Sparticle spectrum and constraints in anomaly mediated supersymmetry breaking models

K. Huitu[a]**, J. Laamanen**[a] **and P.N. Pandita**[a,b,c]

[a]Helsinki Institute of Physics
P.O. Box 64, FIN-00014 University of Helsinki, Finland

[b]Institut für Theoretische Physik und Astrophysik
Universität Würzburg, 97074 Würzburg, Germany

[c]Department of Physics, North Eastern Hill University
Shillong 793 022, India[1]

Abstract

We study in detail the particle spectrum in anomaly mediated supersymmetry breaking models in which supersymmetry breaking terms are induced by the super-Weyl anomaly. We investigate the minimal anomaly mediated supersymmetry breaking models, gaugino assisted supersymmetry breaking models, as well as models with additional residual nondecoupling D-term contributions due to an extra U(1) gauge symmetry at a high energy scale. We derive sum rules for the sparticle masses in these models which can help in differentiating between them. We also obtain the sparticle spectrum numerically, and compare and contrast the results so obtained for the different types of anomaly mediated supersymmetry breaking models.

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1   Introduction

Supersymmetry is at present the only framework in which the Higgs sector of the standard model (SM) is natural. It is, thus, a prominent candidate for physics beyond the SM. Since in nature there are no supersymmetric particles with the same mass as ordinary particles, supersymmetry must be a broken symmetry at low energies. The specific

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[1]Permanent address

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mechanism which breaks supersymmetry is important in determining the sparticle masses and, hence, the experimental signatures of supersymmetry. At present there are several models of supersymmetry breaking. The model of supersymmetry breaking that has been studied most extensively is the gravity mediated [1] supersymmetry breaking model. In this class of models, supersymmetry is assumed to be broken in a hidden sector by fields which interact with the SM particles and their superpartners (the visible particles) only via gravitational interactions. Whereas this mechanism of supersymmetry breaking is simple and appealing, it suffers from the supersymmetric flavor problem. On the other hand, in a different class of models [2], supersymmetry is broken in a hidden sector and transmitted to the visible sector via SM gauge interactions of messenger particles. This mechanism of supersymmetry breaking provides an appealing solution to the supersymmetric flavor problem. Both these types of supersymmetry breaking models have their distinct experimental signatures.

The soft supersymmetry breaking terms in the above breaking mechanisms have contributions originating from the super-Weyl anomaly via loop effects. If gravity and gauge mediation are somehow suppressed, the anomaly mediated contributions can dominate, as may happen, *e.g.*, in brane models [3]. If this happens, then this mechanism of supersymmetry breaking is referred to as anomaly mediated supersymmetry breaking (AMSB). Anomaly mediation is a predictive framework for supersymmetry breaking in which the breaking of scale invariance mediates between hidden and visible sectors.

Since the soft supersymmetry breaking parameters are determined by the breaking of the scale invariance, they can be written in terms of the beta functions and anomalous dimensions in the form of relations which hold at all energies. In the minimal supersymmetric standard model (MSSM), the pure anomaly mediated contributions to the soft supersymmetry (SUSY) breaking parameters M_λ (gaugino mass), m_i^2 (soft scalar mass squared), and A_y (the trilinear supersymmetry breaking coupling, where y refers to the Yukawa coupling) can be written as

$$M_\lambda = \frac{\beta_g}{g} m_{3/2}, \tag{1}$$

$$m_i^2 = -\frac{1}{4}\left(\frac{\partial \gamma_i}{\partial g}\beta_g + \frac{\partial \gamma_i}{\partial y}\beta_y\right)m_{3/2}^2, \tag{2}$$

$$A_y = -\frac{\beta_y}{y}m_{3/2}, \tag{3}$$

where $m_{3/2}$ is the gravitino mass, β's are the relevant β functions, and γ's are the anomalous dimensions of the chiral superfields. An immediate consequence of these relations is that supersymmetry breaking terms are completely insensitive to physics in the ultraviolet. The degrees of freedom that are excitable at a given energy, which determine the anomalous dimensions and beta functions, thus completely specify the soft supersymmetry breaking parameters at that energy. In this way the gaugino masses are proportional to their corresponding gauge group β functions with the lightest supersymmetric (SUSY)

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particle being mainly a wino. Analogously, the scalar masses and trilinear couplings are functions of gauge and Yukawa-coupling β functions. However, it turns out that the pure scalar mass-squared anomaly contribution for sleptons is negative [4]. There are a number of proposals for fixing this problem of tachyonic slepton masses [5, 6, 7, 8, 9]. Additional contributions to the slepton masses can arise in a number of ways, but some of the solutions will spoil the most attractive feature of the anomaly mediated models, $i.e.$, the renormalization group (RG) invariance of the soft terms and the consequent ultraviolet insensitivity of the mass spectrum. Nevertheless, there are various options to cure this problem without reintroducing the supersymmetric flavor problem [4, 5]. A simple phenomenologically attractive way of parametrizing the nonanomaly mediated contributions to the slepton masses, so as to cure their tachyonic spectrum, is to add a common mass parameter m_0 to all the squared scalar masses [10], assuming that such an addition does not reintroduce the supersymmetric flavor problem. As noted above, such an addition of a nonanomaly mediated term destroys the attractive feature of the RG invariance of soft masses. However, the RG evolution of the resulting model, nevertheless, inherits some of the simplicity of the pure anomaly mediated relations.

There are several alternative ways to generate these extra contributions to the soft squared masses. In particular there are models of supersymmetry breaking mediated through a small extra dimension, where SM matter multiplets and a supersymmetry breaking hidden sector are confined to opposite four-dimensional boundaries while gauge multiplets lie in the bulk. In this scenario the soft gaugino mass terms are due to the anomaly mediated supersymmetry breaking. On the other hand, scalar masses get contributions from both anomaly mediation and a tiny hard breaking of supersymmetry by operators on the hidden sector boundary. These operators contribute to scalar masses at one loop and this contribution is dominant, thereby making all squared scalar masses positive. The gaugino spectrum is unaltered, and the model resembles an anomaly mediated supersymmetry breaking model with nonuniversal scalar masses [11].

Another class of models, where the problem with tachyonic slepton masses is solved, is the models with additional residual nondecoupling D-term contributions due to extra U(1)'s at a high energy scale [5, 6, 8, 9]. In these models one can preserve the property of having renormalization group invariant soft terms, at least at the one-loop level [9, 12]. An interesting feature in this type of model is that unlike in the minimal AMSB model, one can have a light stop in the spectrum [9]. Furthermore, if the extra U(1) is anomaly-free, then it can be shown [8] that the ultraviolet insensitivity can be preserved to all orders.

In this paper we consider the mass spectra, and the constraints on this spectra, of the anomaly mediated supersymmetric models. In Section 2 we study the sum rules for the scalar and gaugino masses. In Section 3 we obtain the focus points for the soft scalar masses in the general anomaly mediated supersymmetric models. This will help in determining whether large sparticle masses are possible in these models without violating the constraints of naturalness. In Section 4 we present a detailed numerical study of the sparticle spectrum in different anomaly mediated supersymmetry breaking models and

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compare and contrast them. Section 5 is devoted to a discussion and summary of our results.

2 Sum Rules

The mass spectrum of superparticles in a particular supersymmetric model is determined in terms of soft supersymmetry breaking parameters. These can be obtained at the weak scale by numerical solutions of the relevant RG equations for a particular model with specific boundary conditions at the high scale, usually taken to be the grand unified scale. Since there are more supersymmetric particles than supersymmetry breaking parameters, there are several relations between the sparticle masses, which can be written in terms of the sum rules [13]. These sum rules will, in effect, test the validity of a particular supersymmetry breaking model. Thus by examining the relations between the masses of sparticles, one may be able to distinguish between different supersymmetric models. In this section we shall obtain various sum rules involving sparticle masses for different anomaly mediated supersymmetry breaking models.

2.1 Scalar sector

In the case of MSSM with gravity mediated supersymmetry breaking there are seven physical scalar sparticle masses for the first two generations which can be written in terms of four parameters (for a given $\tan\beta = v_2/v_1, v_1$ and v_2 being the vacuum expectation values of the two Higgs doublets of MSSM). This results in three sum rules for the sparticle masses of the first two generations [13], which can be used to test the various assumptions of MSSM with gravity mediated supersymmetry breaking.

In anomaly mediated supersymmetry breaking models, the anomaly mediated part of the soft masses is not running. Since for the first two generations the Yukawa couplings can be neglected, we do not have any contribution coming from the running of parameters to the masses of the first two generations of squarks and sleptons. For the first two generations, we can, therefore, write the physical masses of the squarks and sleptons at any scale as (in the standard notation)

$$M_{\tilde{u}_L}^2 = c_Q m_0^2 + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta + \left(-\frac{11}{50}g_1^4 - \frac{3}{2}g_2^4 + 8g_3^4\right)\frac{m_{3/2}^2}{(16\pi^2)^2}, \quad (4)$$

$$M_{\tilde{d}_L}^2 = c_Q m_0^2 + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta + \left(-\frac{11}{50}g_1^4 - \frac{3}{2}g_2^4 + 8g_3^4\right)\frac{m_{3/2}^2}{(16\pi^2)^2}, \quad (5)$$

$$M_{\tilde{u}_R}^2 = c_u m_0^2 + \frac{2}{3}\sin^2\theta_W M_Z^2 \cos 2\beta + \left(-\frac{88}{25}g_1^4 + 8g_3^4\right)\frac{m_{3/2}^2}{(16\pi^2)^2}, \quad (6)$$

$$M_{\tilde{d}_R}^2 = c_d m_0^2 - \frac{1}{3}\sin^2\theta_W M_Z^2 \cos 2\beta + \left(-\frac{22}{25}g_1^4 + 8g_3^4\right)\frac{m_{3/2}^2}{(16\pi^2)^2}, \quad (7)$$

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$$M_{\tilde{e}_L}^2 = c_L m_0^2 + \left(-\frac{1}{2} + \sin^2\theta_W\right) M_Z^2 \cos 2\beta + \left(-\frac{99}{50}g_1^4 - \frac{3}{2}g_2^4\right)\frac{m_{3/2}^2}{(16\pi^2)^2}, \tag{8}$$

$$M_{\tilde{\nu}}^2 = c_L m_0^2 + \frac{1}{2} M_Z^2 \cos 2\beta + \left(-\frac{99}{50}g_1^4 - \frac{3}{2}g_2^4\right)\frac{m_{3/2}^2}{(16\pi^2)^2}, \tag{9}$$

$$M_{\tilde{e}_R}^2 = c_e m_0^2 - \sin^2\theta_W M_Z^2 \cos 2\beta + \left(-\frac{198}{25}g_1^4\right)\frac{m_{3/2}^2}{(16\pi^2)^2}, \tag{10}$$

where we have parametrized the nonanomaly mediated contribution to the masses via the parameter m_0, and we have assumed that this contribution could be nonuniversal. Thus the parameters c_Q, c_u, c_d, c_L and c_e could all be different from one another. In the case of minimal anomaly-mediated supersymmetry breaking $c_Q = c_u = c_d = c_L = c_e = 1$. We have also included the D-term contribution to the masses in Eqs. (4) – (10). We can use these equations to relate the masses of squarks and sleptons, via sum rules, for different anomaly mediated supersymmetry breaking models.

However, independently of the model, Eqs. (4), (5) and (8), (9) lead to the sum rules

$$M_{\tilde{d}_L}^2 - M_{\tilde{u}_L}^2 = -\cos 2\beta M_W^2, \tag{11}$$

$$M_{\tilde{e}_L}^2 - M_{\tilde{\nu}}^2 = -\cos 2\beta M_W^2, \tag{12}$$

which relate the masses of squarks and sleptons living in the same $SU(2)_L$ doublet. We note that these sum rules are the same as in the gravity mediated supersymmetry breaking models [13]. These sum rules do not depend on the assumption of universal soft breaking mass m_0, and depend only on the D-term contribution to the squark and slepton masses. They are, thus, independent of the supersymmetry breaking model and test only the gauge structure of the effective low energy supersymmetric model. The other sum rules depend on the soft parameters originating from the supersymmetry breaking mechanism, and are thus model dependent. Depending on the coefficients multiplying m_0 in Eqs. (4) – (10), as specified by different SUSY breaking models, we have three additional sum rules. In this section we consider in addition to the minimal anomaly mediated, the gaugino assisted anomaly mediated SUSY breaking model [11] and an AMSB model with additional U(1) and a light stop [9].

2.1.1 Minimal anomaly mediated supersymmetry breaking model

For the minimal model we have $c_Q = c_u = c_d = c_L = c_e = 1$. A third sum rule can then be obtained by taking a linear combination of Eqs. (5) – (9) and (10)

$$2(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) + (M_{\tilde{d}_R}^2 - M_{\tilde{d}_L}^2) + (M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) = \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta, \tag{13}$$

which is identical to the corresponding sum rule in the gravity mediated models [13]. This sum rule depends only on the assumption of a universal m_0, and is, therefore, a test of the universality of the soft scalar masses in AMSB models as well.

There are four remaining relations between the masses of the first two generations of squarks and sleptons in Eqs. (4) – (10). Two of these can be used to obtain expressions for the input parameters m_0 and $m_{3/2}$ in terms of the squark and slepton masses. Thus Eqs. (6), (7), (8), and (10) give

$$
\begin{aligned}
m_0^2 &= M_{\tilde{e}_R}^2 - \frac{44}{3} \tan^4 \theta_W (M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) - 33 \tan^4 \theta_W (M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) \\
&\quad + \left[-\frac{22}{3} \tan^4 \theta_W + \left(1 + \frac{187}{3} \tan^4 \theta_W \right) \sin^2 \theta_W \right] M_Z^2 \cos 2\beta,
\end{aligned}
\tag{14}
$$

$$
\frac{3}{2} g_2^4 \frac{m_{3/2}^2}{(16\pi^2)^2} = (M_{\tilde{e}_R}^2 - M_{\tilde{e}_L}^2) - \frac{9}{4}(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - \left(\frac{1}{2} - \frac{17}{4} \sin^2 \theta_W \right) M_Z^2 \cos 2\beta.
\tag{15}
$$

The remaining two independent equations can then be converted to two additional sum rules. The first one is obtained from a combination of Eqs. (6), (7), and (10):

$$
\begin{aligned}
(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) &+ \frac{3}{4} \left(3 - \frac{3}{11} \cot^4 \theta_W \right) (M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) \\
&= \left[-\frac{1}{2} + \left(\frac{17}{4} - \frac{9}{44} \cot^4 \theta_W \right) \sin^2 \theta_W \right] M_Z^2 \cos 2\beta.
\end{aligned}
\tag{16}
$$

It is well known that the left and right sleptons are almost degenerate in the minimal AMSB model [10]. This can be easily verified explicitly from Eqs. (8) and (10),

$$
(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) \simeq -0.038 M_Z^2 \cos 2\beta - 0.0078 M_2^2,
\tag{17}
$$

where we have used Eq. (1) for the gaugino mass parameters, and the experimental value of $\sin \theta_W^2 = 0.2312$ [14]. Similarly one notices that in all the squark mass differences $M_{\tilde{q}_i}^2 - M_{\tilde{q}_j}^2$ obtained from Eqs. (4) – (10), the contribution coming from the m_0 part and the strong coupling part cancel in the minimal AMSB model. Therefore such mass differences are small compared to the masses themselves.

The mass difference between the right-handed squarks $\tilde{u}_R$ and $\tilde{d}_R$ from the sum rule (16) is much larger than Eq. (17) for the slepton mass difference,

$$
M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 \simeq -85.2(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) - 3.44 M_Z^2 \cos 2\beta \simeq -0.2 M_Z^2 \cos 2\beta + 0.66 M_2^2,
\tag{18}
$$

where we have used Eq. (17) to obtain the last equality. Although the coefficients of the two terms may not be small, it should be noticed that the part with M_Z is constant and for heavy squarks the mass difference between the $\tilde{u}_R$ and $\tilde{d}_R$ squarks remains rather small.

The other independent sum rule can be obtained by combining Eqs. (4), (5), (7), and (10) to give

$$
\begin{aligned}
(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) &+ \left(\frac{9}{4} - \frac{g_2^4}{2g_3^4} \right) (M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - \frac{3}{16} \frac{g_2^4}{g_3^4} (M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2) \\
&= \left[-\frac{1}{2} + \left(\frac{17}{4} - \frac{5}{8} \frac{g_2^4}{g_3^4} \right) \sin^2 \theta_W \right] M_Z^2 \cos 2\beta.
\end{aligned}
\tag{19}
$$

The sum rules (16) and (19) are unique to the minimal anomaly mediated supersymmetry breaking model.

2.1.2 Gaugino assisted AMSB model

In the gaugino assisted anomaly mediated model, it is assumed that the gauge and gaugino fields reside in the bulk of the extra dimension [11]. The hidden sector is not supposed to contain singlets in which case the anomaly mediated contribution is the dominant contribution to the supersymmetry breaking. As a result of the gaugino wave function renormalization and consequent rescaling of the fields, the scalar soft masses receive contributions proportional to the eigenvalues of the quadratic Casimir operators of the relevant gauge group. In this case the coefficients of m_0 in Eqs. (4) – (10) are [11]

$$c_Q = 21/10, \ c_u = 8/5, \ c_d = 7/5, \ c_L = 9/10, \ c_e = 3/5. \tag{20}$$

In the gaugino assisted AMSB model, the parameters m_0 and $m_{3/2}$ can be written in the form

$$
\begin{aligned}
m_0^2 &= \frac{5}{3}(M_{\tilde{e}_R}^2 + \sin^2\theta_W M_Z^2 \cos 2\beta) + \left(\frac{9}{55}\cot^4\theta_W - 3\right)^{-1}\Big[5M_{\tilde{e}_R}^2 \\
&\quad -4(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) - 9(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) + (-2 + 22\sin^2\theta_W)M_Z^2 \cos 2\beta\Big],
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
6\left(g_2^4 - \frac{33}{5}g_1^4\right)\frac{m_{3/2}^2}{(16\pi^2)^2} &= 5M_{\tilde{e}_R}^2 - 4(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) - 9(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) \\
&\quad +(-2 + 22\sin^2\theta_W)M_Z^2 \cos 2\beta.
\end{aligned}
\tag{22}
$$

In the previous section it was stated that the sum rule (13) is a test of universality of the mass parameter m_0, since in the universal case the dependence on m_0 cancels. Although the extra contributions to the scalar masses are not universal in the gaugino assisted AMSB model, the dependence on the m_0 accidentally cancels, and the sum rule (13) is valid also in the gaugino assisted AMSB model. Thus even if the physical masses satisfy Eq. (13), one needs further confirmation of the universality of the extra contributions. In the present case, a fourth sum rule, corresponding to Eq. (16), can be obtained as

$$3(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - M_{\tilde{e}_R}^2 = 4\sin^2\theta_W M_Z^2 \cos 2\beta. \tag{23}$$

The simple form of this sum rule follows from the fact that the dependence on m_0 vanishes also in this case. Since the experimental lower bound on the selectron and smuon masses is higher than the Z-boson mass, one can deduce from Eq. (23) that the right-handed up-squark is heavier than the right-handed down-squark. This can also be seen from Fig. 1, where the difference $M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2$ is plotted as a function of the selectron mass $M_{\tilde{e}_R}$.
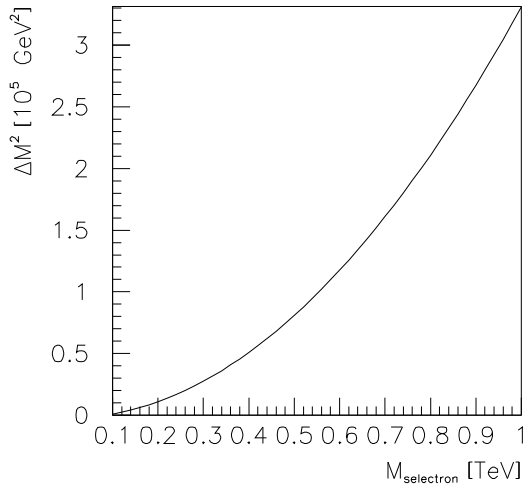
Figure 1: The difference $\Delta M^2 \equiv M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2$ as a function of the right-handed selectron mass $M_{\tilde{e}_R}$. We have plotted the curve for $\tan\beta =10$, but the curves for $\tan\beta > 3$ are all almost identical.

The curves for $\tan\beta > 3$ are all practically identical to the curve shown here, since the dependence on $\tan\beta$ is rather weak. This differs from the minimal AMSB model, where $M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2$ depends on M_2^2 instead of a scalar mass.

The last sum rule, corresponding to the sum rule (19) of the minimal anomaly mediated supersymmetry breaking model, can, in this case, be written as

$$5\left(1+\frac{r}{3}\right)M_{\tilde{e}_R}^2 - 4(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) - (9+2r)(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - \frac{3r}{4}(M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2)$$
$$= \left[2 - \left(\frac{25r}{6}+22\right)\sin^2\theta_W\right]M_Z^2\cos 2\beta, \tag{24}$$

where r is given by $r \equiv (g_2^4 - \frac{33}{5}g_1^4)/(\frac{11}{5}g_1^4 - g_3^4)$.

2.1.3 AMSB model with additional U(1) and a light stop

An interesting possible solution to the tachyonic slepton mass problem is to modify the gauge group of the theory from that of the MSSM gauge group by an additional U(1) factor

group [5, 6, 8, 9]. In the case of MSSM, it is well known that the D-term contributions to the scalar masses are of opposite sign for left- and right-handed particles. However, if the left- and right-handed sleptons both had positive charge under this additional U(1) gauge group, then we could have positive contributions to the slepton masses from D terms, and possibly solve the tachyonic slepton problem. Furthermore, by selecting the charges suitably, one may also have renormalization group invariant soft terms, at least at the one-loop level [9, 12]. Sum rules for a model with Fayet-Iliopoulos D terms due to extra U(1), which are independent of M_W and M_Z, were derived in [8].

In Ref. [9], models with an extra U(1) and RG-invariant sum of the squares of soft SUSY breaking scalar masses were considered. In that paper two models were explicitly constructed, one with a spectrum similar to the minimal AMSB models and another model with a light stop. It turns out that the U(1) charge assignment for getting a light stop is quite constrained. One particular set of the U(1) charges that leads to a light stop is the following:

$$c_Q = 3, \ c_u = -1, \ c_d = -1, \ c_L = 1, \ c_e = 1. \tag{25}$$

Here we will consider the sum rules for this particular light stop model. Since $c_u = c_d$, as well as $c_L = c_e$, the formulas (14) and (15) of the minimal anomaly mediated supersymmetry breaking model are valid in this model as well. Note that the negative soft terms for right-handed squarks are the reason behind the light right-handed stop.

On the other hand, the sum rule (13) is no longer valid. When we take a similar combination as in Eq. (13) of the soft mass parameters, and use the expression for m_0, we find

$$\left(1 - \frac{176}{3}\tan^4\theta_W\right)(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) + \left(2 - 132\tan^4\theta_W\right)(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) + (M_{\tilde{d}_R}^2 - M_{\tilde{d}_L}^2)$$

$$+4M_{\tilde{e}_R}^2 = -\frac{2}{3}\left[-44\tan^4\theta_W + \left(1 + 374\tan^4\theta_W\right)\sin^2\theta_W\right]M_Z^2\cos 2\beta. \tag{26}$$

The sum rule (16) of the minimal AMSB model is valid in this model as well. This is easily verified by taking an appropriate combination of Eqs. (6), (7), and (10), and noticing that in both models $c_u = c_d$, $c_e = 1$, and that the parameter m_0 is determined via Eq. (14) in both these models. The arguments given in section 2.1.1 for the degenerate slepton masses and small mass difference $M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2$ in minimal AMSB are valid in this model as well. From the sum rule (26) one can write approximately

$$M_{\tilde{d}_R}^2 - M_{\tilde{d}_L}^2 \simeq 4.3(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) + 9.9(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - 4M_{\tilde{e}_R}^2 - 2.72 M_Z^2\cos 2\beta. \tag{27}$$

One can easily check that for experimentally allowed parameter values, the right-hand side of Eq. (27) is always negative and not small, and it decreases with the increasing selectron mass.

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To complete the derivation of independent sum rules of this case we give the sum rule corresponding to Eq. (19):

$$(M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2) + \frac{8}{3}(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - \left(\frac{2}{3}\frac{g_3^4}{g_2^4} + \frac{11}{3}\tan^4\theta_W\right)\left[8(M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) + 18(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2)\right]$$

$$+2M_{\tilde{e}_R}^2 = \left[\left(\frac{2}{3}\frac{g_3^4}{g_2^4} + \frac{11}{3}\tan^4\theta_W\right)(4 - 34\sin^2\theta_W) + \frac{4}{3}\sin^2\theta_W\right]M_Z^2\cos 2\beta. \tag{28}$$

2.1.4 Third generation of scalars

So far we have considered the first two generations of squarks and sleptons for which the corresponding Yukawa couplings and their runnings can be neglected. For the third family squarks and sleptons the sum rules are complicated because of the large third generation Yukawa couplings. For small values of $\tan\beta$, we can neglect the effects of the bottom-quark Yukawa coupling, and hence the effects of mixing in the bottom-squark mass matrix. Thus in this limit $\tilde{b}_L$ and $\tilde{b}_R$ are still the mass eigenstates, and $\tilde{b}_R$ is degenerate with $\tilde{d}_R$ to a good approximation. However, since the evolution of $M_{\tilde{b}_L}$ is controlled by the top-quark Yukawa coupling, the situation with respect to hierarchy of the sbottom masses and the squark masses of the first two generations in anomaly mediated supersymmetry breaking models can be predicted only when we have a detailed knowledge of input parameters. In the stop sector, it is easy to see that the sum rules one obtains are independent of the model of supersymmetry breaking, and are, thus, identical to those obtained in MSSM with gravity mediated supersymmetry breaking [13]. Similar observations can be made with respect to the sum rules in the stau sector. Furthermore, when $\tan\beta$ is large, the mixing in the third generation squark and slepton mass matrices becomes important, and the situation becomes complicated. Thus, as far as the third generation squarks and sleptons are concerned, one does not obtain any information which could distinguish the anomaly mediated supersymmetry breaking models from the MSSM with gravity mediated supersymmetry breaking.

2.2 Gaugino sector

In all the models discussed in this work, the gaugino sector remains the same as in the minimal AMSB model, for which the mass difference between the lightest chargino and the neutralino is small. The close proximity of the lightest neutralino and chargino masses is a direct consequence of Eq. (1), which gives for the ratios of the gaugino mass parameters $|M_1| : |M_2| : |M_3| \simeq 2.8 : 1 : 7.1$ after taking into account the next to leading order radiative corrections and the weak scale threshold corrections [10]. Thus the winos are the lightest neutralinos and charginos, and one would expect that the lightest chargino is only slightly heavier than the lightest neutralino in all the models considered in Section 2.1.

It is not feasible to obtain mass sum rules for the neutralino states, since the physical neutralino mass matrix is a 4×4 matrix. However, from the trace of neutralino and chargino mass matrices, one obtains a sum rule, which does not contain the Higgs mixing parameter μ, but which is present in the mass matrices [here the gluino mass $m_{\tilde{g}} = M_3(t_{\tilde{g}})$],

$$2(M_{\tilde{\chi}_1^\pm}^2 + M_{\tilde{\chi}_2^\pm}^2) - (M_{\tilde{\chi}_1^0}^2 + M_{\tilde{\chi}_2^0}^2 + M_{\tilde{\chi}_3^0}^2 + M_{\tilde{\chi}_4^0}^2)$$
$$= \frac{1}{9} \left[\frac{g_2^4}{g_3^4} - \left(\frac{33}{5} \right)^2 \frac{g_1^4}{g_3^4} \right] m_{\tilde{g}}^2 + 4M_W^2 - 2M_Z^2. \tag{29}$$

This average mass squared difference of the charginos and neutralinos differs from the corresponding supergravity (SUGRA) result by the factor $(33/5)^2$ in front of g_1^4 within the square brackets, and by factor $(1/9)$ in front of the square bracket. We note that the ratio of the gluino mass to the other gaugino masses is different in the AMSB models and in the MSSM with gravity mediated supersymmetry breaking,

$$\left. \frac{|m_{\tilde{g}}|}{|M_2|} \right|_{\text{AMSB}} = 3 \left. \frac{|m_{\tilde{g}}|}{|M_2|} \right|_{\text{MSSM}}. \tag{30}$$

We have plotted in Fig. 2 the sum rule (29) both in the AMSB models and the MSSM. The average mass difference in the AMSB models is first positive, but then quickly turns negative (solid line), while in the minimal SUGRA model it is always positive (dashed line). Thus this sum rule could be one of the signatures of the AMSB type of models.

3   Focus points

One of the main motivations for low scale supersymmetry is that it can stabilize the large hierarchy between the weak scale and the unification or Planck scale. This can be realized in a straightforward way if the masses of the supersymmetric partners of the standard model particles are of the order of weak scale. On the other hand the existence of superpartners with masses of the order of weak scale is difficult to reconcile with limits on flavor changing processes, unless one assumes an accurate degeneracy of squarks and sleptons. Even with such a degeneracy, the supersymmetric contributions to CP violation become uncomfortably large, unless squarks and sleptons are heavy. Thus one is forced to examine supersymmetric models with large sparticle masses, thereby coming into conflict with the basic idea of introducing weak scale supersymmetry. Recently, it has been pointed out [15] that one can have large squark and slepton masses (above 1 TeV) without losing the naturalness of the underlying supersymmetric theory. This is achieved by exploiting the existence of focus points in the renormalization group evolution of the soft masses, which make the weak scale insensitive to the variations in the unknown supersymmetry breaking parameters at the high scale. In this approach the squark and slepton masses can be large compared to the weak scale, though gaugino and higgsino masses can be generically lighter.
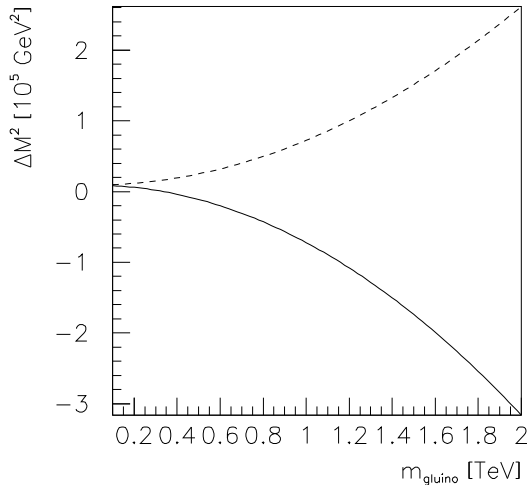
Figure 2: The average mass difference $\Delta M^2 \equiv 2 \sum M^2_{\chi^{\pm}_i} - \sum M^2_{\chi^0_i}$ in the AMSB models (solid line), and in the minimal SUGRA (dashed line) as a function of the gluino mass $m_{\tilde{g}}$.

It is, therefore, important to investigate whether the anomaly mediated supersymmetry breaking models considered in this paper exhibit the focus point behavior so as to admit heavy squark and slepton masses, without violating the principle of naturalness.

In the minimal AMSB model, it has been shown that there is a focus point near the weak scale for the soft supersymmetry breaking Higgs mass parameter $m^2_{H_u}$ if $\tan\beta$ is not very large [16]. The other mass parameters in this model do not have focus points near the weak scale. Since in the more general models studied in this work the soft supersymmetry breaking mass parameters are not universal, it is of considerable interest to find out if there is focus point behavior in these models. In this section we shall consider this question and investigate if these general models exhibit a desirable focus point behavior.

We shall here consider the case of $\tan\beta=10$, for which all couplings other than the top Yukawa coupling can be neglected. Following [16] we denote the supersymmetry breaking bilinear up-type Higgs mass parameter $m^2_{H_u} \equiv m^2_{H_u}|_{AM} + \delta m^2_{H_u}$, where $m^2_{H_u}|_{AM}$ is the pure anomaly mediated value, which does not run, and a similar decomposition for the other scalar mass parameters. Then the coupled renormalization group equations for the

relevant mass parameters are given by

$$\frac{d}{dt}\begin{pmatrix} \delta m_{H_u}^2 \\ \delta m_{U_3}^2 \\ \delta m_{Q_3}^2 \end{pmatrix} = \frac{Y_t^2}{8\pi^2}\begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \delta m_{H_u}^2 \\ \delta m_{U_3}^2 \\ \delta m_{Q_3}^2 \end{pmatrix}. \tag{31}$$

As was done for sfermions in Eqs. (4) – (10), one can define the coefficients c_{H_u} and c_{H_d} which parametrize the nonuniversality for the mass parameters of the two Higgs doublets. Here we need only c_{H_u}, which has a value $c_{H_u} = 1$ in the minimal AMSB model, a value $c_{H_u} = 9/10$ in the gaugino assisted AMSB model, and a value $c_{H_u} = -2$ in the model with an extra U(1) and a light stop. For a set of general initial conditions, $m_0^2(c_{H_u}, c_{U_3}, c_{Q_3})$, the solution can be written as

$$\begin{pmatrix} \delta m_{H_u}^2 \\ \delta m_{U_3}^2 \\ \delta m_{Q_3}^2 \end{pmatrix} = \frac{(c_{H_u} + c_{U_3} + c_{Q_3})}{6}m_0^2 \exp\left[6\int_0^t dt' \frac{Y_t^2}{8\pi^2}\right]\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$
$$+ \frac{m_0^2}{6}\begin{pmatrix} 3(c_{H_u} - c_{U_3} - c_{Q_3}) \\ 2(-c_{H_u} + 2c_{U_3} - c_{Q_3}) \\ (-c_{H_u} - c_{U_3} + 5c_{Q_3}) \end{pmatrix}. \tag{32}$$

It is easy to verify that m_{H_u}, m_{U_3}, and m_{Q_3} can all have focus points only if $c_{H_u} = c_{U_3} = c_{Q_3} = 0$. Thus this case is not of interest here. One can also find conditions that need to be satisfied if two of the mass parameters have focus points. Below we list the two mass parameters that have simultaneous focus points, together with the relevant conditions that need to be satisfied (here $\alpha_t = \exp[6\int_0^t dt' Y_t^2/(8\pi^2)]$):

$$m_{H_u}, m_{U_3}: \quad c_{H_u} = \frac{3}{2}c_{U_3} = \frac{3(1 - \alpha_t)}{5\alpha_t + 1}c_{Q_3},$$

$$m_{H_u}, m_{Q_3}: \quad c_{H_u} = 3c_{Q_3} = \frac{3(1 - \alpha_t)}{2(2\alpha_t + 1)}c_{U_3},$$

$$m_{U_3}, m_{Q_3}: \quad c_{U_3} = 2c_{Q_3} = \frac{2(1 - \alpha_t)}{3(\alpha_t + 1)}c_{H_u}.$$

Obviously these conditions are not realized for the models that we have considered in this paper. Finally, the simple case of a focus point for m_{H_u} is found if $c_{H_u} < c_{U_3} + c_{Q_3}$, for m_{U_3} if $2c_{U_3} < c_{H_u} + c_{Q_3}$, and for m_{Q_3} if $5c_{Q_3} < c_{H_u} + c_{U_3}$. In these inequalities it is assumed that $c_{H_u} + c_{U_3} + c_{Q_3} > 0$. If this sum is negative, then these inequalities should be reversed. We see that for both the minimal and gaugino assisted AMSB models we have a focus point for m_{H_u}. The case of the extra U(1) model is qualitatively different, since $c_{H_u} + c_{U_3} + c_{Q_3} = 0$ by construction in this model, and thus it is not relevant to state at which scale the boundary conditions are given.

In the gaugino assisted AMSB model the expression $\exp\left[6\int_0^t dt' \frac{Y_t^2}{8\pi^2}\right] = 14/23$ at the focus point. In Fig. 3 we depict the running of $m_{H_u}^2$ for the minimal and for the gaugino
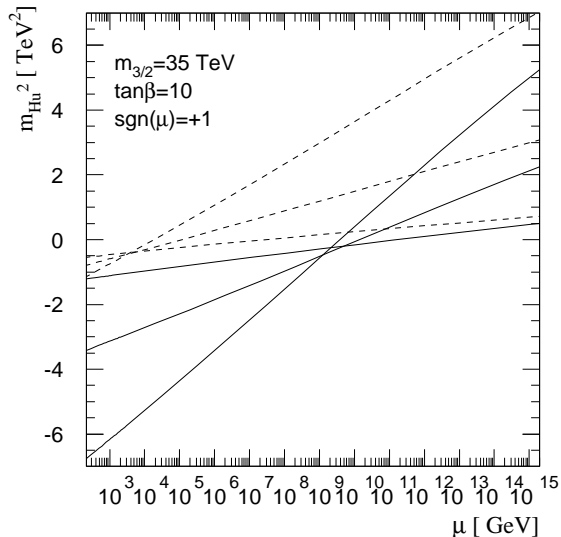
Figure 3: Focus points for the minimal (dashed line) and gaugino assisted (solid line) AMSB models.

assisted AMSB models. Although focus points exist for both these models, only the focus point for the minimal AMSB seems to be physically interesting.

4 Numerical results

So far we have obtained the predictions for the anomaly mediated supersymmetry breaking models in terms of sum rules, which can help in distinguishing between different models. In this section we shall numerically obtain the predictions for the sparticle spectra of these models. For the numerical evaluation of the spectra we have used the program SOFTSUSY [17]. This program uses complete three family mass matrices and Yukawa coupling evolution. Fermion masses and gauge couplings (α, α_s) are evolved to M_Z with the three loop QCD and one loop QED equations. The full MSSM spectrum is taken into account and decoupling threshold effects are taken into account to leading logarithmic order as well as the finite corrections at the scale M_Z. The scalar masses are evolved via one-loop RG equations, and all other functions are evolved via two-loop equations. Parameters are determined iteratively. For the boundary conditions at the GUT scale, we
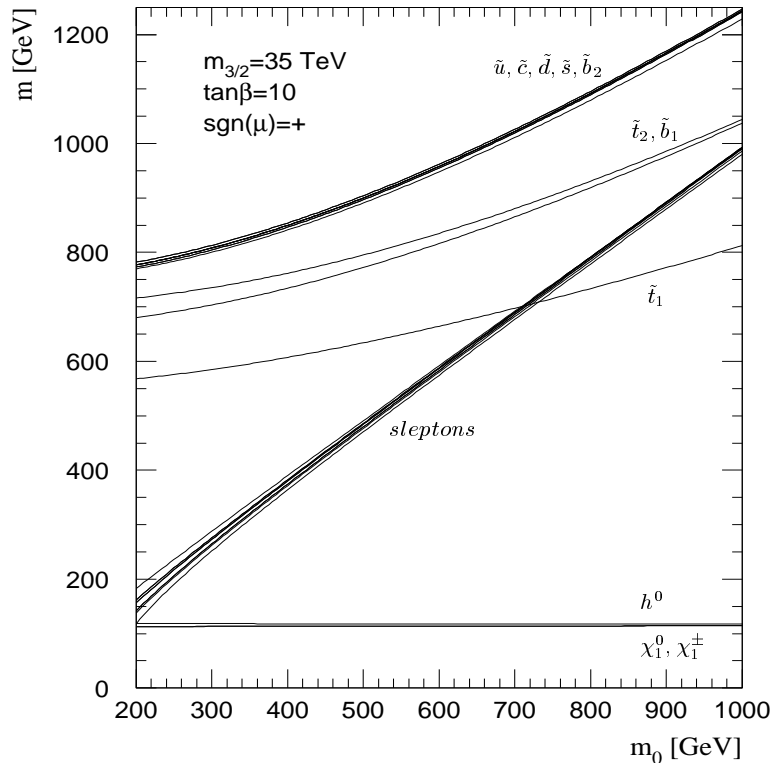
14

Figure 4: The spectrum of the minimal AMSB model.

have used the values given by Eqs. (1), (3), and $m_i^2 + c_i m_0^2$, where m_i^2 is given by Eq. (2), and c_i's are model dependent numerical factors, which are given, *e.g.*, by Eqs. (20) and (25).

In Figs. 4 – 6 we have plotted the spectra for the three different AMSB models. In all three examples we have taken $\tan\beta = 10$ and $m_{3/2} = 35$ TeV. One can easily see the distinguishing features of different models already discussed in terms of the sum rules. In the minimal AMSB model, Fig. 4, the squarks of the first two generations are very closely degenerate in mass. The same is true for the sleptons in this model. The third generation, especially the lighter stop, is considerably lighter than the squarks of the first two generations. For $m_{3/2} = 35$ TeV we see that for a value of $m_0 \sim 730$ GeV, the lightest stop becomes lighter than sleptons.

For the gaugino assisted AMSB model, Fig. 5, the first two generations of sleptons are not degenerate anymore. Recalling that $\tilde{u}_1$ ($\tilde{c}_1$), $\tilde{d}_1$ ($\tilde{s}_1$), and $\tilde{e}_1$ ($\tilde{\mu}_1$) correspond almost exactly to the right-handed squarks and sleptons, we note that the sum rule Eq. (23) is clearly satisfied. On the other hand, the left-handed squarks, corresponding to $\tilde{u}_2$ and $\tilde{d}_2$
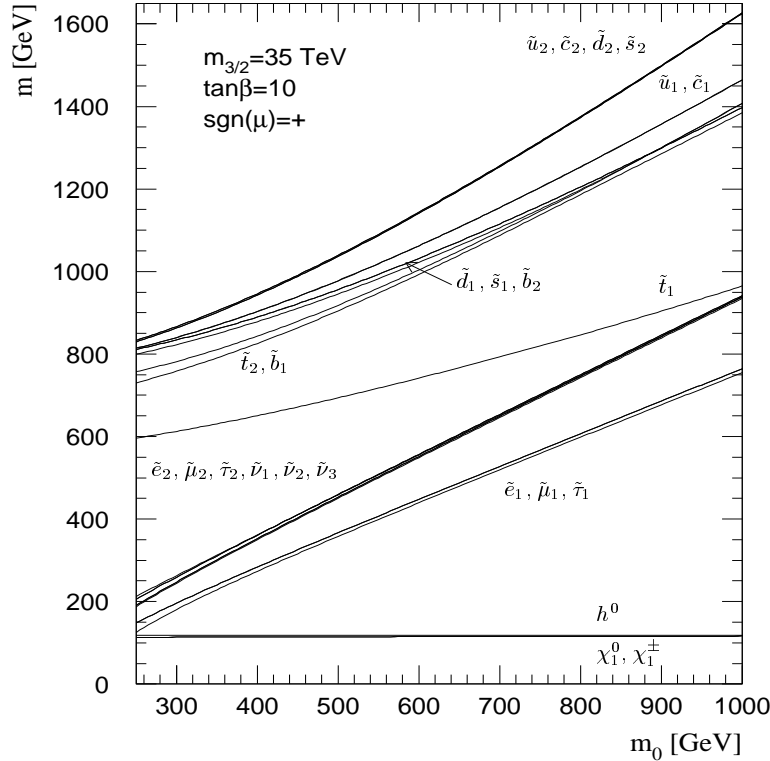
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Figure 5: The spectrum of the gaugino assisted AMSB model.

($\tilde{c}_2$ and $\tilde{s}_2$), or the left-handed sleptons, $\tilde{e}$ and $\tilde{\nu}_1$ ($\tilde{\mu}$ and $\tilde{\nu}_2$), remain close to each other, a reflection of the sum rules Eqs. (11) and (12).

The spectrum of the U(1) model at the weak scale is shown in Fig. 6. It is seen that the spectrum ends with relatively small m_0, since for large enough m_0 the negative charges lead to a tachyonic spectrum. The mass squared difference of squarks $\tilde{d}_L$ and $\tilde{d}_R$ (or $\tilde{s}_L$ and $\tilde{s}_R$) is seen to satisfy the sum rule Eq. (26).

5 Summary and discussion

We have derived sum rules, and obtained the sparticle mass spectra, in different AMSB models in which the problem of tachyonic slepton masses has been solved in qualitatively different ways. Interestingly enough, the sum rules obtained uncover many of the similarities and differences in these models. On the one hand this helps us to tell whether a particular mass spectrum is due to anomaly mediation, and on the other hand it allows us to differentiate between various anomaly mediated models.
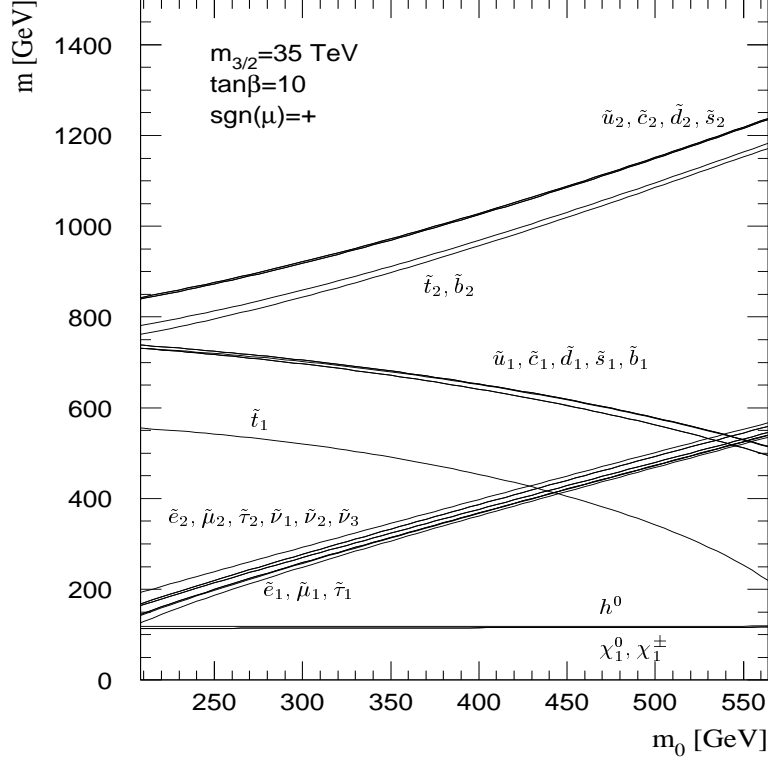
16

Figure 6:   The spectrum of the U(1) model.

Apart from the well known feature of the AMSB gaugino sector, the close mass degeneracy of the lightest chargino and neutralino, the sum rules that we have derived reveal another typical feature of the gaugino sector in these models, namely the average mass difference between charginos and neutralinos. As shown in Fig. 2 the average mass difference between the charginos and neutralinos is typically negative in the AMSB type models. This is contrary to the SUGRA type models, where the average mass difference is positive.

Even if the mass spectrum of sparticles points towards the underlying model being of AMSB type, there could be a wide variety of different possibilities. In such a situation, the sum rules obtained in this paper would help in distinguishing between various models. Comparison of the sfermion mass differences $M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2$ and $M_{\tilde{d}_L}^2 - M_{\tilde{d}_R}^2$ can be crucial in distinguishing between the models considered in this paper. In the minimal AMSB model both these mass differences are small, whereas in the gaugino assisted model slepton mass difference is not small. In the AMSB model with extra U(1) and a light stop, the squark mass difference is considerable, but the slepton mass difference is small. These conclusions

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from the tree-level sum rules of Section 2 have been verified by the one-loop numerical results of Section 4.

In the minimal AMSB model, m_{H_u} has a focus point near the weak scale. We have shown that this attractive feature is, unfortunately, not shared by more general AMSB models.

Finally, we should mention the possibility that the introduction of R-parity violating couplings can solve the tachyonic slepton mass problem. This is an interesting possibility, since if viable, one would have in the supersymmetry breaking sector only those terms which arise due to the anomaly mediation. Although this idea is interesting, it turns out that it is not to easy to find suitable R-parity violating terms for its implementation. In [18] a hierarchy between various R-parity violating couplings was assumed, and it was further assumed that these couplings have quasi fixed points.

However, one of the couplings used contributes to the neutrino masses [19]. Thus it is subject to strict phenomenological bounds, due to which the scenario considered does not seem to be viable in practice. On the other hand, if one assumes that the coupling involved in neutrino masses is tiny, one may still generate large enough masses for all the sleptons with the remaining two couplings used in [18], although the fixed point structure does not hold anymore. Since one has in this case only the two unknown R-parity violating couplings in addition to the mass parameter $m_{3/2}$, one can solve for all these parameters and obtain different sum rules. For the first two generations one has 14 masses, and thus one gets 11 sum rules between the masses. However, we think that this model is not very compelling and do not list these sum rules here.[2]

Acknowledgments

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[2]The couplings and the mass parameter are given by

$$\frac{66}{25}g_1^4\frac{m_{3/2}^2}{(16\pi^2)^2} = -(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) + \sin^2\theta_W M_Z \cos 2\beta,$$

$$2\lambda_{231}\beta(\lambda_{231})\frac{m_{3/2}^2}{(16\pi^2)} = M_{\tilde{e}_R}^2 - 3(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) + 4\sin^2\theta_W M_Z \cos 2\beta,$$

$$\lambda_{132}\beta(\lambda_{132})\frac{m_{3/2}^2}{(16\pi^2)} = M_{\tilde{e}_R}^2 + (M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) - (M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - (M_{\tilde{d}_L}^2 - M_{\tilde{d}_R}^2) + \frac{2}{3}\sin^2\theta_W M_Z \cos 2\beta.$$

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