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SPECIFICS OF DAMPING EVALUATION IN ROTATING MACHINES

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ABSTRACT

Focused on rotor lateral modes, the paper discusses specifics of damping evaluation in rotating machines. Rotors force the fluid trapped in small rotor-to-stator radial clearances to rotate in circumferential fashion. The fluid in circumferential motion generates a tangential force acting in feedback on the rotor. This force direction is opposite to that of damping force. The "effective damping" is, therefore, reduced or even nullified by the fluid interaction effects. The classical measures of damping in mechanical structures, such as Logarithmic Decrement and Amplification Factor, which are used to evaluate machine susceptibility to instability based on documented vibration data, have to be adjusted to include the fluid interaction effects. These measures now represent the measures of Quadrature Dynamic Stiffness (*QDS*). It is shown that they contain the expression defined as Stability Margin (or Nondimensional Stability Margin), derived from the stability condition of the rotor system. A simple model of isotropic rotor lateral vibrations is used to obtain the *QDS* measures.

1. INTRODUCTION

There exist several commonly used measures of damping in mechanical structures. The practical methods of damping evaluation are divided into two fundamental groups: structure damping evaluation based on free vibrations and based on forced vibrations. For linear models of single degree of freedom (the first mode) mechanical systems; all damping measures are comparable (some at first approximation); they differ only by constant coefficients. When considering multimode structures, the damping evaluation methods are usually limited to those based on forced vibrations, as the excitation and identification of higher mode free vibrations becomes impractical. Using the modal approach for the structures with widely spaced natural frequencies, the evaluation of modal damping for each separate mode is reduced to the same methodology as for a single-degree-of-freedom oscillator.

The direct application of all damping evaluation methods mentioned above fails in the case of rotating systems. The evidences of these failures are reflected in many publications (e.g., Vance, 1988, and Lee, 1993) reporting the presence of "negative damping" in the system responses. By nature, damping, defined as a coefficient of the damping force which resists the motion and dissipates energy, is always positive. The reason for inappropriate "negative damping" description lies in inadequate modeling of structures which contain a source of energy. The rotating structures belong to this category. Rotating machines represent a large class of the rotating structures.

In this paper the measures of damping and practical damping evaluation methods for rotating machine rotor lateral modes are discussed. It is shown that the damping itself does not become negative, but, due to rotational motion of the rotor ("source of energy"), a new force is generated. This force acts in opposite direction to the damping, and its magnitude can exceed that of stabilizing damping force. The result, therefore, appears as "negative damping." Since in rotating systems damping is not the only component contributing to the Quadrature Dynamic Stiffness (*QDS*), it is mandatory to use *QDS* in dynamic considerations instead of just "damping." One of the forces generated due to rotor rotation is the fluid-induced tangential force which occurs in small rotor-to-stator radial clearances (such as in fluid-lubricated bearings, in seals, or in blade-tip clearances). This tangential force is the source of rotor instability.

The first section of this paper summarizes the classical measures of damping for a single-mode linear oscillator. The second section discusses the evaluation of Quadrature Dynamic Stiffness and Stability Margin in rotating structures. The results are applicable in the field practice.

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2. CLASSICAL MEASURES OF DAMPING IN MECHANICAL STRUCTURES

The classical damping measures in mechanical structures are based on a mathematical model for a single-degree-of-freedom linear oscillator. The summary of these classical measures of damping is presented in Table 1.

3. MEASURES OF "EFFECTIVE DAMPING" OR "QUADRATURE DYNAMIC STIFFNESS" IN ROTATING STRUCTURES BASED ON ROTOR LATERAL MODES.

In order to relate to principal physical phenomena, the rotating system damping measures will be introduced based on the simple mathematical model; these measures can, however, be directly applied into the machinery vibration test data.

For an adequate description of the lowest lateral modes of rotating machine rotors, two degrees of freedom have to be considered: they correspond to the two rotor orthogonal lateral deflections, orthogonal to the rotor axis of rotation. The coupling forces, namely the gyroscopic and the tangential forces, are due to rotor rotation. The simplest mathematical model of the isotropic rotor first lateral mode is, therefore, as follows:

$$M\ddot{x}+(D+D_s)\dot{x}+I\Omega\dot{y}+Kx+D\lambda\Omega y=F\cos(\omega t+\delta)$$
$$M\ddot{y}+(D+D_s)\dot{y}-I\Omega\dot{x}+Ky-D\lambda\Omega x=F\sin(\omega t+\delta) \quad (1)$$

where $x(t), y(t)$ are rotor lateral displacements in two orthogonal directions (x = "horizontal," y = "vertical"), M, D_s, K are rotor modal mass, damping, and stiffness respectively, D is rotor-surrounding-fluid damping, λ is fluid circumferential average velocity ratio (Muszynska, 1988, Muszynska et al, 1996a), Ω is rotative speed. The product $D\lambda\Omega$ represents the fluid-related tangential force coefficient often referred to as "cross-coupled stiffness." The parameter I, proportional to rotor polar moment of inertia, represents the gyroscopic effect. F,ω,δ are magnitude, frequency, and angular orientation of the externally applied, rotating exciting force. In the general case, its frequency ω is different from rotational frequency Ω and independent from it.

Four eigenvalues of the rotor system (1) can analytically be calculated and are as follows:

$$s_{i\nu}=-\frac{D+D_s}{2M}-\frac{(-1)^i}{\sqrt{2}}\sqrt{-E+\sqrt{E^2+L^2}}+j\left(\frac{I\Omega}{2M}-\frac{(-1)^\nu}{\sqrt{2}}\sqrt{E+\sqrt{E^2+L^2}}\right)\equiv(\mathrm{Re})_i+j\omega_{n\nu}, i,\nu=1,2, j=\sqrt{-1} \quad (2)$$

where

$$E=\frac{K}{M}-\left(\frac{D+D_s}{2M}\right)^2+\left(\frac{I\Omega}{2M}\right)^2, \quad L=\frac{D\Omega}{M}\left(\lambda-\frac{I(1+D_s/D)}{2M}\right)$$

Among these four eigenvalues, pairs of them share the same real parts or imaginary parts. The eigenvalues which predict rotor instability are the ones in which the real part (denoted $(\mathrm{Re})_1$) can become positive. The imaginary parts represent the system natural frequencies and are denoted $\omega_{n\nu}, \nu=1,2$. The rotor stability condition to assure nonpositive real parts is as follows:

$$(SM)\equiv\frac{I\Omega}{2M}+\sqrt{\left(\frac{I\Omega}{2M}\right)^2+\frac{K}{M}}-\frac{\lambda\Omega}{1+D_s/D}\geq 0 \quad (3)$$

which can be transformed into following inequality:

$$\frac{K}{M}-\left(\frac{\Omega}{1+D_s/D}\right)^2\left(\lambda^2-\frac{I\lambda(1+D_s/D)}{M}\right)\geq 0 \quad (4)$$

The inequality (4) can further be solved: For $\lambda^2\geq I\lambda(1+D_s/D)/M$ the stability condition is:

$$\sqrt{\frac{K}{M}}-\frac{\Omega}{1+D_s/D}\sqrt{\lambda^2-\frac{I\lambda(1+D_s/D)}{M}}\geq 0\,, \quad (5)$$

and for $\lambda^2<I\lambda(1+D_s/D)/M$ stability is always assured.

Following Muszynska, 1990, the (SM) in Eq. (3) is referred to as Stability Margin, which will be discussed in section 3.6. The inequalities (3) to (5) are often solved for rotative speed Ω, which results in the "instability threshold."

3.1 Logarithmic Decrement (Log Dec)

The free vibration solution of the isotropic rotor model (1) is $y=\pm jx$, thus the classical definition of the Logarithmic Decrement (Log Dec), $\vartheta=\ln(A_n/A_{n+1})$ (e.g. Lee, 1993), for both rotor lateral modes remains the same (A_n, A_{n+1} are amplitudes of successive free response vibration cycles). For the rotor horizontal response as a particular solution of Eqs. (1):

$$x(t)=Ce^{(\mathrm{Re})_1 t}\cos(\omega_{n1}t+\varepsilon) \quad (6)$$

(where C, ε are constants of integration) the Log Dec is as follows ($T=2\pi/\omega_{n1}$ is the vibration period):

$$\vartheta=\ln\frac{x(0)}{x(T)}=-(\mathrm{Re})_1\frac{2\pi}{\omega_{n1}}=\left(\frac{D+D_s}{2M}-\frac{1}{\sqrt{2}}\sqrt{-E+\sqrt{E^2+L^2}}\right)\frac{2\pi}{\omega_{n1}} \quad (7)$$

Only one eigenvalue, that with the largest real part magnitude and with forward-mode natural frequency, was chosen here. Similarly, Log Dec for other particular solutions including all eigenvalues can be presented. A simple transformation of Eq. (7) using Eq. (2) provides another look of the Log Dec:

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TABLE 1. Classical Measures of Damping for a One-Mode Oscillator $M\ddot{x} + D\dot{x} + Kx = F\cos\omega t$

Parameter / Definition	ζ	ϑ	η	φ	Q	(SL)
Damping Factor $\zeta = \frac{D}{2\sqrt{KM}}$	[zeta]	$\zeta = \frac{\vartheta}{2\pi}$	$\zeta = \frac{\eta}{2}$	$\zeta = \sin\varphi$	$\zeta = \frac{1}{2Q}$	$\zeta = \frac{1}{(SL)\omega_{res}}$
Logarithmic Decrement $\vartheta = \ln\frac{A_n}{A_{n+1}}$	$\vartheta = 2\pi\zeta$	[theta]	$\vartheta = \pi\eta$	$\vartheta = 2\pi\sin\varphi$	$\vartheta = \frac{\pi}{Q}$	$\vartheta = \frac{2\pi}{(SL)\omega_{res}}$
Loss Factor $\eta = \frac{V_n - V_{n+1}}{2\pi V_n}$	$\eta = 2\zeta$	$\eta = \frac{\vartheta}{\pi}$	[eta]	$\eta = 2\sin\varphi$	$\eta = \frac{1}{Q}$	$\eta = \frac{2}{(SL)\omega_{res}}$
Eigenvalue Angle $\varphi = \arctan\frac{-\mathrm{Re}(s)}{\lvert\mathrm{Im}(s)\rvert}$	$\varphi = \arcsin\zeta$	$\varphi = \arcsin\left(\frac{\vartheta}{2\pi}\right)$	$\varphi = \arcsin\left(\frac{\eta}{2}\right)$	[phi]	$\varphi = \arcsin\frac{1}{2Q}$	$\varphi = \arcsin\left(\frac{2}{(SL)\omega_{res}}\right)$
Amplification Factor $Q = \frac{A_{res}}{A_0}$ for $F = const$; $Q = \frac{A_{res}}{A_{(\omega=\infty)}}$ for $F = mr\omega^2$; $Q = \frac{\omega_{res}}{\omega_2 - \omega_1}$ (Half-Power Band-width Method)	$Q = \frac{1}{2\zeta}$	$Q = \frac{\pi}{\vartheta}$	$Q = \frac{1}{\eta}$	$Q = \frac{1}{2\sin\varphi}$	Q	$Q = \frac{(SL)\omega_{res}}{2}$
Phase Slope at Resonance $(SL) = \frac{\Delta\alpha}{\Delta\omega}$	$SL = \frac{1}{\zeta\omega_{res}}$	$SL = \frac{2\pi}{\vartheta\omega_{res}}$	$SL = \frac{2}{\eta\omega_{res}}$	$SL = \frac{2}{\omega_{res}\sin\varphi}$	$SL = \frac{2Q}{\omega_{res}}$	[SLope]

<u>Legend</u>: M,D,K = modal mass, damping, and stiffness respectively, $x(t)$ = displacement, t = time, A = response amplitude, ω = forcing frequency, $A_0, A_{res}, A_{(\omega=\infty)}$ = amplitudes at $\omega = 0$, at resonance and at high frequency (but lower than second resonance), V = potential energy, n = cycle waveform number, F = magnitude of exciting force, m,r = mass and radius of unbalance, ω_{res} = resonance frequency ω_1, ω_2 = "half-power bandwidth" frequencies at response amplitudes equal to $A_{res}/\sqrt{z}$, Δ = increment, α = response phase.

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$$\vartheta = \frac{\pi(D+D_s)^2\left\{\frac{K}{M}-\left(\frac{\Omega}{1+D_s/D}\right)^2\left(\lambda^2-\frac{I\lambda(1+D_s/D)}{M}\right)\right\}}{M^2\omega_{n1}(-(\mathrm{Re})_2)\left(\frac{(D+D_s)^2}{2M^2}+E+\sqrt{E^2+L^2}\right)} \quad (8)$$

In this format the Log Dec (8) contains in the numerator the explicit stability condition (4). If it is satisfied, the Log Dec is positive and the vibration amplitudes decay (or remain constant). If it is not satisfied, the Log Dec is negative, vibration amplitudes increase in time, and the system is unstable.

The Log Dec as a measure of damping is often used in practical impact testing of mechanical structures. If an impact testing used on a *rotating* rotor results in vibrations with increasing amplitudes, thus a negative Log Dec, it does not mean that the "damping is negative." It means that the system *instability threshold was exceeded* due to the existence and high value of the tangential force represented in the model (1) by the ratio λ. As a matter of fact, damping practically has no effect on the instability threshold (usually $D >> D_s$, thus $1+ +D_s/D \approx 1$). The gyroscopic term alone does not affect the stability, but it modifies the system natural frequencies and causes the stability threshold to increase (up to infinity, if for $\lambda > 0$ there is $I > \lambda M/(1+D_s/D)$).

3.2 Loss Factor

The Loss Factor η, as a measure of the system potential energy lost during vibration (see Table 1), is mainly used to evaluate structure elastic member material damping or structural damping in joints, as discussed by Nashif et al, 1985. With approximation, there is $\eta \approx \vartheta/\pi$, where ϑ is the Log Dec. The Loss Factor for the rotor models is not used.

3.3 Eigenvalue Angle

A useful measure of "effective damping," developed from free vibrations and based directly on system eigenvalues, is the angle φ, defined as the arctangent of the ratio of the real and imaginary parts of a specific eigenvalue s, plotted in orthogonal coordinates $(\mathrm{Re}(s), \mathrm{Im}(s))$ (Decay/Growth Rate, Natural Frequency) called the "Root Locus Plane" (Fig. 1):

$$\varphi = \arctan\frac{-\mathrm{Re}(s)}{|\mathrm{Im}(s)|} \quad (9)$$

For the rotor model eigenvalue s_{11} (Eq. (2)) the Eigenvalue Angle is as follows:

$$\varphi = \arctan\left(\frac{-(\mathrm{Re})_1}{\omega_{n1}}\right) = \arctan\left(\frac{\vartheta}{2\pi}\right) \quad (10)$$

where ϑ is the Log Dec, Eq. (7) or (8).

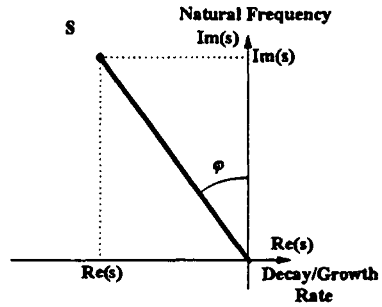

Fig. 1. Root Locus Plane and Eigenvalue Angle φ.

The Eigenvalue Angle is useful for analytical evaluation of a separate mode "effective damping" (Quadrature Dynamic Stiffness) or the Stability Margin, as discussed by Muszynska (1994, 1996b). The Root Locus Technique serves very efficiently in parametric analysis to assess the effect of parameter value changes on the Stability Margin in the system.

3.4 Nonsynchronous Amplification Factor for Direct Resonance at Forward Perturbation

This method is based on the system forced response, excited by an external periodic force with frequency ω independent from the rotative speed Ω. In a particular case, the periodic excitation may come from rotor unbalance, and then $\omega = \Omega$. In the first case the excitation is referred to as "nonsynchronous," and in the particular case as "synchronous" to the rotative speed. In the general case the Amplification Factor should also be referred to as "nonsynchronous" to distinguish it from the synchronous case. For $\omega > 0$ in the model (1) the rotating exciting force has the same direction as the rotor rotation. This type of excitation is referred to as Forward Perturbation. Forward and backward sweep frequency perturbation technique is customarily used for identification of modal parameters of rotors with fluid interaction (Muszynska, 1990, 1995).

The forced response of the rotor model, Eqs. (1), is as follows:

$$x = A\cos(\omega t+\alpha), \qquad y = A\sin(\omega t+\alpha) \quad (11)$$

where A, α are the response amplitude and phase respectively,

$$A = \frac{F}{\sqrt{\left(K+I\Omega\omega-M\omega^2\right)^2+\left[(D+D_s)\omega-D\lambda\Omega\right]^2}},$$

$$\alpha = \delta - \arctan\frac{(D+D_s)\omega-D\lambda\Omega}{K+I\Omega\omega-M\omega^2} \quad (12)$$

The following expressions in Eqs. (12) are defined as components of Complex Dynamic Stiffness (Muszynska, et al, 1990):

$$K+I\Omega\omega-M\omega^2 \equiv \text{Direct Dynamic Stiffness } (DDS) \quad (13)$$

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$(D+D_s)\omega - D\lambda\Omega \equiv$ Quadrature Dynamic Stiffness (QDS) (14)

Note that QDS contains not only damping, as in stationary mechanical structures, but also the tangential force component.

With an approximation, the peak response amplitude A_{res} occurs when $DDS = 0$, i.e., $K + I\Omega\omega - M\omega^2 = 0$, thus

$$\omega = \omega_{res} \equiv \frac{I\Omega}{2M} + \sqrt{\left(\frac{I\Omega}{2M}\right)^2 + \frac{K}{M}} \quad (15)$$

and from Eqs. (12) and (15):

$$A_{res} = \frac{F}{(D+D_s)\omega_{res} - D\lambda\Omega}, \quad \alpha_{res} = \delta - 90^\circ \quad (16)$$

The resonance frequency (15) corresponds to the forward mode natural frequency ω_{n1} given by Eq. (2), and calculated at the instability threshold, when the inequality (3) converts into equality. The real peak amplitude occurs closer to the actual value $\omega = \omega_{n1}$. The amplitude given by Eq. (16) is, therefore, an approximation of the actual peak amplitude. The smaller damping D is in the system, the better amplitude approximation (16) results. With increasing damping D, the actual resonance frequency decreases, and the amplitude peak becomes lower. For a high overcritical damping D the resonance frequency approaches the value

$$\omega_{res_q} = \lambda\Omega / (1 + D_s/D) \approx \lambda\Omega \quad (17)$$

which corresponds to the zero of Quadrature Dynamic Stiffness (14), and for which the peak amplitude (12) is

$$A_{res_q} = \frac{F}{K + I\Omega^2\lambda - M\lambda^2\Omega^2}, \quad \alpha_{resq} = \delta - 0^\circ = \delta \quad (18)$$

The resonance expressed by Eq. (16) is referred to as "Direct" or "Mechanical" resonance, while the one expressed by Eq. (18) is referred to as "Quadrature" or "Fluid-Related" resonance (Muszynska, 1995). The discussion on Amplification Factors for the Quadrature resonance, as well as for the case of Backward Perturbation, will be given in section 3.7.

There are two major definitions of the Amplification Factor, Q: the first one is based on the ratio of resonance peak to nonresonance amplitudes. The second definition is based on the Half-Power Bandwidth Method.* While for the linear model of a single-degree-of-freedom oscillator all definitions converge to the same results (Table 1), their practical applications using vibrational data carry different levels of inaccuracy. The response resonance peak amplitudes of real structures are usually affected by nonlinearities, thus the practical evaluation of the amplitude ratio results in a smaller value. In the amplitude ratio definition, the important information contained in the response phase α is not included. That is why the first definition of the Amplification Factor, since it is less accurate, may be used only in very rough estimates. The expressions of Q for a simplified rotor model (1) $(I = 0, D_s = 0)$ are presented in Table 2.

The definition of Q based on the Half-Power Bandwidth Method,

$$Q = \frac{\omega_{res}}{\omega_2 - \omega_1} \quad (19)$$

is practically insensitive to nonlinearities, and includes both amplitude and phase information. The method can be used for any separate, widely spaced mode of a structure. In Eq. (19) ω_1, ω_2 are frequencies at which the amplitude is $1/\sqrt{2}$ times the peak amplitude A_{res}, and the response phases differ by $\pm 45^\circ$ from the phase α at resonance (16). The Half-Power Bandwidth frequencies ω_1, ω_2 can be calculated from the second Eq. (12):

$$\delta - \arctan\frac{(D+D_s)\omega_i - D\lambda\Omega}{K + I\Omega\omega_i - M\omega_i^2} = \delta - 90^\circ + (-1)^i 45^\circ, \; i = 1,2 \quad (20)$$

From here:

$$\omega_i = (-1)^i \frac{D+D_s}{2M} + \frac{I\Omega}{2M} + \sqrt{\left[(-1)^i \frac{D+D_s}{2M} + \frac{I\Omega}{2M}\right]^2 + \frac{K}{M} - (-1)^i \frac{D\lambda\Omega}{M}}, \; i = 1,2 \quad (21)$$

The difference $\omega_2 - \omega_1$ can be approximated as follows:

$$\omega_2 - \omega_1 \approx \frac{D+D_s}{\sqrt{KM + (I\Omega/2)^2}} \left[\omega_{res} - \lambda\Omega/(1 + D_s/D)\right] \quad (22)$$

The Nonsynchronous Amplification Factor (19) results:

$$Q = \frac{\omega_{res}\sqrt{KM + (I\Omega/2)^2}}{(D+D_s)(SM)} \quad (23)$$

The denominator of Q in Eq. (23) contains the stability condition (3), thus the closer parameters are to the stability

* The name "Half-Power Bandwidth" originated from the voltage response vector of an electrical circuit. This vector amplitude is usually plotted in logarithmic scale. The response amplitudes at which the phases differ by ±45° from the phase at resonance have the value which is a $1/\sqrt{2}$ fraction of the resonance amplitude. Since power is proportional to square of the voltage, these two amplitude points also represent the half-power points of voltage. The voltage amplitude ratio $1/\sqrt{2}$ corresponds to a 3 dB or approximately 30% peak amplitude reduction. Thus these voltage points are called 70% or -3 dB amplitude points, and they define the frequency bandwidth used to calculate the Amplification Factor.

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TABLE 2. Measures of Isotropic Rotor Quadrature Dynamic Stiffness With No Gyroscopic Effect ($I = 0$), and No Rotor Damping ($D_s = 0$), Based on Forced Vibration

Amplification Factor, Q	Nonsynchronous	Direct Resonance Forward	$F = const$ (Amplitude ratio)	$Q = \frac{\sqrt{K^2+D^2\lambda^2\Omega^2}}{D\left(\sqrt{K/M}-\lambda\Omega\right)}$	$= \frac{\sqrt{4\zeta^2\lambda^2\Omega^2+K/M}}{2\zeta\left(\sqrt{K/M}-\lambda\Omega\right)}$	$= \frac{\sqrt{4\zeta^2\lambda^2\Omega^2+K/M}}{2\zeta(NSM)\sqrt{K/M}}$
			Amplitude ratio for $F = mr\omega^2$ and Half-Power Bandwidth Method for both $F=const$ and $F=mr\omega^2$	$Q = \frac{K}{D\left(\sqrt{K/M}-\lambda\Omega\right)}$	$= \frac{1}{2\zeta\left(1-\frac{\lambda\Omega}{\sqrt{K/M}}\right)}$	$= \frac{1}{2\zeta(NSM)}$
		Quadrature Resonance	$F = const$ (Amplitude ratio)	$Q = \frac{\sqrt{K^2+D^2\lambda^2\Omega^2}}{K-M\lambda^2\Omega^2}$	$= \frac{\sqrt{\left(4\zeta^2\lambda^2\Omega^2+K/M\right)K/M}}{K/M-\lambda^2\Omega^2}$	$= \frac{\sqrt{4\zeta^2\lambda^2\Omega^2+K/M}}{(NSM)\left(1+\lambda\Omega/\sqrt{K/M}\right)}$
			$F = mr\omega^2$ (Amplitude ratio)	$Q = \frac{\lambda^2\Omega^2}{K/M-\lambda^2\Omega^2}$	$= \frac{1}{\frac{K/M}{\lambda^2\Omega^2}-1}$	$= \frac{1}{(NSM)\left(\sqrt{K/M}+\lambda\Omega\right)}$
		Direct Resonance Backward	$F = const$ (Amplitude ratio)	$Q = \frac{\sqrt{K^2+D^2\lambda^2\Omega^2}}{D\left(\sqrt{K/M}+\lambda\Omega\right)}$	$= \frac{\sqrt{4\zeta^2\lambda^2\Omega^2+K/M}}{2\zeta\left(\sqrt{K/M}+\lambda\Omega\right)}$	
			Amplitude ratio for $F = mr\omega^2$ and Half-Power Bandwidth Method for both $F=const$ and $F=mr\omega^2$	$Q = \frac{K}{D\left(\sqrt{K/M}+\lambda\Omega\right)}$	$= \frac{1}{2\zeta\left(1+\lambda\Omega/\sqrt{K/M}\right)}$	
	Synchronous ($F = mr\Omega^2$)			$Q_{syn} = \frac{\sqrt{KM}}{D\lvert 1-\lambda\rvert}$		
Phase Slope, SL	Nonsynchronous	Direct Resonance Forward		$SL = \frac{2\sqrt{KM}}{D\left(\sqrt{K/M}-\lambda\Omega\right)}$	$= \frac{1}{\zeta\left(\sqrt{K/M}-\lambda\Omega\right)}$	$= \frac{1}{\zeta(NSM)\sqrt{K/M}}$
		Quadrature Resonance		$SL = \frac{D}{K-M\lambda^2\Omega^2}$	$= \frac{2\zeta\sqrt{K/M}}{K/M-\lambda^2\Omega^2}$	$= \frac{2\zeta}{(NSM)\left(1+\lambda\Omega/\sqrt{K/M}\right)}$
		Direct Resonance Backward		$SL = \frac{2\sqrt{KM}}{D\left(\sqrt{K/M}+\lambda\Omega\right)}$	$= \frac{1}{\zeta\left(\sqrt{K/M}+\lambda\Omega\right)}$	
	Synchronous			$SL = \frac{2M}{D\lvert 1-\lambda\rvert}$	$= \frac{1}{\zeta\lvert 1-\lambda\rvert\sqrt{K/M}}$	
Nondimensional Stability Margin, *NSM*				$(NSM) = 1-\frac{\lambda\Omega}{\sqrt{K/M}}$	Damping Factor	$\zeta = \frac{D}{2\sqrt{KM}}$

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threshold, the larger an Amplification Factor results. At the instability threshold the Amplification Factor becomes infinite. Unlike to "static" mechanical structures, the value of the Amplification Factor is not inversely proportional to damping alone, but to its product with the Stability Margin (3).

3.5 Response Phase Slope at Direct Resonance

The derivative of the rotor response phase (12), which represents the phase slope (*SL*) at direct resonance, ω_{res} (Eq. (15)) is as follows:

$$(SL)=\left.\frac{d\alpha}{d\omega}\right|_{\omega=\omega_{res}}=-\frac{\sqrt{I^2\Omega^2+4KM}}{(D+D_s)\omega_{res}-D\lambda\Omega} \quad (24)$$

The Nonsynchronous Amplification Factor, Eq. (23), can be expressed by using the response phase slope as follows (the minus sign in (24) is dropped):

$$Q=(SL)\frac{\omega_{res}}{2} \quad (25)$$

where the slope, (*SL*), can practically be obtained from the machine response phase data as $SL=\Delta\alpha/\Delta\omega$ (Δ = an increment around the resonance frequency ω_{res}).

3.6 Nondimensional Stability Margin

In sections 3.1 to 3.5 various classical measures of damping in mechanical systems were adapted to the simplest rotor model (1). For rotating systems the classical "measures of damping" must be interpreted as "measures of Quadrature Dynamic Stiffness." While the idea of Direct and Quadrature Dynamic Stiffness applies mainly to the systems with periodic external excitation, it is convenient to use it as a descriptor of rotating system dynamic characteristics.

The rotor stability condition (3) can be interpreted in terms of the Stability Margin. The Stability Margin (*SM*) in Eq. (3) is defined as the difference between zeros (roots) of Direct and Quadrature Stiffnesses as functions of frequency (Muszynska, 1990):

$$(SM)=\omega_{res}-\lambda\Omega/(1+D_s/D) \quad (26)$$

where ω_{res} is the solution (root) (15) of the equation "Direct Dynamic Stiffness (13) equals to zero," and $\lambda\Omega/(1+D_s/D)$ is the root of Quadrature Dynamic Stiffness (14). The Stability Margin (26) has the frequency dimension. In order to be able to compare various systems, it is reasonable to introduce the Nondimensional Stability Margin (*NSM*) as follows:

$$(NSM)=\frac{(SM)}{\omega_{res}}\left[\frac{\dfrac{\lambda\Omega}{1+D_s/D}-\dfrac{I\Omega}{2M}+\sqrt{\left(\dfrac{I\Omega}{2M}\right)^2+\dfrac{K}{M}}}{\sqrt{\dfrac{K}{M}}+\dfrac{\Omega}{1+D_s/D}\sqrt{\lambda^2-\dfrac{I\lambda(1+D_s/D)}{M}}}\right]\equiv$$

$$\equiv\frac{\sqrt{\dfrac{K}{M}}-\dfrac{\Omega}{1+D_s/D}\sqrt{\lambda^2-\dfrac{I\lambda(1+D_s/D)}{M}}}{\omega_{res}} \quad (27)$$

The nondimensionalizing multiplier of (*SM*) within the brackets in Eq. (27) has a positive value, and exists for $\lambda^2>I\lambda/M$ (if $\lambda^2\le I\lambda/M$, then the system is always stable). This multiplier was introduced to simplify the final expression in Eq. (27): the numerator contains the stability condition (5), the denominator contains the Direct Resonance frequency. For stable systems the values of (*NSM*) may vary between one and zero. A negative (*NSM*) means that the system is unstable.

The Nondimensional Stability Margin (*NSM*) better describes and quantifies the system stability situation than the Stability Margin (*SM*). The comparison of the Nondimensional Stability Margin with other Quadrature Dynamic Stiffness measures for the rotor model without gyroscopic effect and rotor external damping $(D_s=0,\ I=0)$ is given in Table 2.

3.7 Amplification Factor for Direct Resonance at Backward Perturbation

The exciting force in the model (1) was assumed rotating in the direction of rotor rotation $(\omega>0)$. If ω has a negative value, backward excitation (perturbation) results (rotating exciting force in the direction opposite to rotation). The response amplitude A_b and phase α_b will now differ from Eqs. (12) by the sign of ω:

$$A_b=\frac{F}{\sqrt{\left(K-I\Omega\omega-M\omega^2\right)^2+\left[(D+D_s)\omega+D\lambda\Omega\right]^2}},$$

$$\alpha_b=\delta+\arctan\frac{(D+D_s)\omega+D\lambda\Omega}{K-I\Omega\omega-M\omega^2} \quad (28)$$

The backward perturbation excites the backward mode associated with the natural frequency ω_{n2} (see Eq. (2)) and results in Direct Resonance when $\omega=\omega_{n2}\approx\omega_{res_b}$. There is no Quadrature Resonance for backward excitation, as the Quadrature Dynamic Stiffness remains always negative, and does not provide any zero value (root).

Calculated by Half-Power Bandwidth Method, the Amplification Factor for the Direct Resonance at backward perturbation is as follows:

$$Q=\frac{\sqrt{KM+(I\Omega/2)}}{D+D_s+D\lambda\Omega/\omega_{res_b}} \quad (29)$$

where from Eqs. (28)

$$\omega_{res_b}=-\frac{I\Omega}{2M}+\sqrt{\left(\frac{I\Omega}{2M}\right)^2+\frac{K}{M}}$$

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The comparison of Amplification Factors for the rotor model without gyroscopic effect and $D_s = 0$ are given in Table 2.

3.8 Phase Slopes for Quadrature Resonance and for Direct Resonance at Backward Perturbation

Similarly to Eq. (24) the phase slope at Quadrature Resonance can be calculated. Instead of $\omega = \omega_{res}$, the substitution now is $\omega = \omega_{res_q} \equiv \lambda\Omega/(1+D/D_s)$:

$$(SL) = \left.\frac{d\alpha}{d\omega}\right|_{\omega = \lambda\Omega/(1+D/D_s)} = -\frac{D+D_s}{K-\lambda\Omega^2(M\lambda - I)} \quad (30)$$

In comparison to Eq. (24) the phase slope for the Quadrature Resonance is less steep.

The phase slope for the Direct Resonance at backward perturbation is as follows:

$$(SL) = \frac{\sqrt{I^2\Omega^2 + 4KM}}{(D+D_s)\omega_{res_b} + D\lambda\Omega} \quad (31)$$

The slope has much lower value than the slope for the forward perturbation, Eq. (24).

5. FINAL REMARKS

The measures of Quadrature Dynamic Stiffness developed from classical measures of damping and adapted to simple isotropic rotor lateral mode model were discussed in this paper. The emphasis was on the Quadrature Dynamic Stiffness, which for stationary structures and simple oscillators equals the damping terms, but for the rotor models must be complemented by the tangential force components. This tangential component most often appears out of phase to damping, so that the entire Quadrature Dynamic Stiffness may become nullified. It is shown that the classical measures of damping adapted for rotors contain an expression similar to the stability condition. A new related expression was introduced, namely, Nondimensional Stability Margin (*NSM*) as a derivative of the stability condition. The Quadrature Dynamic Stiffness measures can efficiently be expressed as functions of the *NSM* (Table 2). These measures apply directly to the rotating machine vibrational data in order to assess the machine susceptibility to instability. More detailed considerations on the topic can be found in the paper by Muszynska (1996b).

In rotors a distinction should be applied for the cases of forward and backward excitation, as well as for cases of Direct (low damping) and Quadrature (high damping) Resonances.

The developed measures apply to the isotropic one-mode lateral model. They may also be applied to higher lateral modes of the rotor, provided they are widely spaced. These Quadrature Dynamic Stiffness measures fail in case of closely spaced modes. One common case in rotor modeling is the anisotropically supported rotor. Its two orthogonal lateral modes have different, but close, frequencies. In synchronous response Bode and polar plots the rotor support anisotropy introduces "split resonances." Such double peaks do not qualify for applying classical measures such as the Amplification Factors, as the peak values are not only functions of the Quadrature Dynamic Stiffnesses and Stability Margins, but also functions of "split" frequency differences. In these cases new, different measures have to be developed.

REFERENCES

Lee, C. W., 1993, "Vibration Analysis of Rotors," Kluwer Academic Publishers.

Muszynska, A., 1988, "Improvements in Lightly Loaded Rotor/Bearing and Rotor/Seal Models," *Trans. of the ASME, Journal of Vibration, Acoustics, Stress and Reliability in Design*, Vol. 110, No. 2, pp. 129-136.

Muszynska, A., Bently, D. E., 1990, "Frequency Swept Rotating Input Perturbation Techniques and Identification of the Fluid Force Models in Rotor/Bearing/Seal Systems and Fluid Handling Machines," *Journal of Sound and Vibration*, Vol. 143, No. 1, pp. 103-124.

Muszynska, A., Hatch, C. T., Bently, D. E., 1994, "Application of Multi-Mode Models and Root Locus Technique for Stability Analysis of Rotor Systems," ISROMAC-5, Hawaii.

Muszynska, A., "Modal Testing of Rotors With Fluid Interaction," 1995, *International Journal of Rotating Machinery*, Vol. 1, No. 2, pp. 83-116.

Muszynska, A., Bently, D. E., 1996a, "Fluid-Induced Instabilities of Rotors: Whirl and Whip – Summary of Results," *Orbit, BNC*, Vol. 17, No. 1.

Muszynska, A., 1996b, "Evaluation of Damping in Nonrotating Structures and Quadrature Dynamic Stiffness in Rotating Structures," BRDRC Report No. 2.

Nashif, A. D., Jones, D. I. G., Henderson, J. P., 1985, "Vibration Damping," John Wiley & Sons.

Vance, J. M., 1988, "Rotordynamics of Turbomachinery," John Wiley & Sons.

NOTATION

D_s, K, M	Rotor modal damping, stiffness, and mass respectively
D	Rotor surrounding fluid damping
$D\lambda\Omega$	Tangential force coefficient ("cross coupled stiffness")
F, ω, δ	Exciting force magnitude, frequency, and angular orientation
m, r	Mass and radius of unbalance respectively
NSM, SM	Nondimensional Stability Margin and Stability Margin respectively
Q, Q_{syn}	Amplification Factors for nonsynchronous and synchronous excitation
SL	Response phase slope (versus excitation frequency) at resonance
ζ	Damping Factor
λ	Fluid circumferential average velocity ratio
Ω	Rotative speed
•	d/dt

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