Specification and Control of Mid-Spatial Frequency Wavefront Errors in Optical Systems

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Abstract: This paper is an introduction to the specification and tolerancing of Mid-spatial frequency (MSF) ripple or waviness. We begin with an introduction to the definition of ripple, spatial frequencies, and MSF ripple as a class of surface error (as opposed to figure or form, roughness, and surface imperfections or defects.) We then cover the derivation of spatial frequency bands of interest, specifications methods and notations, and relative amplitudes for typical manufacturing processes.

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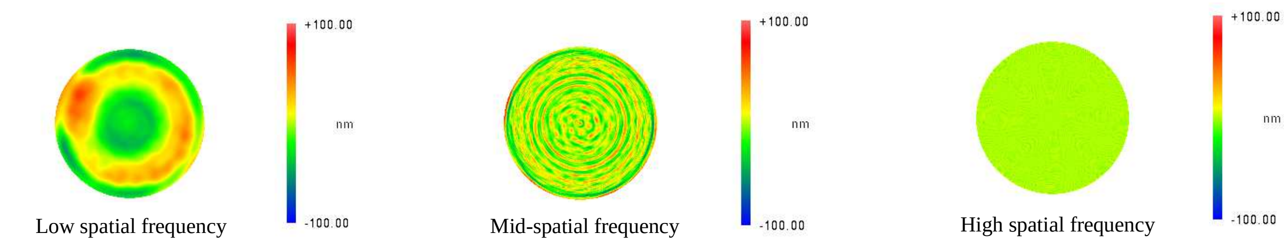
1. Surface ripple overview

Most models of optical systems presume ideal surfaces. The realities of surface fabrication and metrology methods, however, naturally lead to non-ideal optical surfaces. One general way to consider such departures from the ideal, or surface errors, is by the undulations across the surface.

Traditional optical fabrication techniques have part-to-tool geometry that can generate long scale length surface undulations, referred to as form errors, and also surface fracture mechanics that lead short spatial scale length undulations, referred to as microroughness and scatter. Modern sub-aperture and deterministic optical fabrication techniques are more prone to residual periodic surface undulations that are commonly referred to as ripple, or mid-spatial frequency (MSF) errors. [1] Figure 1-1 depicts one surface prepared with a deterministic polishing method analyzed for three different spatial frequency regions (arbitrarily defined by the authors to highlight tool path errors). In this extreme case, the path taken by the deterministic sub-aperture tool is clearly visible in the mid-spatial frequency re



Low spatial frequency region | Mid-spatial frequency region | High spatial frequency region

Figure 1-1 Surface prepared with a deterministic polishing method analyzed in three different spatial frequency regions.

Sub-aperture processing methods have geometries that do not naturally blur out errors in this frequency regime since the work function is smaller than the part. This creates regular periodic errors. Examples of surfaces that require such processing are aspheres and large optics. In many cases, the metrology can also play a part because there may be the need to make multiple measurements to cover a surface and the spatial frequencies of interest. As a rule, the surface form errors of traditionally finished optics and deterministically finished optics are indistinguishable.

Machine tool manufacturers are continually improving tool path algorithms to reduce these errors while tool users make adjustments to reduce residual artifacts, for example by adjusting slurry mechanics and chemistry at the tool/surface interface where possible [2]. Additionally, a final randomized smoothing step may be necessary if a surface does not meet specifications directly after deterministic finishing [3].

2. Mid-spatial frequencies from an application perspective

The term mid-spatial frequency is ambiguous in the literature. For imaging applications low frequency errors, or form errors, retain the peaks and nodes of the point-spread function, while high frequency errors scatter light out of the system, decreasing transmission. Between these two limits, the mid-spatial frequencies erode the peaks and nodes of the point spread function in a broader sense [4]. This broadening and changing of the point spread function can have a deleterious effect on image quality.

The "Fresnel length" of the form errors is very long when compared to the system length [5]. In that regime, aberration theory is applicable and a Zernike decomposition of surface errors is appropriate. On the other hand, the Fresnel lengths of the high spatial frequency surface roughness is very short with respect to the system length, and therefore the scattered light in this part of the spatial frequency spectrum can be treated as loss. Here, we are defining the Fresnel length associated with a given spatial frequency as the distance at which the spatial scale-length (a) of the perturbation has a Fresnel number of 1; that is:

$$L_f = a^2/\lambda \qquad (1)$$

The mid-spatial frequencies can neither be treated as loss, nor can a simple aberration theory apply [6]. We will refer to the MSF regime as the region in the spatial frequency spectrum between the highest frequency of a 37 term Zernike fit of the aberrations, and the spatial frequencies where the Fresnel lengths of the ripples are less than $1/10^{th}$ of the optical path distance from a given surface to the image plane. This factor of 10 in propagation length is conservative, and is based on experience.

3. Considerations for Specifying MSF errors

There are three key questions that should be answered when considering sensitivity and treatment of MSF errors:

A) Is the application and design sensitive to the errors?
B) What errors are expected for a given manufacturing process (considering fabrication and metrology)?
C) How do the errors of a given spatial frequency domain effect performance?

For most systems, these types of errors should not pose any significant difficulties. In the cases where mid-spatial frequencies pose issues, bounds on the allowable errors in those spatial frequency regimes should be provided in the optical prints. There are a number of different means of dealing with bounding such errors by specification in practice. Such methods can include providing spatial frequency bounds on measurements with peak-to-valley specification or bounding with methods that specifically define the spectral content such as power spectral density (PSD) [7]. Note that if MSF errors are not expected to pose an issue for a system, good practice would be to omit such specifications from the prints and avoid needless costs.

Tying the effects of form errors to the performance is generally assessed with aberration theory and sequential raytracing. High frequency assessment has traditionally been achieved by scattering theory and non-sequential raytracing analysis. The mid-frequency regime requires a combination of the two methodologies for a rigorous assessment [8,9].

4. Example: MSF ripple specification derivation for an aspheric using bandpass rms error

We now give an example to show how to derive a mid-spatial frequency ripple specification from a Modulation Transfer Function (MTF) requirement in a design that has sensitivity to mid-spatial frequency errors.

In this example, image data will be evaluated using a threshold to establish the presence or absence of features and blemishes. This drives a requirement for nearly perfect optical performance at high spatial frequencies. The requirements for this lens are summarized in Table 4-1, with highlighting of the driving specification for mid-spatial frequency performance. The derivation of specifications' for the surfaces takes two parts: spatial frequency definition and the bounding of the amplitude of the errors.

For the spatial frequency definitions, one surface will be treated in detail – the aspheric surface. This surface (the first

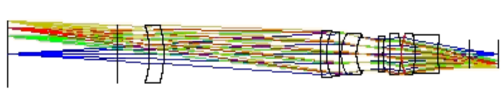


Figure 4-1. Schematic of our example lens. The aspheric is the first surface on the fourth element.

Table 4-1 Requirements for lens design example.

Specification	Requirement	Comment
Wavelength	640nm – 680nm	Diode source
f/number	f/3 or slower at detector	Drives object space f/# to f/8 or so.
Design Residual MTF	49.8% MTF at 120 cycles/mm	Close to diffraction limited perf.
As-built system MTF	45.0% MTF at 120 cycles/mm	Drives mid-spatial freq. requirements
CMOS Camera	1300 x 1000 pixels, 12um centers	10 bit camera readout, 500Hz

surface on the fourth element) is near the stop and hence we will assume the illuminated region corresponds to the lens aperture. Assuming low-frequency errors are comprised of a standard 37 term Zernike polynomial expansion, the low frequency limit of the mid-spatial frequency errors can be estimated to be 5 cycles across the aperture. Since this surface has a clear aperture of 32mm, the spatial period is 6.4mm (or spatial frequency of 0.156mm^{-1}). To calculate the high-spatial frequency cutoff, we determine the scale length whose Frenel distance is one tenth that of the imaging distance from the surface to the image plane. The propagation distance for this surface is approximately equal to the exit pupil distance, or 146mm. Solving equation 1 for the spatial scale length whose L_f is 0.1 x the pupil distance yields:

$$a = (L_f \times \lambda)^{1/2} = (0.1*146\text{mm} \times \lambda)^{1/2} = (0.0096\text{mm}^2)^{1/2} = 0.1\text{mm} \qquad (2)$$

Hence, with this treatment a MSF error tolerance can be placed on the aspheric surface in the spatial period band from 0.1mm to 6.4mm (spatial frequencies from 0.156mm^{-1} to 10mm^{-1}.)

For the amplitude of errors, we consider the design residual MTF at the resolution limit requirement (120 cycles/mm) is 49.8%. In our general tolerance analysis, we have allocated 1% MTF loss at the frequency of interest to mid-spatial frequency errors. The remainder is consumed by tolerance errors (e.g. TIR, form error) which can be treated using conventional tolerancing techniques. The following expression applies for the aggregate MTF degradation from all of the surfaces due to mid-spatial frequency errors:

$$\text{MTF} = \text{MTF}_{dr} \times \exp\left[-\left(\frac{2\pi}{\lambda}\right)^2 \sum_{i=1}^{N} \left(n_i - n_i'\right)^2 \sigma_i^2\right] \qquad (3)$$

where MTF_{dr} is the nominal design MTF, λ is the wavelength, N is the number of surfaces, n_i and n_i' are the index before and after the surface, and σ_i is the rms mid-spatial frequency error of the surface.

For typical 50th wave rms precision optics manufactured using conventional polishing, the surface error in this spatial frequency band is consistently less than 6nm rms. Mid-size optical surfaces polished using deterministic finishing techniques, however, show from 6nm to 18nm of residual MSF error in this band. Typically the spatial frequency region of interest due to subaperture tool path geometry is 0.5 – 4mm. We will assume a tolerance of 6nm for spherical surfaces and 12nm for the asphere for the σ of the respective surfaces. With these predicted errors, the MTF degrades from 49.8% to 48.7% due solely to mid-spatial frequency errors, which is satisfactory. We will assume that the spherical surfaces will meet the 6nm MSF ripple specification without any MSF specification. The asphere drawing will require a mid-spatial frequency specification, however. An example of this in ISO 10110 notation is given in Figure 5-1.

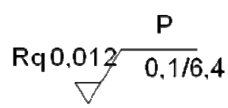


Figure 5-1 Surface texture callout for a 12nm RMS surface error for spatial periods from 0.1mm to 6.4mm

5. References

[1] There are many excellent papers on the subject. See for example J. E. DeGroote, A. E. Marino, K. E. Spencer, and S. D. Jacobs, "Power Spectral Density plots inside MRF spots made with a polishing abrasive-free MR fluid," in *Optifab*, (SPIE, 2005), pp. 134-138.

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