

Spatial structure of the 12-hour wave in the Antarctic as observed by radar

D. M. Riggin¹, D. C. Fritts¹, M. J. Jarvis², and G. O. L. Jones²

¹Colorado Research Associates, Boulder, Colorado, U.S.A.

²British Antarctic Survey, NERC, Cambridge, U.K.

(Received August 17, 1998; Revised April 6, 1999; Accepted April 6, 1999)

We present radar measurements of the 12-hour wave, a zonal wavenumber 1 westward propagating wave that exists in the southern polar mesopause region winds (Hernandez *et al.*, 1993; Forbes *et al.*, 1995). MF radar measurements of the horizontal winds at McMurdo (77.8°S, 166.67°E) show that the 12-hour wave is highly seasonal, occurring during the austral summer solstice. During these seasonal occurrences, the wave is highly intermittent with amplitude peaks of $\approx 30 \text{ m s}^{-1}$. The burst-like occurrences of large 12-hour wave amplitudes are highly correlated between the zonal and meridional direction. The diurnal tide over McMurdo has a more constant amplitude, but it is also an almost exclusively summertime phenomenon. Inertia-gravity wave activity is evident at periods less than 12 hr during the austral winter months. The weakening of gravity wave activity during the summer is probably due to critical layer filtering by the zonal mean wind, 12-hour wave and diurnal tide which are all strong during this season. The 12-hour wave is confined in height to the vicinity of the zero crossing in the zonal winds above the westward jet. Extreme distortion is observed in the vertical phase fronts of the 12-hour wave which could signify either refraction or in situ forcing. The distortion in the phase fronts and localization of the 12-hour wave in time and height is apparently responsible for departures in period from the nominal 12 hours. We do not find the wave period to be systematically different from 12 hours. The association of the 12-hour wave events with shear in the mean wind suggests that refractive effects could conceivably cause a dilation in wave amplitude. However, the shear is of the opposite sign to cause this dilation unless the wave originates at higher altitudes and propagates downward into the mesosphere. Investigations are made of the zonal structure of the 12-hour wave by comparing phases of the 12-hour wind component between McMurdo and the dynasonde at Halley (75.8°S, 26.4°W). The phase is found to be stable and consistent with a westward propagating zonal wavenumber 2 structure during seasons when the 12-hour wave is weak. The migrating semidiurnal tide evidently dominates during these times of the year. During seasons when the 12-hour wave amplitude is large, the zonal structure is highly unstable and there is not an obvious dominant zonal wavenumber.

1. Introduction

Measurements with a Fabry-Perot spectrometer at the South Pole in the early 1990's (Hernandez *et al.*, 1992) first revealed the existence of a 12-hour oscillation in the wind field near mesopause heights above the South Pole. No temperature perturbations were found for this wave which suggests the absence of any 12-hour vertical wind motions (Hernandez *et al.*, 1997). The wave was subsequently shown to have a westward propagating, zonal wavenumber 1 structure (Hernandez *et al.*, 1993). It is therefore different from the migrating semidiurnal tide which has a zonal wavenumber 2 structure. From geometric considerations, the only large-scale, zonally propagating wave structure that can exist in the vicinity of the pole is zonal wavenumber 1, so the existence of a 12-hour zonal wavenumber 1 component of the motion field at the pole is not entirely unexpected. However, the amplitude of this wave at the South Pole is large, up to $\sim 20 \text{ m s}^{-1}$ as measured by spectrometer (Hernandez *et al.*, 1993) and by meteor radar (Portnyagin *et al.*, 1998). The peak amplitude may in fact have been underestimated, since

the meteor radar averaged echoes over the $\sim 10 \text{ km}$ height extent of the meteor decay region and the spectrometer averaged over an $\sim 10 \text{ km}$ thick OH airglow layer (Hernandez *et al.*, 1992). The implication of the large amplitudes is that 12-hour wave is unique to high latitudes and not merely a non-migrating semidiurnal tide. The 12-hour wave is primarily an austral summertime phenomenon, but it can also sometimes be seen during other seasons. Optical measurements cannot be made during full sunlight, and thus the earliest measurements of the 12-hour wave were made during late May (Hernandez *et al.*, 1992) and early August (Hernandez *et al.*, 1993). It is not yet known whether the wave also occurs at high northern latitudes and there is no generally accepted theory for what causes it. For the sake of generality we will avoid referring to it as a tide.

One objective of this paper is to provide further evidence that the 12-hour wave exists at latitudes away from the pole. McMurdo, Scott Base and Halley are the highest latitude sites on the Antarctic continent which currently have the capability to measure mesospheric winds. Data from McMurdo and Halley are used to investigate whether the zonal wavenumber structure of the 12-hour wave is zonal wavenumber 1 (as at the South Pole) or zonal wavenumber 2 (migrating). Other objectives of this paper are to further examine the vertical

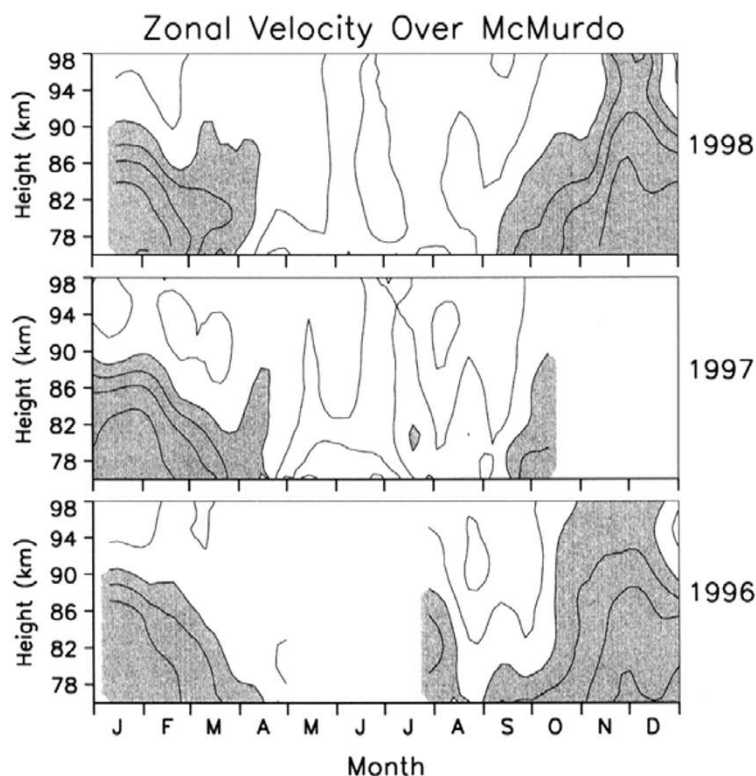


Fig. 1. Mean zonal winds over McMurdo with contours of 10 m s^{-1} (negative shaded) for 1996–98.

structure, phase stability and seasonal dependence of the 12-hour wave.

2. Measurement and Analysis Techniques

The MF radar at McMurdo is a standard design, nearly identical to that described in detail by Fritts and Isler (1994) and has been in operation since January of 1996. The radar operates at a frequency of 1.98 MHz, has a peak power of 25 kW, and employs a transmitted pulse width of $\sim 25 \mu\text{s}$ (corresponding to a range resolution of 3.75 km) with a sampling interval of 2 km, and a temporal resolution of 2 minutes. Most of the results described in the paper were derived with this radar, but in addition we will present some 12-hour wave phase comparisons between the MF radar at McMurdo and the dynasonde at Halley.

The dynasonde, a digital ionospheric sounder (Dudeney *et al.*, 1995), has made ionospheric observations above Halley since 1982. In January 1997 an additional sounding mode and on-line data processing were implemented to allow mesospheric wind measurements to be made using the Imaging Doppler Interferometer (IDI) technique (Jones *et al.*, 1997). The IDI technique identifies radar scattering points in the mesosphere from the consistency of their Doppler phase shifts during a sounding. The angular position of the echoing region is identified by interferometry using four horizontally-spaced receiving antennas, and the echo line-of-sight distance is determined from the time delay between pulse transmission and reception. All echoes from a given sounding are sorted into 5 km height bins and a vector fit is then performed to determine a 3-D velocity representing the neutral wind motion in the mesosphere with

a time resolution of 5 minutes. Halley (75.8°S , 26.4°W) and McMurdo (77.8°S , 166.67°E) are at nearly the same latitude and 193° apart in longitude, nearly ideal locations for making zonal wavenumber estimates. We found that the spatial and temporal patterns of the winds were quite similar at the two sites, but that the wind magnitudes are smaller over Halley. This discrepancy is fortunately not an issue for our analysis since only the correct phase of the 12-hour wave component is needed for accurate zonal wavenumber determination. The mesospheric winds over Halley are further discussed in Charles and Jones (1999).

Our observational results include time-frequency representations of wind amplitudes using the *S*-transform (Stockwell *et al.*, 1996; Fritts *et al.*, 1998). The *S*-transform is a technique for temporal localization of the Fourier transform. It has some similarities to wavelets, but has several unique properties including an absolute phase reference. Although the *S*-transform is not orthogonal, it is fully invertible and collapses in the time domain to give the Fourier spectrum exactly. The localizing function of the *S*-transform is a Gaussian ($\sim e^{-t^2/2d^2}$), where $d = c/f$. The minimum value of $c = 1$ provides a time resolution equal to one period, but larger values of c provide better frequency resolution at the expense of time resolution. The amplitudes are easily interpreted since they are the same as would be derived from least squares fits of sinusoids over Gaussian windows. It can be shown that the choice of a Gaussian window is optimal in the sense of minimizing the uncertainty product $\Delta t \Delta f$ (Papoulis, 1977).

Representations are also made of the 12-hour wave amplitude and phase as a function of time and height. Singular

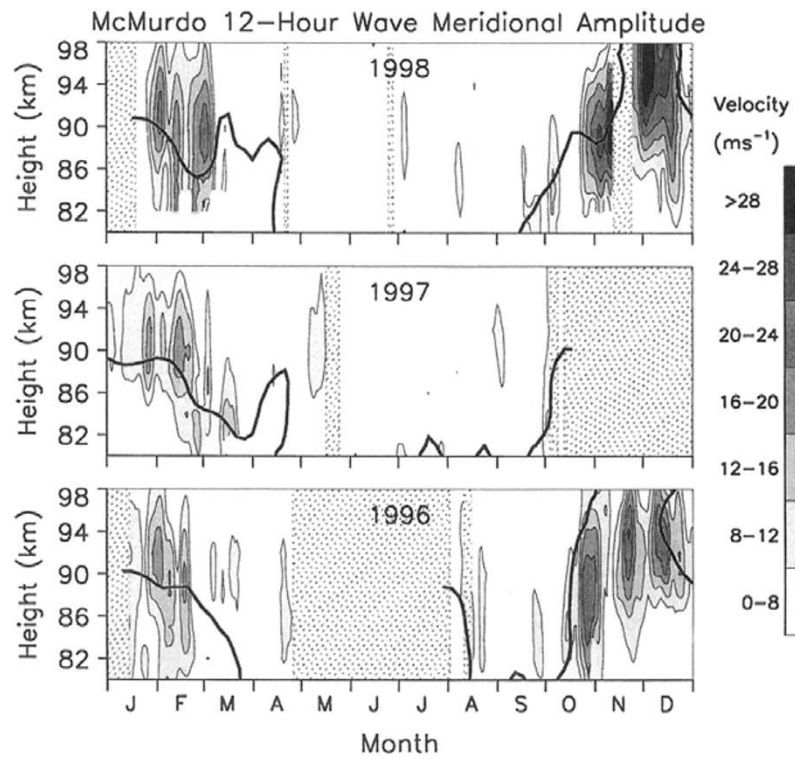


Fig. 2. Amplitude of the meridional component of the 12-hour wave computed from sliding least squares fits and spanning 1996–98. The heavy bold lines indicate the zero zonal wind line.

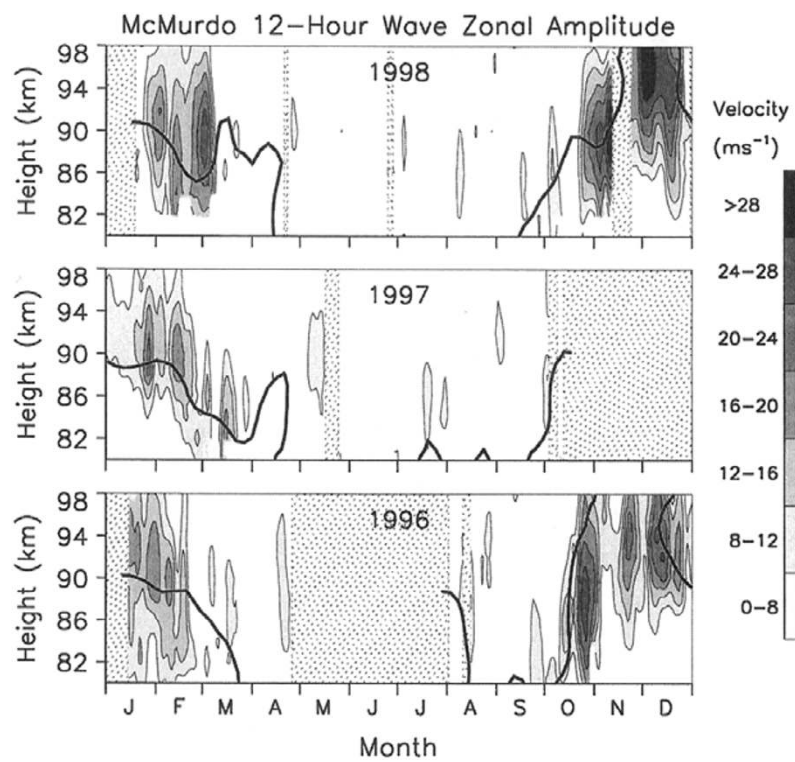


Fig. 3. Same as Fig. 2, but for the zonal component.

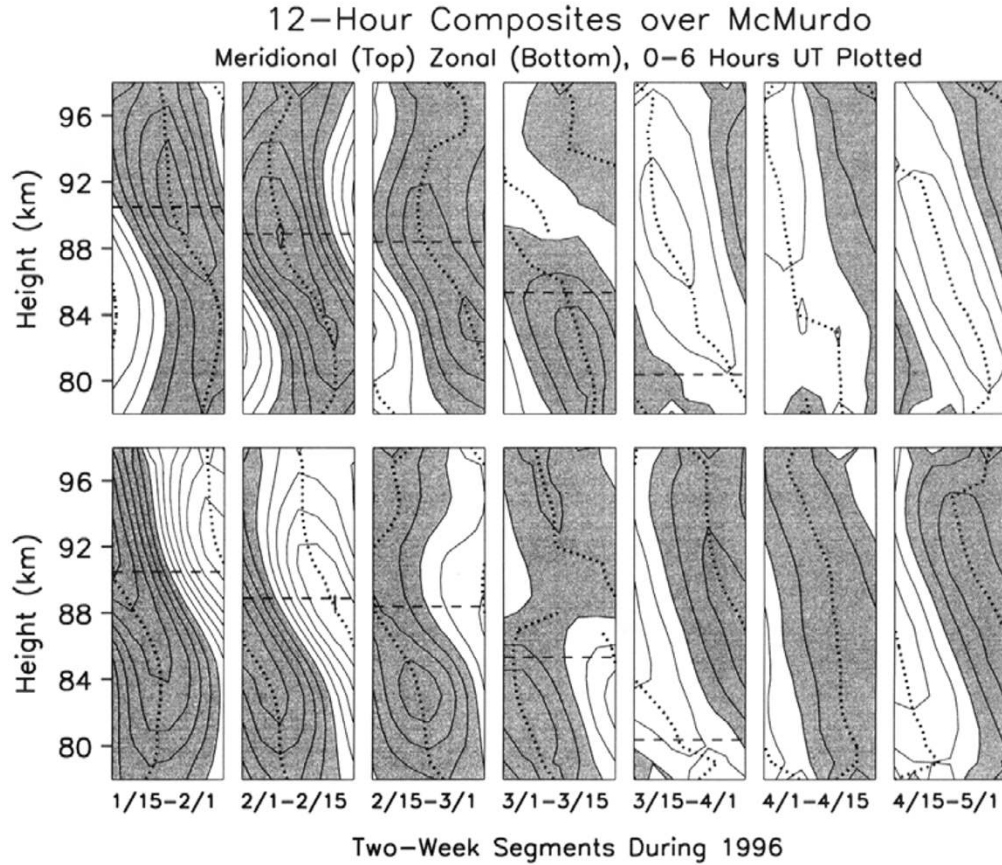


Fig. 4. Composite values of the 12-hour wave (top: meridional, bottom: zonal) each over a two week time segment. The set of composites covers January 15 through May 1, 1996. Only 6 hours of the 12-hour composite are plotted. The contour increments are 4 m s^{-1} and negative values are shaded.

value decomposition (SVD) sinusoidal fits with a Gaussian localizing window were used for these calculations. The Gaussian window was stepped through the data at one-day increments to provide a time series of amplitude and phase for the 12-hour wave at altitudes of 80 to 98 km. The amplitudes and phases over McMurdo determined from this technique provide information on the vertical structure of the 12-hour wave. Time series of 12-hour amplitudes and phases from McMurdo and Halley were used to study the horizontal wavenumber structure of the wave. The amplitude/phase fits can be written in complex form as $\tilde{u}'_j = U_j e^{i\omega t - i\phi_j}$, where j subscript refers to the two stations. From the complex \tilde{u}'_j 's at each height (dropping the t dependence), the complex “cross-fit” can be formed,

$$C_{1,2} = \frac{\langle U_1 e^{-i\phi_1} U_2 e^{+i\phi_2} \rangle}{\langle U_1 \rangle \langle U_2 \rangle}, \quad (1)$$

where the angular brackets denote an average over altitude. Because of the normalization in the denominator, the absolute value of $C_{1,2}$ (or coherence) does not depend on the wave amplitude. The coherence has a range of 0–1, approaching 1 if the wave-front phase structure has the same height dependence at the two sites. The phase of the cross-fit ($\Delta\phi_{1,2} = \Delta\phi_2 - \Delta\phi_1$) is a measure of the phase difference of the wave between the two sites, but is only reliable for larger values of coherence. The technique is further described in Riggins *et al.* (1997).

3. Results

The mean zonal wind over McMurdo is shown in Fig. 1. In spite of the data gaps there is an obvious annual wind variation, the dominant feature of which is an intense westward jet during austral summer. This wind structure is reasonably consistent (via the thermal wind relation) with the well-known temperature minimum in the summer polar mesopause. Figures 2–3 show the meridional and zonal amplitude of the 12-hour wave in a similar time versus height format. The lowest contour is 8 m s^{-1} to highlight the strong seasonal and height dependence of the 12-hour wave. These dependencies are obscured if a lower threshold contour is used since there is a weak 12-hour wave background amplitude that is present throughout the year and which lacks a pronounced height dependence. As further discussed later, we have tentatively identified this weak, diffuse 12-hour contribution with the migrating semidiurnal tide. The heavy solid lines in the figure portray the zero crossing of the zonal wind where the westward jets reverse. The large amplitudes are restricted to austral summer and are also confined to the vicinity of the zero crossing of the zonal wind. Even during the austral summer, the periods of 12-hour wave activity are highly intermittent, but the bursts of activity are highly correlated in the meridional and zonal directions.

The highly intermittent forcing may have implications for the vertical phase structure of the 12-hour wave. This vertical structure is examined using the composites shown in Fig. 4.

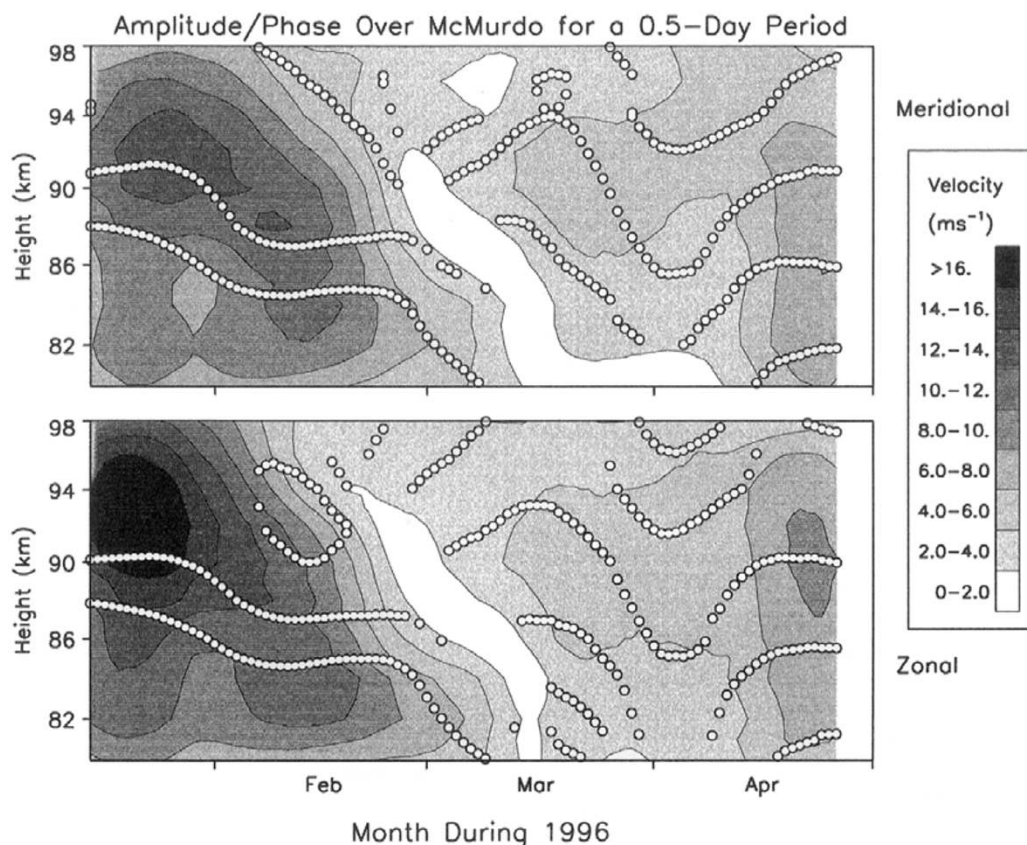


Fig. 5. Contours of meridional (top) and zonal (bottom) 12-hour wave amplitude. The open dots trace out constant phase lines (separated by $\pi/4$ in phase).

To generate these composites first the hourly data are parsed into two week segments and a 24-hour sinusoidal fit is applied to the data at each height and subtracted. Each two-week segment of data is further subdivided into 12-hour sub-segments. The first hour is averaged across all of the sub-segments and similar averages are performed on all the subsequent hours. This yields a 12-hour composite. The final step is to take the negative of the last 6 hours of the composite and average them one-by-one with the first 6 hours. The full set of composites covers January 15 through May 1, 1996. The horizontal axis of each composite covers six hours or one half cycle of the 12-hour wave and dotted lines are used to delineate positive and negative minima in wave perturbation velocity at each height. Note that the meridional composites (top row) are in approximate phase quadrature with the zonal composites (bottom row). During the period shown in Fig. 4, sharp kinks are evident in the vertical phase structure. These kinks seem to be associated with the amplitude maximum of the 12-hour wave and also with the zero crossing of the zonal wind, the height of which is indicated by the horizontal dashed line. The kinks, amplitude maxima and zero crossing of the zonal wind all progressively move downward during the 3.5 month period. As the 12-hour wave weakens, the vertical phase variation becomes more linear. Close examination of the phases reveals that the 12-hour wave is nearly circularly polarized. The meridional and zonal 12-wave components are almost in phase quadrature with anticyclonic rotation with increasing height and time (i.e., the eastward wave component leads the

northward). The sense of phase rotation is consistent with that of an inertia-gravity wave with upward energy propagation. The wave shown in Fig. 4 has a vertical wavelength of ~ 40 km. The optical and meteor radar measurements of 12-hour wave winds at the South Pole were vertically averaged over a ~ 10 km height swath (Hernandez *et al.*, 1992; Forbes *et al.*, 1995). Assuming the vertical structure of the 12-hour wave at the South Pole resembles that at McMurdo, these measurement systems underestimated the wave amplitude by a factor of ~ 2 .

Figure 5 gives another representation of the vertical phase structure during the same period. The analysis is similar to that used for Figs. 2 and 3. However, the Gaussian window width is wider with a standard deviation of ± 5 wave periods, compared with the ± 3 period width in Figs. 2–3. The wider window smoothes the amplitude structure and reduces the amplitudes, but is necessary to make accurate phase estimates. Phase values are portrayed with the white dots in Fig. 5 with the constant phase lines separated by one eighth of a cycle. If the wave had a steady 12-hour period, the phase lines would remain horizontal. Although the phase fluctuates, the meridional and zonal phases are obviously well correlated. The contours of phase are not labeled, but they increase downwards consistent with upward energy propagation. Given downward phase progression, phase lines which trend downwards in time indicate a wave period less than 12 hours. In fact, the power spectra for this 3.5 month period were peaked at a period of 11.94 hours (Fritts *et al.*,

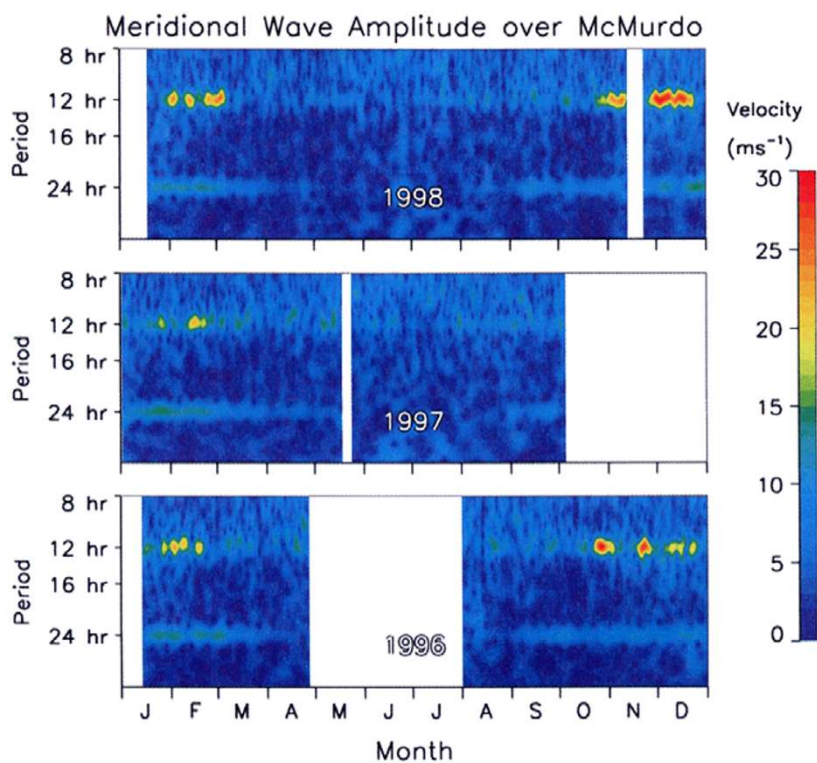


Fig. 6. *S*-transform decomposition of the meridional winds between 8 and 36 hours for 1996–98. Transforms were calculated for heights of 80 to 98 km and then the absolute values averaged over these heights.

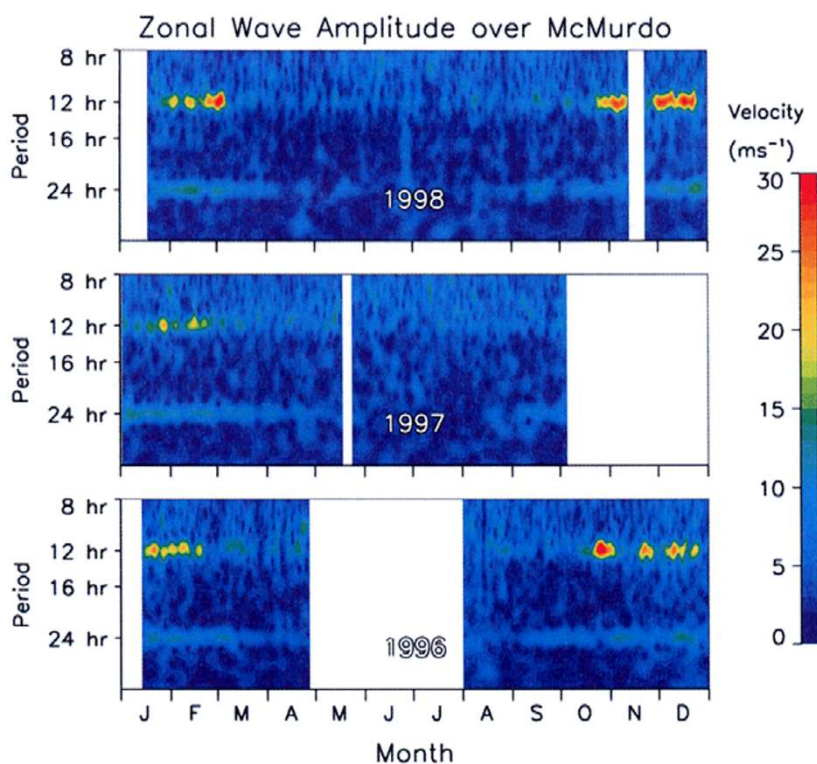


Fig. 7. Same as Fig. 6, but for zonal winds.

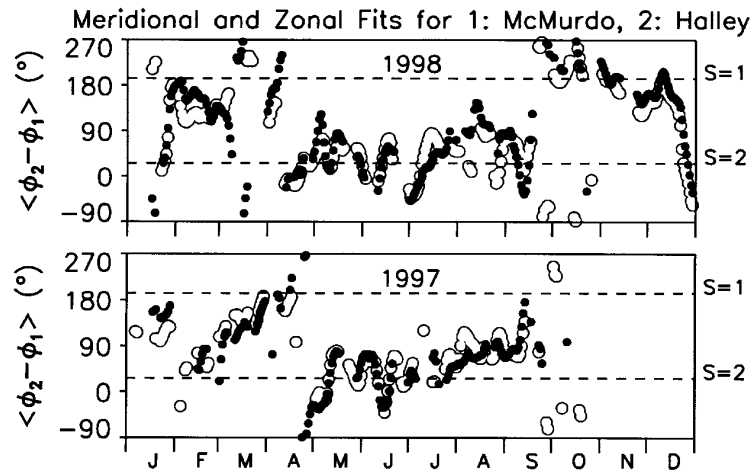


Fig. 8. Phase difference between McMurdo and Halley Base derived from fits to the 12-hour wave. The cross-fits were averaged from 80 to 98 km. Open (solid) dots denote the meridional (zonal) component. The horizontal dashed lines show the phase difference expected for westward zonal wavenumbers of 1 and 2.

1998). As seen in Fig. 5, the phase lines are mostly descending during the times when the wave amplitude is strongest. The power spectra are weighted according to the amplitude squared so this period of descending phase skews the spectral peak to a period less than 12 hours. During early March the phase lines are ascending and a power spectrum computed during this period has a spectral peak at a period longer than 12 hours. Our conclusion is that the phase of the 12-hour wave is unsteady, but the frequency does not systematically deviate from 12 hours.

S -transform decompositions of the meridional and zonal wind components are shown in Figs. 6 and 7, respectively. These figures focus on spectral information between 8 and 36 hours with an S -transform localizing parameter (c) of three. In this presentation, complex S -transforms have been calculated for heights from 80 to 98 km, and then the absolute values averaged over these heights. Note that both the meridional and zonal amplitudes are substantial ($\gtrsim 30 \text{ m s}^{-1}$), in spite of the wide localizing window and considerable height averaging. The diurnal tide and 12-hour wave have nearly identical seasonal dependences, but the diurnal tide lacks the deep amplitude modulation that characterizes the 12-hour wave. There is little wave activity between 12 and 24 hours, but the wave amplitudes are sporadically large at periods less than 12 hours. These waves are short-lived and we found that they do not occur at preferred periods. These characteristics and the restriction of the waves to periods less than the inertial period (12.3 hours at McMurdo) leads us to conclude that they are inertia-gravity waves. These shorter period waves are less prevalent during austral summer when the westward jet, 12-hour wave, and diurnal tide are strong. This anticorrelation is not unexpected since inertia-gravity waves are highly susceptible to critical layer filtering. Comparison of Figs. 6 and 7 demonstrates that the burst-like occurrences of the 12-hour wave are highly correlated in the zonal and meridional directions.

Figure 8 shows the phase angle by which the 12-hour wave signal at Halley lags McMurdo. The cross-fit procedure was

performed using Eq. (1) with altitude averaging between 80 and 98 km. The Gaussian localizing window has a standard deviation width of ± 3 wave periods in this presentation. Although the phase estimates undergo large fluctuations, the meridional phase (open dots) and zonal phase (filled dots) are usually in good agreement. Surprisingly, the phase becomes more stable during the austral mid-winter months when the 12-hour wave amplitudes become weak. The phase during this season is characteristic of a migrating semidiurnal tide ($S = 2$). During times when the 12-hour wave amplitudes are strong the phase is erratic and it is difficult to infer a particular zonal wavenumber. However, there are some tendencies for a westward zonal wavenumber 1 structure during the early part and later part of 1998.

4. Discussion and Conclusions

Climatological data from the Scott Base radar, which is not more than kilometer away from the McMurdo MF site, confirm the McMurdo observations of an austral mid-summer maximum for the 12-hour wave (Portnyagin *et al.*, 1993; Stening *et al.*, 1995). The HRDI instrument aboard the UARS satellite has observed a different seasonal dependence for the migrating semidiurnal tide at $\sim 65^\circ\text{S}$ with an amplitude maximum in May (Burrage *et al.*, 1995). Radar sites at $\sim 68^\circ\text{S}$ also see an April or May maximum (Portnyagin *et al.*, 1998). The radar climatology led Portnyagin *et al.* (1998) to infer that the $S = 1$ structure dominates poleward of $\sim 78^\circ\text{S}$, but dies out and is replaced by $S = 2$ between 68°S and 78°S . Data from the new MF radars at Rothera (68°S , 68°W) and Syowa (69°S , 40°E) will eventually provide further information on the latitudinal extent of the 12-hour ($S = 1$) wave.

Deviations of the 12-hour wave from the nominal 12 hours have been observed over South Pole (e.g., Hernandez *et al.*, 1993; Forbes *et al.*, 1995; Portnyagin *et al.*, 1998) and over McMurdo (Fritts *et al.*, 1998). We further investigated the data segment from which Fritts *et al.* (1998) determined an 11.94 hour period over McMurdo. The constant phase lines of the 12-hour wave were found to episodically ascend and

descend in altitude. Distortions in vertical wavelength can be linked to frequency variations by a conservation of wavefronts expression (Gill, 1982).

$$\partial\omega/\partial z = -\partial m/\partial t. \quad (2)$$

The constant phase (ϕ) lines shown in Fig. 5 are related to m by $m = -\partial\phi/\partial z$. Therefore, we can write $\Delta\omega = \Delta\phi/\Delta t$. Phase generally decreases with increasing z . Therefore, a downward deviation in a constant phase line as a function of time near some fixed height corresponds to decreasing ϕ at that height and this in turn implies a smaller ω . The converse is obviously true for an upward deviation. During this period shown in Fig. 5, the constant phase lines were downward sloping in time in the time-height region where the wave was strongest. Thus, the power spectrum of this time interval (shown in Fritts *et al.* (1998)) is more heavily weighted in the region where the frequency is less than 12 hours. The magnitude of this effect is sufficient to explain the deviations from 12 hour wave period observed at McMurdo and the South Pole. Our conclusion is that the period of the 12-hour unsteady, but not systematically different from 12 hours.

Kinks in the phase fronts of the 12-hour wave were found to be associated with amplitude maxima which were strongly localized in height. These features suggest either in situ excitation of the 12-hour wave or effects due to wave refraction. Portnyagin *et al.* (1998) examined the in situ excitation hypothesis with a numerical model and found it to be plausible. However, the results were not entirely conclusive since the source distribution was ad hoc and tuned to fit the observations. The strong confinement of the 12-hour wave to a region of strong shear above the westward summer jet, suggests a possible dilation due to refractive effects (Riggin *et al.*, 1997). However, a westward traveling wave propagating upwards into the eastward shear above the jet would suffer an amplitude contraction rather than a dilation. Wave refraction is only consistent with the observations if the 12-hour wave had a thermospheric source and propagated down the mesosphere from above. As yet there is no confirmatory experimental evidence for a thermospheric 12-hour wave source, but nonlinear interaction between the migrating semidiurnal tide and a stationary heating source (such as particle precipitation or Joule heating) could conceivably drive such a wave. However, it is difficult to reconcile a thermospheric source for the 12-hour wave with the observed anticyclonic rotation of the wave perturbation winds in height and time.

Acknowledgments. We gratefully acknowledge the strenuous efforts of Mercedes Huaman (who made emergency repairs to the McMurdo radar), Jeanne Kelley (who is operating the radar through the 1998 austral winter), and Joe Pettit (who assisted in the radar

decommissioning). We also acknowledge the British Antarctic Survey's engineering support staff for maintaining the Halley dynamometer. This research was supported by NSF grants OPP-9319068, NSF9709030, and ATM9612743.

References

- Burrage, M. D., D. L. Wu, W. R. Skinner, D. A. Ortland, and P. B. Hays, Latitude and seasonal dependence of the semidiurnal tide observed by the high-resolution Doppler imager, *J. Geophys. Res.*, **100**, 11,313–11,321, 1995.
- Charles, K. and G. O. L. Jones, Mesospheric mean winds and tides observed by the Imaging Doppler Interferometer (IDI) at Halley, Antarctica, *J. Atmos. Sol.-Terr. Phys.*, **61**, 351–362, 1999.
- Dudeney, J. R., A. S. Rodger, A. J. Smith, M. J. Jarvis, and K. Morrison, Satellite experiments simultaneous with Antarctic measurements (SESAME), *Space Sci. Rev.*, **71**, 705–742, 1995.
- Forbes, J. M., N. A. Makarov, and Yu. I. Portnyagin, First results from the meteor radar at South Pole: A large 12-hour oscillation with zonal wavenumber one, *Geophys. Res. Lett.*, **22**, 3247–3250, 1995.
- Fritts, D. C. and J. R. Isler, Mean motions and tidal and two-day structure and variability in the mesosphere and lower thermosphere over Hawaii, *J. Atmos. Sci.*, **51**, 2145–2164, 1994.
- Fritts, D. C., D. M. Riggin, B. B. Balsley, and R. G. Stockwell, Recent results with an MF radar at McMurdo, Antarctica: Characteristics and variability of motions near 12-hour period in the mesosphere, *Geophys. Res. Lett.*, **25**, 297–300, 1998.
- Gill, A. E., *Atmosphere-Ocean Dynamics*, 662 pp., Academic Press, London, 1982.
- Hernandez, G., R. W. Smith, and J. F. Conner, Neutral wind and temperature in the upper mesosphere above South Pole, Antarctica, *Geophys. Res. Lett.*, **19**, 53–56, 1992.
- Hernandez, G., G. J. Fraser, and R. W. Smith, Mesospheric 12-hour oscillation near South Pole, Antarctica, *Geophys. Res. Lett.*, **20**, 1787–1790, 1993.
- Hernandez, G., R. W. Smith, J. M. Kelley, G. J. Fraser, and K. C. Clark, Mesospheric standing waves near South Pole, *Geophys. Res. Lett.*, **24**, 1987–1990, 1997.
- Jones, G. O. L., K. Charles, and M. J. Jarvis, First mesospheric observations using an imaging Doppler interferometer adaptation of the dynamometer at Halley, Antarctica, *Radio Sci.*, **32**, 2109–2122, 1997.
- Papoulis, A., *Signal Analysis*, 431 pp., McGraw-Hill, New York, 1977.
- Portnyagin, Yu. I., J. M. Forbes, G. J. Fraser, R. A. Vincent, S. K. Avery, I. A. Lysenko, and N. A. Makarov, Dynamics of the Antarctic and Arctic mesosphere and lower thermosphere regions—II. The semidiurnal tide, *J. Atmos. Terr. Phys.*, **55**, 843–855, 1993.
- Portnyagin, Yu. I., J. M. Forbes, N. A. Makarov, E. G. Makarov, E. G. Merzlyadov, and S. Palo, The summertime 12-hour wind oscillation with zonal wavenumber $S = 1$ in the lower thermosphere over the South Pole, *Ann. Geophys.*, **18**, 828–837, 1998.
- Riggin, D., D. C. Fritts, T. Tsuda, T. Nakamura, and R. A. Vincent, Radar observations of a 3-day Kelvin wave in the equatorial mesosphere, *J. Geophys. Res.*, **102**, 26,141–26,157, 1997.
- Stening, R., K. Fleming, and G. Fraser, Upper atmosphere semidiurnal tides at Christchurch (44°S) and Scott Base (78°S), *J. Atmos. Terr. Phys.*, **57**, 857–869, 1995.
- Stockwell, R. G., L. Mansinha, and R. P. Lowe, Localization of the complex spectrum: The S transform, *IEEE Trans. Signal Processing*, **44**, 998–1001, 1996.

D. M. Riggin (e-mail: riggin@colorado-research.com), D. C. Fritts, M. J. Jarvis, and G. O. L. Jones