Progress In Electromagnetics Research, PIER 75, 69–84, 2007

SPECTRAL ANALYSIS OF THE FIBONACCI-CLASS ONE-DIMENSIONAL QUASI-PERIODIC STRUCTURES

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Abstract—In this paper, spectral properties of the Fibonacci-class one-dimensional quasi-periodic structures, $FC_J(n)$, as an important optical structure are investigated. Analytical relations for description of the spectral properties of $FC_J(n)$ are used. Fast Fourier Transform (FFT) for investigation of the spectral properties of these structures is proposed. FFT spectrum of the Fibonacci-class one-dimensional quasi-periodic structures contains peaks that are equivalent to photonic bandgaps or multiband reflection filter. Based on the proposed relations and FFT simulation results, the optical bandgap and other properties of these structures are studied. In this paper, the effects of the optical and geometrical parameters on optical properties of the Fibonacci quasi-periodic structures are considered. Our proposed relations show that the spectral contents of the Fibonacci-class one-dimensional quasi-periodic structures have two main terms including the low and high frequency parts. Our results illustrate that the high frequency term depends up on the class order, n, and the width of the layer B, d_b, while the low frequency term depends on the width of the layer A, d_a. According to the proposed method, the spectral contents of $FC_J(n)$ includes multi narrowband peaks multiplied by a quasi periodic envelope function. The number of multi narrow bands within a periods of the envelope function can be controlled by varying d_b and n and also the number of period of envelope function can be manipulated by d_a. Results obtained from our proposed analytical relations and FFT based simulation results are close together.

1. INTRODUCTION

Advancements in high speed signal processing and data communications make narrow and multi band optical filters attractive to optical device engineers and designers. Multilayer structures are the most suitable alternatives for such devices. However, for designing and fabricating narrowband optical filters, one needs a stack containing a great number of layers in which the neighboring layers index differences are exceedingly small. In practice, such structures are hard to fabricate. Nevertheless, inherent properties of the quasi-periodic structures make them alternative potentials, to be used in designing narrowband optical filters.

Quasi-periodic structure was discovered in 1984 [1]. Since then, a considerable amount of research was carried out, considering the optical response of such structures, in particular, based on Fibonacci multilayer [2–14]. Some of these works were focused on studying the localization of light waves within Fibonacci quasi-periodic multilayer structures to create photonic bandgaps similar to those existing in periodic structures [2–5]. It began in 1987, when Kohmoto et al. introduced the first system based on optical Fibonacci multilayers capable of localizing photons [2]. Then, Sibilia et al. have utilized these quasi-periodic structures and demonstrated that the transmission spectrum, produced by such structures, are dense in wavelengths, displaying a self-similar pattern [3]. Next, Gellermann et al. experimentally showed the existence of bandgaps in the spectrum of these structures [4]. Macia used transfer matrix method (TMM) to study Fibonacci dielectric multilayers, numerically [6].

Omnidirectional bandgaps, using Fibonacci quasi-periodic structures, were also reported by Lusk et al. [7]. Peng et al. have observed resonant transmission of light in a symmetric Fibonacci multilayers, characterized by many perfect transmission peaks, useful for narrowband multiwavelength optical filtering applications [8]. They have also studied the light transport through the band-edge states, where they have observed the mode beating and strongly suppressed group velocity. Other research teams have realized the second and third harmonic generations, utilizing such multilayer structures [9] and [10]. Furthermore, complete intensity-dependent switching of pulses were achieved [9] and [10]. Fibonacci sequences were used either as reflectors or transmitters in designing microcavity structures [6, 11, 12]. Useful mathematical expressions for such devices were derived by others [13–15].

Although the Fourier transform has already been used for spectral analysis of some periodic waveguide and grating structures [16, 17], however, so far, we have not seen any published reports on the dependence of the spectral properties of these multilayer's on the physical parameters. In this paper, using the fast Fourier transform (FFT) method, we have studied the spectral properties of the Fibonacci quasi-periodic structures of various dimensions and various refractive index profiles.

Our studies are focused on Fibonacci class $FC_3(n)$. Organization of the paper is as follows. In Section 2, we present the mathematical model describing the quasi-periodic structures, $FC_3(n)$. Then, Fourier transform of the refractive index profile is taken and the spectral properties are extracted. Next, we present the simulation results in Section 3. In Section 4 we describe a few design tools for multiband filters. Finally, we conclude the paper with Section 5.

2. MATHEMATICAL MODEL

According to the mathematical recursive relations the Fibonacci-class quasi periodic structures can be generated using the following two main substitutions that is done for two basic elements A and B [14].

$$B \to B^{n-1}A \quad \text{and} \quad A \to B^{n-1}AB \tag{1}$$

where, n is a positive integer. Based on the proposed substitutions the following relations show the recursive relations for a general case.

$$\begin{cases} FC_1(n) = S_1 = B \\ FC_2(n) = S_2 = B^{n-1}A \\ FC_3(n) = S_3 = (B^{n-1}A)^n B \\ \qquad\vdots \\ FC_j(n) = S_j = S_{j-1}^n S_{j-2} \end{cases} \tag{2}$$

For the purpose of illustration, an explicit form of the proposed general relation for the Fibonacci class $FC_J(n)$ (for $J = 3$) is

$$S_3 = S_2^n S_1 = (B^{n-1}A)^n B = \overbrace{(\underbrace{BBB\dots B}_{n-1}A)\dots(\underbrace{BBB\dots B}_{n-1}A)}^{n}B \quad (3)$$

Now, for realization of the Fibonacci class in the optical domain, we assume that A and B are two optical dielectric layers with constant indices of refractions n_a and n_b and thicknesses d_a and d_b, respectively. So, the index of refraction profile for a one dimensional $FC_3(n)$ multilayer structure can be written as,

$$n(z) = \begin{cases} n_b & 0 < z \leq (n-1)d_b \\ n_a & (n-1)d_b < z \leq (n-1)d_b + d_a \\ n_b & (n-1)d_b + d_a < z \leq 2[(n-1)d_b + d_a] \\ \vdots & \quad\quad\vdots \\ n_b & (n-1)(n-1)d_b + (n-1)d_a < z \leq n(n-1)d_b + (n-1)d_a \\ n_a & n(n-1)d_b + (n-1)d_a < z \leq n[(n-1)d_b + d_a] \\ n_b & n[(n-1)d_b + d_a] < z \leq n[(n-1)d_b + d_a] + d_b \end{cases} \quad (4)$$

where z is the coordinate along which the light propagates.

Now, we take the Fourier transform of the refractive index profile,

$$N(k) = \frac{1}{2\pi}\int_0^L n(z)\exp(jkz)dz, \quad (5)$$

where, k is the wave number in the medium and L is the total structure length. Inserting Eq. (4) into Eq. (5) leads us to

$$\begin{aligned} N(k) = & \int_0^{(n-1)d_b} n_b \exp(jkz)dz + \int_{(n-1)d_b}^{(n-1)d_b+d_a} n_a \exp(jkz)dz \\ & + \int_{(n-1)d_a}^{2(n-1)d_b+d_a} n_b \exp(jkz)dz + \cdots \\ & + \int_{(n-1)(n-1)d_b+(n-1)d_a}^{n(n-1)d_b+(n-1)d_a} n_b \exp(jkz)dz \\ & + \int_{n(n-1)d_b+(n-1)d_a}^{n[(n-1)d_b+d_a]} n_a \exp(jkz)dz \\ & + \int_{n[(n-1)d_b+d_a]}^{n[(n-1)d_b+d_a]+d_b} n_b \exp(jkz)dz \end{aligned} \quad (6)$$

Performing the integrations and applying some algebraic manipulations, we can rewrite $N(k)$ as,

$$N(k) = \frac{-n_b}{jk} + \frac{1}{jk} N'(k), \tag{7}$$

where $N'(k)$ is given by

$$N'(k) = (n_b - n_a) \sum_{i=1}^{n} \{\exp(jk[i(n-1)d_b]) \times (\exp(jk[(i-1)d_a]) - \exp(jk[id_a]))\} + n_b \{(\exp(jk[n(n-1)+1]d_b)) \times (\exp(jknd_a))\} \tag{8}$$

Notice that N' is also a function of the geometric parameters of the structure, n, d_a and d_b. By varying these parameters, we will study the effect of each parameter on the spectral properties of the structure.

3. SIMULATION RESULTS

In this section we first consider a 1-D periodic structure illustrated in Fig. 1 and use FFT to obtain its spectral behavior, for the sake of comparison. Next, we simulate the spectral behavior of a quasi-periodic structure based on Eq. (4). Then, by varying the parameters n, d_a and d_b, we study the effects of these parameters on both the FFT spectrum and the optical property of the structure.

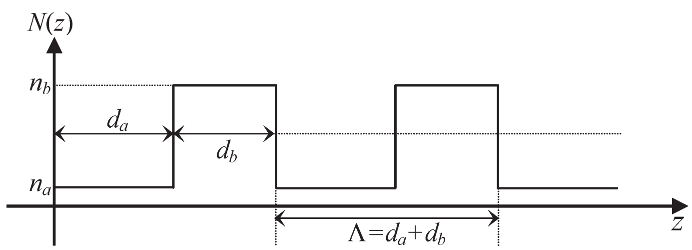


Figure 1. Schematic diagram of a 1-D periodic structure.

3.1. FFT Spectrum of a 1-D Periodic Multilayer Structure

For the structure illustrated in Fig. 1, we have assumed a 1-D periodic multilayer structure with periodicity of $\Lambda = d_a + d_b$ composed of two different materials with thicknesses $d_a = 1\,\mu$m and $d_b = 1\,\mu$m and the corresponding refractive indices of $n_a = 1.5$ and $n_b = 2.5$, respectively. When the Bragg condition for a photon of wavevector $\boldsymbol{k}_{BR}$ is satisfied; i.e., $k_{BR} = \pi/\Lambda$ a band-gap forms which is equivalent to a dip in the transmission spectrum of the structure about the angular frequency $\omega_{BR} = c \cdot k_{BR}$, known as the Bragg frequency, where c is the speed of light in free space. We note the existence of the dominant Fourier component at $k_{BR} = \pi/\Lambda$.

Here, the real-space and the reciprocal space variables are all assumed to be the appropriate optical quantities. The optical response of the 1-D periodic structure using FFT is illustrated in Fig. 2. The peaks in the spectrum occur at $k_{BR} = m\pi/\Lambda$, where $m = 1, 3, 5, \ldots,$ is a positive and odd integer. The largest amplitude in the spectrum occurs at m =1. The FFT spectrum of a 1-D periodic structure gives a qualitative determination of the spectral transmission/reflection characteristics of electromagnetic wave propagating through the structure. It is evident that $\omega_{BR} = c \cdot k_{BR}$ is the frequency at which the dip/peak in the transmission/reflection occurs. We anticipated a similar behavior from the FFT spectra of the quasi-periodic structures, which can be utilized as optical band pass filters.

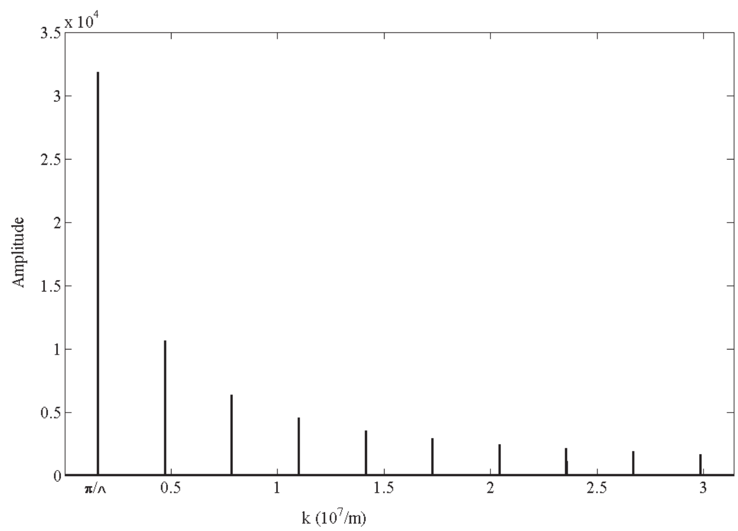


Figure 2. FFT spectrum of a typical periodic structure with $d_a = d_b = 1\,\mu$m, $n_a = 1.5$, $n_b = 2.5$, and Λ =2 μm.

3.2. FFT Spectrum of the $FC_3(n)$ Multilayer Structure

Now we start with a quasi-periodic structure whose refractive index profile is described by Eq. (4). The parameters of the structure are $n = 50$, $d_a = 0.75\,\mu\text{m}$, $d_b = 1\,\mu\text{m}$, $n_a = 1.5$, and $n_b = 2.5$. The length of the structure is $L \approx 2.48\,\text{mm}$. The FFT spectrum of this structure is depicted in Fig. 3. A sample spacing of 10^{-9} was used in our FFT calculations, which is adequate for such dimensions in a quasi-periodic structure. As one can see from the figure, here, there are sets of peaks appearing at equi-distance intervals and are equivalent to sets of corresponding Bragg frequencies. Generalizing the Bragg conditions for the Quasi-periodic structure, for electromagnetic waves propagating through a 1-D quasi-periodic structure, qualitatively, there are sets of dips/peaks in the spectral transmission/reflection characteristics corresponding to Bragg frequencies, resulting in sets of narrow band pass filters. The amplitude of each set decreases with increasing k. Each peak at a given k, in the spectrum is the same as a photonic bandgap at that k. Some properties of the Fourier spectrum are presented in the next section.

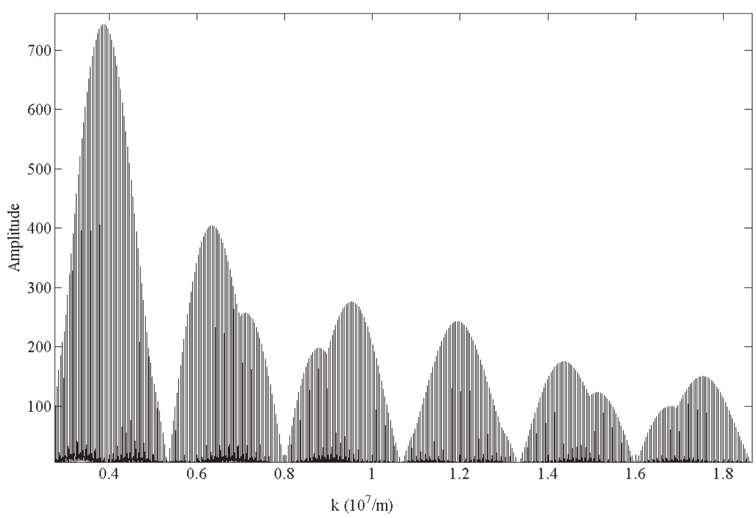
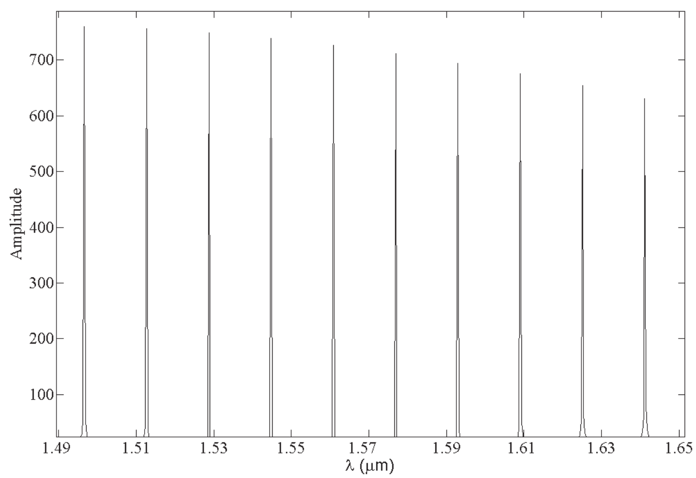


Figure 3. FFT spectrum of a $FC_3(50)$ with $d_a = 0.75\,\mu\text{m}$, $d_b = 1\,\mu\text{m}$, $n_a = 1.5$, $n_b = 2.5$ and $L \approx 2.48\,\text{mm}$.

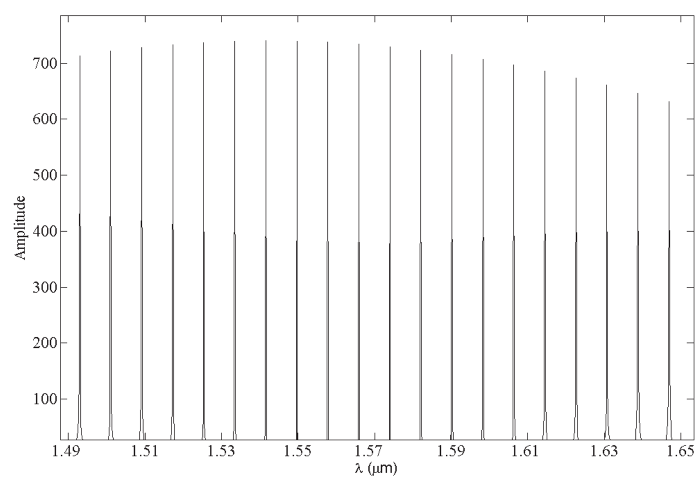
FFT spectrum of the $FC_J(n)$ has sets of peaks located at the Bragg frequencies. Locations of these peaks depends on the geometrical dimensions of the layers and also the Fibonacci's order — i.e., d_b, d_a, and n. Such effects are presented next.

3.3. Effects of Varying d_b

As we have seen earlier from Eq. (8), $N'(k)$ depends on d_b, as $\exp\left(jk\left[i(n-1)d_b\right]\right)$. Since n is, normally, a large number and any change in d_b multiplies this large number. Hence, this term is rapidly oscillating and could be called the high frequency factor. Each sinusoidal term appears as a peak, in the FFT domain, while the peak density is proportional to the size of the argument. Hence, for a given range in the k domain, the peak density increases with increasing d_b.



(a)



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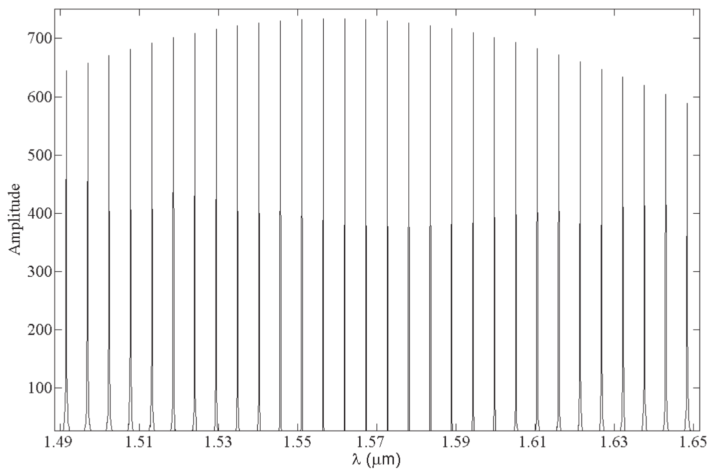


(c)

Figure 4. FFT spectrum of the index profile of three different $FC_3(50)$ with various d_b, depicted in a 160 nm wide window centered around 1550 nm, (a) $d_a = 0.75\,\mu$m, $d_b = 1\,\mu$m, $n_a = 1.5$, $n_b = 2.5$ and $L \approx 2.49$ mm, (b) $d_a = 0.75\,\mu$m, $d_b = 2\,\mu$m and $L \approx 4.95$ mm, (c) $d_a = 0.75\,\mu$m, $d_b = 3\,\mu$m and $L \approx 7.4$ mm.

At the first instance, one expects that any variation in d_b affects the peak distribution density in a given set of the Bragg frequencies.

With all these, our simulation results demonstrated that the small changes (in order of 10 nm) in d_b does not have any considerable effect on the FFT spectrum, while the results for the large variations (in order of μm) have demonstrated significant effects. In our simulations, we have set $d_a = 0.75\,\mu$m, $d_b = 1\,\mu$m and $n = 50$ as the reference dimensions, first. Then, by varying d_b, while keeping the other two parameters fixed, we have studied the possible variations in the resulting spectrums and compared them with the reference spectrum. Fig. 4 compares FFT spectrums of three $FC_3(50)$ structures, with various d_b ($= 1, 2$, and $3\,\mu$m), in a 160 nm wide window centered around 1550 nm. As we have expected, number of the peaks increases as d_b increases, in a linear fashion. Number of the peaks (number of the equivalent PBGs) for $d_b = 1, 2$, and $3\,\mu$m are 10, 20 and 30, respectively. Since each peak is equal to a photonic bandgap, hence the photonic bandgap engineering becomes possible, by varying the peaks positions and/or numbers.

3.4. Effects of Varying d_a

As seen, from Eq. (8), the dependence of $N'(k)$ on d_a is determined by the terms $\exp(jk[(i-1)d_a]) - \exp(jk[id_a])$ in the summation. These terms, contrary to the term determining the dependence of $N'(k)$ on d_b, do not involve the large Fibonacci's order, n. Hence, in comparison, this could be called the low frequency term. Therefore, at the first glance, variations in d_a do not seem to have any significant effect on the spectrum.

However, our simulation results demonstrate that a small variation in d_a, causes a small shift in each set of the Bragg frequencies, in the k domain. Fig. 5 illustrates the simulation results for two structures whose d_a's increased by 30 and 60 nm from that for the structure of Fig. 4(a). A Comparison of the results illustrated in Figs. 5(a) and 5(b) with that depicted in Fig. 4(a), demonstrates that, depending on whether d_a increases or decreases, the peaks in the FFT spectrum will shift to the right or left, correspondingly. As is calculated from the numerical results, the shift is 1.6 nm in wavelength, for a 30 nm variation in d_a. One can take this phenomenon granted as the optical bandgap tunability. That is, the Bragg frequencies can be tuned by tuning d_a. It should be noted that the relation between the variations in d_a and the shifts in the peaks position is not linear.

Next, we study the effects of large variations ($\sim \mu$m) in d_a on the FFT spectrum. Fig. 6 illustrates and compares the FFT spectrum for three different structures with various d_a values. The results demonstrate that, by doubling or tripling the value of d_a, the number of the sets, in a same window, doubles or triples, correspondingly. Although, the number of the peaks in each set is almost reduced 2 or 3 times but the distance between the neighboring peaks in each set remains unchanged, for all three cases.

3.5. Effects of Varying n

As one can expect from Eq. (8), the effects of variations in n should be the same as that for d_b. An increase in n results in an increase in the peak density, in any given range in the k domain. Fig. 7 illustrates the FFT spectrum of a $FC_3(70)$ structure, depicted in a 160 nm window centered around 1550 nm. The rest of the parameters for the structure are the same as those for the structure of Fig. 4(a). A comparison of Fig. 7 with Fig. 4(a), demonstrate that as the Fibonacci's order, n, increases the number of the peaks in the same window increases in a linear fashion. As one can see from Fig. 7, there are 14 peaks occurring at 14 Bragg frequencies, for n=70, while there exist 10 peaks for the case of $n = 50$. Similar to the case of d_b, in this case also by varying

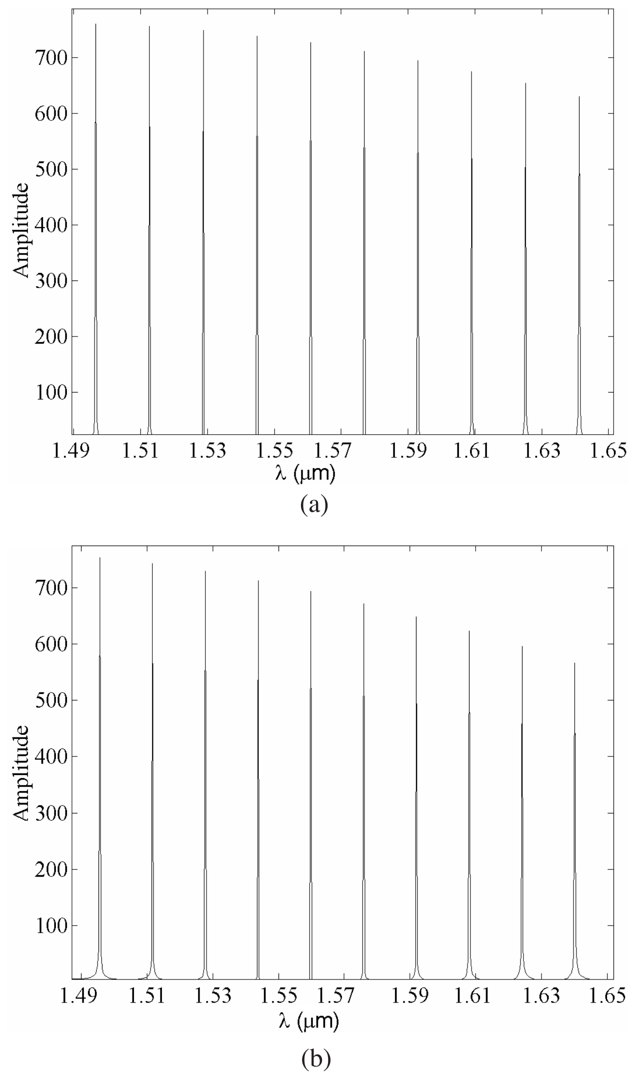
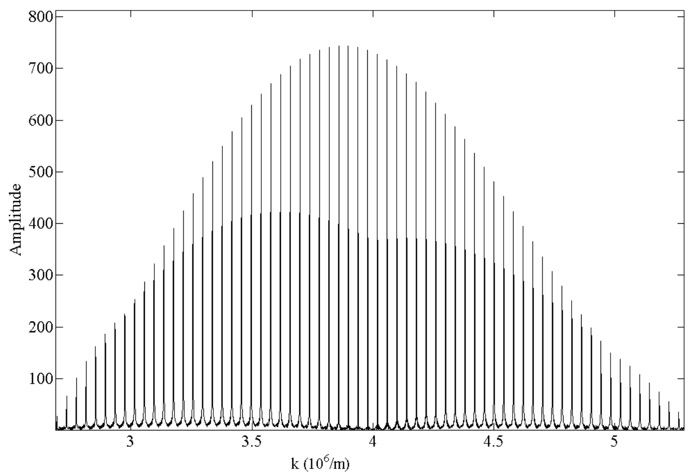


Figure 5. FFT spectrum of the index profile of three different $FC_3(50)$ with with small perturbation in d_a, depicted in a 160 nm wide window centered around 1550 nm, (a) $d_a = 0.78\,\mu$m, $d_b = 1\,\mu$m and $L \approx 2.49$ mm, (b) $d_a = 0.81\,\mu$m, $d_b = 1\,\mu$m and $L \approx 2.48$ mm.

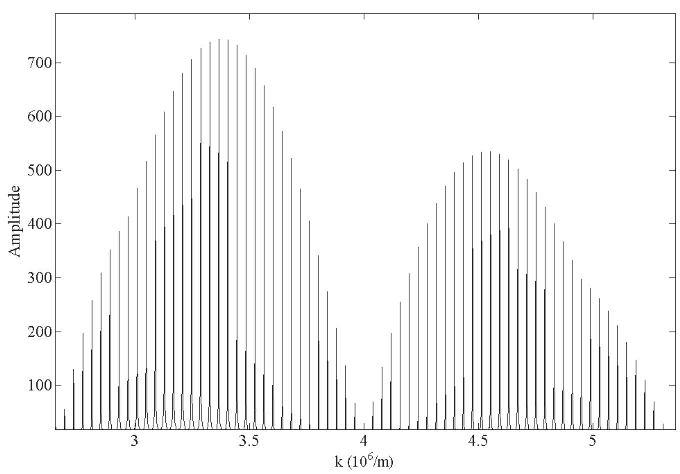
n, while keeping the other parameters fixed, the number and position of the peaks and hence PBGs vary correspondingly. Therefore, by varying n the photonic band gap engineering becomes possible.

4. DESIGN TOOLS FOR MULTIBAND FILTERS

In the previous section, we have demonstrated overall optical properties of $FC_J(n)$ structures and shown that these properties change with the structures parameters. Here, we are going to use such properties to design multiband filters with $FC_3(n)$ structures. Note that each Bragg resonance of the set $\{k_{BR}\}$ acts as a reflecting optical filter. Furthermore, increasing the number of the Bragg resonances per k will result narrower band filters. As an example, the bandwidths of the



(a)



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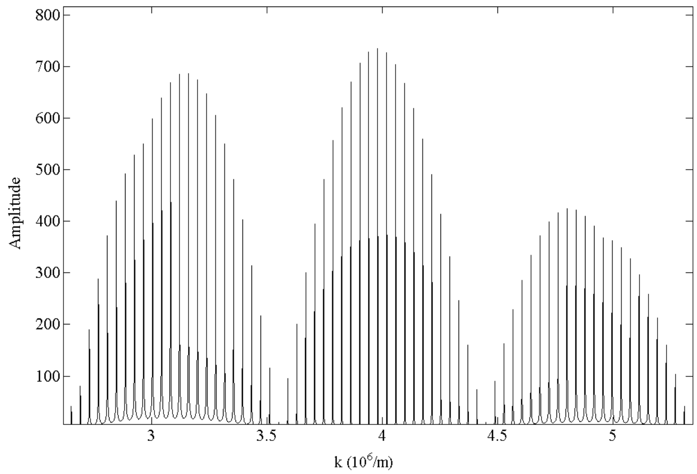

(c)

Figure 6. FFT spectrum of the profile index of the $FC_3(50)$ in the $2.5 \times 10^6\,\mathrm{m}^{-1} \leq k \leq 5.5 \times 10^6\,\mathrm{m}^{-1}$ with large variations in d_a, (a) $d_a = 0.75\,\mu\mathrm{m}$, $d_b = 1\,\mu\mathrm{m}$ and $L \approx 2.49\,\mathrm{mm}$, (b) $d_a = 1.5\,\mu\mathrm{m}$, $d_b = 1\,\mu\mathrm{m}$ and $L \approx 2.53\,\mathrm{mm}$, (c) $d_a = 2.25\,\mu\mathrm{m}$, $d_b = 1\,\mu\mathrm{m}$ and $L \approx 2.56\,\mathrm{mm}$.

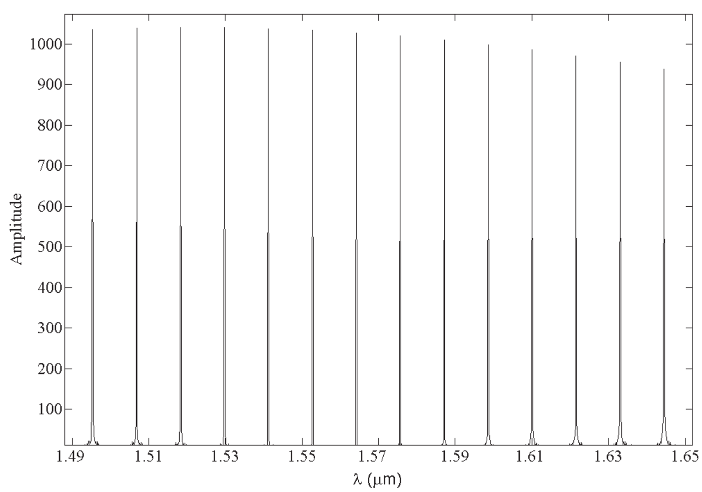

Figure 7. FFT spectrum of the index profile of the $FC_3(70)$ with $d_a = 0.75\,\mu\mathrm{m}$, $d_b = 1\,\mu\mathrm{m}$, $n_a = 1.5$, $n_b = 2.5$, and $L \approx 4.88\,\mathrm{mm}$, depicted in a 160 nm wide window centered around 1550 nm.

filters resulting from each peak in the case of Figs. 4(a), (b) and (c) are 4.8, 2.5 and 1.6 nm, respectively. That is, as d_b is increased, the bandwidths of the filters have decreased. Separations between adjacent peaks in the spectra of Figs. 4(a), (b) and (c) are 17, 8.5 and 4.5 nm, respectively. Also note that the peaks amplitudes appearing in each figure are almost equal. This shows that the dips/peaks appearing in the corresponding transmission/reflection spectra are almost equal.

On the one hand, we have seen that by varying either n or d_b, one can adjust the number of the Bragg frequencies and hence the number of filters and the free spectral range (FSR), in a given window, in the same manner. So, both n and d_b can be used as design tools. On the other hand, we have demonstrated that small variations in d_a cause small displacements in the center frequency of each band. That is, introducing small perturbations in d_a enables one to tune multiband filters. Besides, different values of d_a provide window selectivity in the wavelength domain. Thus, d_a is also another design tool, for the multiband filters.

5. CONCLUSION

In this paper, spectral properties of the Fibonacci-class quasi-periodic structures based on the FFT of the index of refraction have been analyzed. We have shown that the high frequency term in the spectra depends on both n (class order) and d_b (the width of the layer B), while the low frequency term depends on d_a (the width of the layer A). The latter provides envelope functions, which bound the high frequency multi narrow band peaks. These envelope functions vary in a quasi periodic manner. Number of the multi narrow bands within each period of the envelope function can be controlled by varying d_b and n. Furthermore, number of the periods of the envelope functions can be manipulated by varying d_a.

Based on the proposed analysis, multi band optical filters can easily be designed.

ACKNOWLEDGMENT

This work was supported in part by Iran Telecommunication Research Center (ITRC) under grant T500-3679.

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