Spatial Correlation of Solar-Wind Turbulence from Two-Point Measurements

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Interplanetary turbulence, the best studied case of low frequency plasma turbulence, is the only directly quantified instance of astrophysical turbulence. Here, magnetic field correlation analysis, using for the first time only proper two-point, single time measurements, provides a key step in unraveling the space-time structure of interplanetary turbulence. Simultaneous magnetic field data from the Wind, ACE, and Cluster spacecraft are analyzed to determine the correlation (outer) scale, and the Taylor microscale near Earth's orbit.

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The solar wind provides a natural laboratory for study of plasma turbulence at low frequency magnetohydrodynamic (MHD) scales [1,2], with immediate applications in scattering of solar and galactic cosmic rays [3,4], geospace ("space weather") [5], and heating of interplanetary plasma [6,7]. The implications of this best studied case of astrophysical plasma turbulence extend from the coronal heating and origin of the solar wind [8], to rates of star formation [9]. Solar-wind fluctuation properties have been studied in detail [2,10,11]; however, there remains throughout a pervasive ambiguity whenever time-lagged single spacecraft data are used to infer spatial properties. This familiar "frozen-in flow" approximation [12] works because the ordered radial ($\hat{R}$) solar-wind flow (at speed $V_{\rm sw}$) is supersonic and super-Alfvénic. Thus, time lags t are equivalent to spatial lags $\mathbf{r} = V_{\rm sw}\hat{R}t$. That is, convection past the detector occurs in a time short compared to all relevant dynamical time scales. However, the correct way to establish spatial structure is through simultaneous two-point single time measurements. But multipoint data have generally not been available. This situation has been partially alleviated in recent years due to the flotilla of spacecraft currently measuring heliospheric conditions.

Here we report an evaluation of two-point correlation functions using simultaneous measurements from the Wind, ACE, and four Cluster spacecraft, allowing, for the first time, the quantitative verification of several basic solar-wind turbulence results, previously obtained only from single spacecraft observations. We compute estimates of the spatial correlation function and determine both the magnetic outer or correlation length scale, and the Taylor microscale. This permits the empirical determination of an effective magnetic Reynolds number. Other recent multispacecraft studies have focused on time-domain and/or time-lagged optimization of correlations (e.g., [13,14]).

In 1980 the NASA Plasma Turbulence Explorer Panel [15] emphasized the need for simultaneous measurements of plasma and magnetic field fluctuations so that interplanetary MHD turbulence might become as well grounded in observations as classical hydrodynamic turbulence theory. The ambiguities associated with the frozen-in flow approximation are even more problematic in plasma turbulence, where the dispersive waves, and anisotropy, provide complications. These issues have contributed to the persistence of both "wave" and "turbulence" interpretations, that have coexisted in space and astrophysics for four decades. Consequently, a baseline understanding of interplanetary turbulence using multiple spacecraft data acquires a particular importance.

The most basic characteristic of turbulence is that it consists of fluctuations about a mean state. If it is homogeneous in space, then the means, variances, and correlations of fluctuations are independent of the choice of origin of the coordinate system [16]. For a magnetic field $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0 + \mathbf{b}$, the mean is $\langle \mathbf{B} \rangle = \mathbf{B}_0$, the fluctuation (turbulence) is $\mathbf{b} = \mathbf{B} - \mathbf{B}_0$, the variance is $\sigma^2 = \langle |\mathbf{b}|^2 \rangle$ and the two-point correlation function is $R(\mathbf{r}) = \langle \mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}) \rangle$. For homogeneity, R and $\mathbf{B}_0$ are independent of $\mathbf{x}$, but in reality may be weakly dependent on position. Here $\langle \ldots \rangle$ is an ensemble average that is equivalent to a suitably chosen time- or space-averaging procedure. For space and time correlations, the generalization is

$$R(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}, t) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}, t + \tau) \rangle, \tag{1}$$

which depends also upon time lag τ and is time stationary if independent of t. The frozen-in hypothesis makes use of the approximation $R(\hat{z}V_{\rm sw}t, 0) \approx R(0, -t)$ in the presence of a rapid uniform z-aligned flow at velocity $\hat{z}V$. For large $|\mathbf{r}|$, well behaved turbulence become uncorrelated and $R \to 0$. A standard measure of the length scale associated with decorrelation is the outer or correlation scale λ_c^q, defined by a normalized line integral, e.g., $\lambda_c = \int_0^\infty R(s\hat{q})ds/R(0)$ where $\hat{q}$ is a unit vector that selects the direction of integration. One can define a direction-averaged correlation scale λ_c. Another fundamental length scale is the Taylor microscale $\lambda_T = [\langle|\mathbf{b}|^2\rangle/\langle|\nabla\times\mathbf{b}|^2\rangle]^{1/2}$, which is the curvature of $R(r)$ at the origin, and the characteristic length scale of fluctuation gradients. It is sometimes efficient to first estimate the variance $\sigma^2 = \langle|\mathbf{b}|^2\rangle$ and the second order structure function $S(r) = \langle|\mathbf{B}(0) - \mathbf{B}(r)|^2\rangle$, and then reconstruct the correlation function as $R(r) = \sigma^2 - S(r)/2$. Here we present the first systematic determination of $R(r)$, λ_c, and λ_T from multiple spacecraft data.

In the present analysis the magnetic field is measured simultaneously by pairs of spacecraft, and statistics are assembled to estimate the correlation function and associated length scales. All of the data are at a distance $\sim$1 AU from the sun, and essentially on the ecliptic plane. We use either the Advanced Composition Explorer (ACE) spacecraft, paired with the Wind spacecraft, during the period from February, 1998 to December, 2001, or else pairs of Cluster spacecraft in the periods April 1–6, 2003 (Group I) and January 19–February 2, 2004 (Group II). The ACE-Wind interspacecraft spatial separation is usually in the range of 20 to 350 Earth radii (1 $R_E = 6378$ km). The Cluster interspacecraft separation for these periods ranges from $1/40R_E$ to 1 R_E.

The ACE-Wind data are analyzed with a cadence of 1 min, and individual correlation estimates are obtained by averaging over contiguous 24 h periods of data. For each interval I, using the observed magnetic field $\mathbf{B}^I$, we compute a mean magnetic field $\mathbf{B}_0^I$ and the fluctuation $\mathbf{b}^I = \mathbf{B}^I - \mathbf{B}_0^I$.

In classical turbulence theory, one seeks to describe a broader range of phenomena by introducing similarity variables. A standard choice is to express the two-point correlation function as

$$R(r) = \sigma^2\hat{R}(r/\lambda_c), \qquad (2)$$

where $\hat{R}$ is a dimensionless universal function. This is relevant to the solar wind where the turbulence energy density is known to vary with solar rotation, solar cycle, and transient effects [2]. We choose a normalization scheme that takes this into account, and adopt a variance normalization. We compute [17], in each data-interval, normalized correlation functions $R^{\mathrm{norm},I}(r) \equiv \lambda^I\langle\mathbf{b}^I(\mathbf{x})\cdot\mathbf{b}^I(\mathbf{x}+\mathbf{r})\rangle$, where $\lambda^I \equiv \langle\mathbf{b}\cdot\mathbf{b}\rangle/\langle\mathbf{b}^I\cdot\mathbf{b}^I\rangle$, so that $R^{\mathrm{norm},I}(0) = \langle|\mathbf{b}|^2\rangle$ for all intervals I. We present results for $\hat{R} = R^{\mathrm{norm}}/\langle|\mathbf{b}|^2\rangle$. We will not attempt a normalization of the spatial lag r because we will not have independent measurements of λ_c for each interval.

Our first result is an evaluation of the magnetic autocorrelation $R(r)$ from 264 ACE-Wind estimates of normalized correlation amplitude. Figure 1 demonstrates the expected gradual decrease in correlation amplitude as the ACE-Wind spatial separation ranges approximately from 20 to 350 R_E (0.001 AU to 0.015 AU; 1 AU $= 1.5\times10^{13}$ cm). It is notable that the data show a great deal of scatter, even after normalization. Of the various normalizations we have compared, the present approach best organizes the data.

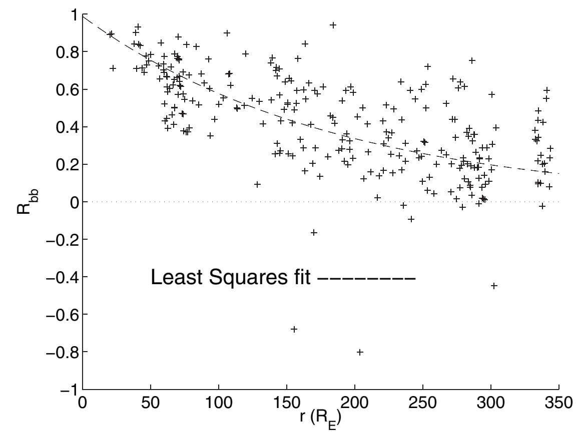

FIG. 1. Estimates of correlation function $R(r)$ from 264 ACE-Wind samples, for separation distances 20–350 R_E. A fit to a constrained $[R_{bb}(0) = 1]$ exponential (dashed line) gives correlation scale $\lambda_c = 186R_E$.

The level of scatter in these estimates appears to be consistent with expectations of the classical random function ergodic theory [18]. Estimates of the correlation function from data of finite duration T act similarly to the estimates σ_T^2 of the variance σ^2, based on an interval of data of length T. The expected statistical variance of a such estimates behaves asymptotically as

$$\Delta_T^2 = \langle(\sigma_T^2 - \sigma^2)^2\rangle \sim 4\sigma^4\frac{T_c}{T}, \qquad (3)$$

provided that the random vector field is stationary with Gaussian (jointly normal) one-point statistics. In our case, $\mathbf{b}$, in the solar wind, is approximately Gaussian [19,20], while the distribution of mean field strength B_0 and turbulence energy σ^2 are broader and roughly log-normal [20,21]. We may estimate the correlation time T_c in the above equations using single spacecraft observations, from which $T_c \approx$ 3–10 h. Using the value $T_c = 6$ h, and the interval length $T = 24$ h used in the ACE-Wind analysis, we conclude that a fractional variability of $\Delta_{24\,\mathrm{h}}^2/\sigma^4 \approx 1$ is the expected size of the scatter in correlation function estimates using this method. This intrinsic variability in no way prevents the *mean* correlation from approaching a stable ensemble average.

A (crude) mean correlation function is extracted from the data by a least squares fit to an assumed exponential form $R(r) = R(0)e^{-r/\lambda}$ constrained to pass through $R(0) \equiv 1$. Fitting to the ACE-Wind data gives the correlation function in Fig. 1. The associated estimate of the (direction-averaged) correlation length is $\lambda_c = 186R_E = 0.0079$ AU. A range λ_c at 1 AU have been reported from frozen-in flow methods using single spacecraft data, e.g., [22,23], with an average of 0.033 AU.

TABLE I. Summary of data intervals used for this analysis.

Data set	Separation (R_E)	Length	Number
ACE-Wind	20–350	2 d	264
Cluster (I)	0.44–1.21	2–16 h	30
Cluster (II)	0.024–0.042	1–35 h	102

The four Cluster spacecraft orbit Earth with varying interspacecraft separation and for a few months per year in the solar wind. This affords an opportunity to supplement the ACE-Wind analysis with independent measurements at spatial separations not otherwise available. Cluster data in both Groups I and II (see Table I) are analyzed beginning with a 0.2 or 0.045 sec cadence, respectively, which are averaged or undersampled down to 4 sec resolution. Intervals that include unwanted magnetospheric wave activity, usually with period around 10 sec, are rejected by inspection. Correlation analysis is carried out on 1 to 35 h samples and in the same way as for the ACE-Wind data, described above. Group I Cluster data has interspacecraft spacings ranging from about 0.5 to 1 R_E, with a mean of about $0.63R_E$, much smaller than the ACE-Wind separations. However these separations are expected to lie in the inertial range of solar-wind fluctuations, as the dissipation scale $\sim 1/k_{\rm diss}$ (dissipation wave number $k_{\rm diss}$) demarcates the short wavelength end of the inertial range, and is estimated to be ≈ 1000 km [24]. Group II Cluster data, from 2004, are at still much smaller separations, around 150–250 km, with a mean around $0.034R_E$. Here we expect to see nonscale invariant effects associated with the termination of the inertial range.

Figure 2 shows correlation estimates from all three sets of data. One can see the convergence of the normalized correlation function towards unity as the separation tends to zero. The spread in each set is of the order of the *deviation* of the correlation from unity, so the groups of estimates tighten up as the correlation gets larger. Anticipating that Cluster group II, with smallest separations, does not correspond to the inertial range, we refine the large (outer) scale fit by including both ACE-Wind and Cluster group I. A constrained exponential fit is carried out, depicted in Fig. 3. The result for the correlation scale is now $193R_E$, rather close to the earlier result that used only ACE-Wind estimates.

The Cluster group II estimates are so highly correlated ($R_{bb} \approx 0.995$) that they are almost certainly associated with the asymptotic approach of the correlation function to unity [16], $\lim_{\epsilon\to 0} R'_{bb}(\epsilon) = 0$ while $\lim_{\epsilon\to 0} R_{bb}(\epsilon) = 1$. For homogeneous turbulence, there is the additional requirement that R_{bb} is an even function of its (vector) argument, so that a power series developed about $r = |\mathbf{r}| = 0$ contains only even powers of r. The Taylor microscale, λ_T, essentially the radius of curvature at the origin, is determined by $R_{bb}(r) \approx 1 - r^2\lambda_T^{-2}/2 + \ldots$. We extract an estimate of λ_T from the analysis by carrying out a fit to a constrained parabolic curve, using the Cluster group II data. The result (Fig. 4) is $\lambda_T = 0.39 \pm 0.11 R_E = 2478 \pm 702$ km $\approx 1.6 \times 10^{-5}$ AU. This is the characteristic scale of the spatial derivatives of solar-wind magnetic field fluctuations.

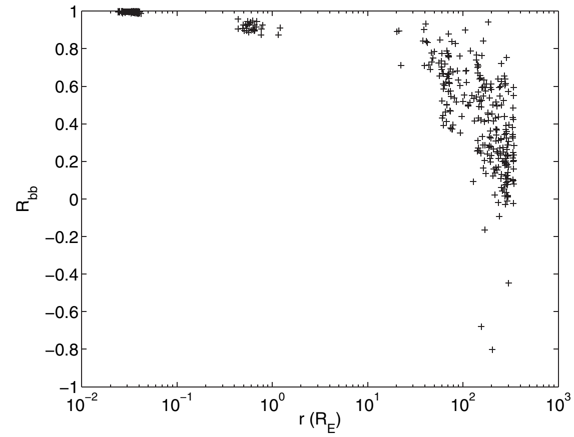

FIG. 2. Estimates of correlation function from ACE-Wind data (as in Fig. 1), supplemented by two sets of Cluster data, a set (1) with separations 0.4–1.2 R_E from data in 2003, and a set (2) with smaller separations 0.02–0.04 R_E, from 2004 data.

In general, $k_{\rm diss}\lambda_T > 1$ in hydrodynamic turbulence [16], and this product $\approx R_m^{1/2}$. However the latter relationship depends upon classical viscous dissipation in the momentum equation, while the exact form of the dissipation function in the collisionless solar wind remains a matter of debate [24,25]. We note here that if the dissipation scale indeed is approximately the ion inertial scale c/ω_{pi}, around 500–700 km at 1 AU in the solar wind (Leamon *et al.*, 1998), then we find that $k_{\rm diss}\lambda_T \approx 4$ for 1 AU conditions.

Given the broadband character of solar-wind turbulence, we suggest that a better estimate of the effective magnetic Reynolds number $R_m^{\rm eff}$ can be obtained using the measured

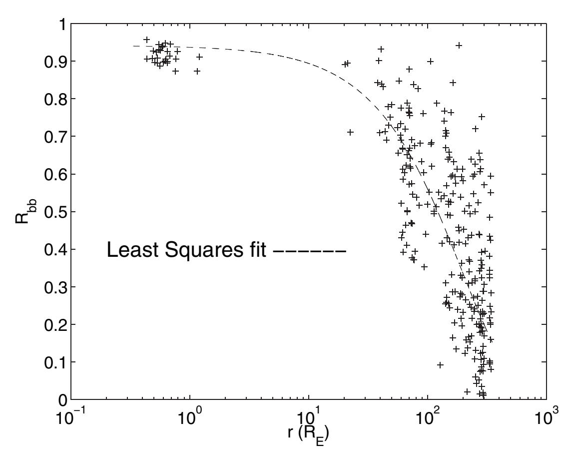

FIG. 3. Constrained exponential fit to ACE-Wind and Cluster set (2) data. This provides an estimate of $\lambda_c = 193R_E$.

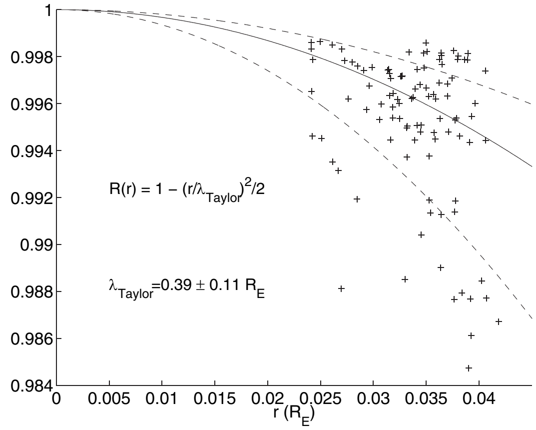

FIG. 4. Parabolic fit to Cluster data set II, providing an estimate of the inner scale, or Taylor microscale, of solar-wind turbulence at 1 AU.

outer scale and Taylor microscale lengths. Using the classical hydrodynamics relationship [16] and the results above, we deduce that

$$R_m^{\text{eff}} = \left(\frac{\lambda_c}{\lambda_T}\right)^2 \approx 230\,000. \quad (4)$$

In conclusion, the use of two-point, single time correlation methods using multispacecraft analysis has enabled the confirmation and possible refinement of several key measurements of space plasma turbulence. We have constrained the correlation function in three bands of spatial separation, and evaluated the correlation scale and the Taylor microscale. These measurements permit evaluation of an effective Reynolds number of 230 000. The above correlation scale is a factor of 2–4 smaller than many reported values based upon frozen-in flow [4,6,26]. This may be due to the high degree of variability of the energy density of solar-wind turbulence at a fixed (1 AU) position, and, in particular, the presence of scale invariant "$1/f$" noise in the magnetic field [27]. Further multispacecraft studies of interplanetary MHD scale turbulence will be required to verify the present results, complementing earlier analyses carried out in the time or frequency domain (e.g., [13,14,23]), and to quantitatively assess possible systematic effects in the frozen-in flow approximation.

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