Spatial SINR Games Combining Base Station Placement and Mobile Association

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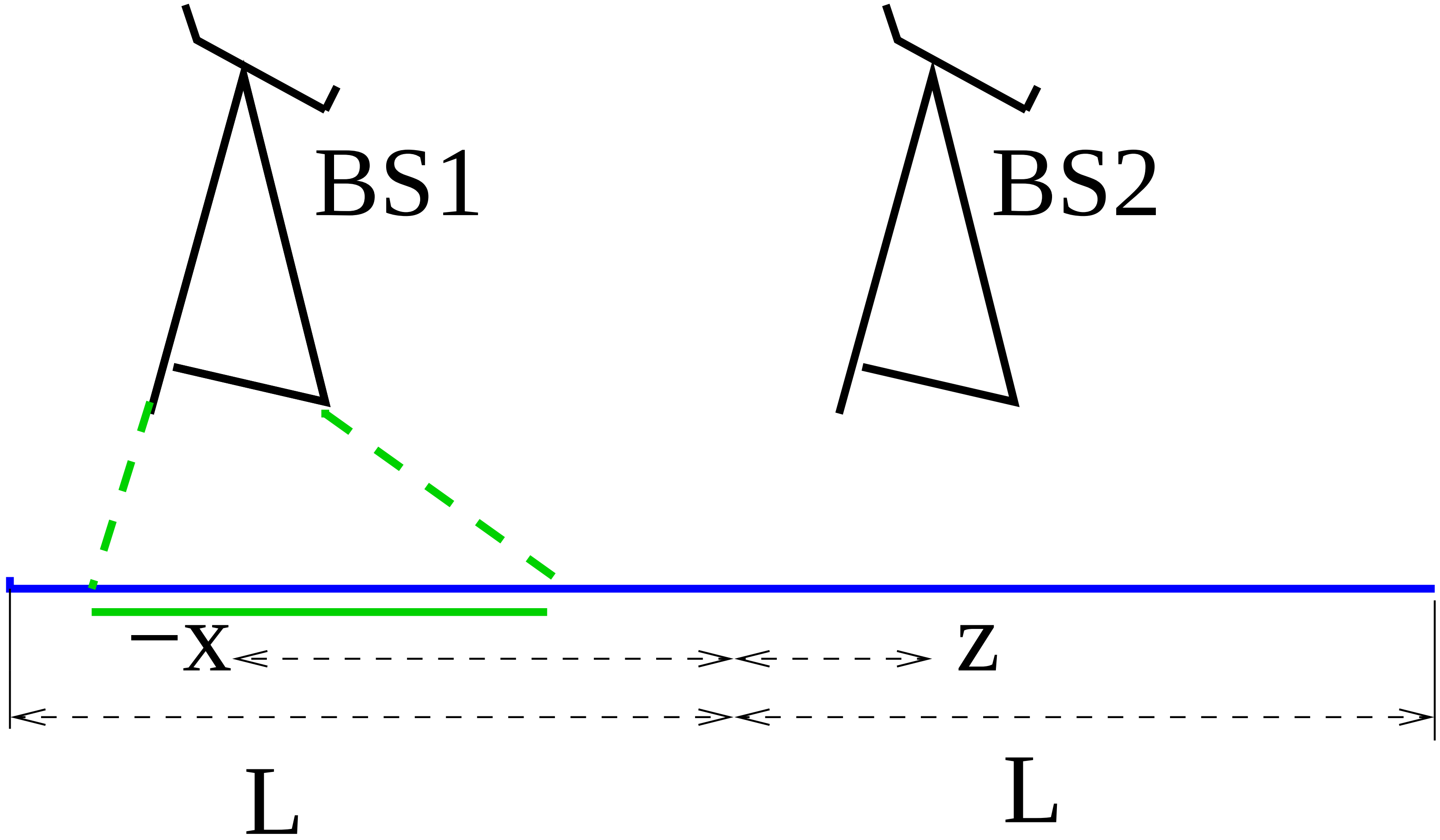

BS1
BS2
−x
z
L
L

1 Background

•Jean J. Gabszewicz and Jacques-Francois Thisse. Location. Aumann and Hart, editors, *Handbook of Game Theory with Economic Applications*.

•Several papers study competition over prices of goods between facilities that have fixed positions, derive the equilibrium allocation of customers.

Such games, as well as hierarchical games in which firms compete for the location or over prices which then determines the customers allocation equilibrium, have already been introduced in 1929 in

Harold Hotelling. Stability in competition. Economic Journal, vol 39, 1929.

•Many references to competition on the line with two firms. Give rise to a separation of the segment into two subsegments ("cells").

•V. Mazalov and M. Sakaguchi. Location game on the plane. *International Game Theory Review*, 2003.

2 Level 1: Association Problem

•A large number (fluid) of mobiles are uniformly located on $[-L.L]$

•2 Base stations. Using the same frequency (channel). Located at $(-x, 1)$ and $(z, 1)$

•Pathloss constant α

•A mobile located at y on the line has a channel gain of $[(y+x)^2+1]^{-\alpha/2}$ to BS 1 and $[(y-z)^2+1]^{-\alpha/2}$ to the other.

2.1 Interference

•Interference at BS i depends on its location, not on the association decisions nor on location of BS j!

•Each of n mobiles transmits at a power of $1/n$. n large:

$$Int_1(\alpha) = \int_{-L}^{L} [(y+x)^2 + 1]^{-\alpha/2} dy,$$

$$Int_2(\alpha) = \int_{-L}^{L} [(y-z)^2 + 1]^{-\alpha/2} dy.$$

At BS 1: $$Int_1 = \begin{cases} arcsinh(L-x) + arcsinh(L+x) & \alpha = 1 \\ arctan(L-x) + arctan(L+x) & \alpha = 2 \end{cases}$$

At BS 2: $$Int_2 = \begin{cases} arcsinh(L+z) + arcsinh(L-z) & \alpha = 1 \\ arctan(L+z) + arctan(L-z) & \alpha = 2 \end{cases}$$

2.2 SINR, Utility and Cells

•Utility of mobile i is a non-decreasing function of its SINR. Association to BS with better SINR.

•SINRs of a mobile located at point y:

$$SINR_1 = \frac{[(y+x)^2+1]^{-\alpha/2}}{Int_1+\sigma^2},$$

$$SINR_2 = \frac{[(y-z)^2+1]^{-\alpha/2}}{Int_2+\sigma^2}$$

•S_i:= cell i: set of locations at which mobiles are connected to BS i

•S_i is an SINR-equilibrium if the following holds: a mobile belongs to S_i if $SINR_i \geq SINR_j$ for $j \neq i$.

2.3 Study of the SINR-association equilibrium

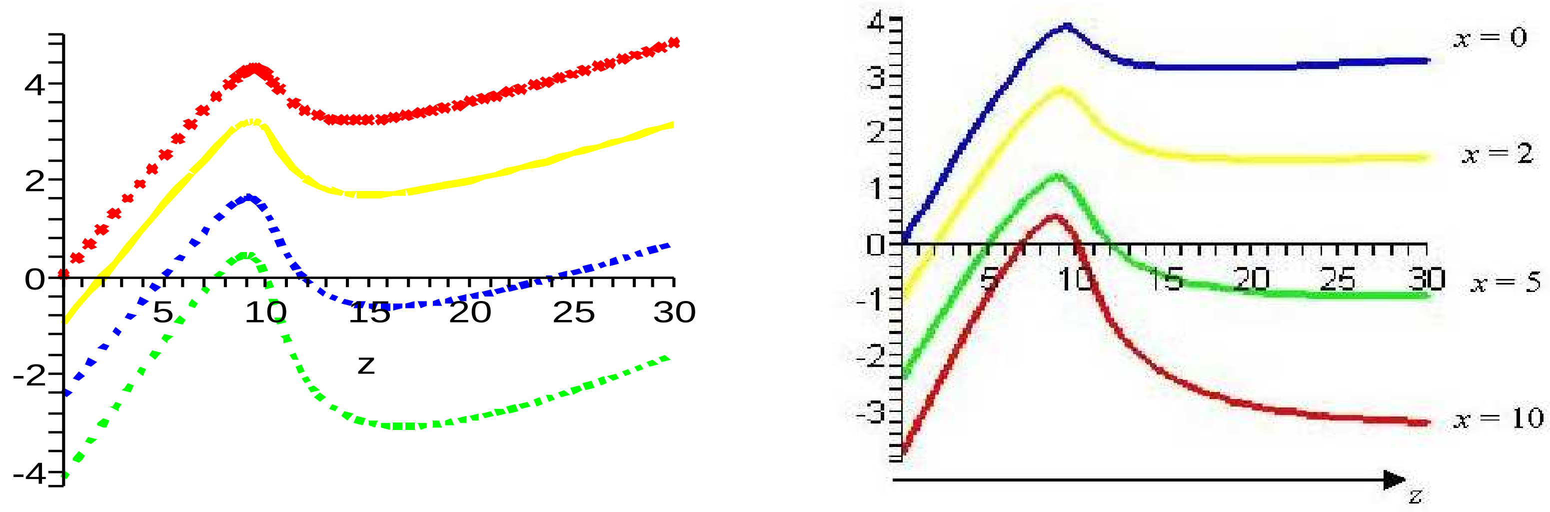


Figure 1: Threshold determining the cell boundaries (vertical axis) as a function of the location of BS2 (horizontal axis) for various locations $-x$ of BS1: $x = 10, 5, 2, 0$. Figure and 1 in the right one. $L = 10, \sigma = 0.3$

•The equilibrium cells in $[-L, L]$ are disjoint intenrvals: $S_1 = [-L, \theta]$, $S_2 = (\theta, L]$

•**Non-monotonicity of θ in z** (and in x).

2.4 Discontinuous Cells

•BS 1 located at $(-15, 1)$. $\alpha = 2, L = 10, \sigma = 0.3$.

•The location of BS 2 is $(z, 1)$ where z is varied from -10 to $+15$.

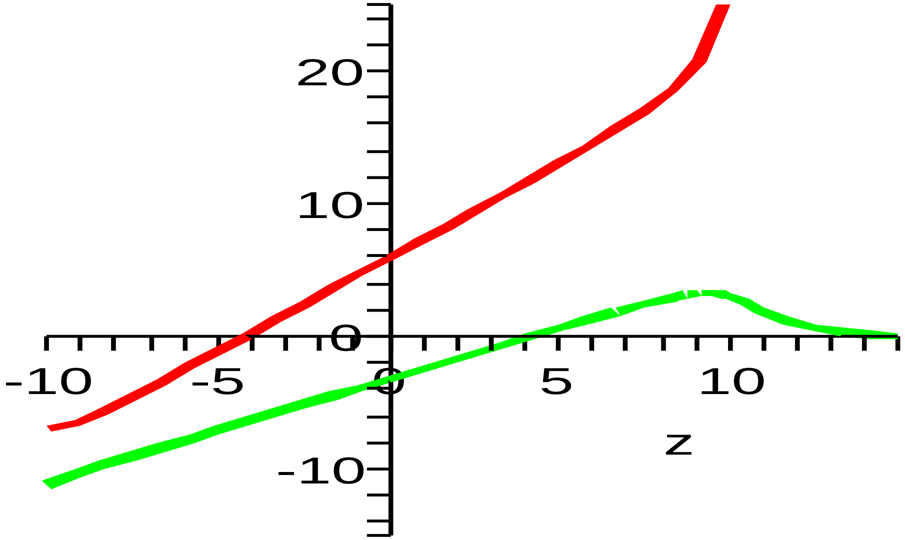


Figure 2: The two boundaries of the the cell corresponding to BS2 as a function nf the location of BS2.

- For each value of z (horizontal axis), we obtain two thresholds, as shown in Figure 2 through the two curves that describe the SINR association equilibrium.

- Call $\theta_1(z)$ the lower curve and $\theta_2(z)$ the higher one.

- The threshold represented by curve θ_1 lies in between the two base stations. The other one lies to the right of BS 2.

- The form of the equilibrium for the SINR-association problem is unusual in the location games.

The reason for the unusual features lies in the SINR criterion:

- When mobile 1 is close to the extreme point $-L$ of the line segment on which mobiles are located and BS 2 is inside the segment, further away from the boundaries, then BS 2 suffers from considerably more interference. (This is even more pronounced when BS 1 lies to the left of $-L$.)
- If a mobile is located sufficiently far from both Base Stations, then the relative difference in the power received at the BSs will be small.
- Thus it will prefer to connect to BS 1 that suffers from less interference. Only when it is sufficiently closer to BS 2 than to BS 1 will it become advantageous for it to connect to BS 2: the stronger gain will then compensate the larger interference.

3 Hierarchical equilibrium: association and optimal BS placement

•Total throughput at BS i:

$$T(x) = \frac{1}{2} \int_{S_i} \frac{g(y+x)}{Int(x)+\sigma^2} \, dy = \frac{1}{2} \frac{\int_{S_i} g(y+x) \, dy}{Int(x)+\sigma^2}$$

$$\text{where} \quad g(y) = [1+y^2]^{-\alpha/2}$$

•In the case of a single BS this reduces to:

$$T(x) = \frac{1}{2} \frac{Int(x)}{Int(x)+\sigma^2}$$

increases with $Int(x)$!

Proposition. Single BS:

•$Int(x)$ is an even function with a unique maximum at $x = 0$. Moreover, $Int(|x|)$ monotonically decreases with $|x|$.

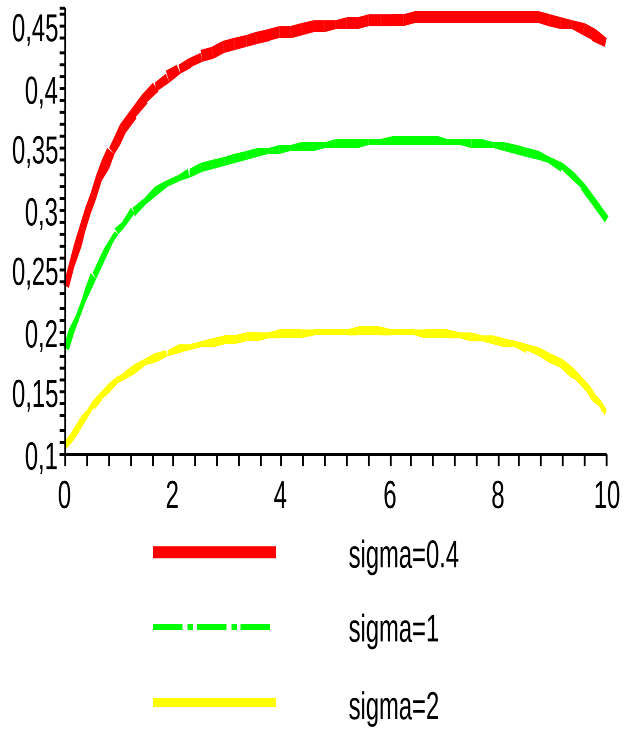
•Best placement at the origin.

3.1 Cooperative placement of 2 BSs

•Optimal placement of base stations for $L = 10, \alpha = 2,\ \sigma =$ 0.4, 1, 2.

•Global throughput obtained by each BS when located symmetrically at points $(-z, 1)$ and $(z, 1)$. We vary the distance z

•The origin and the extreme points (at distance L from the origin) perform bad. This will be in contrast with the case of competing base station.



•The performance close to the optimal location is quite robust to perturbation of the location of the base stations.

•The optimal distance of the BSs from the origin decreases in σ. This is seen from the location optimization results presented in Table 1.

•From further experimentations with larger σ we saw that as $\sigma \rightarrow \infty$, the optimal location of the BSs converges to -5 and 5. Note that at very large σ, the interference does not play a role anymore and the BSs are placed so as to maximize the total received power.

4 Hierarchical Equilibrium: Non-Cooperating BSs

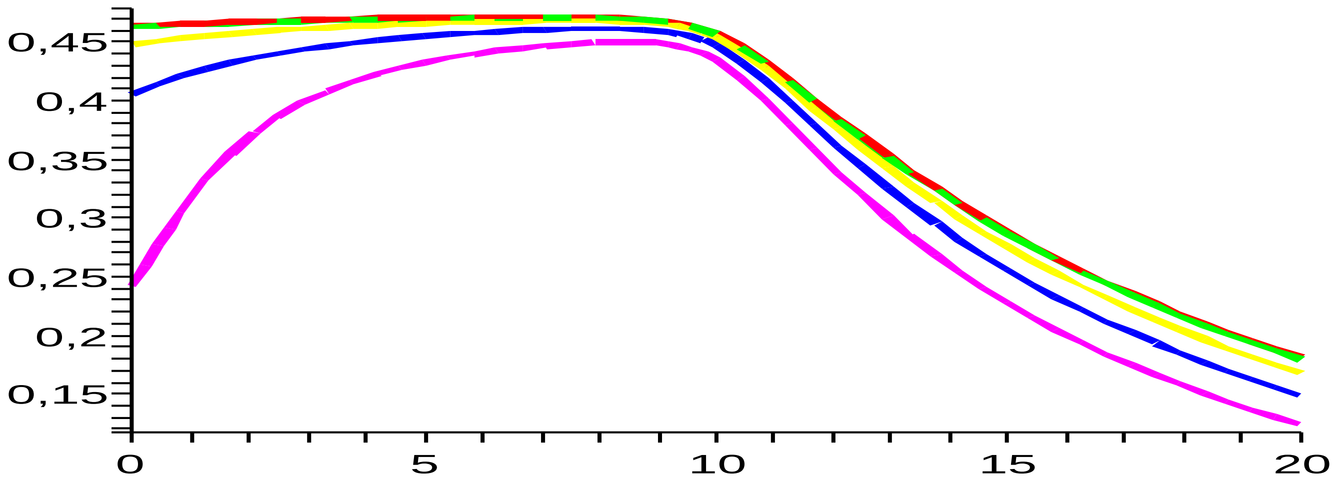


Figure 3: Thp at BS 2 as function of its location when BS 1 is at $-x$ where $x = 0, 3, 6, 9, 10$, $\alpha = 2$, $\sigma = 0.3$

•For all x, the largest throughput at BS 2 is obtained when its location is around $z = 8$. Equilibrium at $x = z = 7.1$.

•At $z = 0$, the thp of BS 2 is increasing for all x. Thus clustering at 0 is not an equilibrium!

Table 1: Optimal and Non-cooperative placement of base station as a function of σ

σ	0.1	0.4	1	2	40
Throughput per BS	0.486	0.460	0.356	0.200	0.000857
Optim. dist. of BSs to 0	8.658	7.745	6.435	5,591	5.002
Eq. dist. of BSs to 0	8.10	6.95	5.50	4,667	4.09

- The equilibria distance decrease with σ
- The equilibria distance is smaller than the optimal one
- The equilibrium distance converges to 4