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in honor of the 60th birthday of

Fon-Che Liu

Preface

This special issue of the Taiwanese Journal of Mathematics contains some selected papers presented in 1999 International Conference on Nonlinear Analysis, held on October 16–20, 1999 at Academia Sinica, Taipei, Taiwan, in honor of Fon-Che Liu who celebrates his sixtieth birthday in December 1999. In the past three decades, Fon-Che Liu has made many distinguished contributions to different areas of analysis, especially on Lusin properties of functions, surface area formula, multiple Fourier series and convex or nonconvex analysis. Over the years, he has supervised the Ph.D. theses of seven students, who in turn carry on his mathematical ideas. As two-time director of the Institute of Mathematics of Academia Sinica and also as president of the Mathematical Society of the Republic of China, he is in a position to oversee and promote the mathematical research activities of the post-Second-World-War generation of mathematicians in Taiwan.

We congratulate him on his birthday and wish him continued success in his scholarly endeavors.

His resumé, including the list of his Ph.D. students and his publications together with a detailed description of his research work, is included in the following pages.

Fon-Che Liu

Born:

December 19, 1939, Taipei, Taiwan

Education:

B.Sc. (1962), National Taiwan University, Taipei, Taiwan

Ph.D. (1968), Purdue University, Lafayette, Indiana, U. S. A., supervised by C. Goffman

Current Positions:

Director (1996–2000) and Research Fellow (1973–), Institute of Mathematics, Academia Sinica

Professor of Mathematics (1974–), National Taiwan University

Positions:

Assistant Professor (1968–70), Wayne State University

Visiting Assistant Professor (1970–71), Purdue University

Associate Research Fellow (1971–73), Institute of Mathematics, Academia Sinica

Acting Director (1971–72, 1973–76, 1977–78), Institute of Mathematics, Academia Sinica

Visiting Professor (1978, Aug.–Oct.), Purdue University

Editor (1978–84), *Bulletin of the Institute of Mathematics, Academia Sinica*

Director (1984–86), Institute of Mathematics, Academia Sinica

Visiting Professor (Fall 1987), Wayne State University

Director (1988–91), Mathematics Research Promotion Center, National Science Council

President (1990–92), The Mathematical Society of the Republic of China

Ph.D. Students:

Narn-Ruey Hsieh, Inequalities of multiply indexed martingales and applications, National Taiwan University, June 1980.

Shiou-Yu Chang, On some properties of KKK-maps and their applications, National Taiwan University, June 1987.

Chiun-Chuan Chen, Removability of singularities for minimizers, National Taiwan University, June 1991.

Jin-Ron Lee, Asymptotic behavior of solutions of semi-linear elliptic partial differential equations and a study on instability, National Taiwan University, June 1993.

Huo-Yan Chen, Boundary value problem of nonlinear systems of second order ordinary differential equations of divergence form, National Taiwan Normal University, June 1994.

Ting-Hsiung Chen, Multiplicity function and surface area of Sobolev mappings, National Taiwan Normal University, June 1996.

Mao-Sheng Chang, An isolated local minimizer of four-phase partition problems in R^2, National Central University, June 1999.

Publications:

1. Approximation-extension type property of continuous functions of bounded variation, *J. Math. Mech.* 19 (1969), 207-218.

2. Discontinuous mappings and surface area, *Proc. London Math. Soc.* (3) 20 (1970), 237–248 (with C. Goffman).

3. On the localization property of square partial sums for multiple Fourier series, *Studia Math.* 44 (1972), 61–69 (with C. Goffman).

4. On the localization of rectangular partial sums for multiple Fourier series, *Proc. Amer. Math. Soc.* 34 (1972), 90–96.

5. On uniform convergence of Fourier series after change of variable, *Tamkang J. Math.* 3 (1972), 49–52.

6. On a theorem of Whitney, *Bull. Inst. Math. Acad. Sinica* 1 (1973), 63–70.

7. Area formula for Sobolev mappings, *Indiana Univ. Math. J.* 25 (1976), 871–876 (with C. Goffman).

8. Essential multiplicity function for a.e. approximately differentiable mappings, in: *C. K. S. Memor. Vol. Acad. Sinica*, 1976, pp. 69–73.

9. A Lusin type property of Sobolev functions, *Indiana Univ. Math. J.* 26 (1977), 645–651.

10. Approximately differentiable mappings and surface area, in: *Studies and Essays in Commemoration of the Golden Jubilee of Academia Sinica*, 1978, pp. 103–111.

11. Hausdorff measures on topological groups, *Rep. Math. Res. Center* 6 (1978), 249–257 (with N.-R. Hsieh & M.-C. Hu).

12. A note on the von Neumann-Sion minimax principle, *Bull. Inst. Math. Acad. Sinica* 6 (1978), 517–524.

13. Lusin type theorem for functions of bounded variation, *Real Anal. Exchange* 5 (1979/80), 261–266 (with C. Goffman).

14. Derivative measures, *Proc. Amer. Math. Soc.* 78 (1980), 218–220 (with C. Goffman).

15. A remark on the spaces $V^p_{\lambda,\alpha}$, *Proc. Amer. Math. Soc.* 82 (1981), 366–368 (with C. Goffman & D. Waterman).

16. Approximation of nonparametric surfaces of finite area, *Chinese J. Math.* 9 (1981), 25–35 (with N. K. Chen).

17. Representation of measurable functions by multiple series, *Proc. London Math. Soc.* (3) 45 (1982), 131–132 (with C. Goffman & R. Zink).

18. A differentiable function for which localization for double Fourier series fails, *Real Anal. Exchange* 8 (1982/83), 223–227 (with C. Goffman & D. Waterman).

19. Embedding distributive lattices in vector lattices, *Bull. Inst. Math. Acad. Sinica* 11 (1983), 459–462.

20. Remark on a theorem of Ky Fan concerning systems of inequalities, *Bull. Inst. Math. Acad. Sinica* 11 (1983), 639–643 (with A. Granas).

21. Lusin property, Proc. SMS, Univ. Montreal on "Méthodes topologiques en analyse nonlinéaire" (1983).

22. Théorèmes de minimax, *C. R. Acad. Sci. Paris Sér. I Math.* 298 (1984), 329–332 (with A. Granas).

23. A general variational inequality and its applications to coincidence theorems, in: *Proc. Summer Colloq. Math. Res. Center*, Taipei, 1984.

24. Some minimax theorems without convexity, Rep. Univ. Montreal, Oct. 1985 (with A. Granas).

25. Remarque sur une application de l'indicatrice de Banach au changement de variables dans une intégrale, Rep. Univ. Montreal, Nov. 1985 (with M. Frigon).

26. Quelques théorèmes de minimax sans convéxité, *C. R. Acad. Sci. Paris Sér. I Math.* 300 (1985), 347–350 (with A. Granas).

27. Representation of lattices and extension of measures, *Contemp. Math.* 42 (1985), 113–117.

28. Coincidences for set-valued maps and minimax inequalities, *J. Math. Pures Appl.* 65 (1986), 119–148 (with A. Granas).

29. Some minimax theorems without convexity, in: *Nonlinear and Convex Analysis*, Proc. in honor of Ky Fan, B.-L. Lin & S. Simons, eds., 1987, pp. 61–75 (with A. Granas).

30. Comparison of linear combinations of systems of functions and applications, *Chinese J. Math.* 17 (1989), 207–220.

31. Approximation-extension properties of functions, in: *Proc. Asian Math. Conference – Hong Kong*, 1990, pp. 296–303.

32. On a form of KKM principle and supinfsup inequalities of von Neumann type and of Ky Fan type, *J. Math. Anal. Appl.* 155 (1991), 420–436.

33. Théorème de minimax sans topoloqie ni convéxité, *Colloq. Math.* 63 (1992), 141–144 (with A. Granas & J. R. Lee).

34. Measure solutions of systems of inequalities, *Topol. Methods Nonlinear Anal.* 2 (1993), 317–331.

35. Approximate Taylor polynomials and differentiation of functions, *Topol. Methods Nonlinear Anal.* 3 (1994), 189–196 (with W. S. Tai).

36. Maximal mean steepness and Lusin type properties, *Ricerche Mat.* 43 (1994), 365–384 (with W. S. Tai).

37. Equilibrium value and measure of systems of functions, *Topol. Methods Nonlinear Anal.* 5 (1995), 255–259 (with Y. J. Chao).

38. Lusin properties and interpolation of Sobolev spaces, *Topol. Methods Nonlinear Anal.* 9 (1997), 163–177 (with W. S. Tai).

Description of work:

(A) Lusin properties of functions

This study originates with Lusin's classical theorem for measurable functions and Whitney's theorem for approximately differentiable functions (*Pacific J. Math.* 1, 143–159). Starting with [6] the question of introducing approximate differentiability of general order and proving the corresponding Lusin property is finally settled in [35]. In [35], a generalization of Rademacher-Stepanoff-Federer theorem for differentiability of functions to that of differentiability of higher order is answered and thus provides an answer to a question raised by Federer in 3.1.17 of his book on geometric measure theory.

The following strong form of Lusin property is proved in [9] for Sobolev functions: Let $u \in W_p^k(G)$, $1 \leq p \leq +\infty$. Then for any $\epsilon > 0$, there is $v \in C^k(G)$ such that $|\{x \in G : u(x) \neq v(x)\}| < \epsilon$ and $\|u - v\|_{k,p} < \epsilon$. If the smallness of the Sobolev norm of $u - v$ is not required , this result is included in [6] and [35] and is first proved by Calderón and Zygmund (*Studia Math.* 20, 171-225).

The result and method of this work have been used by many authors in various directions, e.g., by Acerbi-Fusco to lowersemicontinuity in calculus of variations (*Arch. Rational Mech. Anal.* 86, 125-145), by Giaquinta-Modica-Soucek to Dirichlet integrals for mappings and variational problems for mappings (*Math. Ann.* 294, 325-386; *Ann. Scuola Norm. Sup. Pisa* 16, 393–485) and by Kinderlehrer-Pedregal to equilibrium configurations of crystalline (*Arch. Rational Mech. Anal.* 115, 329–365). This work has been further pursued by J. H. Michael and W. Ziemer and appears in the book *Weakly Differentiable Functions* (Springer-Verlag, 1989) by Ziemer. A special case of this work is also presented in the book *Measure Theory and Fine Properties of Functions* (CRC Press, 1992) by L. C. Evans and R. F. Gariepy. Recently, B. Bojarski and his students have started a systematic study of geometric properties of Sobolev mappings which is also connected with this work (see the survey article by Bojarski in *Function Spaces, Differential Operators and Non-linear Analysis* (Paivarinta, ed., Pitman, 1989).

Closely related to this form of Lusin property are the Lusin properties studied in [37] and [38] which together give quite complete picture of Lusin properties for Sobolev functions and BV functions. In [38] a form of Lusin property which is closely related to interpolations of function spaces is proved for local Sobolev functions in terms of non-increasing rearrangement of functions.

A remarkable Lusin property is established in [1] for continuous functions of bounded variation of a real variable which resembles Lusin property for

Sobolev functions but with C^k-functions replaced by functions whose graph has continuously turning tangents. Although it is believed that this property also holds in higher-dimensional case, but a lack of necessary tools prevents from proving it.

(B) Surface area and area formula

In [2] a satisfactory theory of surface area is first provided for mappings which may not be continuous. The mappings considered are from an oriented cube I in R^m into R^n, $n \geq m$, and are $(m-1)$-continuous mappings; an area functional A is defined for such mappings extending the Lebesgue area functional for continuous mappings. A mapping $T : I \to R^n$ with coordinate functions $f_1, \ldots, f_n$ is called a Sobolev mapping if each $f_i \in W^1_{p_i}$ such that $\sum 1/p_{i_k} \leq 1$ for each $1 \leq i_1 < \cdots < i_m \leq n$. If T is a Sobolev mapping, then its Jacobian $J(T; x)$ is defined for almost all $x \in I$ and is integrable. A natural question is whether the area formula

$$A(T) = \int_I J(T; x) dx$$

holds for $(m-1)$-continuous Sobolev mappings T. It is shown in [2] that area formula always holds when $m = 2$. This includes a classical result of C. B. Morrey for continuous Sobolev mappings from R^2 into R^3 (*Amer. J. Math.* 55, 683-707; 56, 275-293). When $m > 2$ and each $p_i > m - 1$, T is $(m-1)$-continuous. In this case, Goffman and Ziemer show that area formula holds (*Ann. Math.* 92, 482-488). A strictly larger class of $(m-1)$-continuous Sobolev mappings is considered in [7] and area formula is proved for this class. Mappings in this class are called regular Sobolev mappings. Roughly speaking, a mapping is regular if it maps many boundaries of oriented cubes into sets of small m-measure. The method in [7] is different from other methods in that a delicate combination of methods of topological degree with measure theoretic considerations is employed. This combination makes possible a transparent proof of area formula even for C^1 mappings.

In [8] (see also [38]) is constructed a multiplicity function $m(T; y), y \in R^n$ for a.e. approximately differentiable mappings T and is shown the following form of area formula:

$$\int J(T; x) dx = \int m(T; y) dH^m(y),$$

where the Jacobian is defined with approximate partial derivatives of coordinate functions of T and H^m is the m-Hausdorff measure in R^n. This renders possible even a simple proof of area formula for Lipschitz mappings.

(C) Localization of multiple Fourier series

Functions considered are defined on n-torus. In contrast to the case $n = 1$, localization for Fourier series is much more involved when $n > 1$. Indeed, for $n = 2$, there is an everywhere differentiable function for which square sum localization does not hold [18]. Historical developments of multiple Fourier series lead to the consideration of localization for Fourier series of functions in W_p^1. It is shown in [3] that square sum localization holds if $p \geq n - 1$ and for each $1 \leq p < n - 1$ there is a function in W_p^1 for which square sum localization fails. It is also shown in [3] that when $p = n - 1$ there is a function in W_p^1 for which rectangular sum localization does not hold, while the fact that rectangular sum localization holds for W_p^1 if $p > n - 1$ is established in [4]. Thus the question of localization for multiple Fourier series is completely settled with respect to Sobolev space W_p^1. In [4], the fact that Dini-Lipschitz theorem for uniform convergence by rectangular sum holds for general n is also obtained.

(D) Additive set function

S. Bochner and R. S. Phillips have shown that given a finitely additive set function on an algebra L of sets one can represent L by sets in another space so that the given additive set function becomes σ-additive on the new algebra of sets (*Ann. of Math.* 42, 316–324). A method of embedding a distributive lattice L with smallest and largest elements in the Banach lattice B of all valuations of bounded variation on L is introduced in [19] and [27] with the purpose of representing L by subsets of the extreme set of the positive face of the unit ball of the second dual of B. It then follows from this representation of L that all valuations of bounded variation on L can be extended to σ-additive signed measures. In particular, any algebra L of sets can be represented by subsets of a compact Hausdorff space so that all finitely additive probabilities on L can be extended to σ-additive measures on the new algebra of sets. This approach is more natural and gives stronger results than that of Bochner and Phillips.

(E) Convex or nonconvex analysis

Some questions in convex or nonconvex analysis are considered by using topological or non-topological methods. The topological methods are best represented in [12], [28] and [32], while the non-topological methods are best illustrated in [29], [30], [34] and [37]. It is worthwhile to point out that a useful concept of measure solutions of systems of inequalities is introduced in [34] so that Lagrange duality is put in a clearer perspective and can be treated more systematically and generally.