Spectral Anomalies at Wave-Front Dislocations

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In a recent Letter, Gbur, Visser, and Wolf [Phys. Rev. Lett. **88**, 013901 (2002)] predict that remarkable spectral changes can take place in the neighborhood of phase singularities of a diffracted focused wave. We report here the experimental observation of this anomalous spectral behavior and show that this is a general characteristic of optical vortices. Using a high-resolution interferometric technique, we are able to measure directly both the spectral intensity distribution and the spectral phase at the singular points of optical waves.

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Topological singularities of wave-front dislocations have received considerable attention and a completely new field called *singular optics* has recently emerged. Phase singularities are typical manifestations of fields described by complex wave functions and are observed in points where the field amplitude vanishes. They are common characteristics of the phase in any evolving scalar wave field. One notable application of such topological singularities of optical fields is their use as centers for optical trapping and manipulation [1]. The unusual properties of optical singularities are also actively researched as a basis for digital logic in classical and quantum computation [2]. Rich opportunities exist in the case of optical beams where superpositions of propagating waves lead to superoscillations and interesting spatial structures on scales smaller than the wavelength of light. Phase dislocations of the edge type are found in paraxial Gaussian beams [3] as opposed to screw dislocations observed along the axes of Gauss-Laguerre beams [4].

Until now, the vast majority of publications deal with the phase singularities and the associated caustics of monochromatic waves. According to a classification due to Nye and Berry [5], monochromatic waves can possess line, spiral, or combined wave-front dislocations. Colored diffraction catastrophes, however, have been observed, and the type of singularities has been related to color characteristics [6]. Very recently, Gbur, Visser, and Wolf (GVW) have extended the field of "singular optics" and studied the case of polychromatic waves which are spatially fully coherent and are diffracted at an aperture [7]. GVW predicted an anomalous behavior of spectra near phase singularities of such focused waves.

We present the first experimental demonstration of such diffraction-induced spectral modifications in points of phase dislocations and show that this is a rather general behavior of polychromatic wave fields in the vicinity of singular points of the phase. Using a novel method for measuring the complex field structure, i.e., the spectral intensity and phase modifications of a broadband optical field, we are able to visualize remarkable features of phase dislocations in both deterministic and random wave fields.

We consider first the specific characteristics of focused polychromatic fields. Of particular interest is their complicated behavior determined by the phase singularities in the focused region of a converging lens. When the optical field incident on the focusing lens is spatially coherent and has an optical spectrum $S_0(\lambda)$, the spectral intensity in the focal region is given by

$$S(\rho, z, \lambda) = |\psi(\rho, z, \lambda)|^2 S_0(\lambda) = M(\rho, z, \lambda) S_0(\lambda), \quad (1)$$

where $M(\rho, z, \lambda)$ describes the diffraction-induced spectral changes and thus represents a spectral modifier. The function $\psi(\rho, z, \lambda)$ quantifies the spectral modifications in the complex amplitude of an incident field and is, therefore, complex in nature, $\psi(\rho, z, \lambda) = |\psi(\rho, z, \lambda)| \times \exp[i\phi(\rho, z, \lambda)]$.

We examined the case of a polychromatic Gaussian beam of width w that is truncated by an aperture with diameter D and focused by a lens L with the focal length f. The relevant parameter in this case is the Fresnel number $N = D^2/(4\lambda f)$. When the inequality $N \gg 1$ is satisfied, the scalar Debye theory can be used to evaluate the complex field modifier $\psi(\rho, z, \lambda)$ in the region of the geometrical focus [8,9]. For monochromatic Gaussian beams which are spatially limited by apertures, it has been shown that the Airy rings reorganize themselves as the incident beam undergoes a transition to form a plane wave to a truncated Gaussian distribution [3]. Straightforward use of the paraxial diffraction theory for Gaussian fields leads to the calculation of $\psi(\rho, z, \lambda)$; in the geometrical focal plane, this function has the following simple expression:

$$\psi(\rho, 0, \lambda) \propto \frac{\exp(i\Phi)}{\lambda} \int_0^1 J_0(vt) \exp(-\gamma t^2)\, dt, \quad (2)$$

where $\Phi = \pi N(\rho/D)^2$, $v = 4\pi N\rho/D$, and $\gamma = D^2/(4w^2)$. It is evident from Eq. (2) that in the focal region, the spectral density will generally be different from the initial spectrum and will vary with the transversal coordinate ρ. Such diffraction-induced spectral changes have been recently predicted by GVW and studied by numerical simulations for the case of wave fronts with constant amplitude focused by finite size lenses. Spectacular modifications of the initial spectral density take place in the neighborhood of phase singularities.

In order to demonstrate this anomalous behavior, the spectral analysis must overcome some significant challenges. First, since the phase discontinuities in the focal plane produce regions of very low intensity level (Airy rings), the spectral density $S(\rho, z, \lambda)$ has to be measured with high sensitivity over a large dynamic range. Second, these "dark rings" extend over very narrow spatial regions; thus the spectral information has to be spatially resolved with a significant resolution. These requirements cannot be met by conventional dispersive spectrometers and we have implemented a novel high-dynamic-range interferometric technique which permits high sensitivity measurements of the spectral properties of light over narrow spatial regions. In addition, our technique allows us for the first time to perform a direct measurement of the phase in the focal region of optical waves.

The setup is schematically shown in Fig. 1. Light from a broadband source of central wavelength $\lambda_0 = 824$ nm and temporal coherence length $l_c = 30$ μm is coupled into a 2×2 single mode fiber coupler to create a Michelson interferometer. In the reference arm, the light is collimated and reflected by a scanning mirror. On the test arm, the beam is also collimated and subsequently expanded by an afocal system of lenses, such that the field approaches a Gaussian distribution of waist $w = 8.5$ mm. After passing through an aperture of diameter $D = 3.5$ mm, the beam is tightly focused by a lens of focal distance $f = 16$ mm. In order to achieve the spatial resolution required to resolve the diffraction pattern in the focusing region, the plane $z = 0$ is scanned by a metallic sphere of diameter $d = 0.75$ mm, which acts as the second, pointlike mirror of the interferometer. In the Fourier plane, the metallic sphere represents a narrow bandpass filter for the spatial frequencies. The spatial resolution achieved in this setup depends only on the numerical aperture of the optical system, and it has been estimated to be of the order of 1 μm. The metallic sphere is translated along the ρ coordinate using a computerized stage with a precision better than 50 nm. The fields from the reference and test arm are combined to give an interference signal at the detector. The mirror in the reference arm is linearly translated in a periodic fashion—driven by a sawtooth signal—while the time and frequency analysis of the interference signal is synchronized with its motion. The interference term detected in the time domain is proportional with the real part of the cross-correlation function $\Gamma(\rho, T) = \langle E_1(\rho, t) \times E_2^*(t + \tau)\rangle$ of the test field E_1 and reference field E_2, with the angular brackets denoting the time average. Thus, the measured quantity is

$$\Gamma_R(\rho, T) = |\Gamma(\rho, T)| \cos[\Omega_0 T + \Delta\varphi(\rho, T)]. \quad (3)$$

In Eq. (3), Γ_R is the real part of the cross-correlation function, $\Omega_0 = (2v_m/c)\omega_0$ is the Doppler frequency associated with the central optical frequency ω_0, while v_m represents the velocity of the scanning mirror, and c is the speed of light in vacuum; $\Delta\varphi$ is the phase difference between the two fields and the delay T relates to the optical delay τ through $T = (c/2v_m)\tau$.

For each coordinate ρ, it takes 125 ms to acquire the full expression of the cross-correlation function. In order to obtain $\Gamma(\rho, T)$, the carrier frequency Ω_0 is filtered out and the unwrapped phase $\Delta\varphi(\rho, T)$ is simultaneously measured. Since the reference field is constant in amplitude, the intensity profile of the focused beam is proportional to $|\Gamma(\rho, 0)|^2$. The intensity profile across the focused beam has been obtained experimentally by recording the value of the cross-correlation function corresponding to zero delay while the sphere is translated across the beam. In Fig. 2 we show the experimental data together with the result of our calculation for the radial intensity. The first two Airy rings

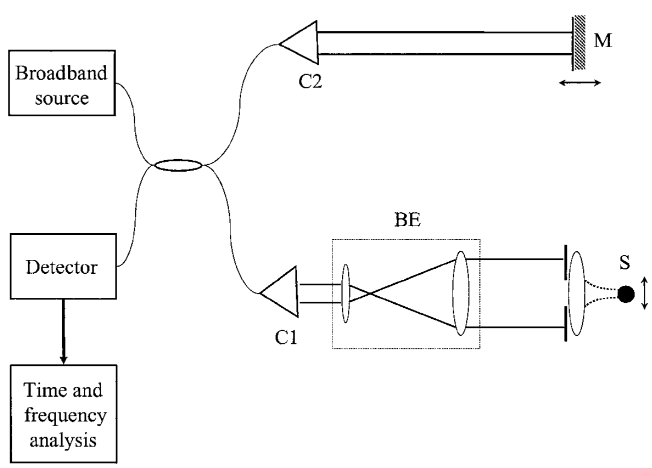

FIG. 1. Schematic representation of the experimental setup. C1, C2, collimators; BE, beam expander; M, plane mirror; and S, spherical mirror.

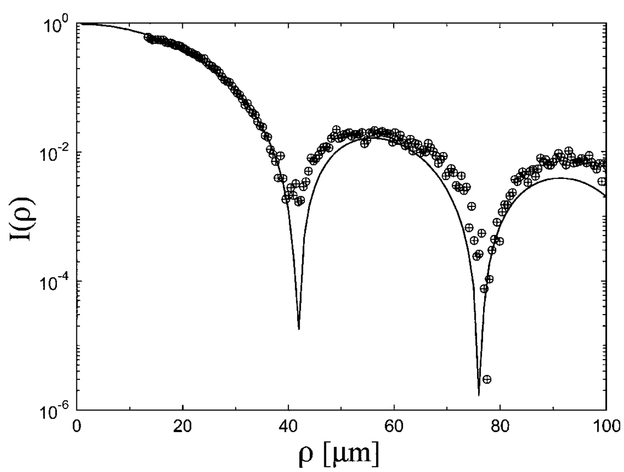

FIG. 2. Radial dependence of intensity in the geometrical focus plane and the corresponding calculation based on the scalar Debye diffraction theory.

are clearly resolved with high contrast indicating that significant beam truncation is introduced at the lens aperture. Note that these sigularities are not present when $w \ll a$, i.e., in the case of Gaussian illumination. The agreement between the Debye theory and the experimental data is remarkable and confirms the significant dynamic range and high spatial resolution of our measurement.

In the frequency domain, the signal recorded by a frequency analyzer is merely the Fourier transform of Eq. (3) and is complex in nature:

$$W(\rho, \Omega) = |W(\rho, \Omega)| e^{i\Delta\varphi(\rho,\Omega)}, \qquad (4)$$

where the frequency Ω relates to the optical frequency ω via $\Omega = 2v/c\omega$ and $\Delta\varphi(\rho, \Omega)$ is the spectral phase difference between the two fields. It becomes apparent that the quantity measured in the frequency domain represents the cross-spectral density of the fields shifted at the Doppler frequency Ω. Thus, the frequency dependence of both the magnitude and the phase associated with the measured function W can be shifted into the optical frequency domain through a simple linear transformation.

In order to obtain the complex field modifier, the cross-spectral density associated with the position $\rho = 0$ was used as a reference. Thus any residual spectral dependencies are removed when the field modifier is evaluated as

$$\psi(\rho, \lambda) = |\psi(\rho, \lambda)| e^{i\phi(\rho,\lambda)} = \frac{W(\rho, \lambda)}{W(0, \lambda)}. \qquad (5)$$

At each point ρ across the focused beam, the spectral phase modification $\phi(\rho, \lambda) = \Delta\varphi(\rho, \lambda) - \Delta\varphi(0, \lambda)$ is obtained from Eq. (5), while the changes brought to the optical spectrum are quantified using Eqs. (1) and (5). Our apparatus can be described as a fiber optics Fourier transform spectrometer, with a continuous scanning of the reference mirror that provides complex spectral information in real time. As in any Fourier transform spectrometer, the spectral resolution achieved is limited by the excursion of the reference mirror and, for our experimental situation, has the value $\delta\Omega/\Omega = \delta\omega/\omega = 5.7 \times 10^{-4}$.

The appealing feature of our method is that the spectrum analyzer provides the cross-spectral density in a spatially resolved manner and one can now further investigate the spectral behavior of the field in the neighborhood of the Airy rings. Figure 3 shows a series of spectra measured with a spatial resolution of 0.1 μm around the second minimum visible in Fig. 2.

As can be seen, the optical spectrum changes dramatically near the phase singularity as predicted by GVW. In comparison with the incident spectral density shown in the lower part of the 3D representation, one can observe a rather complex spectral behavior: depending on the coordinate ρ, the spectrum can be either blue- or redshifted, but it can also be split into two distinct spectral lines. This is the first experimental evidence of such spectral changes induced in the vicinity of phase singularities.

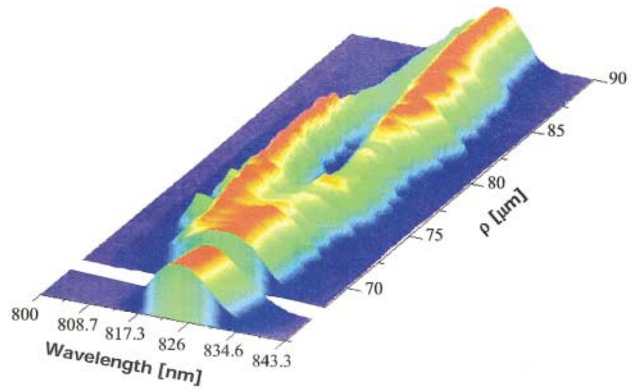

FIG. 3 (color). Spectral modifications in the vicinity of the second Airy ring shown in Fig. 1. Also shown is the initial spectral density of focused radiation.

As shown in Eq. (5), $\psi(\rho, \lambda)$ is a complex quantity the phase of which measures the diffraction-induced changes in the phase of the incident field at the focal region of interest. Figure 4 shows in detail the spectral dependence of the field intensity together with the phase modifier $\phi(\rho, \lambda)$ in the vicinity of the second Airy ring ($\rho = 76.2$ μm). One can clearly see that the discontinuity in the spectral phase produces a significant split of the spectral density. The small discrepancies between the measured behavior of spectra and the calculation based on Debye diffraction theory can be attributed to residual phase aberrations in the optical system.

As mentioned in the introduction, phase singularities are rather general characteristics of complex waves. The natural occurrence of points of phase singularities is not limited to deterministic optical fields although the majority of publications concerned with phase dislocations deal with specific characteristics of optical beams or with singularities of geometrical optics, i.e., optical caustics [10]. Random wave fields—speckle patterns—are natural results of the interaction between coherent or partially coherent waves and inhomogeneous media such as rough surfaces

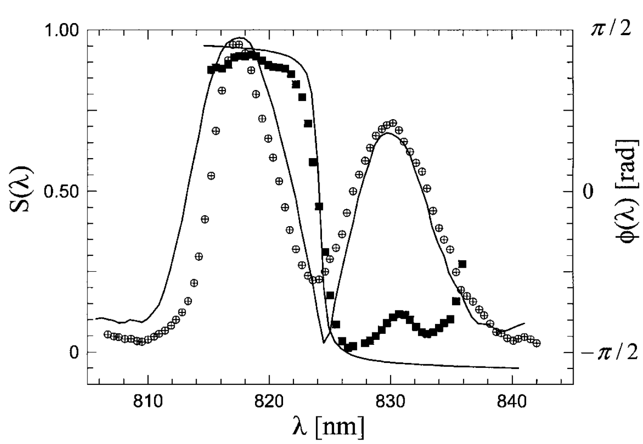

FIG. 4. Optical spectral density and spectral phase modifier measured at $\rho = 76.2$ μm. The continuous lines are the results of the Debye theory for the experimental situation.

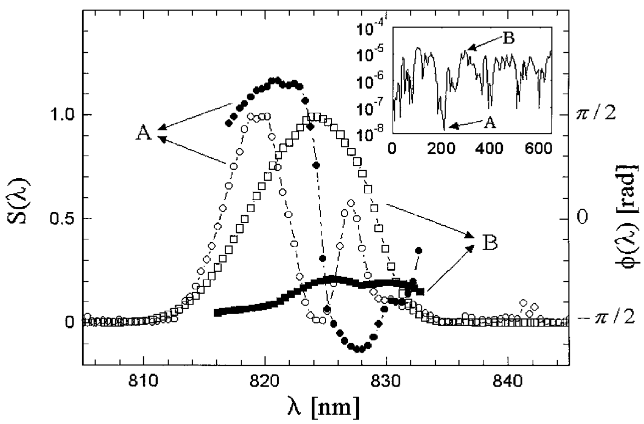

FIG. 5. Optical spectra (open symbols) and spectral phase modifiers (full symbols) associated with two distinct positions of the phase screen, as indicated in the inset. The inset shows the intensity distribution as a function of the screen position over a total displacement of 650 μm.

or multiple scattering volumes. Typical far-field speckles are random distributions of intensity which, in most cases, are well described by Gaussian statistics; the associated phase distributions are complex maps containing local minima, maxima, saddle points, and vortices [11]. We have also investigated such a complex field in a speckle pattern produced at the point $z = \rho = 0$ by placing a thin random phase screen against the lens L in Fig. 1. The screen was translated in a plane perpendicular to the optical axis, while the complex spectral information has been retrieved as a function of the transversal position. Figure 5 shows the results in terms of the optical spectra and phase modifiers for two positions of the phase screen associated with a maximum and a minimum of the total intensity. It can be seen that, in the point of phase discontinuity, i.e., minimum intensity, the spectrum changes dramatically and resembles the diffraction-induced modifications. In the point of phase discontinuity, the spectral phase modifier undergoes a sharp transition around the wavelength where the spectrum splits, while for the point of maximum intensity, it has a continuous behavior.

Interference is the essential feature in producing phase singularities or dislocations. However, the presence of such singular points is not limited to deterministic situations; optical vortices are typical characteristics of general wave fields. In this Letter we have presented experimental evidence for the anomalous spectral behavior found in the focusing region of truncated Gaussian beams as well as in the singular points of random wave fields.

Using a novel procedure for measuring directly the structure of an optical field, i.e., its spectral intensity and also its phase, we have shown that these remarkable spectral modifications are rather general and always take place at points of phase dislocations. The phase dislocations develop usually after a critical propagation length, and the vortex cores are associated with zeros of the spectral intensity. In these points of phase singularities, the flow of energy is yet to be understood in the general case of partially coherent radiation. Moreover, for wave fields which are partially coherent, there is no clear understanding of possible phase discontinuities in points of nonzero intensity. We hope that our findings will stimulate further theoretical and experimental research in the field of singular optics.

Our results demonstrate that new possibilities exist for manipulating optical spectra and, because these spectral changes are specific to critical points of optical fields, fundamental consequences follow for the field reorganization phenomena at subwavelength scales. Understanding the rich behavior of the phase should be of paramount interest for adaptive optics and also for a range of techniques dealing with the interaction between optical waves and matter.

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