# Spatial Filtering of RF Interference in Radio Astronomy

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***Abstract*—We investigate spatial filtering techniques for interference removal in multichannel radio astronomical observations. The techniques are based on the estimation of the spatial signature vector of the interferer from short-term spatial covariance matrices followed by a subspace projection to remove that dimension from the covariance matrix, and by further averaging. The projections will also modify the astronomical data, and hence a correction has to be applied to the long-term average to compensate for this. As shown by experimental results, the proposed technique leads to significantly improved estimates of the interference-free covariance matrix.**

***Index Terms*—Antenna arrays, array signal processing, interference suppression, interferometry, radio astronomy, spatial filtering.**

## I. Introduction

THE contamination of radio astronomical measurements by man-made radio frequency Interference (RFI) is becoming an increasingly serious problem and therefore the application of interference mitigation techniques is essential. Most current techniques address impulsive or intermittent interference and are based on time-frequency detection and blanking, using a single sensor [1], [2] or multiple sensors [3]. A start has been made in applying adaptive filtering techniques using a reference signal [4]–[6].

In this paper, we investigate the efficacy of multichannel spatial filtering for the removal of continuously present radio interference such as TV signals, radio broadcasts, or the GPS satellite system. The proposed technique applies to interferometric radio telescope arrays such as the Westerbork synthesis radio telescope (WSRT) in The Netherlands, the very large array (VLA) in the USA, or future massive phased array telescopes, such as the square kilometer array (SKA) currently in design. Initial results on spatial filtering specifically for the purpose of astronomical *imaging* were published in [7]; this letter presents a refinement which is more generally applicable.

In interferometric radio astronomy the signals from various sensors (telescopes) are usually split into narrow frequency bins (say, 50 kHz), and correlated over 1 to 100 ms to yield short-term correlation matrices. These are then integrated over longer periods of typically 10 to 60 s to yield long-term correlation matrices, which are stored onto tape and constitute the output of the telescope interferometer.

The long-term correlation matrices contain contributions from the astronomical sources in the pointing direction through the main lobe of the telescope, from interferers in the near and far field through the side lobes, and from spatially white receiver noise. The astronomical signals usually have a signal-to-noise ratio (SNR) of $-20$ dB or less, and hence they are too weak to be detected over short integration periods. Harmful interference may range from $-70$ dB up to $+50$ dB with respect to the instantaneous system noise level.

Continually present interferers cannot be cut out in the time-frequency plane and have to be removed using spatial filtering. Assuming that the frequency bins are sufficiently narrow band, we can associate a spatial signature vector to each interferer and estimate these from the short-term correlation matrices. By projecting out the corresponding dimensions, the interference is removed. However, this spatial filtering also modifies the correlation matrix of interest to astronomers and therefore a correction must be applied. The correction is possible under the assumption that the spatial signatures of interferers are sufficiently changing over the 10-s period.

In the next sections, we first introduce the spatial filtering algorithm and discuss the correction that has to be applied. We then show the performance of the algorithm in simulated data, and real data collected at the WSRT.

## II. Data Model

Assume we have a telescope array with $p$ elements. We consider a single frequency bin, with for simplicity at most $q = 1$ interferer present. The array output vector $\mathbf{x}(t)$ is modeled in complex baseband form as

$$\mathbf{x}(t) = \mathbf{a}(t)s(t) + \mathbf{v}(t) + \mathbf{n}(t)$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_p(t)]^T$ is the $p \times 1$ vector of output signals at time $t$ ($^T$ is the transpose operator), $s(t)$ is the interferer signal with spatial signature vector $\mathbf{a}(t)$ which is assumed stationary only over short time intervals, $\mathbf{v}(t)$ is the received sky signal, assumed a stationary Gaussian vector with covariance matrix $\mathbf{R}_v$, and $\mathbf{n}(t)$ is the $p \times 1$ noise vector with independent identically distributed Gaussian entries and covariance matrix $\sigma^2\mathbf{I}$. We assume that $\sigma^2$ is known from a calibration observation, and that $\mathbf{R}_v \ll \sigma^2\mathbf{I}$. Given observations $\mathbf{x}_n = \mathbf{x}(nT_s)$, where $T_s$ is the sampling period, the objective is to estimate $\mathbf{R}_v$.

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## III. Spatial Filtering Algorithm

Given the observations, we first construct short-term covariance estimates $\hat{\mathbf{R}}_k$

$$\hat{\mathbf{R}}_k = \frac{1}{M} \sum_{n=kM}^{(k+1)M} \mathbf{x}_n \mathbf{x}_n^{\mathrm{H}}$$

where $M$ is the number of short-term samples to average, $MT_s$ is of the order of 1 to 100 ms ($\mathrm{H}$ denotes the Hermitian transpose). In the usual procedure, these matrices are then further averaged to obtain a long-term (say, $NMT_s = 10$ s) estimate

$$\hat{\mathbf{R}}^{10\,\mathrm{s}} = \frac{1}{N} \sum_{k=1}^{N} \hat{\mathbf{R}}_k.$$

If there is only an astronomical signal and white Gaussian noise, $\hat{\mathbf{R}}^{10\,\mathrm{s}}$ is an unbiased estimate of the true covariance matrix $\mathbf{R}_0 = \mathbf{R}_v + \sigma^2 \mathbf{I}$.

Consider now the situation where there is an interferer with zero mean, power $\sigma_k^2$ and a spatial signature vector $\mathbf{a}_k$ (normalized to unit norm), assumed constant over the short integration periods. The expected value of the short-term estimates $\hat{\mathbf{R}}_k$ will then be

$$\mathbf{R}_k = \mathbf{R}_0 + \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^{\mathrm{H}} = \mathbf{R}_v + \sigma^2 \mathbf{I} + \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^{\mathrm{H}}.$$

In the construction of the long-term estimate, the interferer contribution will be $(1/N)\sum \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^{\mathrm{H}}$. Depending on the variability of $\mathbf{a}_k$, the contribution will somewhat average out, but if $\sigma_k$ is strong, its influence will be felt: the estimate of $\mathbf{R}_v$ will be biased and also have an increased variance. It is therefore desired to filter the interferer out.

Suppose that the spatial signature $\mathbf{a}_k$ of the interferer is known. We can then form a spatial filter $\mathbf{P}_k$

$$\mathbf{P}_k := \mathbf{I} - \mathbf{a}_k (\mathbf{a}_k^{\mathrm{H}} \mathbf{a}_k)^{-1} \mathbf{a}_k^{\mathrm{H}}$$

which is such that $\mathbf{P}_k \mathbf{a}_k = 0$. Thus, when this spatial filter is applied to the data covariance matrix all the energy due to the interferer will be nulled

$$\tilde{\mathbf{R}}_k := \mathbf{P}_k \hat{\mathbf{R}}_k \mathbf{P}_k \sim \mathbf{P}_k \mathbf{R}_0 \mathbf{P}_k$$

where $\sim$ denotes that the right hand side is the expected value of the left hand side. Note that the astronomical data is modified as well, so that we will have to apply a correction at a later stage.

When the spatial signature of the interferer is unknown, it can be estimated by an eigenanalysis of the sample covariance matrix. More in general, assuming that the noise is white and the astronomical contribution is small, it is well-known that the number of interferers can be detected from the eigenvalues of $\hat{\mathbf{R}}_k$, and that the subspace spanned by the spatial signatures of the interferers can be estimated by the corresponding eigenvectors. This allows us to construct the projection matrix $\mathbf{P}_k$ [3].

When we average the modified covariance matrices $\tilde{\mathbf{R}}_k$, we obtain the long-term estimate

$$\tilde{\mathbf{R}}^{10\,\mathrm{s}} := \frac{1}{N} \sum_{k=1}^{N} \tilde{\mathbf{R}}_k = \frac{1}{N} \sum_{k=1}^{N} \mathbf{P}_k \hat{\mathbf{R}}_k \mathbf{P}_k.$$

We now discuss the correction that has to be applied to $\tilde{\mathbf{R}}^{10\,\mathrm{s}}$ to recover an unbiased estimate of $\mathbf{R}_0$, assuming that the interferer has been projected out completely. We employ the matrix identity $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{B})$ where $\mathrm{vec}(\cdot)$ denotes a stacking of the columns of a matrix, and $\otimes$ the Kronecker product. This gives

$$\begin{aligned}\mathrm{vec}(\tilde{\mathbf{R}}^{10\,\mathrm{s}}) &= \frac{1}{N} \sum_{k=1}^{N} (\mathbf{P}_k^T \otimes \mathbf{P}_k) \mathrm{vec}(\hat{\mathbf{R}}_k) \\ &\sim \left\{ \frac{1}{N} \sum_{k=1}^{N} (\mathbf{P}_k^T \otimes \mathbf{P}_k) \right\} \mathrm{vec}(\mathbf{R}_0) \\ &= \mathbf{C}\mathrm{vec}(\mathbf{R}_0)\end{aligned}$$

where $\mathbf{C} := (1/N)\sum_{k=1}^{N} (\mathbf{P}_k^T \otimes \mathbf{P}_k)$. Thus, we can obtain an unbiased estimate of $\mathbf{R}_0$ by applying the inverse of $\mathbf{C}$ to $\mathrm{vec}(\tilde{\mathbf{R}}^{10\,\mathrm{s}})$

$$\hat{\mathbf{R}}^{10\,\mathrm{s}} := \mathrm{unvec}\left(\mathbf{C}^{-1} \mathrm{vec}(\tilde{\mathbf{R}}^{10\,\mathrm{s}})\right) \sim \mathbf{R}_0 = \mathbf{R}_v + \sigma^2 \mathbf{I}. \quad (1)$$

In short, to obtain the covariance matrix due to the astronomical sources, we can average the projected short-term covariance matrices as usual to long-term averages, but have to apply the correction matrix $\mathbf{C}$ which is formed in the same way by averaging $\mathbf{P}_k^T \otimes \mathbf{P}_k$.

At this point, we can make several remarks.

- The invertibility of $\mathbf{C}$ is crucial to be able to recover $\mathbf{R}_0$. If all $\mathbf{P}_k$ are the same ($\mathbf{a}_k$ is stationary), then $\mathbf{C}$ will not be invertible. One can show that an average of only a few different $\mathbf{P}_k$ is needed to ensure invertibility. Thus we need $\mathbf{a}_k$ to be sufficiently variable over the long integration period. This is expected since a) ground-based mobile interferers are often subject to multipath fading, which limits the coherency time of $\mathbf{a}_k$ to 10 to 100 ms, b) satellite and airplane interferers are moving, and moreover, c) the telescopes are slowly rotating while tracking a point in the sky, and continuously compensate the changing baseline lengths by delay tracking and a phase rotation (fringe correction) in the order of a hertz. This causes even stationary interferers (TV stations) to have some change in spatial signature over a period of 10 s. The effect is stronger at small declinations of the astronomical field of interest (the telescopes rotate faster), but will be absent at high declinations (e.g., pointing at the North Celestial Pole). Spatial filtering will also not work (and $\mathbf{C}$ will be not invertible) if the interference is entering only on a single telescope.
- The amount of residual interference is determined by the accuracy of the interferer spatial signature estimate. A good estimate can be obtained only if it is sufficiently strong. Thus, we propose to detect the presence of an interferer using a standard test on the eigenvalues, and to apply a projection only if the interferer is detected.

## IV. Simulation Results

In a computer simulation, we considered a scenario in which there are $p = 4$ telescopes, a weak astronomical signal ($-20$ dB), and a single interferer of varying power and random

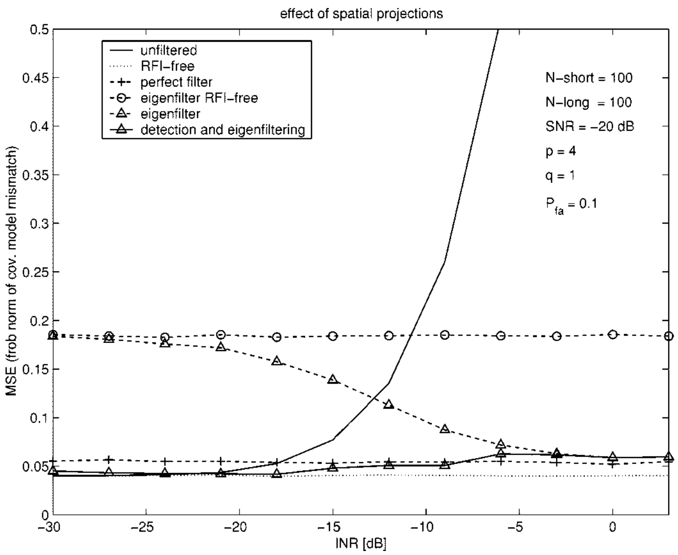


Fig. 1. MSE as function of interferer power.

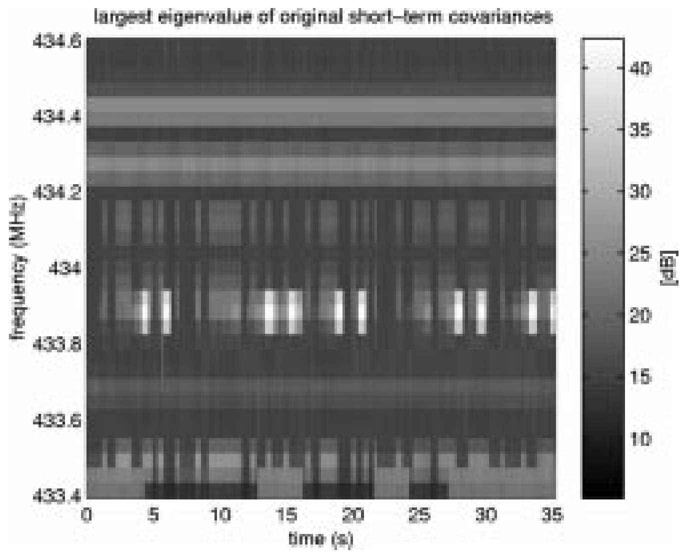


Fig. 2. Amateur broadcast interference, both continuous and intermittent, recorded at the WSRT. Spectrogram of the largest eigenvalue of $\hat{\mathbf{R}}_k$.

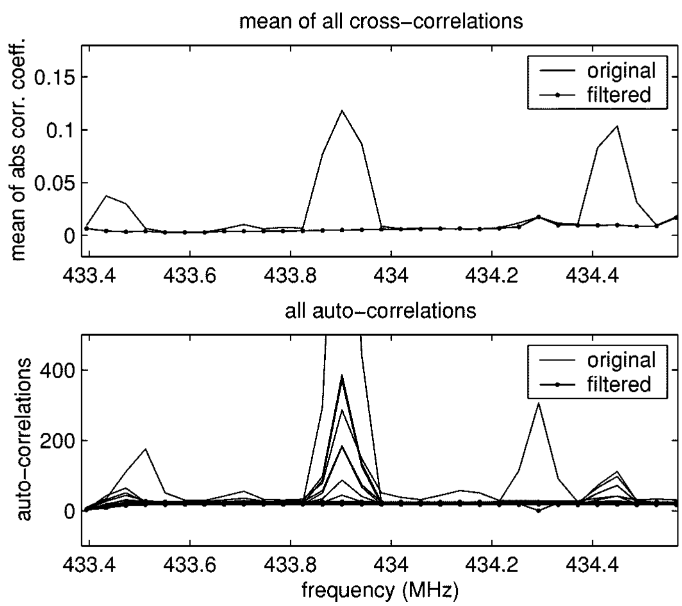


Fig. 3. Top: mean cross-correlation spectrum and bottom: all auto-correlation spectra, before and after the spatial projection algorithm.

unit-norm $\mathbf{a}_k$. The received data is correlated over $M = 100$ samples, the projection is applied, and the result further averaged over $N = 100$ such matrices. The performance measure is the mean-squared estimation error $\mathrm{MSE} = \mathrm{E}\{\|\hat{\mathbf{R}}^{10\,\mathrm{s}} - \mathbf{R}_0\|_F\}$. Fig. 1 shows the MSE curves for several cases.

- *Unfiltered Interference:* The long-term covariance estimate is computed as traditionally done.
- *RFI-free:* The data does not contain interference and the covariance is estimated as traditionally done.
- *Perfect Filter:* Assumes that the spatial signatures of the interferer are perfectly known.
- *Eigenfilter:* The spatial signatures are estimated from the eigenvalue decomposition of the short-term data covariance matrices.
- *Detection + Eigenfilter:* First, it is seen whether the interference is observable in the data using a standard likelihood test (white-noise test with known $\sigma$) on the short-term covariance estimates [8]

$$L = 2M\left[\mathrm{tr}(\hat{\mathbf{R}}_k/\sigma^2) - p - \log\det(\hat{\mathbf{R}}_k/\sigma^2)\right] \gtrless \gamma$$

  where $\gamma$ is a detection threshold. (In Fig. 1, the detection threshold was selected to obtain a false alarm probability of 0.1.) If an interferer is detected, then the spatial projection is applied as before.

For reference, we also show the result of applying the eigenfiltering algorithm to RFI-free data.

The results indicate that for INRs above $-15$ dB,[1] it is essential to apply the spatial filter. If the spatial signatures of the interferer are perfectly known, then the final estimate is almost as good as in the RFI-free case. If the spatial signatures are estimated from the data, then it is important first to detect if there is an interferer, otherwise for weak interferers the final covariance estimate is biased. In combination with detection, it is seen that the covariance estimate is very close to the interference-free result.

[1]This level depends on the number of telescopes $p$, the number of short-term samples $M$, and on the selected false alarm rate.

## V. Experimental Results

We applied the spatial filtering technique to a data set containing time continuous and intermittent interference observed at the WSRT. The data set is a $p = 8$-channel recording of a 1.25-MHz-wide band at 434 MHz containing signals from the astronomical source 3C48 (white noise signal) contaminated by narrow-band amateur radio broadcasts. The data was partitioned into 32 frequency bins (each processed separately), the short term averaging period was 10 ms ($M = 781$), and the number of time intervals was $N = 1000$.

Fig. 2 shows the largest eigenvalue, on a logarithmic scale, of the correlation matrix $\hat{\mathbf{R}}_k$ as a function of time and frequency. We see that there is continuous interference at 434.3 MHz and 434.4 MHz, as well as at least four intermittent sources at other frequencies in the band. Comparing the eigenvalues to a frequency-dependent threshold showed that in most cases, about one or two eigenvalues were affected and needed to be projected out. Occasionally (the channel at 433.9 MHz), up to about six eigenvalues were affected.

The results of the spatial filtering algorithm are shown in Fig. 3. The upper graph shows the mean of all cross-correlations, before and after applying the spatial filter, and the lower graph shows the eight auto-correlations. It is seen that at most frequency bins both the time-continuous and intermittent interference is suppressed significantly, and the resulting spectrum is flat with a cross-correlation of about 0.01 indicative of the astronomical source.

The condition number of $\mathbf{C}$ determines the amount of noise amplification due to the correction in (1). In the experiment, it was small over almost all frequency bins, in the range of three to occasionally 20. This shows that the interference usually has sufficient spatial fluctuations due to multipath fading or the fringe correction. Only at 434.3 MHz, the condition number was extremely large (order 200), which explains the relatively poor filtering performance at that frequency. For this frequency, the lower graph shows that only a single telescope received the strong continuous interference, hence the corresponding $\mathbf{a}_k$-vector was nearly stationary. We conclude that for interferometric radio telescope arrays, the proposed spatial filtering algorithm provides a very interesting and practical technique for interference mitigation.

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