of spatio-temporal approaches

be inferred that the 2-D CS outperforms tensor completion at low CMRs. However, at higher CMR, the system becomes more observable and tensor completion perform better than 2-D CS since the smoothness across raw measurements is more pronounced and better exploited.

B. CASE II- ONLY POWER MEASUREMENTS

In distribution grids, the substation bus is selected as the slack bus and the remaining buses are usually modeled as PQ buses [43]. In reality, sensors with voltage measuring capability may not be deployed because of additional infrastructure costs. Therefore, voltage measurements at non-slack buses are normally unavailable. To simulate this scenario, we compare the performance of DSSE methods with traditional measurements [44] (only power measurements at the non-slack bus). It can be inferred from Fig. 5 and Fig. 6 that the matrix completion method delivers a poor performance since the entries of voltage columns in the observed matrix are all unfilled. Similarly, tensor completion in Fig. 7 and Fig. 8 also performs poorly in estimating the voltage states. Both 1-D CS and 2-D CS based methods, however, reconstruct the states with high fidelity even at 40% CMRs.

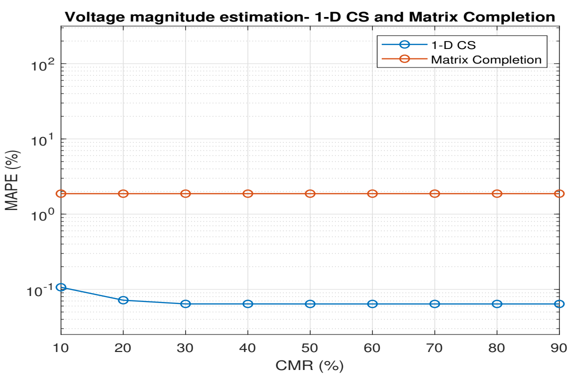

FIGURE 5. Case II : Voltage magnitude performance of spatial approaches

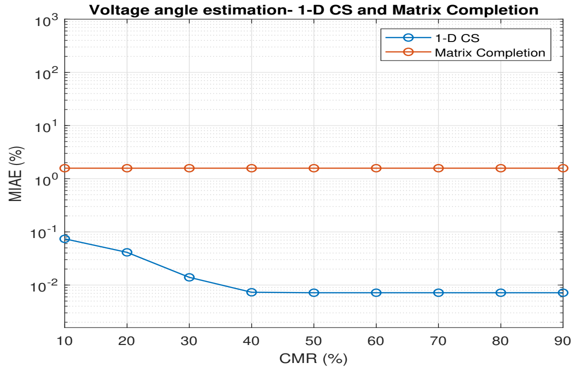

FIGURE 6. Case II : Voltage angle performance of spatial approaches

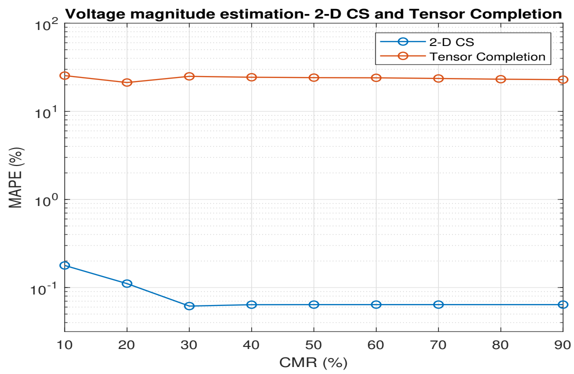

FIGURE 7. Case II : Voltage magnitude performance of spatio-temporal approaches

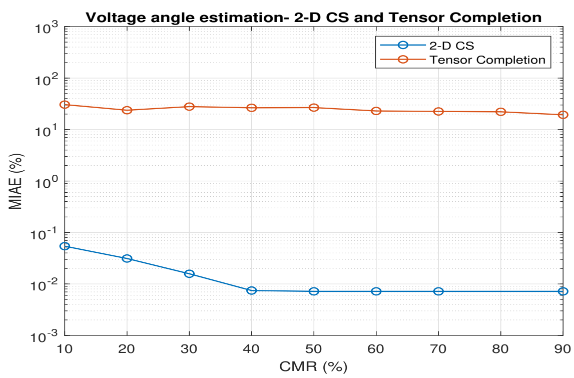

FIGURE 8. Case II : Voltage angle performance of spatio-temporal approaches

C. CASE III - ROBUSTNESS TO BAD DATA

The robustness of DSSE methods is evaluated on the IEEE 37 bus test system with power and voltage measurement data. The percentage of bad data is increased from 0% to 10% of the total measurements with a fixed CMR of 75%. For each percentage, 50 cases are generated with different randomly generated bad data. We assume that the measurement noise distribution has mean zero and standard deviation of 1% of the actual parameter value, and the standard deviation of bad measurements is 100% of the actual value. The performance

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3035378, IEEE Access

of WLS, robust 1-D CS and matrix completion is illustrated in the presence of bad data and measurement noise in Fig. 9 and Fig. 10. It can be inferred that WLS is more sensitive to bad data than the sparse-aware approaches. For instance, with the presence of 10% of bad data, the magnitude performance of the matrix completion approach is 94% better compared to the WLS method.

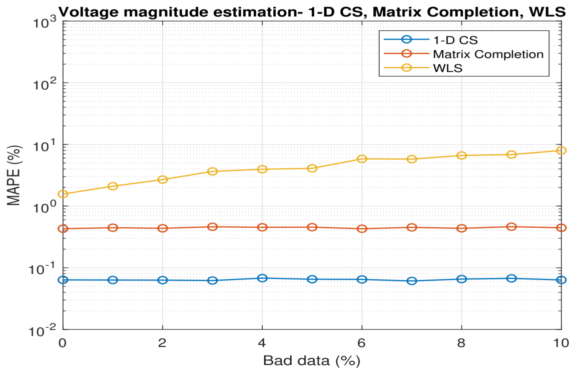

FIGURE 9. Case III: Voltage magnitude performance of spatial approaches for different bad data

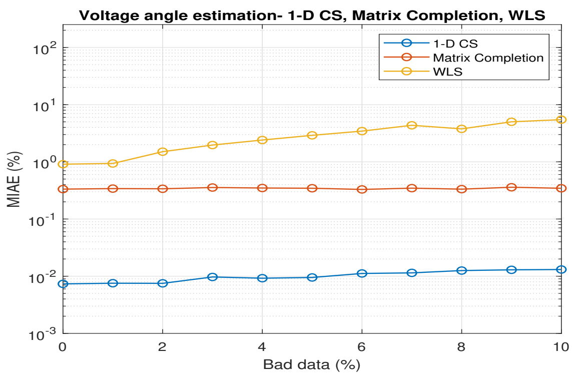

FIGURE 10. Case III: Voltage angle performance of spatial approaches for different bad data

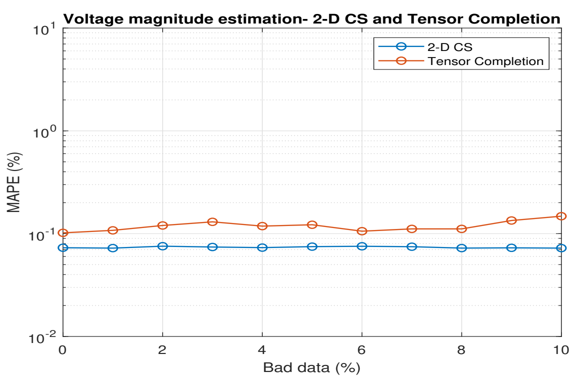

FIGURE 11. Case III: Voltage magnitude performance of spatio-temporal approaches for different bad data

Fig 11 and Fig 12 illustrate the performance of robust versions of tensor completion and 2-D CS based DSSE. These

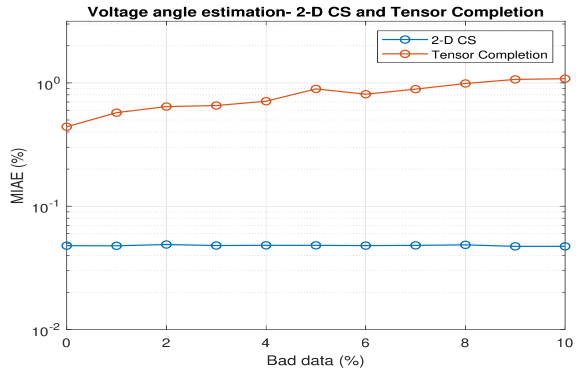

FIGURE 12. Case III: Voltage angle performance of spatio-temporal approaches for different bad data

methods deliver comparable performance in terms of voltage magnitude estimation. However, 2-D CS based method has improved accuracy in voltage angle estimation, compared to tensor completion. This is because, tensor completion focuses more on voltage magnitude error reduction relative to voltage angle accuracy due to the different scales of those constituent tensor elements.

D. CASE IV - ROBUSTNESS TO NETWORK PARAMETER UNCERTAINTY

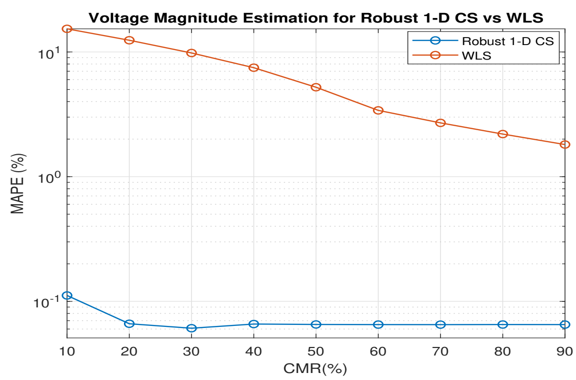

FIGURE 13. Case IV: Voltage magnitude performance of spatial approaches in presence of network parameter uncertainty

The performance of the robust formulation in (27) is compared against WLS for different CMRs in Fig. 13 and Fig. 14. The measurement set includes real power ($3m$), reactive power ($3m$) and voltage phasors ($3m$). The states to be estimated are voltage magnitude ($3m$) and voltage angle ($3m$). Since implementing the WLS approach requires a full rank jacobian matrix, i.e., the number of measurements needs to be equal to the number of states, this approach is not feasible with less than 66% measurements. Therefore, pseudo-measurements based on historical data of the network are added to make the system observable for CMR less than 66%. To simulate network parameter uncertainties, the resistance in the branch 2-3 for phase A is intentionally made erroneous by 10 p.u. Also, the reactance in the branch 12-13 for phase

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3035378, IEEE Access

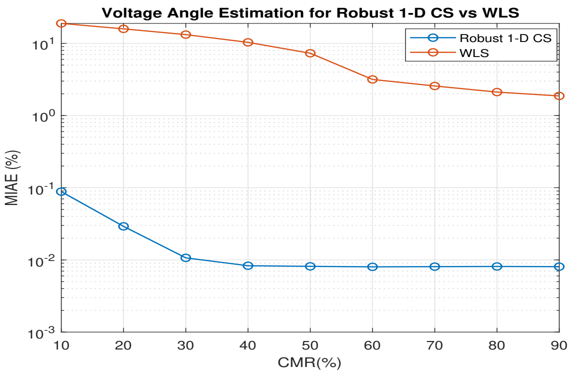

FIGURE 14. Case IV: Voltage angle performance of spatial approaches in presence of network parameter uncertainty

B is modified from 0.29 p.u to 4.65 p.u. The measurements are generated with this perturbed model whereas the model known to the estimator remains unchanged. It can be inferred that the states estimated using the WLS approach is not robust to parameter uncertainty. However, the 1-D CS based approach reconstructs the states with high fidelity even at low CMR values. Also, the WLS estimates are poor as the pseudo-measurements are inaccurate compared to the real-time measurements.

E. CASE V - IEEE 123 BUS TEST SYSTEM

To demonstrate the scalability of the four sparsity-based approaches, they are implemented on the IEEE 123 bus test system. In this case, both power and voltage measurements are assumed available and the accuracy of DSSE using spatial and spatio-temporal approaches are depicted in Fig. 15-16 and Fig. 17-18 respectively. It can be inferred that 1-D CS performs better at all observability conditions compared to matrix completion. Tensor completion performs better for higher data availabilities than 2-D CS. These results are consistent with the findings for the IEEE 37 bus test system.

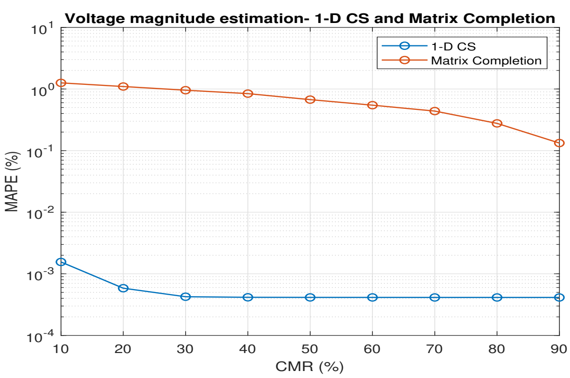

FIGURE 15. Case V: Voltage magnitude performance of spatial approaches

The time required for each of these sparsity-based methods for one Monte-Carlo simulation and one CMR is tabulated in Table 1. These time calculations are performed on intel i5 core, 8 GB RAM processor with CVX Mosek package. In accordance with the complexity analysis in section IV, the time complexity of 1-D CS is lowest than all other methods. Matrix completion and tensor completion methods have higher time complexity than 1-D CS but lower than 2-D CS.

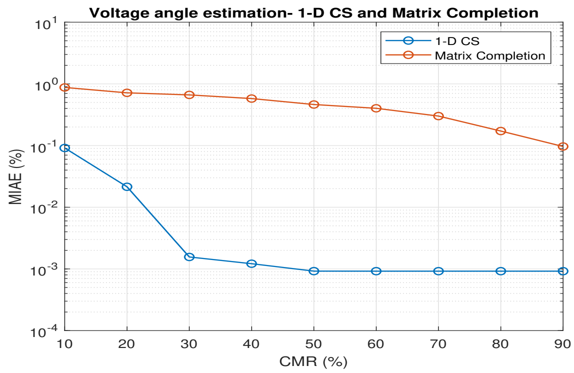

FIGURE 16. Case V: Voltage angle performance of spatial approaches

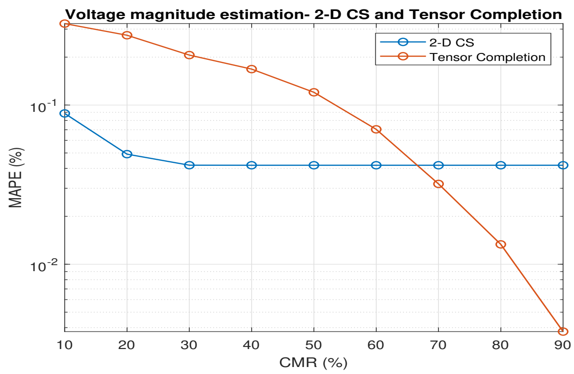

FIGURE 17. Case V: Voltage magnitude performance of spatio-temporal approaches

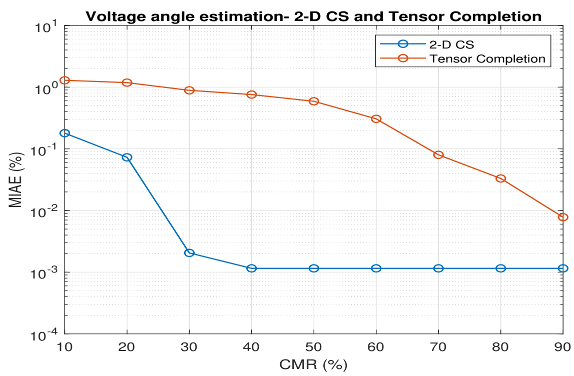

FIGURE 18. Case V: Voltage angle performance of spatio-temporal approaches

VI. CONCLUSION

This paper provides a comparative study of four sparsity-based DSSE approaches. In addition to the comparison of

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3035378, IEEE Access

TABLE 1. Time requirement for 1 run, 1 CMR

Sparsity-based methods	Time (IEEE 37 bus)	Time (IEEE 123 bus)
1-D CS	3.6s	20.5s
Matrix Completion	10.7s	65.1s
2-D CS	83.9s	411.9s
Tensor Completion	21.1s	132.2s

error performance, new robust formulation for these approaches are derived to tackle the presence of bad data and network parameter uncertainties. Numerical studies show the superior performance of compressive sensing based approaches, compared to matrix completion and tensor completion based methods. The performance gains of CS based methods are especially pronounced in low observability conditions. Furthermore, the proposed robust optimization formulations of sparse-aware approaches are compared with conventional WLS based estimation method and shown to offer significant performance improvements. Future efforts will focus on developing low complexity and distributed implementations of 2-D CS and tensor completion based DSSE methods.

REFERENCES

[1] J. Wu, Y. He, and N. Jenkins, "A robust state estimator for medium voltage distribution networks," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1008–1016, 2012.

[2] K. Dehghanpour, Z. Wang, J. Wang, Y. Yuan, and F. Bu, "A survey on state estimation techniques and challenges in smart distribution systems," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 2312–2322, 2018.

[3] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on information theory*, vol. 46, no. 2, pp. 388–404, 2000.

[4] S. S. Alam, B. Natarajan, and A. Pahwa, "Distribution grid state estimation from compressed measurements," *IEEE Transactions on Smart Grid*, vol. 5, no. 4, pp. 1631–1642, 2014.

[5] R. Madbhavi, H. S. Karimi, B. Natarajan, and B. Srinivasan, "Tensor completion based state estimation in distribution systems," *11th Conference on Innovative Smart Grid Technologies, Washington DC, Feb*, 2020.

[6] E. Manitsas, R. Singh, B. C. Pal, and G. Strbac, "Distribution system state estimation using an artificial neural network approach for pseudo measurement modeling," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 1888–1896, 2012.

[7] K. A. Clements, "The impact of pseudo-measurements on state estimator accuracy," in *2011 IEEE Power and Energy Society General Meeting*. IEEE, 2011, pp. 1–4.

[8] H. S. Karimi and B. Natarajan, "Compressive sensing based state estimation for three phase unbalanced distribution grid," in *GLOBECOM 2017-2017 IEEE Global Communications Conference*. IEEE, 2017, pp. 1–6.

[9] A. Joshi, L. Das, B. Natarajan, and B. Srinivasan, "A framework for efficient information aggregation in smart grid," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 4, pp. 2233–2243, 2018.

[10] H. S. Karimi and B. Natarajan, "Recursive dynamic compressive sensing in smart distribution systems," in *2020 IEEE Power & Energy Society Innovative Smart Grid Technologies Conference (ISGT)*. IEEE, 2020, pp. 1–5.

[11] M. Babakmehr, M. G. Simões, M. B. Wakin, and F. Harirchi, "Compressive sensing-based topology identification for smart grids," *IEEE Transactions on Industrial Informatics*, vol. 12, no. 2, pp. 532–543, 2016.

[12] P. L. Donti, Y. Liu, A. J. Schmitt, A. Bernstein, R. Yang, and Y. Zhang, "Matrix completion for low-observability voltage estimation," *IEEE Transactions on Smart Grid*, 2019.

[13] X. P. Li, L. Huang, H. C. So, and B. Zhao, "A survey on matrix completion: perspective of signal processing," *arXiv preprint arXiv:1901.10885*, 2019.

[14] B. Liu, H. Wu, Y. Zhang, R. Yang, and A. Bernstein, "Robust matrix completion state estimation in distribution systems," *arXiv preprint arXiv:1902.02009*, 2019.

[15] M. Liao, D. Shi, Z. Yu, W. Zhu, Z. Wang, and Y. Xiang, "Estimate the lost phasor measurement unit data using alternating direction multipliers method," in *2018 IEEE/PES Transmission and Distribution Conference and Exposition (T&D)*. IEEE, 2018, pp. 1–9.

[16] V. Thadikemalla, I. Srivastava, S. Bhat, and A. Gandhi, "Data loss mitigation mechanism using compressive sensing for smart grids," in *2020 IEEE International Conference on Power Electronics, Smart Grid and Renewable Energy (PESGRE2020)*. IEEE, 2020, pp. 1–6.

[17] A. S. Zamzam, Y. Liu, and A. Bernstein, "Model-free state estimation using low-rank canonical polyadic decomposition," *arXiv preprint arXiv:2004.05741*, 2020.

[18] A. Monticelli, "Electric power system state estimation," *Proceedings of the IEEE*, vol. 88, no. 2, pp. 262–282, 2000.

[19] M. Göl and A. Abur, "Lav based robust state estimation for systems measured by pmus," *IEEE Transactions on Smart Grid*, vol. 5, no. 4, pp. 1808–1814, 2014.

[20] M. K. Celik and A. Abur, "A robust wlav state estimator using transformations," *IEEE Transactions on Power Systems*, vol. 7, no. 1, pp. 106–113, 1992.

[21] A. Abur and A. G. Exposito, *Power system state estimation: theory and implementation*. CRC press, 2004.

[22] W.-H. Liu, F. F. Wu, and S.-M. Lun, "Estimation of parameter errors from measurement residuals in state estimation (power systems)," *IEEE Transactions on power systems*, vol. 7, no. 1, pp. 81–89, 1992.

[23] W.-H. Liu and S.-L. Lim, "Parameter error identification and estimation in power system state estimation," *IEEE Transactions on power systems*, vol. 10, no. 1, pp. 200–209, 1995.

[24] A. Bernstein and E. Dall'Anese, "Linear power-flow models in multiphase distribution networks," in *2017 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe)*. IEEE, 2017, pp. 1–6.

[25] E. J. Candès and M. B. Wakin, "An introduction to compressive sampling," *IEEE signal processing magazine*, vol. 25, no. 2, pp. 21–30, 2008.

[26] J. Liu, P. Musialski, P. Wonka, and J. Ye, "Tensor completion for estimating missing values in visual data," *IEEE transactions on pattern analysis and machine intelligence*, vol. 35, no. 1, pp. 208–220, 2012.

[27] B. Chen, H. Li, and B. Zhou, "Real-time identification of false data injection attacks: A novel dynamic-static parallel state estimation based mechanism," *IEEE Access*, vol. 7, pp. 95 812–95 824, 2019.

[28] M. I. Oozeer and S. Haykin, "Cognitive risk control for mitigating cyber-attack in smart grid," *IEEE Access*, vol. 7, pp. 125 806–125 826, 2019.

[29] D. An, Q. Yang, W. Liu, and Y. Zhang, "Defending against data integrity attacks in smart grid: A deep reinforcement learning-based approach," *IEEE Access*, vol. 7, pp. 110 835–110 845, 2019.

[30] M. E. Bento, D. Dotta, R. Kuiava, and R. A. Ramos, "A procedure to design fault-tolerant wide-area damping controllers," *IEEE Access*, vol. 6, pp. 23 383–23 405, 2018.

[31] D. Duan, L. Yang, and L. L. Scharf, "Phasor state estimation from pmu measurements with bad data," in *2011 4th IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*. IEEE, 2011, pp. 121–124.

[32] W. Xu, M. Wang, J.-F. Cai, and A. Tang, "Sparse error correction from nonlinear measurements with applications in bad data detection for power networks," *IEEE Transactions on Signal Processing*, vol. 61, no. 24, pp. 6175–6187, 2013.

[33] V. Kekatos and G. B. Giannakis, "Distributed robust power system state estimation," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1617–1626, 2012.

[34] Z. Zhou, X. Li, J. Wright, E. Candes, and Y. Ma, "Stable principal component pursuit," in *2010 IEEE international symposium on information theory*. IEEE, 2010, pp. 1518–1522.

[35] D. Goldfarb and Z. Qin, "Robust low-rank tensor recovery: Models and algorithms," *SIAM Journal on Matrix Analysis and Applications*, vol. 35, no. 1, pp. 225–253, 2014.

[36] Y. Lin and A. Abur, "Robust state estimation against measurement and network parameter errors," *IEEE Transactions on power systems*, vol. 33, no. 5, pp. 4751–4759, 2018.

[37] S. Boyd, S. P. Boyd, and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[38] R. G. Baraniuk, "Compressive sensing [lecture notes]," *IEEE signal processing magazine*, vol. 24, no. 4, pp. 118–121, 2007.

[39] M. F. Duarte and R. G. Baraniuk, "Kronecker compressive sensing," *IEEE Transactions on Image Processing*, vol. 21, no. 2, pp. 494–504, 2011.

[40] T. Bouwmans and E. H. Zahzah, "Robust pca via principal component pursuit: A review for a comparative evaluation in video surveillance," *Computer Vision and Image Understanding*, vol. 122, pp. 22–34, 2014.
[41] K. Schneider, B. Mather, B. Pal, C.-W. Ten, G. Shirek, H. Zhu, J. Fuller, J. L. R. Pereira, L. F. Ochoa, L. R. de Araujo *et al.*, "Analytic considerations and design basis for the ieee distribution test feeders," *IEEE Transactions on power systems*, vol. 33, no. 3, pp. 3181–3188, 2017.
[42] M. Grant and S. Boyd, "Cvx: Matlab software for disciplined convex programming, version 2.1," 2014.
[43] S. Bhela, V. Kekatos, and S. Veeramachaneni, "Enhancing observability in distribution grids using smart meter data," *IEEE Transactions on Smart Grid*, vol. 9, no. 6, pp. 5953–5961, 2017.
[44] G. N. Korres and N. M. Manousakis, "State estimation and bad data processing for systems including pmu and scada measurements," *Electric Power Systems Research*, vol. 81, no. 7, pp. 1514–1524, 2011.

SHWETA DAHALE received the B.Tech. degree in Electrical Engineering from College of Engineering, Pune, India, in 2016, and the M. Tech. degree in electrical engineering from Indian Institute of Technology Gandhinagar, Gujarat, India, in 2018. She is currently a Ph.D. candidate at the Kansas State University, Manhattan, KS, USA. Her research interests include optimization, machine learning and state estimation in smart grids.

HAZHAR SUFI KARIMI received his B.Sc. degree in Electrical Engineering from Sahand University of Technology in 2012, and the M.Sc. degree in Electrical Engineering (Control) from Tarbiat Modares University, Iran, in 2014. He is currently a Ph.D. candidate at the Kansas State University, Manhattan, KS, USA. His research interests include statistical signal processing, estimation theory, machine learning, and power distribution systems.

KEXING LAI (M'19) received the Ph.D. degree in electrical and computer engineering from The Ohio State University, Columbus, OH, in 2019. He is currently a Research Assistant Professor at Kansas State University, Manhattan, KS. His research focuses on power system operation and planning, power system protection, application of multi-agent and machine learning in power systems, state estimation and resilience analysis of power systems.

BALASUBRAMANIAM NATARAJAN (SM'08) received the B.E. degree (Hons.) in electrical and electronics engineering from Birla Institute of Technology and Science, Pilani, India, Ph.D. degree in electrical engineering from Colorado State University, Fort Collins, CO, USA, Ph.D. degree in Statistics from Kansas State University, Manhattan, KS, USA, in 1997, 2002, and 2018, respectively. He is currently a Clair N. Palmer and Sara M. Palmer Endowed Professor and the Director of the Wireless Communication and Information Processing Research Group. His research interests include statistical signal processing, stochastic modeling, optimization, and control theories. He has worked on and published extensively on modeling, analysis and networked estimation and control of smart distribution grids and cyber physical systems in general. He has published over 200 refereed journal and conference articles and has served on the editorial board of multiple IEEE journals including IEEE Transactions on Wireless Communications.

• • •