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Spectra of Discrete Symplectic Eigenvalue Problems with Separated Boundary Conditions

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Abstract—In this paper we consider a discrete symplectic eigenvalue problem with separated boundary conditions and obtain formulas for the number of eigenvalues on a given interval of the variation of the spectral parameter. In addition, we compare the spectra of two symplectic eigenvalue problems with different separated boundary conditions.

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1. Introduction. Consider the eigenvalue problem for a discrete symplectic system

$$y_{i+1}(\lambda) = W_i(\lambda)y_i, \;\; i = 0, \dots, N, \;\; y_i(\lambda) = [x_i(\lambda)u_i(\lambda)]^\top \in \mathbb{R}^{2n} \tag{1}$$

with separated boundary conditions

$$R_0^* x_0(\lambda) - R_0 u_0(\lambda) = 0, \;\; R_{N+1}^* x_{N+1}(\lambda) + R_{N+1} u_{N+1}(\lambda) = 0, \tag{2}$$

where $R_0^* R_0^\top = R_0 R_0^{*\top}$, $R_{N+1}^* R_{N+1}^\top = R_{N+1} R_{N+1}^{*\top}$, and $\operatorname{rang}[R_0^* - R_0] = \operatorname{rang}[R_{N+1}^* R_{N+1}] = n$. In particular, for $R_1 = I$ and $R_2 = 0$ these conditions are the Dirichlet boundary conditions

$$x_0(\lambda) = x_{N+1}(\lambda) = 0. \tag{3}$$

The matrix of system (1) satisfies the conditions

$$W_i(\lambda) = \begin{bmatrix} I & 0 \\ -\lambda \mathcal{W}_i & I \end{bmatrix} S_i, \;\; S_i^\top J S_i = J, \quad J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \quad \mathcal{W}_i = \mathcal{W}_i^\top, \quad \mathcal{W}_i \geq 0, \tag{4}$$

where I and 0 are, respectively, the identical and null $n \times n$ matrices. It is symplectic for any $i = 0, \dots, N$ and $\lambda \in \mathbb{R}$. A special case of systems (1), (4) are Hamiltonian systems of difference equations. These systems are discrete analogs of the following canonical system of differential equations:

$$J^\top Y'(t) = \begin{bmatrix} \lambda \mathcal{W}(t) - \mathcal{C}(t) & \mathcal{A}(t)^\top \\ \mathcal{A}(t) & \mathcal{B}(t) \end{bmatrix} Y(t), \quad \mathcal{W}(t) = \mathcal{W}(t)^\top, \quad \mathcal{W}(t) \geq 0, \tag{5}$$

where $\mathcal{B}(t) = \mathcal{B}(t)^\top$ and $\mathcal{C}(t) = \mathcal{C}(t)^\top$. Note that the spectral theory for system (5) with general self-adjoint boundary conditions is studied rather thoroughly in [1] (Chap. VI) and in [2, 3]. The corresponding theory for discrete symplectic eigenvalue problems is being actively developed recently [4–6]. The main tool of these research is the *discrete oscillation theory* for problem (1) [7, 8]. This theory includes (as a special case) the classical oscillation theory for scalar and vector Sturm–Liouville

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