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Spatial Power Spectrum of the Main Geomagnetic Field, and Extrapolation to the Core

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Summary

The spatial 'power' spectrum of the main geomagnetic field has been estimated for harmonics up to $n = 500$. It is shown to consist of two components, long wavelengths being dominated by fields originating in the core, and short wavelengths by fields originating in the crust; the cross-over occurs at $n \geqslant 11$, a wavelength $\leqslant 3600$ km.

The core field is often approximated by a set of spherical harmonic coefficients. It is shown that at present main field coefficients for $n \gtrsim 9$, and secular variation coefficients for $n \gtrsim 6$, are not known with significant accuracy. Estimates are made of the standard deviations of the IGRF coefficients, and the standard deviation of the IGRF field deduced. This field is known to about 0·5 per cent at the surface but only to about 10 per cent at the core. Its time variation is known only to about 20 per cent at the surface, and is very uncertain at the core.

1. Introduction

This paper considers the contribution to the mean square geomagnetic field of internal origin (i.e. the mean of $\mathbf{H}.\mathbf{H}$ over the Earth's surface) from different spatial frequencies (or, equivalently, wavelengths). As we are concerned with a closed-spherical surface only certain frequencies have physical meaning—those corresponding to wavelengths $2\pi a/n$ where a is the Earth's radius and n is an integer. The 'spectrum' is therefore discrete rather than continuous, though the distinction is not important for large values of n.

Fourier analysis separates a function $f(\lambda)$ known on a circle of radius a into orthogonal components such that

$$f(\lambda) = \sum_{n=0}^{\infty} (A_n \cos n\lambda + B_n \sin n\lambda)$$

$$= \sum C_n \cos (n\lambda + \varepsilon_n).$$

The values of ε_n and hence of A_n and B_n depend on the (arbitrary) choice of origin, but the values of $C_n = (A_n^2 + B_n^2)^{\frac{1}{2}}$ are independent of origin. Because of the orthogonality, each harmonic component contributes independently to the mean square value of $f(\lambda)$ over the circle; the total contribution of components of wavelength $2\pi a/n$ is

$$\tfrac{1}{2}C_n^2 = \tfrac{1}{2}(A_n^2 + B_n^2).$$

Similarly, spherical harmonic analysis separates a function $f(\theta, \lambda)$ known on a spherical surface into orthogonal components such that

$$f(\theta, \lambda) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \Theta_n^m(\theta)(A_n^m \cos m\lambda + B_n^m \sin m\lambda)$$

$$= \sum \sum \Theta_n^m(\theta) C_n^m \cos (m\lambda + \varepsilon_n^m).$$

Now not only do the values of ε_n^m, and hence of A_n^m and B_n^m, depend on the (arbitrary) choice of origin of λ, but also for a given n the relative values of the C_n^m depend on the (arbitrary) choice of the $\theta = 0$ axis. Again, because of the orthogonality each harmonic component contributes independently to the mean square value of $f(\theta, \lambda)$ over the sphere; however to determine fully the contribution from wavelengths $2\pi a/n$ to the mean square, all the harmonics of that n must be considered. This contribution is proportional to the sum

$$\sum_{m=0}^{n} (C_n^m)^2 = \sum [(A_n^m)^2 + (B_n^m)^2],$$

which is independent of the choice of axes. (The constant of proportionality depends on the normalization of the functions $\Theta_n^m(\theta)$.)

In this paper $f(\theta, \lambda)$ is the vector geomagnetic field **H**, which can be specified near the Earth's surface by its scalar potential

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} P_n^m(\cos \theta)(g_n^m \cos m\lambda + h_n^m \sin m\lambda); \tag{1}$$

this expression is valid only for fields of internal origin, but in practice fields of external origin are almost entirely eliminated by using yearly means.

For the semi-normalized $P_n^m(\cos \theta)$ used in geomagnetism, the mean square value over the surface of the field **H** produced by harmonics of a given n (i.e. of wavelength $2\pi a/n$), which will be called R_n, is (Lowes 1966)

$$R_n = (n+1) \sum_{m=0}^{n} [(g_n^m)^2 + (h_n^m)^2]. \tag{2}$$

By analogy with a time varying 'signal', a plot of 'power' R_n against 'frequency' n is called the spatial power spectrum.

Most of the field observed at the Earth's surface comes from electric currents in the core, and we would like to extrapolate this field down to the core–mantle boundary. However such an extrapolation using equation (1) is valid only if there are no sources (magnetization or electric current) in the mantle or crust. We know that there are small but significant contributions to the surface field from the magnetization of crustal rocks, and such crustal fields must be removed (or shown to be negligible) before extrapolation is feasible.

This paper extends and amplifies the work of Bullard (1967) and Booker (1969), who used only the radial component of the core field, and considers in more detail the random and systematic errors involved.

In Section 2 the spectrum of the crustal field is considered, in Section 3 the spectrum of the core field, and in Section 4 the (spatial) spectrum of the time variation of the core field. Section 5 considers extrapolation to the core, and in Section 6 the various results of this paper are discussed.

2. Spectrum of the crustal field

Because of its shallow origin the crustal field can be expected to consist largely of high harmonics, for which there are no estimates from spherical harmonic analysis. However a reasonable indication of the higher frequency spectrum can be obtained from the Fourier analysis by Alldredge, van Vooris & Davis (1963) of a round-the-world profile of the magnitude of the magnetic field. Because only one profile was analysed the individual coefficients are very scattered, but Fig. 1 gives the smoothed version of the power spectrum obtained by Bullard (1967). Bullard suggested that there was a distinct change in the slope at about $n = 25$, corresponding to a wavelength of about 1600 km, and that this represented the separation between fields of core and crustal origin.

The spectrum from $n = 25$ to $n = 500$ is in fact fitted quite well by the line

$$y = 50 \exp(-0{\cdot}004\, n), \tag{3}$$

and the departures from the line are probably not significant.

For this (essentially) plane geometry, extrapolation downawrds involves the factor $\exp(2zn/a)$ so the observed spectrum is consistent with sources giving a 'white' power spectrum at depth 9 km (12 km below the flight level). Clearly, whatever the actual distribution of sources, the spectrum beyond $n = 25$ comes from shallow, crustal, sources. Assuming that for these sources the spectrum of equation (3) holds for all n, summation from $n = 1$ to ∞ would give the mean square crustal field. (Only 10 per cent of this m.s. field would come from $n < 25$, and only 13 per cent from $n > 500$, so in fact the shape of the spectrum at the two ends is not very important.)

However the Alldredge *et al.* analysis differs in two significant ways from a spherical harmonic analysis of the vector field.

First, it was a Fourier analysis along a (nearly) great circle track. Algebraically, the situation is very complicated; the weights given to individual spherical harmonic coefficients depend on the orientation of the track, and the harmonic contributions to the mean square field are not separable. Physically, however, we have a one-dimensional cross-section of a two-dimensional pattern; the details may well be

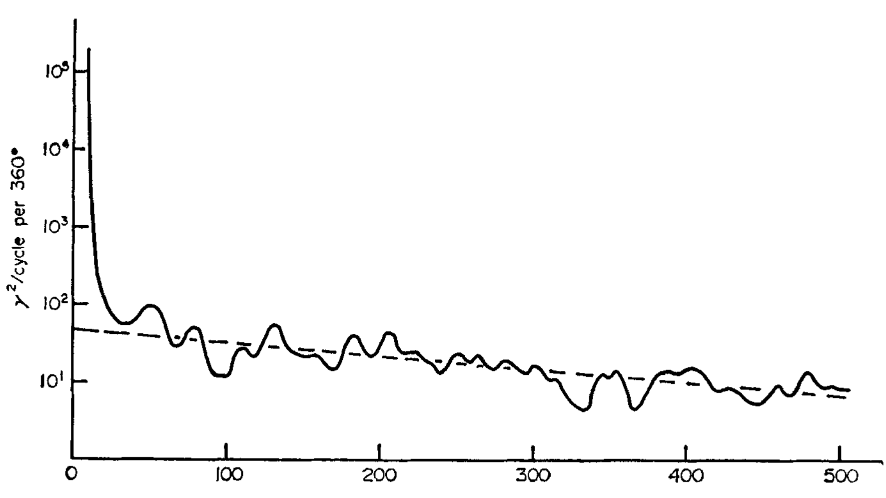

FIG. 1. Smoothed power spectrum of the round-the-world magnetic intensity profile of Alldredge *et al.* (1963). (After Bullard (1967). Bullard plotted C_n^2, not $\frac{1}{2}C_n^2$, and his ordinate scale was wrong by a factor 5.)
$1\gamma = 1$ nanotesla

altered, particularly at low frequencies, but it is unlikely that the general nature of the higher frequency spectrum, or its total power, will be much changed.

Secondly, what was analysed was not the (three components of the) vector field, but the magnitude of the field. In general this results in a very complicated distortion of the spectrum. However, the higher harmonics of the crustal field are effectively only a small perturbation on the much larger, only slowly varying, main field. In such a situation it can be shown that the main field acts as a directional filter, picking out that part of the perturbation field which is along the local direction of the main field. In the resultant power spectrum the powers of perturbations locally parallel to the main field remain almost unchanged at their fundamental frequencies, while perpendicular perturbations have almost all their power transferred to zero frequency; harmonic and intermodulation terms are small, and would tend to average out. Thus for a random, small, high frequency, crustal field, uncorrelated with the large, low frequency, main field, we would see on average one-third of the crustal power.

We must therefore expect the Alldredge *et al.* spectrum to be significantly different from a spherical harmonic spectrum at low frequencies, but to be similar at high frequencies apart from a factor of about 3. We can therefore use the spectrum to estimate the total rms crustal field to be about 200γ. This value compares well with those of 300γ for (continental) Canada and about 120γ for (oceanic) Bermuda, obtained by Serson & Hannaford (1957) for about 3000 km of vector magnetometer track in each case, and values ranging from 160 to 230γ (when corrected by the $3^{\frac{1}{2}}$ factor) for NE Atlantic and Indian Ocean towed magnetometer profiles totalling about 3000 km, obtained by Neidell (1964).

As was pointed out by Bullard, in the spectrum of Fig. 1 there is no absence of power at intermediate frequencies, as had been suggested by Alldredge & van Hooris (1961) and Alldredge *et al.* (1963). It is however consistent with the existence of only two sources of field, one in the crust and one in the core. If, as seems likely, the spectrum of equation (3) is valid also for low frequency crustal fields, then for all values of n for which reliable spherical harmonic coefficients are at present available ($n \lesssim 10$) the crustal contribution can be ignored.

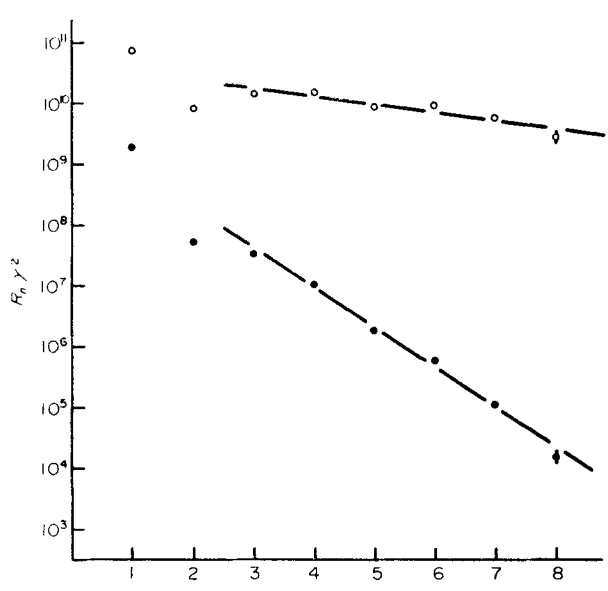

FIG. 2. Total mean square contribution to vector field by all harmonics of degree n. IGRF 1965·0. Closed circles, at Earth's surface; open circles, at core surface.

3. Spectrum of the core field

Up to $m \leqslant n \leqslant 8$ the field of deep origin is specified approximately by the 80 spherical harmonic coefficients of the IGRF 1965.0 (IAGA Working Group 1971). Although the individual coefficients have a wide range of values, if we, in effect, average them by performing the summation of equation (2) we obtain the 'power' spectrum (independent of co-ordinate system) given by the points plotted in the lower part of Fig. 2. Apart from those for = 1 and 2, the points are fitted remarkably well by the line

$$R_n = 4{\cdot}0 \times 10^9 \, (4{\cdot}5)^{-n}. \tag{4}$$

Although the departures from a smooth curve are significantly greater than the errors of the points, it does seem likely that the source mechanism gives a smooth spectrum on average.

Up to $n = 7$ the points for the different analyses considered for the IGRF do not depart significantly from each other. For $n > 8$ analyses which have been taken to higher degree differ significantly, but all give points very much above the line, and give the impression of an upward curvature of the spectrum (see e.g. Fig. 2 of Bullard (1967)). However these values of R_n are almost certainly very much overestimated. This is because *random* errors in the individual coefficients lead on average to a *systematic* increase in the sum of their squares, and hence of R_n; in fact coefficients consisting only of 'noise' can give quite large values of R_n.

That these analyses do have large errors in their coefficients is indicated by the fact that the three analyses for which the separation into internal and external parts is possible all give large magnitude high harmonic fields of external origin (and, for Fanselau & Kautzleben, of non-potential origin); see Table 1. It is most unlikely that such large external fields of long time scale do exist, and most probable that the external R_n values are simply an indication of the errors of analysis. It can then be easily shown that the external R_n give a direct estimate of the systematic errors of the internal R_n. It follows that for these three analyses none of the coefficients for $n \geqslant 11$ are significantly above error level, and we are left with four, very uncertain, values, three for $n = 9$ and one for $n = 10$, which are scattered about the line. The same argument almost certainly holds for the results of the other high degree analyses at present available.

Table 1

Values of $R_n = (n+1) \sum_{m=0}^{n} [(g_n^m)^2 + (h_n^m)^2]$ *for the internal and external coefficients of the analyses of Fanselau & Kautzleben* (1958), *Vestine* et al. (1963), *and Hurwitz* et al. (1966).

Unit $10^4 \gamma^2$.

n	R_n F & K Int.	F & K Ext.	Vestine Int.	Vestine Ext.	Hurwitz Int.	Hurwitz Ext.
6	77·36	4·01	70·45	9·95	62·66	0·90
7	13·63	2·66	20·08	6·09	8·90	0·76
8	4·32	2·91	8·69	6·60	1·78	0·40
9	3·56	2·30	6·36	3·43	0·56	0·12
10	1·71	2·13	3·67	2·86	0·39	0·11
11	1·95	0·58	3·27	2·93	0·06	0·11
12	1·33	1·25	1·92	1·99	0·10	0·13
13	0·08	1·02				
14	1·00	1·18				
15	1·14	1·00				

Table 2

Contributions of harmonics of degree n *to surface field*

(a) *Main field, IGRF* 1965·0. *Unit:* 1 *gauss* = $10^5\gamma = 10^{-4}$ *tesla*

n	rms field	rms SD				
	At Earth's surface					
1	0·4378	0·0006	0·4378	0·0006		
2	0·0739	0·0007				
3	0·0581	0·0007				
4	0·0332	0·0008				
5	0·0138	0·0008	0·1010	0·0020	0·4492	0·0020
6	0·0079	0·0007				
7	0·0034	0·0007				
8	0·0013	0·0006				
>8			~0·0008			
	At core surface					
1	2·683	0·003	2·683	0·003		
2	0·829	0·008				
3	1·193	0·017				
4	1·24	0·03				
5	0·95	0·05	2·5	0·3	3·7	0·3
6	0·99	0·09				
7	0·78	0·16				
8	0·5	0·3				
>8			~1·1			

(b) *Secular variation field, IGRF* 1956·0. *Unit:* γ */yr*

(For explanation of correction see text)

n	rms field	Corrected field	rms SD				
	At Earth's surface						
1	25	25	4	25	4		
2	55	55	5				
3	29	28	5			69	13
4	15	14	5½	65	12		
5	13	12	5½				
6	9	7	5½				
7	5	—	5½				
8	4	—	5½	~6			
>8							
	At core surface						
1	150	150	25	150	25		
2	620	610	60				
3	590	580	110			1600	840
4	550	510	210	1590	840		
5	920	830	390				
6	1170	920	710				
7	1190	—					
8	1590	—		?			
>8							

The second column gives the (rms) average values (over the sphere) of the synthesized vector fields; this is $R_n^{\frac{1}{2}}$ for the main field, and $Q_n^{\frac{1}{2}}$ for the secular variation.

The standard deviations tabulated are the (rms) average values (over the sphere) of the standard deviation of the synthesized vector field; they are $[(n+1)\,(2n+1)]^{\frac{1}{2}}\sigma_n$ for the σ_n of Table 3. (Those given in the conference abstract of Lowes (1972) were the standard deviations of the values of $R_n^{\frac{1}{2}}$ and $Q_n^{\frac{1}{2}}$, *i.e.* $(n+1)^{\frac{1}{2}}\sigma_n$, using somewhat different values of σ_n.)

In Section 5 it is shown that the spectrum of equation (4) is consistent with the field originating in the core, and in the absence of any reliable data for higher harmonics this spectrum will be assumed to hold for all values of n. The total rms contribution for harmonics with $n > 8$ can then be estimated, and this value, together with those for $n \leqslant 8$, is given in the first part of Table 2a.

However these rms field contributions, and the field synthesized from the coefficients, are both subject to error, as the IGRF coefficients will not be exactly those appropriate to the Earth's field. An indication of the uncertainty of the coefficients is given by Fig. 3(a), which shows the scatter of the eight sets of coefficients which were submitted for the IGRF (Cain & Cain 1971). It is to be expected that the standard deviations of the coefficients will vary only with n and not with m (Lowes, to be published), and the observed scatter is reasonably fitted by standard deviations σ_n varying smoothly from 23γ for $n = 1$, to 5γ for $n = 8$ (Table 3).

These eight analyses used data sets having considerable overlap, so the scatter of their coefficients will underestimate their errors. On the other hand, the IGRF coefficient set is a weighted mean of four sets, so would be expected to have somewhat smaller errors than individual sets. On the assumption that these two effects cancel, the standard deviations of Table 3 have been attached to the IGRF coefficients; assuming that covariances can be ignored then leads to the average standard deviations of the synthesized field which are given in Table 2a.

The values of σ_n given in Table 3 are considerably larger than those obtained from the internal consistency of individual analyses. For example Leaton, Malin & Evans (1965) obtained values $\lesssim 11\gamma$, and Cain *et al.* (1967) values $\lesssim 3\gamma$. However, that they are reasonable estimates of error to be attached to the IGRF coefficients is confirmed by the figures given by Cain & Cain (1971) for the rms differences between the IGRF and various data sets. When extrapolated to an altitude of 1000 km the figures of Table 2 predict a rms difference of 86γ, while the observed rms difference between the IGRF and the total intensity observations of OGO 4 (which were not used in the analyses) was 57γ, corresponding to about 92γ for the vector field. For surface data we would expect about 300γ (composed of rms contributions of 200γ from the coefficient errors, 80γ from harmonics for $n > 8$, and 200γ from the crustal field (see Section 2), while that observed was about 200γ per component, corresponding to about 350γ for the vector field. The observed residuals include a contribution from the errors in the secular variation coefficients of the IGRF; the differences between expected and observed figures correspond to a few years of the errors estimated in the next Section.

From Table 2a we see that, in terms of a spherical harmonic expansion, at the surface we know the dipole field to about 0·1 per cent, the non-dipole field to about 2 per cent, and the total field to about 0·5 per cent.

4. Spectrum of the secular variation

Practically all the observed, long period, time variation—the secular variation—is due to variation of the core field; there is no significant contribution from the crust. Applying the analysis of Section 3 to the IGRF 1965.0 secular variation coefficients gives the (uncorrected) values of Q_n—the mean square secular variation field for each n—shown in Fig. 4 and Table 2b.

Fig. 3(b) shows the very large scatter of the eight sets of secular variation coefficients submitted for the IGRF. However the POGO (3/68) set, based on only 1·8 years data, considerably increases the scatter; it is also significantly different from the revised POGO (10/68) set (Cain & Langel 1968). It was therefore omitted, and the scatter of the other seven sets found to be fitted reasonably well by standard deviations varying smoothly from $3{\cdot}2\gamma$/yr for $n = 1$, to $0{\cdot}9\gamma$/yr for $n = 8$, again considerably larger than internal estimates of error.

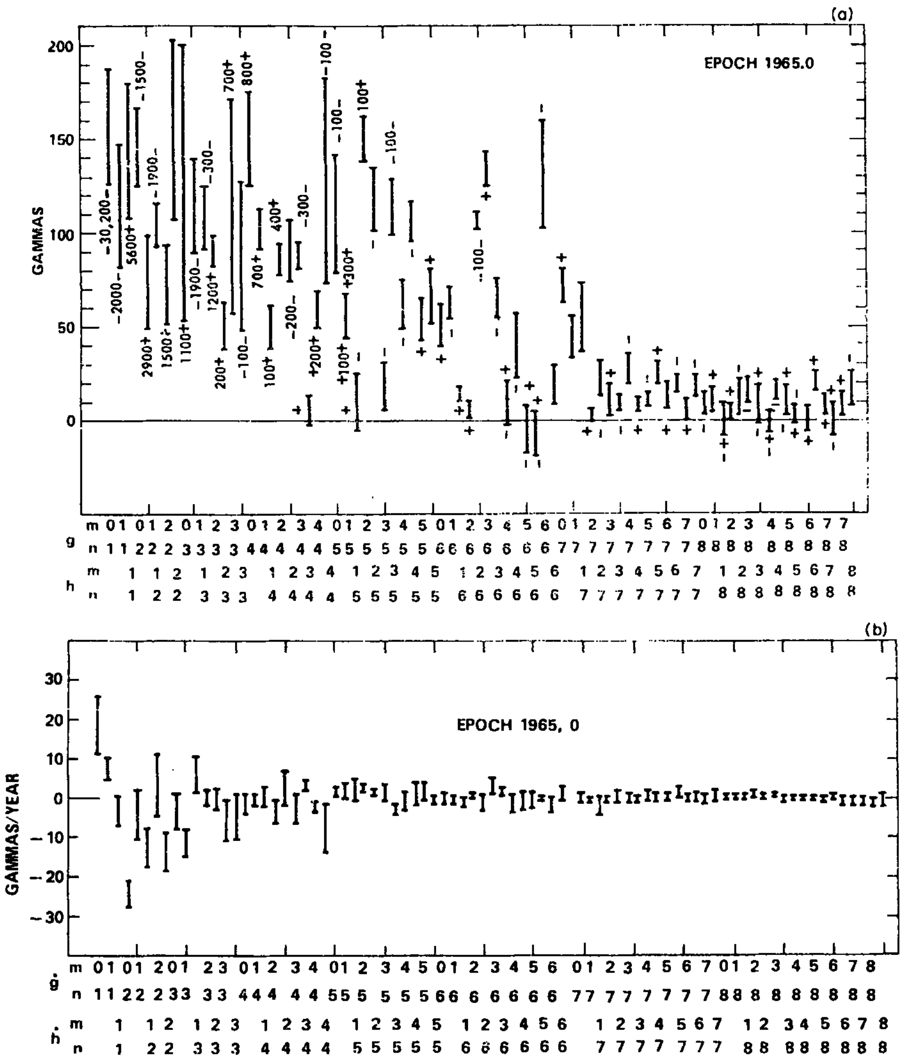

Fig. 3. Range of the eight sets of harmonic coefficients submitted for the IGRF. (a) Main field; (b) secular variation. (Reproduced by permission from Zmuda 1971.)

Table 3

Estimates of standard deviations σ_n of IGRF coefficients for main field and secular variation field

n	σ_n	
1	23γ	$1{\cdot}6\gamma$/yr
2	17	1·2
3	14	1·0
4	12	0·8
5	9	0·7
6	8	0·6
7	6	0·5
8	5	0·45

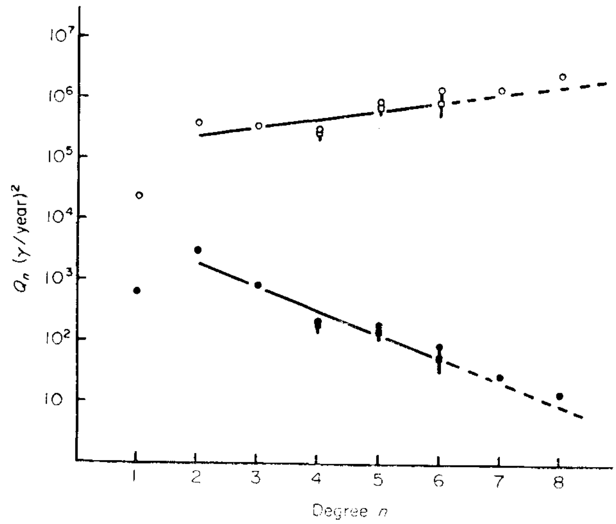

FIG. 4. Total mean square contribution to vector secular variation field by all harmonics of degree n. IGRF 1965·0. Closed circles, uncorrected values at Earth's surface; open circles, uncorrected values at core surface. Squares, corrected values (see text).

The IGRF secular variation coefficients were obtained by averaging those of five sets. Three of these were obtained directly from secular variation data (two from observatory data and one from a world chart). In the other two the secular variation coefficients were derived (simultaneously with main field and second derivative coefficients) from a much larger data set (of which the observatory data was only a small fraction). So in this case the observed scatter is probably more nearly typical of the standard deviations of individual coefficients. For simplicity the standard deviations of the IGRF coefficients have been estimated by halving the scatter figures; they are given in Table 3 and lead to the standard deviations given in Table 2b.

Because of the large random errors in the secular variation coefficients the Q_n values will, as explained above, be systematically overestimated. It is easily shown that the average systematic error is the square of the average standard deviation of Table 2b; corrected values of Q_n are also given in Table 2b and shown in Fig. 4. It is clear that the values for $n = 7$ and 8 are not significant, and that the field for $n = 6$ is very uncertain.

In this case there is no independent world-wide test that the estimates of standard deviations given in Table 2b are valid for the IGRF. In Scandinavia the difference between the IGRF secular variation and the observed values has a vector rms of 23γ/yr (Barraclough 1971), but this represents a very small part of the surface.

That the secular variation is very uncertain is not surprising. The standard deviations (Table 3) of the main field coefficients correspond to 2–10 years of secular variation. Also the field does not vary linearly with time; if large intervals are used to reduce the error in the mean rate of change, then the second derivative becomes important.

It is clear that Q_n decreases much less rapidly with n than does R_n. There is in fact reason to expect that the secular variation spectrum will be less steep than the field spectrum, because a substantial part of the secular variation is due to the westward drift of the non-dipole field pattern with respect to the Earth's surface. Two alternative suggestions are:

(a) *Part* of the secular variation is due to the drift at about $\omega = 0{\cdot}2°$/yr of *all* the non-dipole field (Bullard *et al.* 1950); or

(b) *All* of the secular variation is due to the drift at about $0{\cdot}4°$/yr of *part* of the non-dipole field (Yukutake & Tachinaka 1969).

In either case the secular variation produced is proportional to the longitudinal gradient of the appropriate field, so that shorter wavelengths are emphasized compared with longer wavelengths. It turns out that, on average, we would expect a given main field spectrum to be multiplied by a factor of $n(2n+1)\omega^2/6$ to give the westward drift secular variation spectrum; a contribution of this nature would considerably decrease the slope at small n. We would therefore expect an extrapolation of the line drawn on Fig. 4 to overestimate the contribution of higher harmonics.

From Table 2b we see that, in terms of a spherical harmonic expansion, the secular variation field at the surface is known only up to $n = 6$, and only to about 20 per cent.

5. Extrapolation to the core

Extrapolation downwards involves multiplying the spectrum by the factor $(a/r)^{2n+4}$. The observed logarithmic main field spectrum for $n = 3$ to 8 is nearly a straight line, corresponding to sources giving a 'white' spectrum at radius $0{\cdot}47a$ (500 km inside the core); there is some indication that the spectrum is in fact concave downwards, corresponding to random dipole sources at radius $0{\cdot}35a$. Extrapolation of the field down to the core boundary is therefore reasonable, and the result is shown on the upper part of Fig. 2 and the second part of Table 2a. (If the spectrum were in fact curved the estimates of the $n > 8$ field would need to be reduced.)

(Nagata (1965) obtained a source radius of $0{\cdot}42a$. However he plotted $[(g_n^m)^2+(h_n^m)^2]/(2n+1)$ against n, and used as his criterion of source depth that these *individual* contributions to the mean square *potential* should on average be equal at the source; he therefore obtained a greater depth than that obtained here by considering the *total* contribution of degree n to the mean square *field*. Nagata's suggestion of a different source for higher harmonics was based on the coefficients of Vestine and Fanselau & Kautzleben which have been shown above to be spurious.)

Although individual harmonics have the same 'signal/noise' ratio at both surface and core, the less accurate higher harmonics are relatively more important at the core; also the unknown high harmonics now contribute significantly. At the core boundary we therefore still know the dipole field to $0{\cdot}1$ per cent, but only know the $n = 2$ to 8 non-dipole field to about 10 per cent, the total non-dipole field to about 40 per cent, and the total field to about 10 per cent.

If the secular variation is also extrapolated to the core we obtain the results given in the upper part of Fig. 4 and the second part of Table 2b. The increase of Q_n with n is probably due to the production of a large part of the secular variation by westward drift; if so Q_n would start to decrease at about $n = 8$. The combined field of the first six degrees is known only to about 50 per cent, and from Fig. 4 we cannot estimate how much the higher degrees contribute. However a lower estimate of the magnitude can be obtained by assuming that the main field drifts bodily westward at $0{\cdot}2°$/yr; this then contributes 1000γ/yr for $n = 7$ and 8. Extrapolating the spectrum of equation (4) give another 2500γ/yr for $n > 8$, but this figure would be considerably reduced if the spectrum were concave downwards.

Extrapolation to the core using equation (1) would not be valid if there were significant electric currents in the intervening mantle. There probably are currents 'leaking' into the, slightly conducting, mantle from the, highly conducting, core. These currents will give toroidal magnetic fields which will reduce to zero at the surface; although at the core boundary these toroidal fields may be as large as the poloidal fields we are considering, there is no direct way of measuring them. Fortunately, such toroidal magnetic fields would not affect the extrapolation of the observed (poloidal) magnetic fields.

However, extrapolation of the poloidal magnetic fields might be affected by their time variation, as this will induce electric currents which will reduce the magnitude of the variations seen at the surface. The integrated time constant of the mantle is thought to be about 4 years, so that fields with periods of about 30 years or less will be significantly attenuated (Currie 1967). Unfortunately we do not know enough about the time spectrum of the field to be able to estimate the importance of this attenuation on the spatial spectrum. (Booker (1969) estimated typical periods τ_n by dividing R_n by Q_n, and concluded that attenuation was not significant; however his conclusion would not be valid if a substantial part of a given R_n had periods shorter than τ_n.) Certainly, using equation (1) to extrapolate can only underestimate the magnitude of the poloidal secular variation field at the core, but the same cannot be said for the main field, at least part of which is effectively the time integral of the secular variation.

MacDonald (1957) has shown that, for a given period, the 'physical' attenuation due to induced currents is smaller for short wavelength spatial harmonics than for long wavelengths. However it is likely that short wavelength fields will tend to vary more rapidly than long wavelength fields, so it is not possible to say if this will affect the secular variation spatial spectrum.

6. Discussion

It has been shown that at present the known world-wide logarithmic spatial power spectra of both the long wavelength and short wavelength fields are well fitted by straight lines. If these lines are extrapolated and combined we obtain the spectrum of Fig. 5. This somewhat idealized spectrum probably represents the present surface field spectrum to better than about 30 per cent. Any physically plausible crustal field must have a spectrum which decreases at small n, so the extrapolation of equation (3) to small n is not really valid. However it appears that the core field is greater than the crustal field to at least $n = 11$.

To estimate the accuracy of a field approximately specified by a finite set of spherical harmonic coefficients, we need to know both the magnitude of the field of the unspecified high harmonics, and the errors of the specified coefficients.

The spectrum of equation (4) can be used to estimate the effect of high harmonics; the results are given in Table 4 for the Earth's surface and also for two typical satellite altitudes. (The figures would need to be reduced somewhat if the logarithmic spectrum is in fact concave downwards.) Corresponding figures for the crustal field (using the spectrum of equation (3) with the $\times 3$ correction) would be about 190, 20 and 4γ, but the last two values are really only upper limits as they depend critically on the spectrum remaining flat at small n. It is clear however that, at the Earth's surface, evaluating more coefficients will have very little effect on the residual field.

The estimates made in this paper of the errors of the IGRF coefficients are inevitably somewhat subjective, but it is thought that they are probably correct within a factor of two.

Using these error estimates we see that the core field, about $45\,000\gamma$ rms, is reasonably well known at the surface, the ($n \leqslant 8$) IGRF representing it in 1965 to about 200γ. An extrapolation of Table 2a indicates that it would just about be worth-

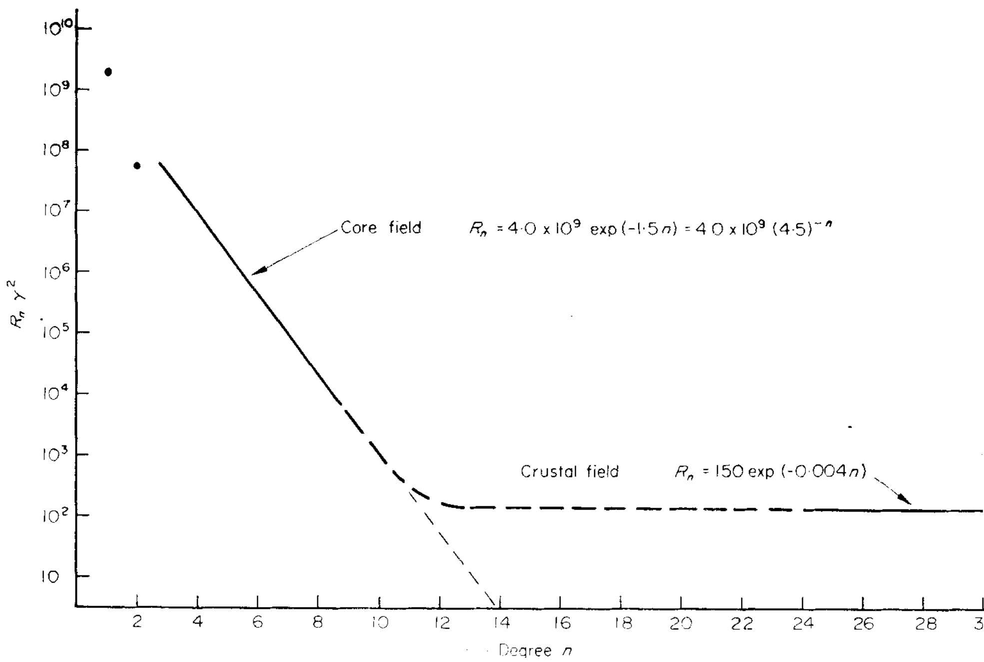

FIG. 5. Idealized surface power spectrum, 1965.

Table 4

Estimate of magnitude of core field from harmonics with $n > n_{max}$

n_{max}	rms field of higher harmonics at altitude		
	0 km	350 km	1000 km
8	81γ	45	16
9	38	20	6
10	18	9	3
11	8	4	1
12	4	2	

while to determine the $n = 9$ coefficients as well, but there would be no point in going further unless smaller errors could be obtained; this conclusion is valid for all depths and altitudes.

The IGRF secular variation coefficients are a very much poorer fit to the actual secular variation; with the error estimates of this paper only the coefficients for $n \leqslant 5$ are known at all reliably, and the coefficients for $n = 7$ and 8 are quite meaningless. At the surface the secular variation, about 70γ/yr rms, is known only to about 15γ/yr.

Because some of the contributing coefficient sets were the result of joint solutions, the errors of the secular variation coefficients are not necessarily statistically independent of those of the main field coefficients (i.e. the covariances may be significant). Ignoring this, we find that the very large uncertainty of the secular variation coefficients means that the deviation between the IGRF and the core field will double, from 200 to 400γ rms, in about 14 years. Clearly, any improvement in an IGRF to be used for about 10 years will depend more on a better understanding of the secular variation than on any increase in the number of coefficients specified.

Extrapolation down to the core boundary shows that the total rms poloidal field there is about 3·9G. If the extrapolation is valid, we know the 3·7G coming from $n \leqslant 8$ to about 0·3G, but there is also about 1G coming from higher harmonics.

The extrapolated secular variation is very poorly known; the field from harmonics with $n \leqslant 6$ is known only to about 50 per cent, and we cannot do more than guess at the total magnitude. Also, any shielding by induced currents in the mantle will have relatively much more effect on the secular variation than on the main field.

It is clear that attempts to use the extrapolated secular variation to investigate, for example, the conservation of magnetic flux from the core (Booker 1969), or the core surface velocity pattern (Ball, Kahle & Vestine 1969; Malin & Saunders 1973), will be subject to very large uncertainties.

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References

Alldredge, L. R. & van Vooris, G., 1961. Depth to sources of geomagnetic anomalies, *J. geophys. Res.*, **66**, 3793–3800.

Alldredge, L. R., van Vooris, G. D. & Davis, T. M., 1963. A magnetic profile round the world, *J. geophys. Res.*, **68**, 3679–3692.

Ball, R. H., Kahle, A. B. & Vestine, E. H., 1969. Determination of surface motions of the Earth's core, *J. geophys. Res.*, **74**, 3659–3680.

Barraclough, D. R., 1971. Secular variation in Scandinavia, *Nature Phys. Sci.*, **233**, 94–98. (With correction in **236**, 96, 1972.)

Booker, J. R., 1969. Geomagnetic data and core motions, *Proc. R. Soc.*, **A369**, 27–40.

Bullard, E. C., 1967. The removal of trend from magnetic surveys, *Earth Planet. Sci. Letters*, **2**, 293–300.

Bullard, E. C., Freedman, C., Gellman, H. & Nixon, J., 1950. The westward drift of the Earth's magnetic field, *Phil. Trans. R. Soc.*, **A243**, 67–92.

Cain, J. C. & Cain, S. J., 1971. Derivation of the International Geomagnetic Reference Field (IGRF 10/68), *Nasa Technical Note* TN D-6237.

Cain, J. C., Hendricks, S. J., Langel, R. A. & Hudson, W. V., 1967. A proposed model for the International Geomagnetic Reference Field—1965, *J. Geomagn. Geoelect.*, **19**, 335–355.

Cain, J. C. & Langel, R. A., 1968. The geomagnetic survey by the polar orbiting geophysical observatories OGO-2 and OGO-4 1965–1967, *Goddard Space Flight Center Report* X-612-68-502.

Currie, R. G., 1967. Magnetic shielding properties of the Earth's mantle, *J. geophys. Res.*, **72**, 2623–2633.

Fanselau, G. & Kautzleben, H., 1958. Die analytische Darstelling des geomagnetischen Feldes, *Geofiz. pura Appl.*, **41**, 33–72.

Hurwitz, L., Knapp, D. G., Nelson, J. H. & Watson, D. E., 1966. Mathematical model of the geomagnetic field for 1965, *J. geophys. Res.*, **71**, 2373–2383.

IAGA Working Group, 1969. International Geomagnetic Reference Field 1965·0, *J. geophys. Res.*, **74**, 4407–4408.

Leaton, B. R., Malin, S. R. C. & Evans, J., 1965. An analytical representation of the estimated geomagnetic field and its secular change for the epoch 1965·0, *J. Geomagn. Geoelect.*, **17**, 187–194.

Lowes, F. J., 1966. Mean-square values on sphere of spherical harmonic vector fields, *J. geophys. Res.*, **71**, 2179.

Lowes, F. J., 1972. The recent geomagnetic field and its time variation, *Trans. Am. geophys. Un.*, **53**, 610–612.

McDonald, K. L., 1957. Penetration of the geomagnetic secular variation through a mantle with variable conductivity, *J. geophys. Res.*, **62**, 117–141.

Malin, S. R. C. & Saunders, I., 1973. Rotation of the Earth's magnetic field, *Nature*, **245**, 25–26.

Nagata, T., 1965. Convergency of the spherical harmonic coefficients of the geomagnetic field, *J. Geomagn. Geoelect.*, **17**, 153–155.

Neidell, N. S., 1964. *Spectral studies of marine geophysical profiles*, Ph.D. Dissertation, Cambridge University.

Serson, P. H. & Hannaford, W. L. W., 1957. A statistical analysis of magnetic profiles, *J. geophys. Res.*, **62**, 1–18.

Vestine, E. H., Sibley, W. L., Kern, J. W. & Carlstedt, J. L., 1963. Integral and spherical harmonic analyses of the geomagnetic field for 1955. Part I, *J. Geomagn. Geoelect.*, **15**, 47–72.

Yukutake, T. & Tachinaka, H., 1969. Separation of the Earth's magnetic field into the drifting and the standing parts, *Bull. Earthq. Res. Inst.*, **47**, 65–97.

Zmuda, A. J., 1971. The International Reference Field 1965·0. Introduction, in *World Magnetic Survey*, ed., A. J. Zmuda, *IAGA Bull.*, No. 28, 148–152.