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Spatially Piecewise Fuzzy Control Design for Sampled-Data Exponential Stabilization of Semi-linear Parabolic PDE Systems

Jun-Wei Wang, Shun-Hung Tsai, Han-Xiong Li, and Hak-Keung Lam

Abstract—This paper employs a Takagi-Sugeno (T-S) fuzzy partial differential equation (PDE) model to solve the problem of sampled-data exponential stabilization in the sense of spatial L^∞ norm $\|\cdot\|_\infty$ for a class of nonlinear parabolic distributed parameter systems (DPSs), where only a few actuators and sensors are discretely distributed in space. Initially, a T-S fuzzy PDE model is assumed to be derived by the sector nonlinearity method to accurately describe complex spatiotemporal dynamics of the nonlinear DPSs. Subsequently, a static sampled-data fuzzy local state feedback controller is constructed based on the T-S fuzzy PDE model. By constructing an appropriate Lyapunov–Krasovskii functional candidate and employing vector-valued Wirtinger's inequalities, a variation of vector-valued Poincaré–Wirtinger inequality in 1D spatial domain, as well as a vector-valued Agmon's inequality, it is shown that the suggested sampled-data fuzzy controller exponentially stabilizes the nonlinear DPSs in the sense of $\|\cdot\|_\infty$, if sufficient conditions presented in term of standard linear matrix inequalities (LMIs) are fulfilled. Moreover, an LMI relaxation technique is utilized to enhance exponential stabilization ability of the suggested sampled-data fuzzy controller. Finally, the satisfactory and better performance of the suggested sampled-data fuzzy controller are demonstrated by numerical simulation results of two examples.

Index Terms—Sampled-data control, exponential stability, distributed parameter systems, Agmon's inequality, Takagi-Sugeno fuzzy partial differential equation model.

I. INTRODUCTION

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MUCH research has been done concerned with sampled-data control systems over the past few decades, leading to considerable results for analysis and synthesis of sampled-data control systems (e.g., [1]- [13] and the references therein). Note that the aforementioned results are mainly developed for dynamic systems whose behavior only depends on time and mathematical models take the form of ordinary/delay differential equations (O/DDEs). In real industrial applications, however, most processes are spatiotemporal in nature so that their behavior must depend on time as well as spatial position, for example, thermal diffusion processes, pipe flow, chemical processes, and flexible mechanical structure, etc [14]- [17]. These spatiotemporal processes are modeled as distributed parameter systems (DPSs) described by partial differential equations (PDEs). By the model reduction techniques (e.g., Galerkin's method), a finite-dimensional sampled-data boundary control design was developed in [18] for a linear parabolic DPS and a finite-dimensional observer-based active fault-tolerant control design was reported in [19] for nonlinear DPSs with sampled-data measurements in time. But an exact performance of closed-loop DPSs is difficultly achieved due to truncation before control design. To achieve exact and better control performance, some infinite-dimensional robust design results have been developed in [20]- [23] for linear sampled-data controllers of nonlinear parabolic DPSs directly on the basis of their original semi-linear PDE model.

On the other hand, fuzzy control based on the Takagi-Sugeno (T-S) fuzzy model [24] offers a conceptually simple, systematic and effective framework to support nonlinear control design of complex dynamic systems and has become increasingly popular during the past few decades (see, e.g., [25]- [27]). In the last decade, many fruitful results of fuzzy-model-based sampled-data control design have been reported for nonlinear O/DDE systems [28]- [38]. With the help of the Galerkin's method with the singular perturbation approach, finite-dimensional guaranteed cost and H_∞ fuzzy-model-based sampled-data control designs have been recently proposed in [39] and [40] for semi-linear parabolic PDE (SLP-PDE) systems. Although fuzzy-model-based control approach has been successfully extended in [41]- [46] for infinite-dimensional nonlinear control design of SLPPDE systems by proposing T-S fuzzy PDE model, the suggested results mainly focus on exponential stabilization of SLPPDE systems via continuous-time fuzzy controllers. To the best of authors' knowledge, the result on infinite-dimensional fuzzy-model-based control design has not been reported yet for sampled-

data exponential stabilization of SLPPDE systems with a finite number of actuators and sensors, which motivates this study.

This paper discusses the problem of infinite-dimensional fuzzy-model-based sampled-data control design in the sense of spatial L^∞ norm $\|\cdot\|_\infty$ for a class of SLPPDE systems with a finite number of actuators and sensors discretely distributed over the spatial domain. The objective of this paper is to develop a conceptually simple but effective infinite-dimensional design method of a sampled-data fuzzy controller only using state information taken from sensors located at some known local areas of the spatial domain (i.e., local piecewise state information), such that the resulting closed-loop PDE system is exponentially stable in the sense of $\|\cdot\|_\infty$. In the proposed design method, a T-S fuzzy PDE model is first assumed to be constructed [45], [46] to accurately describe the complex spatiotemporal dynamics of the SLPPDE system. Based on the fuzzy PDE model, a sampled-data fuzzy controller is then constructed only using local piecewise state information. It is shown by constructing an appropriate Lyapunov-Krasovskii functional candidate and using vector-valued Wirtinger's inequalities, a variation of vector-valued Poincaré–Wirtinger inequality in 1D spatial domain, as well as a vector-valued Agmon's inequality that the suggested sampled-data fuzzy controller exponentially stabilizes the SLPPDE system in the sense of $\|\cdot\|_\infty$. Moreover, exponential stabilization ability of the suggested sampled-data fuzzy controller is enhanced by providing an LMI relaxation technique. The main results of this paper are presented in terms of standard LMIs, which are directly solved by the solvers in MATLAB's LMI Control Toolbox [47]. Finally, the effectiveness and merit of the suggested sampled-data fuzzy controller are demonstrated by numerical simulation results of two examples.

Main contribution and novelty of this paper are summarized as follows: (i) A variation of vector-valued Poincaré–Wirtinger inequality in 1D spatial domain is provided by first mean value theorem for definite integrals; (ii) A vector-valued Agmon's inequality is introduced from the scalar one; (iii) An LMI-based infinite-dimensional design method of static sampled-data fuzzy controller is developed for exponential stabilization of the SLPPDE system in the sense of $\|\cdot\|_\infty$. In comparison to existing finite-dimensional fuzzy sampled-data control designs in [39] and [40] for the SLPPDE systems, the main difference of the proposed design method of this paper lies in that finite-dimensional fuzzy sampled-data control designs in [39] and [40] only ensure exponential stabilization in the sense of spatial L^2 norm $\|\cdot\|_2$, while the infinite-dimensional sampled-data fuzzy control design of this paper guarantees exponential stabilization in the sense of $\|\cdot\|_\infty$.

The organization of the rest of this paper is given as follows. Section II provides preliminaries and problem formulation. Section III gives a static sampled-data fuzzy control design. Numerical simulation results are provided in Section IV to show satisfactory and better performance of the suggested sampled-data fuzzy controller than the existing ones. Finally, Section V offers some brief concluding remarks.

Notation: The set of all real numbers, n-dimensional Euclidean space with the norm $\|\cdot\|$ and the set of all $m \times n$ matrices are denoted by $\Re$, $\Re^n$ and $\Re^{m \times n}$, respectively. $|\varsigma|$

denotes absolute value of any real number ς. $\boldsymbol{I}$ stands for an identity matrix of appropriate dimension. For a matrix $\boldsymbol{M} \in \Re^{n \times n}$, $\boldsymbol{M} > (<, \leq)0$ means that it is symmetric and positive definite (negative definite, negative semi-definite, respectively). For a square matrix $\boldsymbol{A}$, its minimum and maximum eigenvalues are denoted by $\lambda_{\min}(\boldsymbol{A})$ and $\lambda_{\max}(\boldsymbol{A})$, respectively. For a given scalar $L > 0$, $\mathcal{L}_n^2([0, L]) \triangleq \mathcal{L}^2([0, L]; \Re^n)$ is a Hilbert space of square integrable vector functions $\boldsymbol{\omega}(x) : [0, L] \to \Re^n$ with the norm $\|\boldsymbol{\omega}(\cdot)\|_2 \triangleq \sqrt{\int_0^L \boldsymbol{\omega}^T(x)\boldsymbol{\omega}(x)dx}$. This norm is also referred to as ***spatial L^2 norm***. Given an integer $\bar{k}$ and a constant $L > 0$, $\mathcal{H}_n^{\bar{k}}((0, L)) \triangleq \mathcal{W}^{\bar{k}, 2}((0, L); \Re^n)$ is a Sobolev space of absolutely continuous vector functions $\boldsymbol{\omega}(x) : (0, L) \to \Re^n$ with square integrable derivatives $\frac{d^{\bar{k}}\boldsymbol{\omega}(x)}{dx^{\bar{k}}}$ of the order $\bar{k}$ and with the norm $\|\boldsymbol{\omega}(\cdot)\|_{\mathcal{H}_n^{\bar{k}}((0, L))} \triangleq \sqrt{\int_0^L \sum_{i=0}^{\bar{k}} \frac{d^i \boldsymbol{\omega}^T(x)}{dx^i} \frac{d^i \boldsymbol{\omega}(x)}{dx^i} dx}$. For any $\boldsymbol{y}(\cdot, t) \in \mathcal{H}_n^1((0, L))$, ***spatial L^∞ norm*** is defined as $\|\boldsymbol{y}(\cdot, t)\|_\infty \triangleq \max_{x \in [0, L]} \|\boldsymbol{y}(x, t)\|$. The subscripts x and t of $\boldsymbol{y}(x, t)$ are partial derivatives with respect to x and t, i.e., $\boldsymbol{y}_t(x, t) = \partial \boldsymbol{y}(x, t)/\partial t$, $\boldsymbol{y}_x(x, t) = \partial \boldsymbol{y}(x, t)/\partial x$, and $\boldsymbol{y}_{xx}(x, t) = \partial^2 \boldsymbol{y}(x, t)/\partial x^2$, respectively. The transpose of a vector or a matrix is denoted by the superscript 'T'. The symbol '$*$' in matrix expressions denotes an ellipsis induced by symmetry.

II. Preliminaries and Problem Formulation

A. System description

The class of nonlinear DPSs under consideration is described by the following SLPPDE in 1D spatial domain

$$
\begin{cases}
\boldsymbol{y}_t(x, t) = \boldsymbol{\Theta}\boldsymbol{y}_{xx}(x, t) + \boldsymbol{f}(\boldsymbol{y}(x, t)) \\
\qquad\quad + \boldsymbol{G}(x)\boldsymbol{u}(t), \ x \in (0, L), \ t > 0, \\
\boldsymbol{y}(0, t) = 0, \ \boldsymbol{y}_x(x, t)|_{x=L} = 0, \ t > 0, \\
\boldsymbol{y}(x, 0) = \boldsymbol{y}_0(x), \ x \in [0, L],
\end{cases} \tag{1}
$$

where $\boldsymbol{y}(x, t) \triangleq [y_1(x, t) \quad y_2(x, t) \quad \cdots \quad y_n(x, t)]^T \in \mathbb{D}$ is state ($\mathbb{D} \triangleq \{\boldsymbol{y}(\cdot, t) \in \mathcal{L}_n^2([0, L]) | \ \|y_i(\cdot, t)\|_\infty \leq \phi_i, \ i \in \{1, 2, \cdots, n\}\} \subset \mathcal{L}_n^2([0, L])$ is a given local domain containing the equilibrium profile $\boldsymbol{y}(\cdot, t) = 0$, $\phi_i > 0$, $i \in \{1, 2, \cdots, n\}$ are real known scalars), $x \in [0, L] \subset \Re$ and $t \geq 0$ are spatial position and time variables, respectively. $0 < \boldsymbol{\Theta} \in \Re^{n \times n}$ is a known diffusivity coefficient matrix and controls how fast the masses can spread in the media, the term $\boldsymbol{\Theta}\boldsymbol{y}_{xx}(x, t)$ describes diffusion phenomenon and that is a spontaneous dispersion of mass in the spatial domain from the higher density/concentration/temperature area to the lower density/concentration/temperature area, $\boldsymbol{f}(\boldsymbol{y}(x, t))$ is sufficiently smooth in $\boldsymbol{y}(x, t)$ and satisfies $\boldsymbol{f}(0) = 0$. $\boldsymbol{u}(t) \triangleq [u_1(t) \quad u_2(t) \quad \cdots \quad u_m(t)]^T \in \Re^m$ is control input provided by m actuators, which are discretely distributed over the spatial domain $(0, L)$. $\boldsymbol{G}(x)$ is a known square integrable matrix function of x, in which the i-th column describes the i-th actuator's distribution. Here, $\boldsymbol{G}(x)$ is chosen as $\boldsymbol{G}(x) \triangleq [\boldsymbol{g}_1(x) \quad \boldsymbol{g}_2(x) \quad \cdots \quad \boldsymbol{g}_m(x)] \in \Re^{n \times m}$, where

$$
\boldsymbol{g}_v(x) \triangleq \begin{cases} \boldsymbol{g}_v & x \in [x_v, x_{v+1}) \\ 0 & \text{otherwise,} \end{cases} \quad v \in \mathcal{M}, \tag{2}
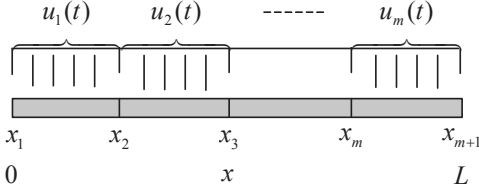$$

Fig. 1: Spatial piecewise control form [46]

with $[x_v, x_{v+1})$ is the v-th actuator coverage area and $\mathcal{M} \triangleq \{1, 2, \cdots, m\}$. This form $\boldsymbol{G}(x)$ will produce m zones of *spatial piecewise control* over the intervals $[x_v, x_{v+1})$, $v \in \mathcal{M}$ [46], which is illustrated in Fig. 1. It is easily seen from Fig. 1 that $0 = x_1 < x_2 < \cdots < x_m < x_{m+1} = L$.

We introduce the following definitions of exponential stability in the sense of $\|\cdot\|_2$ and $\|\cdot\|_\infty$:

Definition 1: (*Exponential stability in the sense of* $\|\cdot\|_2$, [46]) The SLPPDE system (1) with $\boldsymbol{u}(t) \equiv 0$ is said to be *exponentially stable in the sense of* $\|\cdot\|_2$, if there exist two constants $\sigma \geq 1$ and $\rho > 0$ such that $\|\boldsymbol{y}(\cdot, t)\|_2 \leq \sigma \|\boldsymbol{y}_0(\cdot)\|_2 \exp(-\rho t)$ is fulfilled for any $t \geq 0$.

Definition 2: (*Exponential stability in the sense of* $\|\cdot\|_\infty$) The SLPPDE system (1) with $\boldsymbol{u}(t) \equiv 0$ is said to be *exponentially stable in the sense of* $\|\cdot\|_\infty$, if there exist two constants $\alpha \geq 1$ and $\beta > 0$ such that $\|\boldsymbol{y}(\cdot, t)\|_\infty \leq \alpha \exp(-\beta t)$ is fulfilled for any $t \geq 0$.

Remark 1: Clearly, Definitions 1 and 2 are not equivalent and Definition 2 is stronger than Definition 1 as if the system (1) is exponentially stable in the sense of $\|\cdot\|_\infty$, it must be exponentially stable in the sense of $\|\cdot\|_2$, but not vice versa.

B. T-S fuzzy PDE model and problem formulation

For the convenience of fuzzy control design, by following the main idea of the sector nonlinearity method for semi-linear PDE systems [45] and [46], we assume that the following T-S fuzzy PDE model is constructed by the sector nonlinearity method to exactly describe complex spatiotemporal dynamics for the SLPPDE of (1) in a given operating domain $\mathfrak{S}$:

Plant Rule i:
IF $\xi_1(x, t)$ is F_{i1} and $\cdots$ and $\xi_l(x, t)$ is F_{il}, **THEN**

$$\begin{aligned} \boldsymbol{y}_t(x, t) &= \boldsymbol{\Theta}\boldsymbol{y}_{xx}(x, t) + \boldsymbol{A}_i \boldsymbol{y}(x, t) \\ &+ \boldsymbol{G}(x)\boldsymbol{u}(t), \ x \in (0, L), \ t > 0, \ i \in \mathbb{S} \end{aligned} \quad (3)$$

where F_{ij}, $i \in \mathbb{S} \triangleq \{1, 2, \cdots, r\}$, $j \in \{1, 2, \cdots, l\}$ are fuzzy sets, $\boldsymbol{A}_i \in \Re^{n \times n}$, $i \in \mathbb{S}$ are known matrices, and $r = 2^l$ is the number of IF-THEN fuzzy rules. The premise variables $\xi_1(x, t), \xi_2(x, t), \cdots, \xi_l(x, t)$ are assumed to be functions of $\boldsymbol{y}(x, t)$. The overall dynamics of the fuzzy parabolic PDE (3) is expressed as

$$\begin{aligned} \boldsymbol{y}_t(x, t) &= \boldsymbol{\Theta}\boldsymbol{y}_{xx}(x, t) + \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(x, t))\boldsymbol{A}_i \boldsymbol{y}(x, t) \\ &+ \boldsymbol{G}(x)\boldsymbol{u}(t), \ x \in (0, L), \ t > 0, \end{aligned} \quad (4)$$

where $\boldsymbol{\xi}(x, t) \triangleq [\xi_1(x, t) \ \xi_2(x, t) \ \cdots \ \xi_l(x, t)]^T$ and $h_i(\boldsymbol{\xi}(x, t)) \triangleq \frac{\prod_{j=1}^{l} F_{ij}(\xi_j(x, t))}{\sum_{i=1}^{r} \prod_{j=1}^{l} F_{ij}(\xi_j(x, t))}$, $i \in \mathbb{S}$, $F_{ij}(\xi_j(x, t))$ is

grade of the membership of $\xi_j(x, t)$ in F_{ij} for $i \in \mathbb{S}$. In this paper, it is assumed that $\prod_{j=1}^{l} F_{ij}(\xi_j(x, t)) \geq 0$, $i \in \mathbb{S}$ and $\sum_{i=1}^{r} \prod_{j=1}^{l} F_{ij}(\xi_j(x, t)) > 0$ for $x \in (0, L)$ and $t > 0$. Hence, $h_i(\boldsymbol{\xi}(x, t))$, $i \in \mathbb{S}$ have the following property for $x \in (0, L)$ and $t > 0$:

$$h_i(\boldsymbol{\xi}(x, t)) \geq 0, \ i \in \mathbb{S} \ \text{and} \ \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(x, t)) = 1. \quad (5)$$

From above analysis, we know that the SLPPDE in (1) is equivalent to the fuzzy PDE (4) in the operating domain $\mathfrak{S}$. Generally, the operating domain $\mathfrak{S}$ is chosen as $\mathbb{D} \subset \mathfrak{S}$ to guarantee the *robustness* of the fuzzy PDE model (4).

This study considers a static sampled-data fuzzy local state feedback controller of the following form by the fuzzy PDE model (4):

$$\begin{aligned} u_v(t) &= \sum_{i=1}^{r} h_{vi}(t_k) \frac{\boldsymbol{k}_{vi}^T}{\Delta \tilde{x}_v} \int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \boldsymbol{y}(x, t_k) dx, \ v \in \mathcal{M}, \\ &\quad t \in [t_k, t_{k+1}), \ k \in \wp \triangleq \{0, 1, 2, \cdots\}, \end{aligned} \quad (6)$$

where

$$\begin{aligned} h_{vi}(t_k) &\triangleq \frac{1}{\Delta \tilde{x}_v} \int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} h_i(\boldsymbol{\xi}(x, t_k)) dx, \\ \Delta \tilde{x}_v &\triangleq \tilde{x}_{2v} - \tilde{x}_{2v-1}, \ v \in \mathcal{M}, \end{aligned} \quad (7)$$

and $\boldsymbol{k}_{vi} \in \Re^n$, $v \in \mathcal{M}$, $i \in \mathbb{S}$ are control gain parameters to be determined, and t_k, $k \in \wp$ are sampling instants and satisfy $t_{k+1} - t_k \leq T_h$, $k \in \wp$ ($T_h > 0$ is the maximum sampling step in time given in advance). From (5) and (7), we get

$$h_{vi}(t_k) \geq 0, \ i \in \mathbb{S} \ \text{and} \ \sum_{i=1}^{r} h_{vi}(t_k) = 1, \quad (8)$$

for any sampling instants t_k, $k \in \wp$ and $v \in \mathcal{M}$. Clearly, the sampled-data fuzzy control law (6) with (7) is a weighted average of sampled-data linear control laws $\frac{\boldsymbol{k}_{vi}^T}{\Delta \tilde{x}_v} \int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \boldsymbol{y}(x, t_k) dx$, $v \in \mathcal{M}$, $i \in \mathbb{S}$, $k \in \wp$, where the mean value of the sampled-data state $\boldsymbol{y}(x, t_k)$, $k \in \wp$ is utilized and the weights are determined by the functions $h_{vi}(t_k)$, $v \in \mathcal{M}$, $i \in \mathbb{S}$, $k \in \wp$ (i.e., the mean value of fuzzy membership functions $h_i(\boldsymbol{\xi}(x, t))$, $i \in \mathbb{S}$ on the subintervals $[\tilde{x}_{2v-1}, \tilde{x}_{2v}]$, $v \in \mathcal{M}$ at the sampling instants t_k, $k \in \wp$).

By substituting the sampled-data fuzzy controller (6) into the fuzzy PDE (4), considering the boundary conditions and the initial condition in (1), and using the definition of $\boldsymbol{G}(x)$ given in (2), the resulting closed-loop sampled-data fuzzy PDE system is given as follows:

$$\begin{cases} \boldsymbol{y}_t(x, t) = \boldsymbol{\Theta}\boldsymbol{y}_{xx}(x, t) + \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(x, t))\boldsymbol{A}_i \boldsymbol{y}(x, t) \\ \quad + \boldsymbol{g}_v \sum_{i=1}^{r} h_{vi}(t_k) \frac{\boldsymbol{k}_{vi}^T}{\Delta \tilde{x}_v} \int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \boldsymbol{y}(x, t_k) dx, \\ \quad k \in \wp, \ x \in [x_v, x_{v+1}), \ t > 0, \ v \in \mathcal{M}, \\ \boldsymbol{y}(0, t) = 0, \ \boldsymbol{y}_x(x, t)|_{x=L} = 0, \ t > 0, \\ \boldsymbol{y}(x, 0) = \boldsymbol{y}_0(x), \ x \in [0, L]. \end{cases} \quad (9)$$

The fuzzy-model-based sampled-data control design addressed in this paper is formally defined as follows:

For the SLPPDE system (1), the objective of this study is to develop an LMI-based design of the static sampled-data fuzzy controller (6) such that the resulting closed-loop SLPPDE system is exponentially stable in the sense of $\|\cdot\|_\infty$.

C. Important lemmas

Lemma 1 (Vector-valued Wirtinger's inequalities): Let $\boldsymbol{y}(\cdot, t) \in \mathcal{H}_n^1((0, L))$ be a vector function with $\boldsymbol{y}(0, t) = 0$ or $\boldsymbol{y}(L, t) = 0$, $t \geq 0$. Then, for any matrix $0 \leq \boldsymbol{S} \in \Re^{n \times n}$, we have for any $t \geq 0$

$$\int_0^L \boldsymbol{y}^T(x, t) \boldsymbol{S} \boldsymbol{y}(x, t) dx \leq \frac{4L^2}{\pi^2} \int_0^L \boldsymbol{y}_x^T(x, t) \boldsymbol{S} \boldsymbol{y}_x(x, t) dx. \quad (10)$$

Moreover, for a vector function $\boldsymbol{y}(\cdot, t) \in \mathcal{H}_n^2((0, L))$ with $\boldsymbol{y}_x(0, t) = 0$ or $\boldsymbol{y}_x(L, t) = 0$, $t \geq 0$, we have the following inequality for any matrix $0 \leq \boldsymbol{S} \in \Re^{n \times n}$

$$\int_0^L \boldsymbol{y}_x^T(x, t) \boldsymbol{S} \boldsymbol{y}_x(x, t) dx$$
$$\leq \frac{4L^2}{\pi^2} \int_0^L \boldsymbol{y}_{xx}^T(x, t) \boldsymbol{S} \boldsymbol{y}_{xx}(x, t) dx, \ t \geq 0. \quad (11)$$

Proof: The proof of this lemma is easily done by Theorem 2 in [48] and thus omitted here. ∎

Lemma 2 (A variation of vector-valued Poincaré–Wirtinger inequality in 1D spatial domain): Let $\boldsymbol{y}(\cdot, t) \in \mathcal{H}_n^1((0, L))$, $t \geq 0$ be a vector function. Then, for any matrix $0 \leq \boldsymbol{S} \in \Re^{n \times n}$, we have

$$\int_0^L (\boldsymbol{y}(x, t) - \boldsymbol{y}_l(t))^T \boldsymbol{S} (\boldsymbol{y}(x, t) - \boldsymbol{y}_l(t)) dx$$
$$\leq 4\phi \pi^{-2} \int_0^L \boldsymbol{y}_x^T(x, t) \boldsymbol{S} \boldsymbol{y}_x(x, t) dx, \ t \geq 0 \quad (12)$$

where $\boldsymbol{y}_l(t) \triangleq (l_2 - l_1)^{-1} \int_{l_1}^{l_2} \boldsymbol{y}(x, t) dx$, $[l_1, l_2] \subset [0, L]$, and $\phi \triangleq \max\{l_2^2, (L - l_1)^2\}$.

Proof: See the Appendix A ∎

Lemma 3 (Vector-valued Agmon's inequality): Let $\boldsymbol{y}(\cdot, t) \in \mathcal{H}_n^1((0, L))$ be a vector function with $\boldsymbol{y}(0, t) = 0$ or $\boldsymbol{y}(L, t) = 0$, $t \geq 0$. The following inequality is fulfilled:

$$\|\boldsymbol{y}(\cdot, t)\|_\infty^2 \leq 2\|\boldsymbol{y}(\cdot, t)\|_2 \|\boldsymbol{y}_x(\cdot, t)\|_2 \leq \|\boldsymbol{y}(\cdot, t)\|_{\mathcal{H}_n^1((0, L))}^2, \ t \geq 0. \quad (13)$$

Proof: See the Appendix B ∎

Note that the membership functions $h_{vi}(t_k)$, $v \in \mathcal{M}$, $i \in \mathbb{S}$, $k \in \wp$ in the sampled-data fuzzy controller (6) are different from the ones $h_i(\boldsymbol{\xi}(x, t))$, $i \in \mathbb{S}$ in the fuzzy PDE model (4). This characteristic prevents direct use of the parameterized LMI technique [41] to reduce control design conservativeness. To overcome this drawback, this paper will provide an LMI relaxation technique. By considering (5) and (8), we get

$$|h_i(\boldsymbol{\xi}(x, t)) - h_{vi}(t_k)| \leq 1, \ x \in [x_v, x_{v+1}), \ v \in \mathcal{M} \quad (14)$$

for any $t \in [t_k, t_{k+1})$, $k \in \wp$, and $i \in \mathbb{S}$. Using (14), the following lemma provides an LMI relaxation technique:

Lemma 4: Consider $n \times n$ matrices $\boldsymbol{\Lambda}_v(x, t, t_k) \triangleq \sum_{i=1}^r \sum_{j=1}^r h_i(\boldsymbol{\xi}(x, t)) h_{vj}(t_k) \boldsymbol{\Lambda}_{vij}$, $x \in [x_v, x_{v+1})$, $v \in \mathcal{M}$, $t \in [t_k, t_{k+1})$, $k \in \wp$. The matrix inequalities $\boldsymbol{\Lambda}_v(x, t, t_k) < 0$, $v \in \mathcal{M}$ are fulfilled for $x \in [x_v, x_{v+1})$, $v \in \mathcal{M}$, $t \in [t_k, t_{k+1})$, $k \in \wp$, if there exist matrices $0 > \boldsymbol{\Gamma}_{vij} \in \Re^{n \times n}$, $0 > \boldsymbol{\Upsilon}_{vij} \in$

$\Re^{n \times n}$, $\boldsymbol{\Omega}_{vij} = \boldsymbol{\Omega}_{vji}^T \in \Re^{n \times n}$, and $\boldsymbol{\Omega}_{vi(j+r)} = \boldsymbol{\Omega}_{v(j+r)i}^T \in \Re^{n \times n}$, $v \in \mathcal{M}$, $i, j \in \mathbb{S}$ such that

$$\boldsymbol{\Delta}_{vij} + \boldsymbol{\Delta}_{vji} \leq \boldsymbol{\Omega}_{vij} + \boldsymbol{\Omega}_{vji}, \ v \in \mathcal{M}, \ i, j \in \mathbb{S} \quad (15)$$

$$\boldsymbol{\Lambda}_{vij} - 2\boldsymbol{\Delta}_{vij} - \sum_{k=1}^r (\boldsymbol{\Delta}_{vik}^+ + \boldsymbol{\Delta}_{vkj}^+)$$
$$\leq \boldsymbol{\Omega}_{vi(j+r)} + \boldsymbol{\Omega}_{v(j+r)i}, \ v \in \mathcal{M}, \ i, j \in \mathbb{S} \quad (16)$$

$$\boldsymbol{Y}_v \triangleq \begin{bmatrix} \boldsymbol{Y}_{v11} & \boldsymbol{Y}_{v12} \\ * & \boldsymbol{Y}_{v11} \end{bmatrix}, \ v \in \mathcal{M} \quad (17)$$

where $\boldsymbol{\Delta}_{vij} \triangleq \boldsymbol{\Gamma}_{vij} - \boldsymbol{\Upsilon}_{vij}$, $\boldsymbol{\Delta}_{v,ij}^+ \triangleq \boldsymbol{\Gamma}_{vij} + \boldsymbol{\Upsilon}_{vij}$, $i, j \in \mathbb{S}$,

$$\boldsymbol{Y}_{v,11} \triangleq \begin{bmatrix} \boldsymbol{\Omega}_{v11} & \cdots & \boldsymbol{\Omega}_{v1r} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Omega}_{vr1} & \cdots & \boldsymbol{\Omega}_{vrr} \end{bmatrix}, \ v \in \mathcal{M},$$

$$\boldsymbol{Y}_{v,12} \triangleq \begin{bmatrix} \boldsymbol{\Omega}_{v1(r+1)} & \cdots & \boldsymbol{\Omega}_{v1(2r)} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Omega}_{vr(r+1)} & \cdots & \boldsymbol{\Omega}_{vr(2r)} \end{bmatrix}, \ v \in \mathcal{M}.$$

Proof: The proof of this lemma is easily done via Lemma 2 in [46] and thus omitted here. ∎

Remark 2: LMI relaxation techniques have been proposed in [45] and [46] by making assumptions on fuzzy membership functions to reduce the fuzzy control design conservativeness. These assumptions to some extent restrict the application of the control design in [45] and [46]. In this paper, an LMI relaxation technique is developed in Lemma 4 by removing these assumptions made in [45] and [46]. Lemma 4 is a special case of the LMI relaxation technique in [46], where the positive parameters $\varsigma_{v,i}$ are chosen as 1.

III. STATIC SAMPLED-DATA FUZZY CONTROL DESIGN

Let us consider a Lyapunov–Krasovskii functional candidate of the following form for the system (9):

$$V(t) = V_1(t) + V_2(t) + V_3(t), \ t \in [t_k, t_{k+1}), \ k \in \wp, \quad (18)$$

where

$$V_1(t) = \int_0^L \boldsymbol{y}^T(x, t) \boldsymbol{P}_1 \boldsymbol{y}(x, t) dx, \quad (19)$$

$$V_2(t) = \int_0^L \boldsymbol{y}_x^T(x, t) \boldsymbol{P}_2 \boldsymbol{y}_x(x, t) dx, \quad (20)$$

$$V_3(t) = T_h \int_0^L \int_{t_k}^t (s - t + T_h) \boldsymbol{y}_s^T(x, s) \boldsymbol{P}_3 \boldsymbol{y}_s(x, s) ds dx, \quad (21)$$

with $0 < \boldsymbol{P}_1 \in \Re^{n \times n}$, $0 < \boldsymbol{P}_2 \in \Re^{n \times n}$, and $0 < \boldsymbol{P}_3 \in \Re^{n \times n}$ are Lyapunov matrices to be determined. Obviously, the function $V(t)$ given in (18) is continuous in time from the right for $\boldsymbol{y}(x, t)$ satisfying the system (9) and absolutely continuous for $t \neq t_k$, $k \in \Im$. Along the jumps t_k, $k \in \Im$ $V(t)$ satisfies $V(t_k) \leq V(t_k^-) \triangleq \lim_{t \to t_k^-} V(t)$, $k \in \Im$ since $V_3(t_k^-) \triangleq \lim_{t \to t_k^-} V_3(t) \geq 0$ and $V_3(t_k) = 0$ after the jumps

because $\boldsymbol{y}(x,t)|_{t=t_k} = \boldsymbol{y}(x,t_k)$. Moreover, $V(t)$ satisfies the following inequality for any $t \in [t_k, t_{k+1})$, $k \in \wp$:

$$\rho_1 \|\boldsymbol{y}(\cdot,t)\|_{\mathcal{H}_n^1((0,L))}^2 \leq V(t) \leq \lambda_{\max}(\boldsymbol{P}_1) \int_0^L \boldsymbol{y}^T(x,t)\boldsymbol{y}(x,t)dx$$

$$+ \lambda_{\max}(\boldsymbol{P}_2) \int_0^L \boldsymbol{y}_x^T(x,t)\boldsymbol{y}_x(x,t)dx$$

$$+ \lambda_{\max}(\boldsymbol{P}_3)T_h^2$$

$$\times \int_0^L \int_{t_k}^t \boldsymbol{y}_s^T(x,s)\boldsymbol{y}_s(x,s)dsdx, \quad (22)$$

where $\rho_1 \triangleq \min\{\lambda_{\min}(\boldsymbol{P}_1), \lambda_{\min}(\boldsymbol{P}_2)\}$. For $t = 0$, the following inequality can be obtained from (22)

$$V(0) \leq \rho_2 \|\boldsymbol{y}_0(\cdot)\|_{\mathcal{H}_n^1((0,L))}^2 \quad (23)$$

where $\rho_2 \triangleq \max\{\lambda_{\max}(\boldsymbol{P}_1), \lambda_{\max}(\boldsymbol{P}_2)\}$.

Note that the proposed Lyapunov-Krasovskii functional candidate of the form (18)-(21) is different from that reported in [3]- [6], [9]- [11], and [28]- [38] for the sampled-data control design of the systems which are modeled by O/DDEs, due to that the system (1) is spatiotemporal and includes the term $\boldsymbol{\Theta}\boldsymbol{y}_{xx}(x,t)$. Although the problem of sampled-data fuzzy control design of nonlinear parabolic DPSs has been addressed in [39] and [40], the proposed Lyapunov-Krasovskii functional candidate is constructed based on the low-dimensional ODE approximations obtained from the singular perturbation formulation of Galerkin's method. On the other hand, the Lyapunov-Krasovskii functional candidate of the form (18)-(21) is also different from the ones used in [20]- [22] for sampled-data control design of semi-linear parabolic PDE systems, where an exponential term $\exp(2\alpha(s-t))$ (α is a given positive constant) is introduced in construction of Lyapunov-Krasovskii functional candidates.

Theorem 1: Consider the SLPPDE system (1) and the T-S fuzzy PDE model (4). For given constants $L > 0$, $T_h > 0$, an integer $m > 0$, and parameters $\tilde{x}_{2v-1}, \tilde{x}_{2v}$, $v \in \mathcal{M}$, $x_2, x_3, \cdots, x_m$, if there exist matrices $0 < \boldsymbol{X}_1 \in \Re^{n \times n}$, $0 < \boldsymbol{X}_2 \in \Re^{n \times n}$, $0 < \boldsymbol{X}_3 \in \Re^{n \times n}$ and vectors $\boldsymbol{z}_{vj} \in \Re^n$, $v \in \mathcal{M}$, $j \in \mathbb{S}$ such that the following LMIs are fulfilled:

$$\boldsymbol{\Phi}_{vij} \triangleq \begin{bmatrix} \boldsymbol{\Phi}_{1vij} & \boldsymbol{\Phi}_{2vij} \\ * & \boldsymbol{\Phi}_{3vij} \end{bmatrix} < 0, \ v \in \mathcal{M}, \ i,j \in \mathbb{S} \quad (24)$$

where

$$\boldsymbol{\Phi}_{1vij} \triangleq \begin{bmatrix} [\boldsymbol{A}_i\boldsymbol{X}_1 + *] - \frac{\pi^2}{4\varphi_v}\boldsymbol{\Phi}_1 & \frac{\pi^2}{4\varphi_v}\boldsymbol{\Phi}_1 + \boldsymbol{g}_v\boldsymbol{z}_{vj}^T \\ * & -\frac{\pi^2}{4\varphi_v}\boldsymbol{\Phi}_1 \end{bmatrix},$$

$$\boldsymbol{\Phi}_{2vij} \triangleq \begin{bmatrix} -\boldsymbol{X}_1\boldsymbol{A}_i^T & \boldsymbol{X}_1\boldsymbol{A}_i^T & -\boldsymbol{g}_v\boldsymbol{z}_{vj}^T \\ -\boldsymbol{z}_{vj}\boldsymbol{g}_v^T & \boldsymbol{z}_{vj}\boldsymbol{g}_v^T & 0 \end{bmatrix},$$

$$\boldsymbol{\Phi}_{3vij} \triangleq \begin{bmatrix} -[\boldsymbol{\Theta}\boldsymbol{X}_2 + *] & \boldsymbol{X}_2\boldsymbol{\Theta} & \boldsymbol{g}_v\boldsymbol{z}_{vj}^T \\ * & T_h^2\boldsymbol{X}_3 - 2\boldsymbol{X}_1 & -\boldsymbol{g}_v\boldsymbol{z}_{vj}^T \\ * & * & -\frac{\Delta\tilde{x}_v}{\Delta x_v}\boldsymbol{X}_3 \end{bmatrix},$$

in which

$$\boldsymbol{\Phi}_1 \triangleq [\boldsymbol{\Theta}\boldsymbol{X}_1 + *],$$
$$\varphi_v \triangleq \max\{(\tilde{x}_{2v} - x_v)^2, (x_{v+1} - \tilde{x}_{2v-1})^2\},$$
$$\Delta x_v \triangleq x_{v+1} - x_v, \ v \in \mathcal{M}, \quad (25)$$

then there exists a static sampled-data fuzzy controller (6) such that the resulting closed-loop system is exponentially stable in the sense of $\|\cdot\|_\infty$, where the control gain parameters $\boldsymbol{k}_{vj}$, $v \in \mathcal{M}$, $j \in \mathbb{S}$ are given by

$$\boldsymbol{k}_{vj}^T = \boldsymbol{z}_{vj}^T\boldsymbol{X}_1^{-1}, \ v \in \mathcal{M}, \ j \in \mathbb{S}. \quad (26)$$

Proof: See the Appendix C. ∎

By the Lyapunov-Krasovskii functional candidate (18) with (19)-(21) and making use of integration by parts and Lemmas 1-3, Theorem 1 provides an LMI-based design of static sampled-data fuzzy controller (6) exponentially stabilizing the SLPPDE system (1) in the sense of $\|\cdot\|_\infty$. The corresponding control gains $\boldsymbol{k}_{vi}$, $v \in \mathcal{M}$, $i \in \mathbb{S}$ are constructed as (26) from feasible solutions to the LMIs (24), which are directly solved by *feasp* solver in Matlab's LMI Control Toolbox [47].

Based on Theorem 1 and Lemma 4, we will give a less conservative LMI-based design for static sampled-data fuzzy controller (6).

Theorem 2: For given constants $L > 0$, $T_h > 0$, an integer $m > 0$, and parameters $\tilde{x}_{2v-1}, \tilde{x}_{2v}$, $v \in \mathcal{M}$, $x_2, x_3, \cdots, x_m$, consider the SLPPDE system (1) and the T-S fuzzy PDE model (4). If there exist matrices $0 < \boldsymbol{X}_1 \in \Re^{n \times n}$, $0 < \boldsymbol{X}_2 \in \Re^{n \times n}$, $0 < \boldsymbol{X}_3 \in \Re^{n \times n}$, vectors $\boldsymbol{z}_{vj} \in \Re^n$, and matrices $0 > \boldsymbol{\Pi}_{vij} \in \Re^{5n \times 5n}$, $0 > \boldsymbol{N}_{vij} \in \Re^{5n \times 5n}$, $\boldsymbol{\Sigma}_{vij} = \boldsymbol{\Sigma}_{vji}^T \in \Re^{5n \times 5n}$, $\boldsymbol{\Sigma}_{vi(j+r)} = \boldsymbol{\Sigma}_{v(j+r)i}^T \in \Re^{5n \times 5n}$, $v \in \mathcal{M}$, $i,j \in \mathbb{S}$ such that the following LMIs hold:

$$\bar{\boldsymbol{X}}_{vij} + \bar{\boldsymbol{X}}_{vji} \leq \boldsymbol{\Sigma}_{vij} + \boldsymbol{\Sigma}_{vji}, \ v \in \mathcal{M}, \ i,j \in \mathbb{S} \quad (27)$$

$$\boldsymbol{\Phi}_{vij} - 2\bar{\boldsymbol{X}}_{vij} - \sum_{k=1}^r (\bar{\boldsymbol{X}}_{vik}^+ + \bar{\boldsymbol{X}}_{vkj}^+)$$
$$\leq \boldsymbol{\Sigma}_{vi(j+r)} + \boldsymbol{\Sigma}_{v(j+r)i}, \ v \in \mathcal{M}, \ i,j \in \mathbb{S} \quad (28)$$

$$\begin{bmatrix} \boldsymbol{U}_{v11} & \boldsymbol{U}_{v12} \\ * & \boldsymbol{U}_{v11} \end{bmatrix} < 0, \ v \in \mathcal{M} \quad (29)$$

where $\bar{\boldsymbol{X}}_{vij} \triangleq \boldsymbol{\Pi}_{vij} - \boldsymbol{N}_{vij}$, $\bar{\boldsymbol{X}}_{vij}^+ \triangleq \boldsymbol{\Pi}_{vij} + \boldsymbol{N}_{vij}$, $\boldsymbol{\Phi}_{vij}$ (given in Theorem 1), $v \in \mathcal{M}$, $i,j \in \mathbb{S}$,

$$\boldsymbol{U}_{v11} \triangleq \begin{bmatrix} \boldsymbol{\Sigma}_{v11} & \cdots & \boldsymbol{\Sigma}_{v1r} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}_{vr1} & \cdots & \boldsymbol{\Sigma}_{vrr} \end{bmatrix}, \ v \in \mathcal{M},$$

$$\boldsymbol{U}_{v12} \triangleq \begin{bmatrix} \boldsymbol{\Sigma}_{v1(r+1)} & \cdots & \boldsymbol{\Sigma}_{v1(2r)} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}_{vr(r+1)} & \cdots & \boldsymbol{\Sigma}_{vr(2r)} \end{bmatrix}, \ v \in \mathcal{M},$$

then there exists a static sampled-data fuzzy controller (6) for the SLPPDE system (1) guaranteeing exponential stability of the resulting closed-loop system in the sense of $\|\cdot\|_\infty$, where the control gains $\boldsymbol{k}_{vj}$, $v \in \mathcal{M}$, $j \in \mathbb{S}$ are given by (26).

Proof: The proof of this theorem is easily done with the aid of Theorem 1 and Lemma 4 and thus omitted. ∎

By Lemma 4, the exponential stabilization ability of static sampled-data fuzzy controller (6) designed by Theorem 2 is enhanced, which will be verified by Example 1 in Section IV.

Remark 3: The main results (i.e., Theorems 1 and 2) are also applicable for the case of mixed Neumann-Dirichlet boundary conditions $\boldsymbol{y}_x(x,t)|_{x=0} = \boldsymbol{y}(L,t) = 0$. This is because both the inequality (11) (it requires $\boldsymbol{y}_x(x,t)|_{x=0} = 0$ or

$\boldsymbol{y}_x(x,t)|_{x=L} = 0$) in Lemma 1 and the vector-valued Agmon's inequality (it requires $\boldsymbol{y}(0,t) = 0$ or $\boldsymbol{y}(L,t) = 0$ in Lemma 3 are employed in the development of these main results.

Remark 4: By Remark 1, the main results (i.e., Theorems 1 and 2) are also directly employed to solve static sampled-data fuzzy control design for the SLPPDE system (1) in the sense of $\|\cdot\|_2$. In this situation, these main results are also applicable for the case of Neumann boundary conditions $\boldsymbol{y}_x(x,t)|_{x=0} = \boldsymbol{y}_x(x,t)|_{x=L} = 0$ due to that they can be derived in the absence of the vector-valued Agmon's inequality given in Lemma 3.

Remark 5: Although the main results (i.e., Theorems 1 and 2) only address the case of single input for each actuator's active area (see Eq. (2)), they are also revised for the case of multi-input for this active area. On the other hand, by using the technique of spatial differential linear matrix inequality (SDLMI) reported in [41] and [42], the main results of this paper can be revised to address the problem of sampled-data fuzzy control design of the SLPPDE system (1) where $\boldsymbol{f}(\boldsymbol{y}(x,t))$ is replaced by $\boldsymbol{f}(\boldsymbol{y}(x,t),x)$.

Remark 6: In comparison to most recent results in [44]-[46] for fuzzy control design of nonlinear DPSs whose control input is continuous in time, one of the main differences of this paper lies in that fuzzy control design is developed in this paper for semi-linear parabolic PDE systems, where the control input is sampled-data and discontinuous in time. Apart from the sampled-data case, another main difference is that by a variation of vector-valued Poincar-Wirtinger inequality in 1D space (i.e., Lemma 2) and a vector-valued Agmon's inequality (i.e., Lemma 3), the suggested fuzzy control design proposed in this paper is addressed in the sense of $\|\cdot\|_\infty$, which is stronger than the control design results in [44]-[46] discussed in the sense of $\|\cdot\|_2$ by Remark 1. Moreover, the implementation of the proposed fuzzy controller of this paper requires the sensors only active over some partial areas of the spatial domain, while the sensors are required to be distributed continuously over the entire spatial domain for the implementation of the fuzzy controller proposed in [44] and located at some specified points of the spatial domain for the fuzzy controllers reported in [45] and [46].
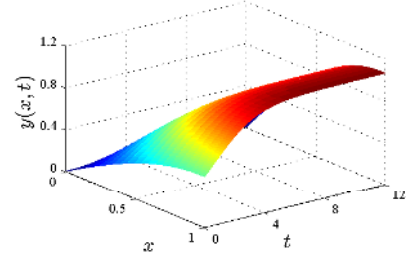
IV. NUMERICAL SIMULATION

To illustrate the satisfactory and better performance of the proposed design method, this section considers two semi-linear parabolic PDE systems: the first one is a scalar one subject to mixed Dirichlet-Neumann boundary conditions and the second one is a multi-variable one subject to mixed Neumann-Dirichlet boundary conditions.

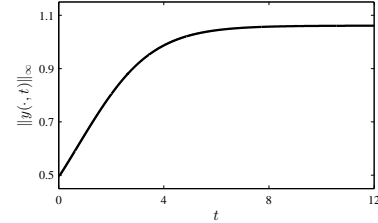*Example 1:* Consider a semi-linear parabolic PDE system:

$$\begin{cases} y_t(x,t) = y_{xx}(x,t) + f(y(x,t)) + \boldsymbol{g}^T(x)\boldsymbol{u}(t) \\ y(0,t) = y_x(x,t)|_{x=1} = 0, \ y(x,0) = y_0(x), \end{cases} \quad (30)$$

where $y(x,t)$ is state, $\boldsymbol{u}(t) \triangleq [u_1(t) \quad u_2(t)]^T \in \Re^2$ is manipulated control input, $\boldsymbol{g}(x) \triangleq [g_1(x) \quad g_2(x)]^T$ is control influence function describing distribution of piecewise actuators. Set $f(y(x,t)) = 3\sin(y(x,t))$, $g_1(x) = \begin{cases} 1 & x \in [0, 0.5) \\ 0 & \text{otherwise} \end{cases}$,

$g_2(x) = \begin{cases} 2 & x \in [0.5, 1) \\ 0 & \text{otherwise} \end{cases}$, $\boldsymbol{u}(t) = 0$ and $y_0(x) =$

(a)

(b)

Fig. 2: Open-loop numerical simulation results: (a) profile of evolution of $y(\cdot, t)$ and (b) trajectory of $\|y(\cdot, t)\|_\infty$

$0.5\sin(0.5\pi x)$, $x \in [0, 1]$, Fig. 2 provides open-loop numerical simulation results: profile of evolution of $y(\cdot, t)$ and trajectory of $\|y(\cdot, t)\|_\infty$. It is obvious from Fig. 2 that the equilibrium profile $y(\cdot, t) = 0$ of the open-loop system of (30) is unstable in the sense of $\|\cdot\|_\infty$.

According to Example 1 in [46], when $y(x,t) \in (-\pi, \pi)$, $x \in [0, 1]$, the semi-linear system (30) is exactly represented by the following T-S fuzzy PDE system of two rules:

Plant Rule 1:
IF $y(x,t)$ is "about 0", **THEN**

$$\begin{cases} y_t(x,t) = y_{xx}(x,t) + a_1 y(x,t) + \boldsymbol{g}^T(x)\boldsymbol{u}(t) \\ y(0,t) = y_x(x,t)|_{x=1} = 0, \ y(x,0) = y_0(x) \end{cases}$$

Plant Rule 2:
IF $y(x,t)$ is "about $-\pi$ or π", **THEN**

$$\begin{cases} y_t(x,t) = y_{xx}(x,t) + a_2 y(x,t) + \boldsymbol{g}^T(x)\boldsymbol{u}(t) \\ y(0,t) = y_x(x,t)|_{x=1} = 0, \ y(x,0) = y_0(x) \end{cases}$$

where $a_1 = 3$ and $a_2 = 3\varpi$. The overall fuzzy PDE system is written as

$$\begin{cases} y_t(x,t) = y_{xx}(x,t) \\ \qquad + \sum_{i=1}^2 h_i(y(x,t)) a_i y(x,t) + \boldsymbol{g}^T(x)\boldsymbol{u}(t) \quad (31) \\ y(0,t) = y_x(x,t)|_{x=1} = 0, \ y(x,0) = y_0(x), \end{cases}$$

where

$$h_1(y(x,t)) = \begin{cases} \frac{\sin(y(x,t)) - \varpi y(x,t)}{(1-\varpi)y(x,t)} & y(x,t) \neq 0 \\ 1 & y(x,t) = 0 \end{cases}$$

$$h_2(y(x,t)) = 1 - h_1(y(x,t)), \quad (32)$$

with $\varpi \triangleq 0.01/\pi$.

Set $\tilde{x}_1 = 0.2$, $\tilde{x}_2 = 0.3$, $\tilde{x}_3 = 0.7$, and $\tilde{x}_4 = 0.8$, we thus have $\varphi_1 = \varphi_2 = 0.09$. Let $T_h = 0.1360$, $\boldsymbol{\Theta} = 1$, $\boldsymbol{g}_1 = 1$, $\boldsymbol{g}_2 = 2$, and $\boldsymbol{A}_i = a_i$, $i \in \{1, 2\}$. By Theorem 2, solving the LMIs (27)-(29) by *feasp* solver [47] and using
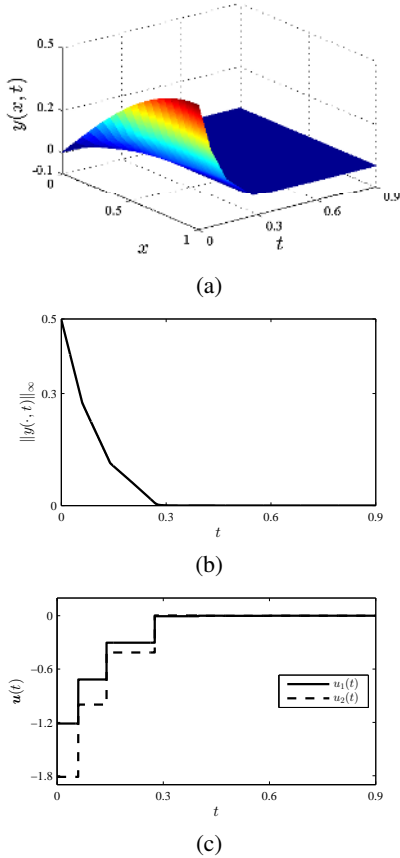
(a)

(b)

(c)

Fig. 3: Closed-loop numerical simulation results: (a) profile of evolution of $y(x, t)$, (b) trajectory of $\|y(\cdot, t)\|_\infty$, and (c) trajectory of sampled-data fuzzy control input $\boldsymbol{u}(t)$

(26), we obtain control parameters k_{vi}, $v \in \{1, 2\}$, $i \in \{1, 2\}$ as $k_{11} = -3.4965$, $k_{12} = -3.4935$, $k_{21} = -1.7482$, and $k_{22} = -1.7467$. Applying the sampled-data fuzzy controller (6) with the aforementioned control parameters to the system (30), the resulting closed-loop numerical simulation results: profile of evolution of $y(x, t)$, trajectory of $\|y(\cdot, t)\|_\infty$, and the corresponding trajectory of the sampled-data fuzzy control input $\boldsymbol{u}(t)$ are shown in Fig. 3. What is apparent from Fig. 3 is that the suggested sampled-data fuzzy controller (6) stabilizes the system (30) in the sense of $\| \cdot \|_\infty$. On the other hand, by Theorem 1, when $T_h = 0.1360$, it has been verified that the LMIs (24) are infeasible. Setting $T_h = 0.03654$, solving the LMIs (24) and using (26), we can get the control parameters k_{vi}, $v \in \{1, 2\}$, $i \in \{1, 2\}$ as $k_{1i} = -3.2784$ and $k_{2i} = -1.6392$, $i \in \{1, 2\}$. In this situation, the sampled-data fuzzy controller (6) will be reduced to a linear one. Obviously, the exponential stabilization ability of the sampled-data fuzzy controller (6) designed by Theorem 2 is significantly enhanced by Lemma 4 for this example.

Next, we will provide a comparison study for the system (30) between the sampled-data fuzzy controller (6) and a sampled-data fuzzy modal-feedback controller from [39] and [40]. By following the main idea of the control design in [39] and [40], we construct a sampled-data fuzzy modal-feedback
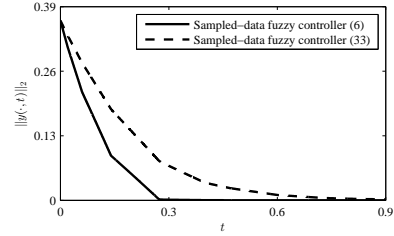

Fig. 4: Closed-loop trajectory of $\|y(\cdot, t)\|_2$ using sampled-data fuzzy controllers (6) and (33)

controller as follows:

$$u_v(t) = \sum_{i=1}^{2} h_i(x_s(t_k)) k_{vi} x_s(t_k),$$
$$v \in \{1, 2\}, \ t \in [t_k, t_{k+1}), \ k \in \Im, \quad (33)$$

where $\nu(x) = \sqrt{2} \sin(0.5\pi x)$, $x_s(t) \triangleq \int_0^1 \nu(x) y(x, t) dx$, $k_{11} = -5.1$, $k_{12} = -5.2$, $k_{21} = -2.4$, $k_{22} = -2.3$, and

$$h_1(x_s(t)) = \begin{cases} \frac{\varpi_2 x_s(t) - f(x_s(t))}{(\varpi_2 - \varpi_1) x_s(t)} & x_s(t) \neq 0 \\ \frac{\varpi_2 - 1}{\varpi_2 - \varpi_1} & x_s(t) = 0 \end{cases}$$
$$h_2(x_s(t)) = 1 - h_1(x_s(t)),$$

with $f(x_s(t)) = \int_0^1 \nu(x) \sin(x_s(t)\nu(x)) dx$, $df(x_s(t)) = \int_0^1 \nu^2(x) \cos(x_s(t)\nu(x)) dx$, $\varpi_1 \triangleq \min_{x_s(t) \in [-0.4, 1.2]} \{df(x_s(t))\}$, and $\varpi_2 \triangleq \max_{x_s(t) \in [-0.4, 1.2]} \{df(x_s(t))\}$.

Applying the sampled-data fuzzy modal-feedback controller (33) to the system (30), the closed-loop trajectory of $\|y(\cdot, t)\|_2$ is shown in Fig. 4. According to Remark 4, the sampled-data fuzzy controller (6) can also exponentially stabilize the system (30) in the sense of $\|y(\cdot, t)\|_2$. The closed-loop trajectory of $\|y(\cdot, t)\|_2$ is also given in Fig. 4 for the system (30) driven by the sampled-data fuzzy controller (6). It is clear from Fig. 4 that in comparison to the sampled-data fuzzy modal-feedback controller from [39] and [40], the suggested sampled-data fuzzy controller (6) provides a better control performance.

To further illustrate the improvement of Lemma 4, we provide a numerical comparison among Lemma 2 in [45], Lemma 2 in [46] and Lemma 4. It is easily observed from the fuzzy membership functions $h_1(y(x, t))$ and $h_2(y(x, t))$ defined in (32) that Lemma 2 in [45] cannot be utilized to reduce the conservativeness of the fuzzy control design due to that $h_2(y(x, t)) = 0$ when $y(x, t) = 0$. By Theorem 1 and Lemma 2 in [46] with $\varsigma_{v,i} = 1.2$, $v, i \in \{1, 2\}$, it has been verified that the corresponding LMI conditions are infeasible for $T_h = 0.1360$. That is, the improvement of Lemma 4 is better than that of Lemma 2 in [46] with $\varsigma_{v,i} = 1.2$, $v, i \in \{1, 2\}$.

Example 2: Consider piecewise sampled-data control of multi-variable parabolic PDE systems with piecewise control
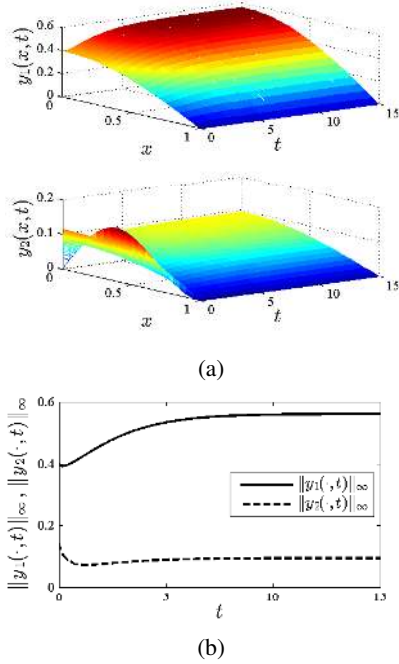
(a)

(b)

Fig. 5: Open-loop numerical simulation results: (a) profiles of evolution of $y_1(\cdot, t)$ and $y_2(\cdot, t)$, and (b) trajectories of $\|y_1(\cdot, t)\|_\infty$ and $\|y_2(\cdot, t)\|_\infty$

inputs and mixed Neumann-Dirichlet boundary conditions

$$\begin{cases} y_{1,t}(x,t) = y_{1,xx}(x,t) + 3y_1(x,t) \\ \qquad - y_2(x,t) - y_1^3(x,t) + \bar{\boldsymbol{g}}^T(x)\boldsymbol{u}(t), \\ y_{2,t}(x,t) = y_{2,xx}(x,t) + 0.45y_1(x,t) - 0.1y_2(x,t) \\ y_{i,x}(x,t)|_{x=0} = y_i(1,t) = 0, \ i \in \{1,2\} \\ y_i(x,0) = y_{i,0}(x), \ i \in \{1,2\} \end{cases}$$
(34)

where $y_i(x,t)$, $i \in \{1,2\}$ are state variables, $\boldsymbol{u}(t) \triangleq [u_1(t) \ u_2(t)]^T \in \Re^2$ is manipulated input, $\bar{\boldsymbol{g}}(x) \triangleq [\bar{g}_1(x) \ \bar{g}_2(x)]^T$ is influence function describing distribution of piecewise actuators, where $\bar{g}_1(x) = \begin{cases} 1 & x \in [0, 0.5) \\ 0 & \text{otherwise} \end{cases}$ and $\bar{g}_2(x) = \begin{cases} 2 & x \in [0.5, 1) \\ 0 & \text{otherwise} \end{cases}$. $y_{i,0}(x)$, $i \in \{1,2\}$ are initial conditions. The PDE in the system (34) is a FitzHugh-Nagumo (FHN) equation that is widely applied to model wavy behavior in excitable media in biology [49] and chemistry [50], where the state variables $y_i(x,t)$, $i \in \{1,2\}$ are usually termed as the concentrations of "*activator*" and "*inhibitor*", respectively. The initial conditions in (34) are assumed to be $y_{1,0}(x) = 0.4\cos(0.5\pi x)$ and $y_{2,0}(x) = 0.2\cos(0.5\pi x)\sin(\pi x)$. Fig. 5 shows open-loop profiles of evolution of $y_1(x,t)$ and $y_2(x,t)$, and open-loop trajectories of $\|y_1(\cdot,t)\|_\infty$ and $\|y_2(\cdot,t)\|_\infty$. It is easily seen from Fig. 5 that operating steady profiles $y_i(\cdot, t) = 0$, $i \in \{1,2\}$ of the system (34) are unstable in the sense of $\|\cdot\|_\infty$.

Let $\boldsymbol{y}(x,t) \triangleq [y_1(x,t) \ y_2(x,t)]^T$, $\boldsymbol{g}_1 = [1 \ 0]^T$, and $\boldsymbol{g}_2 = [2 \ 0]^T$, the system (34) is rewritten as the form (1), where $\boldsymbol{\Theta} = \boldsymbol{I}$, $m = 2$, $x_1 = 0$, $x_2 = 0.5$, $x_3 = 1$, and

$$\boldsymbol{f}(\boldsymbol{y}(x,t)) = \begin{bmatrix} 3y_1(x,t) - y_2(x,t) - y_1^3(x,t) \\ 0.45y_1(x,t) - 0.1y_2(x,t) \end{bmatrix}.$$

It is clear from Fig. 5 that $y_1(\cdot,t) \in [-0.6, 0.6]$, $t \geq 0$. The operating domain $\mathfrak{S}$ is chosen as $\mathfrak{S} = [-1.2, 1.2]$. By the T-S fuzzy PDE modeling approach in Section IV [41], when $y_1(\cdot, t) \in \mathfrak{S}$, the system (34) can be exactly represented by the following T-S fuzzy PDE system of two rules:

Plant Rule 1:
IF $\xi(x,t)$ is "Big", **THEN**

$$\begin{cases} \boldsymbol{y}_t(x,t) = \boldsymbol{\Theta}\boldsymbol{y}_{xx}(x,t) + \boldsymbol{A}_1\boldsymbol{y}(x,t) + \boldsymbol{G}(x)\boldsymbol{u}(t) \\ \boldsymbol{y}_x(x,t)|_{x=0} = \boldsymbol{y}(1,t) = 0, \ \boldsymbol{y}(x,0) = \boldsymbol{y}_0(x) \end{cases}$$

Plant Rule 2:
IF $\xi(x,t)$ is "Small", **THEN**

$$\begin{cases} \boldsymbol{y}_t(x,t) = \boldsymbol{\Theta}\boldsymbol{y}_{xx}(x,t) + \boldsymbol{A}_2\boldsymbol{y}(x,t) + \boldsymbol{G}(x)\boldsymbol{u}(t) \\ \boldsymbol{y}_x(x,t)|_{x=0} = \boldsymbol{y}(1,t) = 0, \ \boldsymbol{y}(x,0) = \boldsymbol{y}_0(x) \end{cases}$$

where $\xi(x,t) = y_1^2(x,t)$,

$$\boldsymbol{A}_1 = \begin{bmatrix} 3 - \vartheta & -1 \\ 0.45 & -0.1 \end{bmatrix} \text{ and } \boldsymbol{A}_2 = \begin{bmatrix} 3 & -1 \\ 0.45 & -0.1 \end{bmatrix}$$

with $\vartheta \triangleq \max_{y_1(\cdot,t)\in\mathfrak{S}} y_1^2(\cdot, t) = 1.44$. The overall fuzzy model is written as follows:

$$\begin{cases} \boldsymbol{y}_t(x,t) = \boldsymbol{\Theta}\boldsymbol{y}_{xx}(x,t) \\ \qquad + \sum_{i=1}^2 h_i(\xi(x,t))\boldsymbol{A}_i\boldsymbol{y}(x,t) + \boldsymbol{G}(x)\boldsymbol{u}(t) \\ \boldsymbol{y}_x(x,t)|_{x=0} = \boldsymbol{y}(1,t) = 0, \ \boldsymbol{y}(x,0) = \boldsymbol{y}_0(x), \end{cases}$$

where the fuzzy membership functions $h_1(\xi(x,t))$ and $h_2(\xi(x,t))$ are chosen as

$$h_1(\xi(x,t)) = \vartheta^{-1}\xi(x,t) \text{ and } h_2(\xi(x,t)) = 1 - h_1(\xi(x,t)).$$

For more details, please refer to [41].

Set $\tilde{x}_1 = 0.2$, $\tilde{x}_2 = 0.3$, $\tilde{x}_3 = 0.7$, and $\tilde{x}_4 = 0.8$, we thus have $\varphi_1 = \varphi_2 = 0.09$. Let $T_h = 0.14$. By solving LMIs (27)-(29) and using (26), the control gain parameters $\boldsymbol{k}_{vi}^T$, $v \in \{1,2\}$, $i \in \{1,2\}$ are given as:

$$\begin{bmatrix} \boldsymbol{k}_{11}^T \\ \boldsymbol{k}_{21}^T \end{bmatrix} = \begin{bmatrix} -3.4059 & 0.2105 \\ -1.7029 & 0.1053 \end{bmatrix},$$

and

$$\begin{bmatrix} \boldsymbol{k}_{12}^T \\ \boldsymbol{k}_{22}^T \end{bmatrix} = \begin{bmatrix} -3.4073 & 0.2104 \\ -1.7036 & 0.1052 \end{bmatrix}.$$

By applying the suggested sampled-data fuzzy controller (6) with above control gain parameters, closed-loop numerical simulation results: profiles of evolution of $y_1(x,t)$ and $y_2(x,t)$, trajectories of $\|y_1(\cdot,t)\|_\infty$ and $\|y_2(\cdot,t)\|_\infty$, and trajectory of sampled-data fuzzy control input $\boldsymbol{u}(t)$ are shown in Fig. 6. What is apparent from Fig. 6 is that the suggested sampled-data fuzzy controller (6) can stabilize the system (34) in the sense of $\|\cdot\|_\infty$.

V. CONCLUSIONS

This paper has proposed an LMI-based static sampled-data fuzzy local state feedback control design for a class of SLPPDE systems in the sense of $\|\cdot\|_\infty$. In the proposed design development, three types of spatial integral inequalities are utilized, i.e., vector-valued Wirtinger's inequalities, a variation of vector-valued Poincaré–Wirtinger inequality in 1D spatial domain, and a vector-valued Agmon's inequality, respectively.
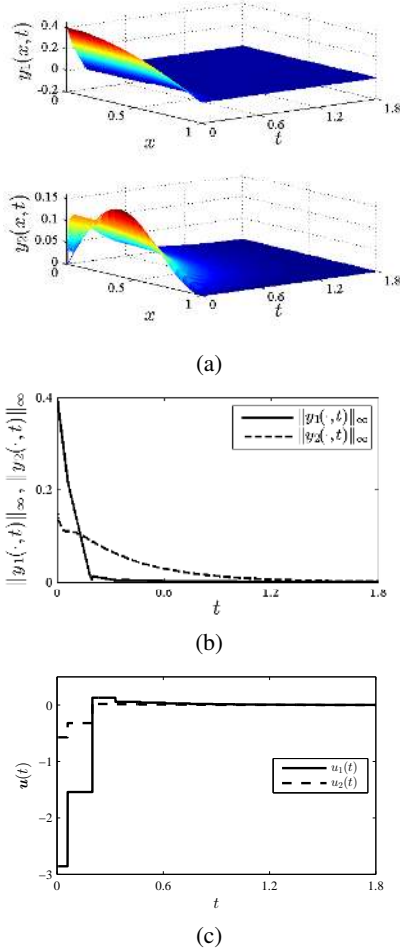
(a)

(b)

(c)

Fig. 6: Closed-loop numerical simulation results: (a) profiles of evolution of $y_1(x,t)$ and $y_2(x,t)$, (b) trajectories of $\|y_1(\cdot,t)\|_\infty$ and $\|y_2(\cdot,t)\|_\infty$, (c) trajectory of sampled-data fuzzy control input $\boldsymbol{u}(t)$

It has been shown by constructing an appropriate Lyapunov–Krasovskii functional candidate and utilizing integration by parts and these three integral inequalities that the suggested sampled-data fuzzy controller exponentially stabilizes the PDE system in the sense of $\|\cdot\|_\infty$, if the LMI-based sufficient conditions are satisfied. Moreover, the exponential stabilization ability of the suggested sampled-data fuzzy controller is enhanced by proposing an LMI relaxation technique. Finally, the satisfactory and better performance of the suggested sampled-data fuzzy controller are demonstrated by numerical simulation results of two examples.

Note that main results of this paper are developed by Lemma 2. To utilize this lemma, we assume $0 < \boldsymbol{\Theta} \in \Re^{n\times n}$ in the SLPPDE system (1) such that the inequality $[\boldsymbol{P}_1\boldsymbol{\Theta}+] > 0$ (see Eq. (49)) is fulfilled for any $\boldsymbol{P}_1 > 0$. Indeed, this assumption limits the scope of the application of the main results. In the further study, we will study the problem of sampled-data fuzzy control design for the SLPPDE system (1) with any general matrix $\boldsymbol{\Theta} \in \Re^{n\times n}$.

APPENDIX A
PROOF OF LEMMA 2

Proof: In the light of the eigendecomposition for the matrix $\boldsymbol{S} \geq 0$, we get

$$\boldsymbol{S} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T, \tag{35}$$

where $\boldsymbol{U}$ is an orthogonal matrix and $0 \leq \boldsymbol{\Lambda} \triangleq \mathrm{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ whose entries are eigenvalues of $\boldsymbol{S}$. By using (35) and setting

$$\tilde{\boldsymbol{z}}(x,t) \triangleq \boldsymbol{U}^T\boldsymbol{y}(x,t) \text{ and } \tilde{\boldsymbol{z}}_l(t) \triangleq \boldsymbol{U}^T\boldsymbol{y}_l(t), \tag{36}$$

where $\tilde{\boldsymbol{z}}(x,t) = [\tilde{z}_1(x,t) \quad \tilde{z}_2(x,t) \quad \cdots \quad \tilde{z}_n(x,t)]^T$ and $\tilde{\boldsymbol{z}}_l(t) = [\tilde{z}_{l,1}(t) \quad \tilde{z}_{l,2}(t) \quad \cdots \quad \tilde{z}_{l,n}(t)]^T$, with

$$\tilde{z}_{l,i}(t) = \frac{1}{l_2 - l_1}\int_{l_1}^{l_2}\tilde{z}_i(x,t)dx, \ i \in \{1,2,\cdots,n\}. \tag{37}$$

By first mean value theorem for definite integrals [51], for any $\boldsymbol{y}(\cdot,t) \in \mathcal{H}_n^1((0,L))$ ($\tilde{\boldsymbol{z}}(\cdot,t) \in \mathcal{H}_n^1((0,L))$, obviously), there exists scalars $\bar{l}_{i,t} \in [l_1,l_2]$, $i \in \{1,2,\cdots,n\}$ such that

$$\int_{l_1}^{l_2}\tilde{z}_i(x,t)dx = (l_2-l_1)\tilde{z}_i(\bar{l}_{i,t},t), \ i \in \{1,2,\cdots,n\} \tag{38}$$

where $\tilde{z}_i(\bar{l}_{i,t},t)$ is called *mean value* of $\tilde{z}_i(x,t)$ on $[l_1,l_2]$. The scalars $\bar{l}_{i,t}$, $i \in \{1,2,\cdots,n\}$ such that the Eq. (38) is fulfilled are not unique and their accurate value is unknown in general. From (36)-(38), we get

$$\int_0^L(\boldsymbol{y}(x,t)-\boldsymbol{y}_l(t))^T\boldsymbol{S}(\boldsymbol{y}(x,t)-\boldsymbol{y}_l(t))dx$$
$$= \int_0^L(\tilde{\boldsymbol{z}}(x,t)-\tilde{\boldsymbol{z}}_l(t))^T\boldsymbol{\Lambda}(\tilde{\boldsymbol{z}}(x,t)-\tilde{\boldsymbol{z}}_l(t))dx$$
$$= \sum_{i=1}^n\int_0^L\lambda_i(\tilde{z}_i(x,t)-\tilde{z}_i(\bar{l}_{i,t},t))^2dx, \tag{39}$$

For each $i \in \{1,2,\cdots,n\}$, by applying extended scalar Wirtinger's inequality [48] and considering $\bar{l}_{i,t} \in [l_1,l_2]$, the following inequality is easily derived

$$\int_0^L(\tilde{z}_i(x,t)-\tilde{z}_i(\bar{l}_{i,t},t))^2dx$$
$$\leq \frac{4\max\{\bar{l}_{i,t}^2,(L-\bar{l}_{i,t})^2\}}{\pi^2}\int_0^L\tilde{z}_{i,x}^2(x,t)dx$$
$$\leq \frac{4\phi}{\pi^2}\int_0^L\tilde{z}_{i,x}^2(x,t)dx. \tag{40}$$

It is deduced from (36), (40) and $\lambda_i \geq 0$, $i \in \{1, 2, \cdots, n\}$ that

$$\sum_{i=1}^{n} \int_0^L \lambda_i (\tilde{z}_i(x,t) - \tilde{z}_i(\bar{l}_{i,t}, t))^2 dx$$
$$\leq \frac{4\phi}{\pi^2} \sum_{i=1}^{n} \int_0^L \lambda_i \tilde{z}_{i,x}^2(x,t) dx$$
$$= \frac{4\phi}{\pi^2} \int_0^L \tilde{z}_x^T(x,t) \mathbf{\Lambda} \tilde{z}_x(x,t) dx$$
$$= \frac{4\phi}{\pi^2} \int_0^L \mathbf{y}_x^T(x,t) \mathbf{S} \mathbf{y}_x(x,t) dx. \qquad (41)$$

The inequality (12) is directly derived from (39) and (41). ∎

APPENDIX B
PROOF OF LEMMA 3

Proof: Using integration by parts and considering $\mathbf{y}(0,t) = 0$, we have for any $t \geq 0$

$$\int_0^x \mathbf{y}^T(\varsigma,t) \mathbf{y}_\varsigma(\varsigma,t) d\varsigma = \mathbf{y}^T(x,t) \mathbf{y}(x,t)$$
$$- \int_0^x \mathbf{y}_\varsigma^T(\varsigma,t) \mathbf{y}(\varsigma,t) d\varsigma. \qquad (42)$$

In the light of Cauchy–Schwarz inequality, the expression (42) is rewritten as for any $x \in [0, L]$ and $t \geq 0$

$$\mathbf{y}^T(x,t) \mathbf{y}(x,t) = 2 \int_0^x \mathbf{y}^T(\varsigma,t) \mathbf{y}_\varsigma(\varsigma,t) d\varsigma$$
$$\leq \sqrt{\int_0^x \|\mathbf{y}(\varsigma,t)\|^2 d\varsigma} \sqrt{\int_0^x \|\mathbf{y}_\varsigma(\varsigma,t)\|^2 d\varsigma}$$
$$\leq \|\mathbf{y}(\cdot,t)\|_2 \|\mathbf{y}_x(\cdot,t)\|_2. \qquad (43)$$

As it is fulfilled for any $x \in [0, L]$, the inequality (43) is further written as $\max_{x \in [0,L]} \|\mathbf{y}(x,t)\|^2 \leq 2\|\mathbf{y}(\cdot,t)\|_2 \|\mathbf{y}_x(\cdot,t)\|_2$, $t \geq 0$, which implies the first inequality in (13) for the case of $\mathbf{y}(0,t) = 0$. By using the triangle inequality and considering the definition of $\|\cdot\|_{\mathcal{H}_n^1((0,L))}$, we have $2\|\mathbf{y}(\cdot,t)\|_2 \|\mathbf{y}_x(\cdot,t)\|_2 \leq \|\mathbf{y}(\cdot,t)\|_{\mathcal{H}_n^1((0,L))}^2$, $t \geq 0$, which is the second inequality in (13). The inequality (13) is easily derived for the case of $\mathbf{y}(L,t) = 0$ in a similar manner. ∎

APPENDIX C
PROOF OF THEOREM 1

Proof: Assume that the LMIs (24) are fulfilled for matrices $0 < \mathbf{X}_1 \in \Re^{n \times n}$, $0 < \mathbf{X}_2 \in \Re^{n \times n}$, $0 < \mathbf{X}_3 \in \Re^{n \times n}$ and vectors $\mathbf{z}_{vj} \in \Re^n$, $v \in \mathcal{M}$, $j \in \mathbb{S}$. By applying integration by parts and considering the boundary conditions in (1), we get for any $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{\Theta}$

$$\int_0^L \mathbf{y}^T(x,t) \mathbf{P}_1 \mathbf{\Theta} \mathbf{y}_{xx}(x,t) dx = -\int_0^L \mathbf{y}_x^T(x,t) \mathbf{P}_1 \mathbf{\Theta} \mathbf{y}_x(x,t) dx, \qquad (44)$$

$$\int_0^L \mathbf{y}_x^T(x,t) \mathbf{P}_2 \mathbf{y}_{xt}(x,t) dx = -\int_0^L \mathbf{y}_{xx}^T(x,t) \mathbf{P}_2 \mathbf{y}_t(x,t) dx. \qquad (45)$$

Define

$$\tilde{\mathbf{y}}_v(x,t) \triangleq \mathbf{y}(x,t) - \frac{1}{\Delta \tilde{x}_v} \int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \mathbf{y}(x,t) dx, \ v \in \mathcal{M}, \qquad (46)$$

$$\bar{\mathbf{y}}_v(t,t_k) \triangleq \int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \int_{t_k}^{t} \mathbf{y}_s(x,s) ds dx, \ v \in \mathcal{M}, \ k \in \wp, \qquad (47)$$

where $\Delta \tilde{x}_v$, $v \in \mathcal{M}$ are defined by (7). Using (44) and (47), the time derivative of $V_1(t)$ in (19) along the solution to the system (9), we get for any $t \in [t_k, t_{k+1})$, $k \in \wp$

$$\dot{V}_1(t) = -\int_0^L \mathbf{y}_x^T(x,t)[\mathbf{P}_1 \mathbf{\Theta} + *]\mathbf{y}_x(x,t) dx$$
$$+ \int_0^L \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(x,t)) \mathbf{y}^T(x,t)[\mathbf{P}_1 \mathbf{A}_i + *]\mathbf{y}(x,t) dx$$
$$+ 2 \sum_{v=1}^{m} \int_{x_v}^{x_{v+1}} \mathbf{y}^T(x,t) \mathbf{P}_1 \mathbf{g}_v dx$$
$$\times \sum_{j=1}^{r} h_{vj}(t_k) \frac{\mathbf{k}_{vj}^T}{\Delta \tilde{x}_v} \int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \mathbf{y}(x,t) dx$$
$$- 2 \sum_{v=1}^{m} \int_{x_v}^{x_{v+1}} \mathbf{y}^T(x,t) \mathbf{P}_1 \mathbf{g}_v dx$$
$$\times \sum_{j=1}^{r} h_{vj}(t_k) \frac{\mathbf{k}_{vj}^T}{\Delta \tilde{x}_v} \bar{\mathbf{y}}_v(t,t_k). \qquad (48)$$

It can be obtained from $\mathbf{P}_1 > 0$ and $\mathbf{\Theta} > 0$ that

$$\mathbf{\Psi} \triangleq [\mathbf{P}_1 \mathbf{\Theta} + *] > 0. \qquad (49)$$

By using the inequality (12) in Lemma 2 and considering (49) and $[\tilde{x}_{2v-1}, \tilde{x}_{2v}] \subset [x_v, x_{v+1}]$, we get for any $v \in \mathcal{M}$,

$$-\int_{x_v}^{x_{v+1}} \mathbf{y}_x^T(x,t) \mathbf{\Psi} \mathbf{y}_x(x,t) dx$$
$$\leq -\frac{\pi^2}{4\varphi_v} \int_{x_v}^{x_{v+1}} \tilde{\mathbf{y}}_v^T(x,t) \mathbf{\Psi} \tilde{\mathbf{y}}_v(x,t) dx, \qquad (50)$$

where φ_v, $v \in \mathcal{M}$ are defined by (25). Substituting (50) into (48) and considering $\bigcup_{v=1}^{m} [x_v, x_{v+1}] = [0, L]$, we get

$$\dot{V}_1(t) \leq \sum_{v=1}^{m} \int_{x_v}^{x_{v+1}} \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\boldsymbol{\xi}(x,t)) h_{vj}(t)$$
$$\times \hat{\mathbf{y}}_v^T(x,t) \bar{\mathbf{\Psi}}_{1vij} \hat{\mathbf{y}}_v(x,t) dx$$
$$- 2 \sum_{v=1}^{m} \int_{x_v}^{x_{v+1}} \mathbf{y}^T(x,t) \mathbf{P}_1 \mathbf{g}_v dx$$
$$\times \sum_{j=1}^{r} h_{vj}(t_k) \frac{\mathbf{k}_{vj}^T}{\Delta \tilde{x}_v} \bar{\mathbf{y}}_v(t,t_k), \qquad (51)$$

where $\hat{\mathbf{y}}_v(x,t) \triangleq [\mathbf{y}^T(x,t) \quad (\Delta \tilde{x}_v)^{-1} \int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \mathbf{y}^T(x,t) dx]^T$ and

$$\bar{\mathbf{\Psi}}_{1vij} \triangleq \begin{bmatrix} [\mathbf{P}_1 \mathbf{A}_i + *] - \frac{\pi^2}{4\varphi_v} \mathbf{\Psi} & \frac{\pi^2}{4\varphi_v} \mathbf{\Psi} + \mathbf{P}_1 \mathbf{g}_v \mathbf{k}_{vj}^T \\ * & -\frac{\pi^2}{4\varphi_v} \mathbf{\Psi} \end{bmatrix},$$
$$v \in \mathcal{M}, \ i,j \in \mathbb{S}.$$

Similarly, using (45), (47), and $\bigcup_{v=1}^{m}[x_v, x_{v+1}] = [0, L]$, for $t \in [t_k, t_{k+1})$, $k \in \wp$, the time derivative of $V_2(t)$ defined in (20) along the solution to the system (9) is given as follows:

$$
\begin{aligned}
\dot{V}_2(t) = & -\int_0^L \boldsymbol{y}_{xx}^T(x,t)[\boldsymbol{P}_2\boldsymbol{\Theta} + *]\boldsymbol{y}_{xx}(x,t)dx \\
& - 2\int_0^L \sum_{i=1}^r h_i(\boldsymbol{\xi}(x,t))\boldsymbol{y}_{xx}^T(x,t)\boldsymbol{P}_2\boldsymbol{A}_i\boldsymbol{y}(x,t)dx \\
& - 2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \boldsymbol{y}_{xx}^T(x,t)\boldsymbol{P}_2\boldsymbol{g}_v dx \\
& \times \sum_{j=1}^r h_{vj}(t_k)\frac{\boldsymbol{k}_{vj}^T}{\Delta\tilde{x}_v}\int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \boldsymbol{y}(x,t)dx \\
& + 2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \boldsymbol{y}_{xx}^T(x,t)\boldsymbol{P}_2\boldsymbol{g}_v dx \\
& \times \sum_{j=1}^r h_{vj}(t_k)\frac{\boldsymbol{k}_{vj}^T}{\Delta\tilde{x}_v}\bar{\boldsymbol{y}}_v(t,t_k). \quad (52)
\end{aligned}
$$

Based on the Jensen's inequality [52], we get for any $t \in (t_k, t_{k+1})$, $\boldsymbol{P}_3 > 0$ and $x \in [0, L]$

$$
\begin{aligned}
& -\int_{t_k}^t \boldsymbol{y}_s^T(x,s)\boldsymbol{P}_3\boldsymbol{y}_s(x,s)ds \\
& \leq -T_h^{-1}\int_{t_k}^t \boldsymbol{y}_s^T(x,s)ds\boldsymbol{P}_3\int_{t_k}^t \boldsymbol{y}_s(x,s)ds. \quad (53)
\end{aligned}
$$

When $t = t_k$, the inequality (53) is also fulfilled obviously due to that for any $x \in [0, L]$,

$$
\begin{aligned}
\lim_{t\to t_k}\frac{1}{t-t_k}\int_{t_k}^t \boldsymbol{y}_s^T(x,s)ds\boldsymbol{P}_3\int_{t_k}^t \boldsymbol{y}_s(x,s)ds \\
= 2\lim_{t\to t_k}\int_{t_k}^t \boldsymbol{y}_s^T(x,s)ds\boldsymbol{P}_3\boldsymbol{y}_t(x,t) = 0.
\end{aligned}
$$

Utilizing $\bigcup_{v=1}^{m}[x_v, x_{v+1}] = [0, L]$ and (53), for $t \in [t_k, t_{k+1})$, $k \in \wp$, the time derivative of $V_3(t)$ defined in (21) is obtained as follows

$$
\begin{aligned}
\dot{V}_3(t) \leq & T_h^2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \boldsymbol{y}_t^T(x,t)\boldsymbol{P}_3\boldsymbol{y}_t(x,t)dx \\
& -\sum_{v=1}^m \int_{x_v}^{x_{v+1}}\int_{t_k}^t \boldsymbol{y}_s^T(x,s)ds\boldsymbol{P}_3\int_{t_k}^t \boldsymbol{y}_s(x,s)dsdx. \quad (54)
\end{aligned}
$$

With the help of the Jensen's inequality again and $\bigcup_{v=1}^{m}[\tilde{x}_{2v-1}, \tilde{x}_{2v}] \subset [0, L] = \bigcup_{v=1}^{m}[x_v, x_{v+1}]$, we know for any $v \in \mathcal{M}$ and $t \in [t_k, t_{k+1})$, $k \in \wp$

$$
\begin{aligned}
& -\int_{x_v}^{x_{v+1}}\int_{t_k}^t \boldsymbol{y}_s^T(x,s)ds\boldsymbol{P}_3\int_{t_k}^t \boldsymbol{y}_s(x,s)dsdx \\
& \quad < -(\Delta\tilde{x}_v)^{-1}\bar{\boldsymbol{y}}_v^T(t,t_k)\boldsymbol{P}_3\bar{\boldsymbol{y}}_v(t,t_k). \quad (55)
\end{aligned}
$$

Substitution of (55) into (54), we rewrite (54) as

$$
\begin{aligned}
\dot{V}_3(t) < & T_h^2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \boldsymbol{y}_t^T(x,t)\boldsymbol{P}_3\boldsymbol{y}_t(x,t)dx \\
& -\sum_{v=1}^m \frac{1}{\Delta x_v \Delta\tilde{x}_v}\int_{x_v}^{x_{v+1}} \bar{\boldsymbol{y}}_v^T(t,t_k)\boldsymbol{P}_3\bar{\boldsymbol{y}}_v(t,t_k)dx, \quad (56)
\end{aligned}
$$

where Δx_v, $v \in \mathcal{M}$ are defined by (25).

On the other hand, by considering $\bigcup_{v=1}^{m}[x_v, x_{v+1}] = [0, L]$, from (9) and (47), it is clearly seen for any $t \geq 0$ that

$$
\begin{aligned}
0 = & 2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \boldsymbol{y}_t^T(x,t)\boldsymbol{P}_1\boldsymbol{\Theta}\boldsymbol{y}_{xx}(x,t)dx \\
& + 2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \sum_{i=1}^r h_i(\boldsymbol{\xi}(x,t))\boldsymbol{y}_t^T(x,t)\boldsymbol{P}_1\boldsymbol{A}_i\boldsymbol{y}(x,t)dx \\
& + 2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \boldsymbol{y}_t^T(x,t)\boldsymbol{P}_1\boldsymbol{g}_v dx \\
& \times \sum_{j=1}^r h_{vj}(t_k)\frac{\boldsymbol{k}_{vj}^T}{\Delta\tilde{x}_v}\int_{\tilde{x}_{2v-1}}^{\tilde{x}_{2v}} \boldsymbol{y}(x,t)dx \\
& - 2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \boldsymbol{y}_t^T(x,t)\boldsymbol{P}_1\boldsymbol{g}_v dx \sum_{j=1}^r h_{vj}(t_k)\frac{\boldsymbol{k}_{vj}^T}{\Delta\tilde{x}_v}\bar{\boldsymbol{y}}_v(t,t_k) \\
& - 2\sum_{v=1}^m \int_{x_v}^{x_{v+1}} \boldsymbol{y}_t^T(x,t)\boldsymbol{P}_1\boldsymbol{y}_t(x,t)dx. \quad (57)
\end{aligned}
$$

By using (51), (52), (56), and (57), for $t \in [t_k, t_{k+1})$, $k \in \wp$, the time derivative of $V(t)$ defined in (18) along the solution to the system (9) is given as follows:

$$
\begin{aligned}
\dot{V}(t) < & \sum_{v=1}^m \int_{x_v}^{x_{v+1}} \sum_{i=1}^r \sum_{j=1}^r h_i(\boldsymbol{\xi}(x,t))h_{v,j}(t) \\
& \times \boldsymbol{\zeta}_v^T(x,t,t_k)\bar{\boldsymbol{\Psi}}_{vij}\boldsymbol{\zeta}_v(x,t,t_k)dx, \quad (58)
\end{aligned}
$$

where

$$
\boldsymbol{\zeta}_v(x,t,t_k) \triangleq \left[\hat{\boldsymbol{y}}_v^T(x,t) \quad \boldsymbol{y}_{xx}^T(x,t) \quad \boldsymbol{y}_t^T(x,t) \quad \frac{\bar{\boldsymbol{y}}_v^T(t,t_k)}{\Delta\tilde{x}_v}\right]^T,
$$

$$
\bar{\boldsymbol{\Psi}}_{vij} \triangleq \left[\begin{array}{cc} \bar{\boldsymbol{\Psi}}_{1vij} & \bar{\boldsymbol{\Psi}}_{2vij} \\ * & \bar{\boldsymbol{\Psi}}_{3vij} \end{array}\right], \quad v \in \mathcal{M}, \ i,j \in \mathbb{S}
$$

in which

$$
\bar{\boldsymbol{\Psi}}_{2vij} \triangleq \left[\begin{array}{ccc} -\boldsymbol{A}_i^T\boldsymbol{P}_2 & \boldsymbol{A}_i^T\boldsymbol{P}_1 & -\boldsymbol{P}_1\boldsymbol{g}_v\boldsymbol{k}_{vj}^T \\ -\boldsymbol{k}_{vj}\boldsymbol{g}_v^T\boldsymbol{P}_2 & \boldsymbol{k}_{vj}\boldsymbol{g}_v^T\boldsymbol{P}_1 & 0 \end{array}\right],
$$

$$
\bar{\boldsymbol{\Psi}}_{3vij} \triangleq \left[\begin{array}{ccc} -[\boldsymbol{P}_2\boldsymbol{\Theta}+*] & \boldsymbol{\Theta}\boldsymbol{P}_1 & \boldsymbol{P}_2\boldsymbol{g}_v\boldsymbol{k}_{vj}^T \\ * & T_h^2\boldsymbol{P}_3 - 2\boldsymbol{P}_1 & -\boldsymbol{P}_1\boldsymbol{g}_v\boldsymbol{k}_{vj}^T \\ * & * & -\frac{\Delta\tilde{x}_v}{\Delta x_v}\boldsymbol{P}_3 \end{array}\right].
$$

Let

$$
\begin{aligned}
& \boldsymbol{X}_1 = \boldsymbol{P}_1^{-1}, \ \boldsymbol{X}_2 = \boldsymbol{P}_2^{-1}, \ \boldsymbol{X}_3 = \boldsymbol{P}_1^{-1}\boldsymbol{P}_3\boldsymbol{P}_1^{-1}, \\
& \boldsymbol{z}_{vj}^T = \boldsymbol{k}_{vj}^T\boldsymbol{X}_1, \ v \in \mathcal{M}, \ j \in \mathbb{S}. \quad (59)
\end{aligned}
$$

By pre- and post-multiplying the LMIs (24) with a block-diagonal matrix $\mathbb{P} \triangleq \text{diag}\{\boldsymbol{P}_1, \boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_1, \boldsymbol{P}_1\}$, respectively, and using (59), we obtain

$$
\bar{\boldsymbol{\Psi}}_{vij} < 0, \ v \in \mathcal{M}, \ i,j \in \mathbb{S}. \quad (60)
$$

One can find an appropriate scalar $\rho > 0$ such that

$$
\bar{\boldsymbol{\Psi}}_{vij} + \rho\boldsymbol{I} \leq 0, \ v \in \mathcal{M}, \ i,j \in \mathbb{S}. \quad (61)
$$

Substituting (61) into (58) and considering (5) and (8), we obtain for $t \in [t_k, t_{k+1})$, $k \in \wp$

$$\dot{V}(t) < -\rho \int_0^L \mathbf{y}^T(x,t)\mathbf{y}(x,t)dx$$
$$- \rho \int_0^L \mathbf{y}_{xx}^T(x,t)\mathbf{y}_{xx}(x,t)dx - \rho \int_0^L \mathbf{y}_t^T(x,t)\mathbf{y}_t(x,t)dx. \tag{62}$$

By applying the inequality (11) in Lemma 1 and considering $\mathbf{y}_x(x,t)|_{x=L} = 0$, we get for any $t > 0$

$$\int_0^L \mathbf{y}_x^T(x,t)\mathbf{y}_x(x,t)dx \leq \frac{4L^2}{\pi^2} \int_0^L \mathbf{y}_{xx}^T(x,t)\mathbf{y}_{xx}(x,t)dx. \tag{63}$$

By following proofs of Theorem 3 in [48] and Theorem 1 in [53], it is easily shown from the inequalities (62) and (63) that for any $t_f \geq 0$ there exists a constant $\kappa > 0$ such that

$$V(t_f) \leq V(0)\exp(-\kappa t_f). \tag{64}$$

By (22) and (23), the inequality (64) is written as $\|\mathbf{y}(\cdot, t_f)\|_{\mathcal{H}_n^1((0,L))}^2 \leq \rho_2 \rho_1^{-1} \|\mathbf{y}_0(\cdot)\|_{\mathcal{H}_n^1((0,L))}^2 \exp(-\kappa t_f)$. Based on Lemma 3, we further get $\|\mathbf{y}(\cdot, t_f)\|_\infty \leq \sqrt{\rho_2 \rho_1^{-1}} \|\mathbf{y}_0(\cdot)\|_{\mathcal{H}_n^1((0,L))} \exp(-0.5\kappa t_f)$. Hence, we conclude from the above inequality and Definition 2 that the system (9) is exponentially stable in the sense of $\|\cdot\|_\infty$. Consider $\mathbb{D} \subset \mathfrak{S}$ and the equivalence between the SLPPDE in (1) and the fuzzy PDE (4) in the operating domain $\mathfrak{S}$, the suggested sampled-data fuzzy controller (6) can locally exponentially stabilize the system (1) in the sense of $\|\cdot\|_\infty$. From (59), we obtain (26). ∎

REFERENCES

[1] J. Ackermann, *Sampled-Data Control Systems: Analysis and Synthesis, Robust System Design*, Berlin, Heidelberg: Springer-Verlag, 1985.

[2] T. Chen and B. Francis, *Optimal Sampled-Data Control Systems*, 2nd Ed. London: Springer-Verlag, 1996.

[3] D. Nesic and A. R. Teel, Sampled-data conrol of nonlinear systems: An overview of recent results, *Perspectives Robust Control*, vol. 268 pp. 221–239 2001.

[4] A. Dabroom and H. Khalil, Output feedback sampled-data control of nonlinear systems using high-gain observers, *IEEE Trans. Automatic. Control*, vol. 46, no. 11, pp. 1712-1725, 2001.

[5] L.-S. Hu, J. Lam, Y.-Y. Cao and H.-H. Shao, A linear matrix inequality (LMI) approach to robust H_2 sampled-data control for linear uncertain systems, *IEEE Trans. Syst., Man, Cybern.: Part B*, vol. 33, no. 1, pp. 149-155, 2003.

[6] E. Fridman, A. Seuret J. P. Richard, Robust sampled-data stabilization of linear systems: An input delay approach, *Automatica*, vol. 40 no. 8 pp. 1441–1446 2004.

[7] P. Naghshtabrizi, J. Hespanha, and A. Teel, Exponential stability of impulsive systems with application to uncertain sampled-data systems, *Syst. Control & Lett.*, vol. 57, no. 5, pp. 378–385, 2008.

[8] H. Fujioka, A discrete-time approach to stability analysis of systems with a periodic sample-and-hold devices, *IEEE Trans. Automatic. Control*, vol. 54, no. 9, pp. 2440–2445, 2009.

[9] E. Fridman, A refined input delay approach to sampled-data control, *Automatica*, vol. 46, no. 8, pp. 421-427, 2010.

[10] W.-H. Chen and W.-X. Zheng, An improved stabilization method for sampled-data control systems with control packet loss, *IEEE Trans. Automat. Control*, vol. 57, no. 9, pp. 2378–2384, 2012.

[11] W.-H. Chen, Z. Wang, D. Wei, and X. Lu, Robust sampled-data H_∞ control of uncertain singularly perturbed systems using time-dependent Lyapunov functionals, *Int. J. Systems Science*, vol. 46, no. 15, pp. 2832–2852, 2015.

[12] T. Ahmed-Ali, E. Fridman, F. Giri, L. Burlion, and F. Lamnabhi-Lagarrigue, Using exponential time-varying gains for sampled-data stabilization and estimation, *Automatica*, vol. 67, pp. 244–251, 2016.

[13] A. Seuret and E. Fridman, Wirtinger-like Lyapunov-Krasovskii functionals for discrete sampled-data systems, *IMA J. of Math. Control and Information*, doi:10.1093/imamci/dnx002, 2017.

[14] W. H. Ray, *Advanced Process Control*. New York: McGraw-Hill, 1981.

[15] P. D. Christofides, *Nonlinear and Robust Control of PDE Systems: Methods and Applications to Transport-Reaction Processes*. Boston, MA: Birkhauser, 2001.

[16] H.-X. Li and C. Qi, *Spatio-Temporal Modeling of Nonlinear Distributed Parameter Systems: A Time/Space Separation Based Approach*, New York: Springer-Verlag, 2011.

[17] W. He, S. S. Ge, B. V. How, et al, *Dynamics and Control of Mechanical Systems in Offshore Engineering*, London: Springer-Verlag, 2014.

[18] M.-B. Cheng, V. Radisavljevic, C.-C. Chang, et al., A sampled-data singularly perturbed boundary control for a heat conduction system with noncollocated observation, *IEEE Trans. Automatic. Control*, vol. 54, no. 5, pp. 1305-1310, 2009.

[19] S. Ghantasala and N. H. EI-Farra, Active fault-tolerant control of sampled-data nonlinear distributed parameter systems, *Int. J. Robust Nonlinear Control*, vol. 22, no. 1, pp. 22-42, 2012.

[20] E. Fridman and A. Blighovsky, Robust sampled-data control of a class of semilinear parabolic systems, *Automatica*, vol. 48, no. 5, pp. 826-836, 2012.

[21] E. Fridman and N. Bar Am, Sampled-data distributed H_∞ control of transport reaction systems, *SIAM J. Control Optim.*, vol. 51, no. 2, pp. 1500-1527, 2013.

[22] A. Selivanov and E. Fridman, Sampled-data relay control of diffusion PDEs, *Automatica*, vol. 82, pp. 59–68, 2017.

[23] W.-H. Chen, S. Luo and W.-X. Zheng, Sampled-data distributed H_∞ control of a class of 1-D parabolic systems under spatially point measurements, *J. Franklin Inst.*, vol. 354, no. 1, pp. 197-214, 2017.

[24] T. Takagi and M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, *IEEE Trans. Syst. Man, Cybern.*, vol. 15, no. 1, pp. 116–132, 1985.

[25] K. Tanaka and H. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. New York: Wiley, 2001.

[26] G. Feng, *Analysis and Synthesis of Fuzzy Control Systems: A Model Based Approach*, Boca Raton, FL: CRC, 2010.

[27] R.-E. Precup and H. Hellendoorn, A survey on industrial applications of fuzzy control, *Comput. Ind.*, vol. 62, no. 3, pp. 213–226, 2011.

[28] H. Gao and T. Chen, Stabilization of nonlinear systems under variable sampling: a fuzzy control approach, *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 5, pp. 972-983, 2007.

[29] H. K. Lam and F. H. F. Leung, Sampled-data fuzzy controller for time-delay nonlinear systems: fuzzy-model-based LMI approach, *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 37, no. 3, pp. 617–629, 2007.

[30] C. Peng, Q.-L. Han, D. Yue, and E. Tian, Sampled-data robust H_∞ control for T-S fuzzy systems with time delay and uncertainties, *Fuzzy Sets Syst.*, vol. 179, no. 1, pp. 20–33, 2011.

[31] X.-L. Zhu, B. Chen, D. Yue and Y. Wang, An improved input delay approach to stabilization of fuzzy systems under variable sampling, *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 330-341, 2012.

[32] H. K. Lam, Stabilization of nonlinear systems using sampled-data output-feedback fuzzy controller based on polynomial-fuzzy-model-based control approach, *IEEE Trans. Syst., Man, Cybern.: Part B*, vol. 42, no. 1, pp. 258–267, 2012.

[33] Z.-G. Wu, P. Shi, H. Su, and J. Chu, Network-based robust passive control for fuzzy systems with randomly occurring uncertainties, *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 5, pp. 966–971, 2013.

[34] Z.-G. Wu, P. Shi, H. Su, and J. Chu, Sampled-data fuzzy control of chaotic systems based on a T-S fuzzy model, *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 153–163, 2014.

[35] Z.-P. Wang and H.-N. Wu, On fuzzy sampled-data control of chaotic systems via a time-dependent Lyapunov functional approach, *IEEE Trans. Cyber.*, vol. 45, no. 4, pp. 819-829, 2015.

[36] Z.-G. Wu, P. Shi, H. Su, and R. Lu, Dissipativity-based sampled-data fuzzy control design and its application to truck-trailer system, *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 5, pp. 1669–1679, 2015.

[37] Y. Wang, Y. Xia, and P. Zhou, Fuzzy-model-based sampled-data control of chaotic systems: A fuzzy time-dependent Lyapunov-Krasovskii functional approach, *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 6, pp. 1672–1684, 2017.

[38] H. Kim, J. Park, and Y.-H. Joo, A fuzzy Lyapunov-Krasovskii functional approach to sampled-data output-feedback stabilization of polynomial

fuzzy systems, *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 1, pp. 366–373, 2018.

[39] Z.-P. Wang and H.-N. Wu, Finite dimensional guaranteed cost sampled-data fuzzy control for a class of nonlinear distributed parameter systems, *Infor. Sci.*, vol. 327, pp. 21–39, 2016.

[40] H.-N. Wu and Z.-P. Wang, Observer-based H_∞ sampled-data fuzzy control for a class of nonlinear parabolic PDE systems, *IEEE Trans. Fuzzy Syst.*, Available online, doi:10.1109/TFUZZ.2017.2686337, 2017.

[41] J.-W. Wang, H.-N. Wu and H.-X. Li, Distributed proportional-spatial derivative control of nonlinear parabolic systems via fuzzy PDE modeling approach, *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 3, pp. 927–938, 2012.

[42] H.-N. Wu, J.-W. Wang and H.-X. Li, Exponential stabilization for a class of nonlinear parabolic PDE systems via fuzzy control approach, *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 318–329, 2012.

[43] S.-J. Ho and B.-S. Chen, Low design-cost fuzzy controllers for robust stabilization of nonlinear partial differential systems, *IEEE Trans. Fuzzy Syst.* vol. 24, no. 6, pp. 1378–1394, 2016.

[44] J.-W. Wang, H.-N. Wu, Y. Yu, and C.-Y. Sun, Mixed H_2/H_∞ fuzzy proportional-spatial integral control design for a class of nonlinear distributed parameter systems, *Fuzzy Sets Syst.*, vol. 306, pp. 26–47, 2017.

[45] J.-W. Wang and H.-N. Wu, Exponential pointwise stabilization of semi-linear parabolic distributed parameter systems via the Takagi-Sugeno fuzzy PDE model, *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 1, pp. 155-173, 2018.

[46] J.-W. Wang, H.-X. Li and H.-N. Wu, A membership-function-dependent approach to fuzzy pointwise state feedback control design for nonlinear parabolic distributed parameter systems with spatially discrete actuators, *IEEE Trans. Syst., Man, and Cybern.: Syst.*, vol. 47, no. 7, pp. 1486–1499, 2017.

[47] P. Gahinet, A. Nemirovskii, A. J. Laub, and M. Chilali, *LMI Control Toolbox for Use with Matlab, Natick*, MA: The Math.Works Inc., 1995.

[48] J.-W. Wang and H.-N. Wu, Some extended Wirtinger's inequalities and distributed proportional-spatial integral control of distributed parameter systems with multi-time delays, *J. Franklin Inst.*, vol. 352, no. 10, pp. 4423–4445, 2015.

[49] J. Keener and J. Sneyd, *Mathematical Physiology*, New York: Springer-Verlag, 1998.

[50] K. Theodoropoulos, Y.-H. Qian, and I. G. Kevrekidis, 'Coarse' stability and bifurcation analysis using timesteppers: a reaction diffusion example, *Proc. Natl. Acad. Sci.*, vol. 97, no. 18, pp. 9840–9843, 2000.

[51] T. M. Apostol, *Mathematical Analysis*, 2nd ED. Massachusetts: Addison-Wesley Publishing Company, 1974.

[52] K. Gu, An integral inequality in the stability problem of time-delay systems, in: *Proc. 39th IEEE Conf. Decision Control*, Sydney, Australia, 2000, pp. 2805–2810.

[53] J.-W. Wang, H.-X. Li and H.-N. Wu, Fuzzy guaranteed cost sampled-data control of nonlinear systems coupled with a scalar reaction-diffusion process, *Fuzzy Sets Syst.*, vol. 302, pp. 121–142, 2016.

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