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Spatiotemporal coherence resonance of phase synchronization in weakly coupled chaotic oscillators

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We study phase synchronization (PS) of coupled chaotic oscillators as a result of an interplay between local coupling and global noise. In the weak coupling region, noise first significantly enhances spatiotemporal PS, but it spoils the phase coherence of chaotic oscillation and reduces the degree of synchronization at large intensities. The spatiotemporal behavior exhibits coherence resonance. Noise enhanced phase synchronization is of great relevance to ecology.

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Nontrivial effects of noise in nonlinear systems, such as stochastic resonance (SR) [1], and coherence resonance (CR) [2] have been of significant recent interest. By CR, pure noise without an external signal can induce maximal coherent motion, observed mainly in excitable systems. The frontier of interest has shifted to spatiotemporal systems recently, such as spatiotemporal SR [3] and array-enhanced SR [4]. Pure spatially independent noise can induce traveling waves [5] or global oscillations [6] in excitable media. Noise-induced coherence can be significantly enhanced when the noisy excitable elements are coupled [7], known as array-enhanced CR [8].

Most previous investigations considered the effect of spatially independent noise on homogeneous excitable media. However, natural systems can hardly be fully homogeneous; moreover, chaotic oscillation and spatial correlation of noise are of great relevance in many situations. In ecology, population oscillations can be well described by Rössler type chaotic food-web models [9]. The chaotic population oscillators over large geographical regions are generically nonidentical due to demographic heterogeneity; however, they are often affected by similar environmental fluctuations [10]. They are also weakly coupled due to the migration among populations [9]. It is thus important to study how coupling and noise together affect the collective behavior of extended heterogeneous chaotic systems. Observations have shown synchronous fluctuation of populations over large geographical regions [10]. Basically, there are two explanations for this synchronization behavior. The first one, known as the Moran effect [11], suggests that two separated populations may become correlated when exposed to similar environmental fluctuations. However, the Moran effect is only well understood in linear and simple systems, but not in nonlinear ones [12]; in particular, it was pointed out that for chaotic models the Moran effect alone cannot synchronize populations [13]. The second explanation is based on phase synchronization (PS) [14–17] of nonidentical chaotic oscillators in a weak coupling regime [9]. However, weak coupling by migration alone may not be sufficient for population synchronization in real situations, and moreover the effects of common environmental fluctuations have not been taken into account.

We study PS in spatially extended heterogeneous chaotic oscillators as a combined result of common (global) noise and weak (local) coupling. The model is a lattice of $N\times N$ ($N=100$) Rössler chaotic oscillators with periodic boundaries and a global noise:

$$\dot{x}_{ij}=-\omega_{ij}y_{ij}-z_{ij}+g\sum_{kl}(x_{kl}-x_{ij})+D\xi_1, \qquad (1)$$

$$\dot{y}_{ij}=\omega_{ij}x_{ij}+0.15y_{ij}+D\xi_2, \qquad (2)$$

$$\dot{z}_{ij}=0.4+z_{ij}(x_{ij}-8.5), \qquad (3)$$

where each oscillator is connected to its four nearest neighbors, the parameters ω_{ij} are randomly and uniformly distributed in [0.96,0.98], g is the coupling strength, and D is the intensity of independent Gaussian noises ξ_k ($k=1,2$) with $\langle\xi_k(t)\xi_l(t-\tau)\rangle=\delta_{k,l}\delta(\tau)$. The main properties of the system reported in the following are similar for coupling extended to more neighbors as well as for correlated ξ_1 and ξ_2. Noise is not applied to the z variable, because it would easily make the system unstable. The lattice has random initial conditions on the chaotic attractors before coupling and noise are applied. The phase ϕ_{ij} of each oscillator is defined as $\phi_{ij}=\arctan(y_{ij}/x_{ij})$ [15].

In this extended system of chaotic oscillators subjected to noise, we characterize both temporal and spatial PS. In perfect synchronization of two noiseless periodic oscillators, the phases are locked to a constant difference, while in noisy chaotic oscillators, the phase difference $\Delta\phi(t)$ fluctuates around a certain preferred value and undergoes many noise-induced 2π phase slips. To distinguish the preferred phase difference from noise-induced phase slips, we consider the cyclic phase difference $[\Delta\phi(t) \bmod 2\pi]$ so that $\Delta\phi$ and $\Delta\phi+2\pi$ represent the same direction in the phase space. In the distribution of the cyclic phase difference constructed with a histogram of M bins [18], preferred phase differences are manifested by peaks. The sharpness of the distribution characterizes the degree of phase synchronization, and is quantified by the entropy $S=-\Sigma_k^M p(k)\ln p(k)$ [17]. In the lattice, we randomly choose an oscillator as a reference oscillator and compute the entropy $S(ij)$ of the ijth oscillator with respective to the reference one. By averaging over space and normalizing with the entropy of the uniform distribution $S_m=\ln(M)$, we get the temporal PS index

$$\rho_t=(S_m-\langle S\rangle)/S_m. \qquad (4)$$

ρ_t shows no sensitive dependence on the reference oscillator. The degree of PS is higher for larger ρ_t. The degree of

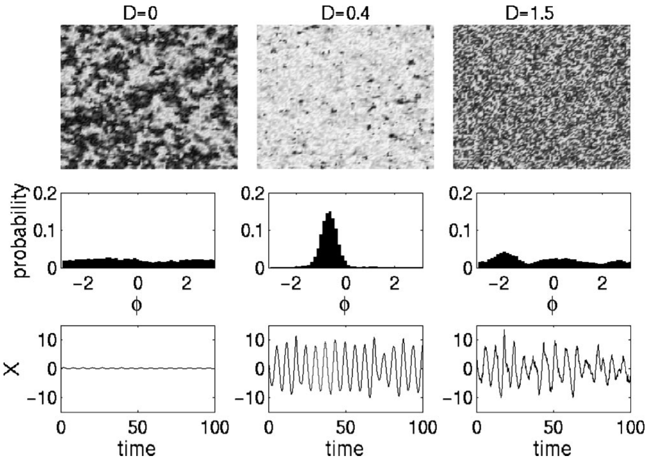

FIG. 1. Behavior of a weakly coupled lattice ($g=0.01$) with a global noise of various intensities. The upper panel: a snapshot of the spatial patterns of x_{ij} in gray scales (the scale range is the same for the three patterns, with white representing maximal and black minimal values) The middle panel: the corresponding distributions of phases ϕ_{ij} on $[-\pi,\pi]$ The lower panel: the corresponding mean field X vs time.

spatial PS can be manifested by the fluctuation amplitude of the mean field $X=\langle x_{ij}\rangle_s$ [16]. We calculate the variance $\sigma_X^2=\langle(X-\langle X\rangle_t)^2\rangle_t$ of X.

Before considering the effects of noise, we briefly describe the behavior of the system with increasing coupling strength g. Without noise, the lattice achieves a global frequency locking when the coupling strength $g>0.04$, which, however, does not immediately lead to large scale spatial synchronization. The lattice does not display a visible macroscopic mean field ($X\sim 0$) until $g>0.165$, and for $g<0.165$, it exhibits typically chaotic spiral waves [19] with increasing length scales for larger g.

We are especially interested in the effects of noise in the weak coupling region where the coupling alone is not sufficient to achieve a high degree of PS. Typical behavior of the lattice in this region is shown in Fig. 1. Without noise, the pattern is rather random, only displaying phase clusters with small length scales. Accordingly, the phases ϕ_{ij} are almost uniformly distributed and the lattice does not have a macroscopic mean field ($X\sim 0$). With a global noise of intensity $D=0.4$, the pattern becomes rather uniform and the phases are sharply distributed most of time. In the phase space (x,y), the states of the oscillators form a cloud of points around a direction corresponding to the peak value, which rotates during the evolution of the system. As a result of this collective behavior over large spatial scales, a coherent macroscopic mean field emerges. However, when the noise is much stronger, most of the time the pattern becomes fairly random again and the phases are not sharply distributed. The mean field X still has large amplitudes but its temporal behavior is rather noisy, because strong noise has spoiled the phase coherence of the chaotic oscillations.

Now we examine the temporal PS behavior. Figure 2 illustrates the phase properties of two randomly chosen oscillators in the lattices. Without noise, the phases are generally not synchronized, as seen by increasing phase differences

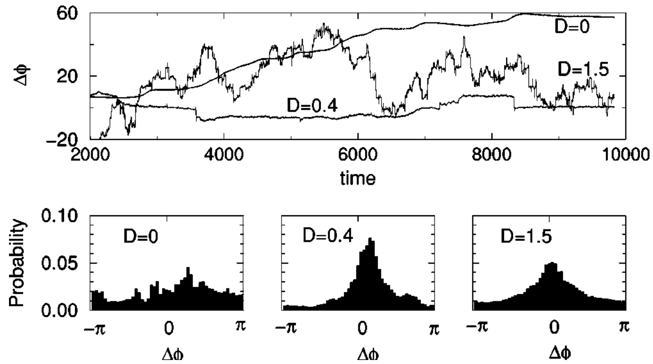

FIG. 2. Temporal phase synchronization behavior between two randomly chosen oscillators in weakly coupled lattices ($g=0.01$). The upper panel displays time series of lifted phase differences (defined on the whole real axis). The lower panel shows the corresponding distributions of the cyclic phase differences modulated into $[-\pi,\pi]$.

and a broad distribution of the cyclic phase differences; however, the weak coupling has already been manifested by the presence of small peaks in the distribution. When adding a global noise with intensity $D=0.4$, fairly long phase synchronization epochs (hundreds of oscillations) are observed, and the distribution has a sharp peak. In the presence of a larger noise ($D=1.5$), the phases become strongly incoherent and the phase differences perform random-walk fluctuations. Nevertheless, there are a lot of phase synchronization epochs. The distribution also has a peak, but it is not as pronounced as that for $D=0.4$. The picture is similar for other pairs of oscillators in the lattices.

Now we have seen clearly that global noise plays a significant role in spatiotemporal PS of weakly coupled chaotic oscillators. When the noise is weak, phase coherence of the oscillation is only spoiled slightly; however, PS can be enhanced drastically and the whole lattice achieves a large scale collective motion. Although larger noise introduces stronger long-range interactions into the lattice, the dynamics becomes rather noisy and phase coherence is seriously spoiled, which reduces the degree of PS. As a result of the competition between reduced phase coherence and enhanced long-range interaction, there should exist an optimal amount of noise which induces the most coherent spatiotemporal motion in the system.

To characterize the coherence of the spatiotemporal motion, we employ the time series of the mean field X. Its amplitude and temporal coherence reflect the spatial and temporal coherence in the lattice, respectively. A combination measure of the spatiotemporal coherence based on X can be defined as [2]

$$\beta=H\frac{\omega_p}{\Delta\omega}, \tag{5}$$

where ω_p is the frequency of the main peak in the spectrum of X, H is the peak height mainly depending on the amplitude of X, and $\Delta\omega$ is the half-width of the peak, reflecting temporal randomness of X. In Fig. 3, β is shown as a function of the noise intensity D for various coupling strengths in

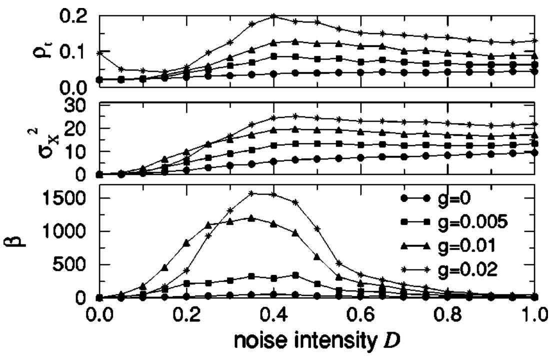

FIG. 3. Coherence resonance features of the spatiotemporal behavior of the lattice. (a) The temporal phase synchronization index ρ_t; (b) the variance σ_X^2 of the mean field X; and (c) the coherence factor β of X.

the weak coupling region, along with the variance σ_X^2 of the mean field X and the temporal phase synchronization index ρ_t.

We find that when the oscillators are not coupled ($g=0$), the global noise alone leads to a slight enhancement of PS; however, the degrees of both spatial and temporal PS are rather low. Fairly strong noise can induce a visible macroscopic mean field, but the coherence β has only small values. This is consistent with previous investigations that the Moran effect alone is not sufficient to synchronize chaotic ecological models [13], while if a weak coupling is introduced, which alone is not strong enough to induce sufficient spatial and temporal PS, the interplay between the global noise and local coupling is able to achieve a high degree of spatiotemporal PS. A combination of rather weak coupling and noise is sufficient to generate large scale and coherent collective motion in the lattice. With the increases of the noise intensity, the coherence β of the spatiotemporal behavior increases, reaches a maximum, and decreases at large noise intensity, displaying the typical features of CR [2,7,8]. It is important to emphasize that the mechanism of spatiotemporal CR as a result of competition between noise-enhanced PS and noise-induced phase incoherence in the present system is different from that in excitable systems where noise-induced excitation plays the major role [2,7,8]. Noise-enhanced PS here is also due to a different mechanism from that in excitable media subjected to spatially uncorrelated noise [20].

The interplay between global noise and local coupling can result in another resonantlike behavior as a function of coupling strength g (Fig. 4). Without noise, the lattice exhibits a complicated wave structure with larger length scales for increasing g (Fig. 5, $D=0$). A weak global noise has the effect of enhancing *phase clustering* in the present system (Fig. 1),

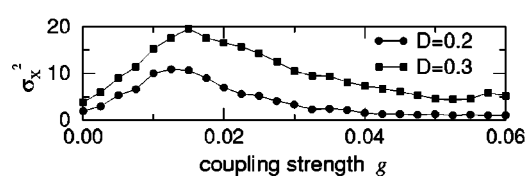

FIG. 4. Resonantlike feature in the lattice shown by σ_X^2 vs g. The coherence β exhibits similar behavior.

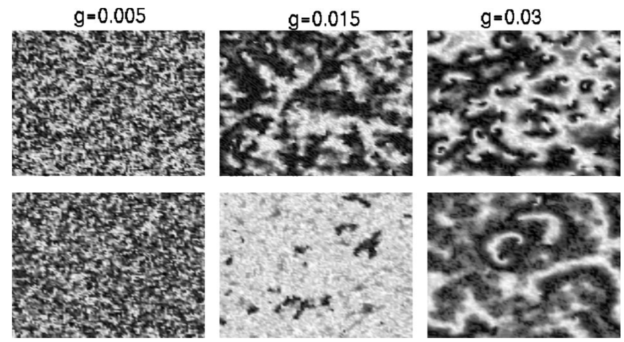

FIG. 5. Snapshots of patterns of x_{ij} at different coupling strength g and noise intensity $D=0$ (upper panel) and $D=0.3$ (lower panel). Gray scales as in Fig. 1.

similar to clustering behavior in globally coupled systems [21]. However, noise-enhanced phase clustering occurs over the largest length scales at a certain intermediate coupling strength g whose value depends on the noise intensity D (Fig. 4). At stronger coupling, the wave structure can partly survive in the presence of noise, which prevents phase clustering over large length scales; but the sizes of the wave structure (Fig. 5, $D=0.3$) are much larger than those in the noise-free case. To characterize the two different scenario of phase clustering behavior without or with noise in the weak coupling regime, we define a length scale dependent order parameter of phase clustering as the fraction of the number of pairs of oscillators with phase difference $|\Delta\phi| \bmod \pi < 0.1\pi$ [22] over a spatial window of $N_1 \times N_1$ oscillators. The mean value $R(N_1)$ obtained by averaging over moving windows and a large number of snapshots of patterns is shown in Fig. 6. R is called the mean phase clustering ratio. Without noise, phase clustering occurs only at small length scales, which increase with g. The global noise induces phase clustering over large length scales at intermediate g values (e.g., $g=0.015$). At larger g (e.g., $g=0.03$), phase clustering occurs at length scales much larger than those in the noise-free case, reflecting the noise-enhanced wave size. However, R becomes smaller again at large N_1. This length scale dependent order parameter is very useful for the analysis of clustering behavior in systems having both local coupling and global driving signals.

We have shown that noise can play a very constructive role is enhancing spatiotemporal PS in a lattice of weakly coupled chaotic oscillators when it acts commonly on all oscillators. We have demonstrated a different mechanism of CR, here in chaotic oscillatory media rather than in excitable systems. An optimal amount of noise generates the most co-

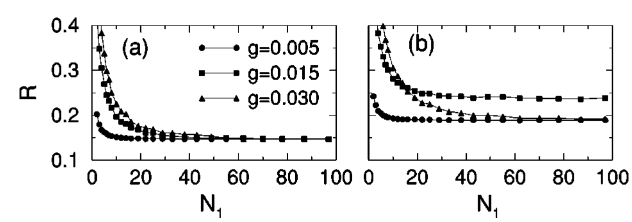

FIG. 6. Phase clustering ratio R as a function of spatial window length N_1 for $D=0$ (a) and $D=0.3$ (b).

herent spatiotemporal motion in the system. As a result of the interplay between global noise and local coupling, the system may establish maximal sensitivity to external random forcing by adapting the coupling strength to the regime of largest length scales of phase clustering. These nontrivial features are of significance especially in ecology: while the Moran effect or migration alone is not sufficient to synchronize populations, a cooperative interplay between them can achieve population synchronization over large geographical scales.

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[22] In globally coupled identical systems, a cluster is defined by vanishing difference among the elements, i.e., $|\mathbf{x}_i - \mathbf{x}_j| = 0$. In clustering of phases in noisy nonidentical systems, we consider a threshold δ and define clusters as $|\Delta\phi| \bmod \pi < \delta$. The properties are similar for different δ values.