Spatial Competition with Heterogeneous Firms

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► Vogel 2008, JPE Vol. 116, no. 3 pp. 423-465 <u>solves</u> a stylized spatial competition model with endogenous product characteristics choice to confront this issue.

Model Set Up 1

Market is represented by a circle with unit circumference.
Locations are indexed by $z \in [0, 1)$ and consumers (of Mass L) are
uniformly distributed along circle. Each consumer buys either one
unit of output from one of the firms (N set of firms) or no unit at
all (in that case utility is zero). In case of buying from firm i he
derives utility

$$u(z, i) = v - p_i - t \times D(z, i) \tag{1}$$

with v common valuation of output, p_i price of firm i and $D(z, i)$
shortest path in circle from z to i. $t > 0$ is the marginal
"shopping" cost. Solution to consumer's problem is

$$i \in \underset{j \in N}{\operatorname{argmin}} \{p_j + tD(z, j)\} \quad and \quad p_i + tD(z, i) \leq v$$

Tiebreak rule : if indifferent between two firms, buy from nearest
one. We assume v is so large that every costumer always buys one
good.

Model Set Up 2

There are $n \geq 2$ firms. Firm i-s costs of supplying a consumer at z are

$$k_i + 2\tau D(z, i) \tag{2}$$

with k_i marginal costs of production, $\bar{k}$ the average of marginal costs and 2τ "shipping" costs with $\tau \in [0, t)$. The game consists of two stages.

- **First Stage** Location stage : firms simultaneously choose their locations $z_i \in [0, 1)$ with $\mathbf{z} \equiv (z_0, \dots, z_{n-1})$
- **Second Stage** Price Stage : firms observe $\mathbf{z}$ and choose prices p_i.

Strategy : Choice of probab. distribution over $[0, 1)$ and for each given $z \in [0, 1)$ choice of a probab. distribution over prices. Solution concept SPNE : each location is optimal given continuation game and each price distribution chosen in each continuation game is optimally chosen given strategies of other firms.

Model Set Up 3

▶ Market shares with no undercutting (i.e. there exists some indifferent consumer between all pairs of adjacent firms in the price stage, i.e. firm has positive market share) :

$$x_i^n = x_{i,i-1} + x_{i,i+1} = \frac{1}{2t}[p_{i-1} + p_{i+1} - 2p_i + t(d_{i-1,i} + d_{i,i+1})] \tag{3}$$

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- General market shares :

$$x_i = \frac{1}{2t}[p_j + p_{j'} - 2p_i + t(D(i,j) + D(i,j'))]$$
(4)

if j and j' are closest neighbors with some indiff. consumer inbetween. If $\exists j$ s.t. $p_i > p_j + tD(i,j)$ then of course $x_i = 0$. Also $x_i = 1$ if $\forall j \neq i \quad p_i < p_j + tD(i,j)$. Note : $x_i = x_i^n$ if no undercutting occurs.

Equilibrium Analysis 1

There is no pure strategy SPNE of this simple game !

Reason : Profits are not globally quasiconcave in own price, nor continuous.
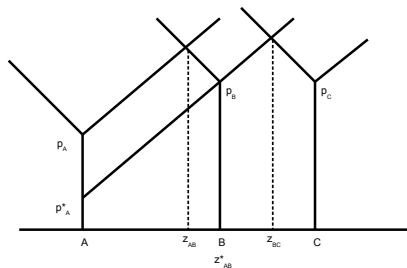

Figure 1: - Market shares are discontinuous in prices.

Equilibrium Analysis 2

Approach in Paper to find mixed SPNE :

▶ Define auxiliary game : same as original game, only that now market shares are given by (3) and not (4). Profit function becomes continuous and quasiconcave in own price.

Figure 2: - Market shares in auxiliary and real game.

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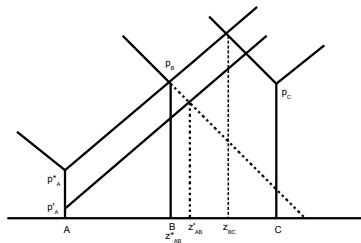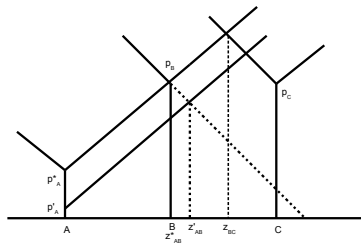 function becomes continuous and quasiconcave in own price.

Figure 2: - Market shares in auxiliary and real game.

▶ Auxiliary game has pure SPNE !

Equilibrium Analysis 3

Proposition 1.

For any set of parameters $\theta \equiv (n, t, \tau, L)$ and $k \geq 0$ there exists a $\phi(\theta, k) > 0$ such that if $k_i \in [k, k + \phi(\theta, k)]$ for all i, then the set of SPNE-s is nonempty. Moreover the following properties hold for a subset O^* of the SPNE-s:

1. *Strategies are pure along the equilibrium path for every equilibrium in O^*.*
2. *For any order of firms around the circle there exists a corresponding SPNE in O^*.*
3. *Equilibrium in O^* is characterized by*

$$
\begin{aligned}
d_{i,i+1}^* &= \frac{1}{n} + \frac{2}{3t+2\tau}\left(\bar{k} - \frac{k_i + k_{i+1}}{2}\right), \\
p_i^* &= (t+\tau)\left(\frac{1}{n} + \frac{2}{3t+2\tau}\bar{k}\right) + \frac{t}{3t+2\tau}k_i, \\
x_i^* &= \frac{1}{n} + \frac{2}{3t+2\tau}(\bar{k} - k_i), \\
\pi_i^* &= Lt(x_i^*)^2.
\end{aligned}
\tag{5}
$$

Equilibrium Analysis 4

Proof Idea : Let π_i^{A*} , π_i^* be respectively firm i-s profit in the auxiliary and real game along equilibrium path and $\pi_i^{A'}$, $E[\pi_i^{'}]$ firm i-s highest payoff from unilateral deviation from equilibrium path in auxiliary and real game. Author shows that if marginal costs of firms are similar enough then :

- $\pi_i^{A*} = \pi_i^*$ for all i
- $\pi_i^{A*} \geq \pi_i^{A'}$ for all i if $\tau \geq 0$ and $\pi_i^{A*} > \pi_i^{A'}$ for all i if $\tau > 0$.
- Either $\pi_i^{A'} \geq E[\pi_i^{'}]$ for all i or $\pi_i^* > E[\pi_i^{'}]$ for all i

This implies that in any case $\pi_i^* \geq E[\pi_i^{'}]$ which is the SPNE-condition in the real game ! Regarding <u>uniqueness</u> :

Proposition 2.

If $\tau > 0$ and the k_i-s are similar enough, then any SPNE of the real game is strict along equilibrium path if and only if it is part of the set O^* of Proposition 1

Uniqueness important for comparative statics !

Comments, Intuition 1

- ▶ Price and market share depend on costs of competitors only
  through $\bar{k}$: each firm locates at center of its market share to
  minimize shipping costs $\implies$ location-adjusted prices of
  indifferent consumers are equal accross firms $\implies$ profits
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- A firm's profit increases in its isolation $d_{-1,i} + d_{i,i+1}$. i.e. there are two effects from a fall in k_i, prices are lower since costs are lower (classic argument) and additionally now less productive firms position themselves away from the more productive ones $\implies$ more productive firms are more isolated, hence have more market power, hence higher profits. This tends to limit the extent of price fall. Unproductive firms specialize in niches. Implication for tariff example ?

Comments, Intuition 2

- Uniqueness result means uniqueness in outcomes, since in fact, even for $\tau > 0$ multiple equilibria exist if firms have assymetric marginal costs.

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▶ Uniqueness result means uniqueness in outcomes, since in fact, even for $\tau > 0$ multiple equilibria exist if firms have assymetric marginal costs.

▶ As t or τ increase, differences in marginal costs become less important for competition $\implies$ market shares and profits approach symmetric case.

▶ Recalling formula for $d^*_{i,i+1}$ two neighbors produce varieties that are separated by more than $\frac{1}{n}$ iff their average cost is less than market average. I.e. there is a negative relationship between average costs of two direct competitors and their location in space.

Extension : Horizontal and Vertical Differentiation

The model above can be applied to markets with :

- ▶ Homogeneous goods which are differentiated by geographic locations
- ▶ Heterogeneous goods of different varieties, each costumer has its ideal variety z on the circle, its position.

$\implies$ Horizontal Differentiation only !

One can model additional $\boxed{\text{vertical differentiation}}$ of goods by specifying anew the utility function :

$$u(z, i) = v - 1 + q_i^{\gamma} - p_i - tD(z, i) \quad \text{with } \gamma \in [0, 1) \qquad (6)$$

where q_i is the quality of the good produced by firm i. At first game stage firms now choose both position and quality. Basic insights are similar to first model, except that now more productive firms (in either quality or costs) don't necessarily offer lower prices.

End

Thank you for your attention !