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# NASA TECHNICAL MEMORANDUM

NASA TM X-64842

(NASA-TM-X-64842) SPECTRAL IRRADIANCE  
CURVE CALCULATIONS FOR ANY TYPE OF SOLAR  
ECLIPSE (NASA) ~~26~~ p HC \$3.25 CSCL 03B

23

N74-26266

Unclas  
G3/29 39956

## SPECTRAL IRRADIANCE CURVE CALCULATIONS FOR ANY TYPE OF SOLAR ECLIPSE

By Adarsh Deepak and J. E. Merrill  
Aero-Astroynamics Laboratory



February 1, 1974

**NASA**

*George C. Marshall Space Flight Center  
Marshall Space Flight Center, Alabama*

1. REPORT NO. NASA TM X-64842	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.	
4. TITLE AND SUBTITLE Spectral Irradiance Curve Calculations for Any Type of Solar Eclipse		5. REPORT DATE February 1, 1974	6. PERFORMING ORGANIZATION CODE S&E-AERO-AF, 7905
7. AUTHOR(S) Adarsh Deepak* and J. E. Merrill**		8. PERFORMING ORGANIZATION REPORT #	
9. PERFORMING ORGANIZATION NAME AND ADDRESS George C. Marshall Space Flight Center Marshall Space Flight Center, Alabama 35812		10. WORK UNIT NO.	11. CONTRACT OR GRANT NO.
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D. C. 20546		13. TYPE OF REPORT & PERIOD COVERED Technical Memorandum	
15. SUPPLEMENTARY NOTES Prepared by Aero-Astroynamics Laboratory, Science and Engineering *National Academy of Sciences Postdoctoral Research Associate **Department of Physics and Astronomy, University of Florida, Gainesville, FL		14. SPONSORING AGENCY CODE	
16. ABSTRACT <p>This paper describes a simple procedure for calculating the eclipse function (EF), <math>\alpha</math>, and hence the spectral irradiance curve (SIC), <math>(1-\alpha)</math>, for any type of solar eclipse: namely, the occultation (partial/total) eclipse and the transit (partial/annular) eclipse. The SIC (or the EF) gives the variation of the amount (or the loss) of solar radiation of a given wavelength reaching a distant observer for various positions of the moon across the sun. The problem of computing the SIC is complicated by the presence of the solar limb-darkening. Our scheme is based on the theory of light curves of eclipsing binaries, the results of which are tabulated in Merrill's Tables, and is valid for all wavelengths for which the solar limb-darkening obeys the cosine law: <math>J = J_c (1 - x + x \cos y)</math>. As an example of computing the SIC for an occultation eclipse which may be total, the calculations for the March 7, 1970, eclipse are described in detail. The calculations for the transit eclipses can be carried out in a similar manner.</p>			
17. KEY WORDS	18. DISTRIBUTION STATEMENT Unclassified-unlimited <i>J. A. Evingood</i> J. A. Evingood Director, Aero-Astroynamics Laboratory		
19. SECURITY CLASSIF. (of this report) Unclassified	20. SECURITY CLASSIF. (of this page) Unclassified	21. NO. OF PAGES 24	22. PRICE NTIS

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## LIST OF SYMBOLS

Symbol	Definition
$D_m$	mean geocentric distance of the moon; $D_m = 60.268 R_E$
$i$	inclination of the effective apparent orbit of the moon about the sun as seen by the observer; $i = 90$ deg for an eclipse which becomes central
$J$	surface brightness of the sun at any point of surface
$J_c$	surface brightness of the sun at center of disk
$k$	ratio of the angular radius of the smaller disk to that of the larger disk; $k = r_s / r_g$
$p$	geometrical depth of the eclipse at any instant (since it varies with time); it is defined as the distance from the limb of the larger disk (angular radius, $r_g$ ) to the center of the smaller disk (angular radius, $r_s$ ) in terms of the angular radius of the smaller disk, namely, $p = \left( \frac{\delta - r_g}{r_s} \right)$
$R_E$	radius of earth
$r_m$	angular radius of moon
$r_{sn}$	angular radius of sun; $r_{sn} < r_m$ for the occultation solar eclipse at any stage (partial or total); $r_{sn} > r_m$ for the transit eclipse at any stage (partial or annular)
$x$	coefficient of limb-darkening; $J = J_c (1 - x + x \cos \gamma)$
$x_{\alpha}^{oc}$	$\alpha^{oc}$ during the eclipse for any value of $x$

## LIST OF SYMBOLS (Concluded)

Symbol	Description
$x$ $\alpha^{\text{tr}}$	$\alpha^{\text{tr}}$ during the eclipse for any value of $x$
$\alpha$	ratio of light loss at any time to light loss at internal tangency; $\alpha = \alpha(k, p)$ ; often referred to as the Russell Function or the eclipse function
$\alpha^{\text{oc}}$	$\alpha$ during the occultation eclipse
$\alpha_0^{\text{oc}}$	$\alpha^{\text{oc}}$ at mideclipse; maximum $\alpha$
$\alpha^{\text{tr}}$	$\alpha$ during the transit eclipse
$\alpha_0^{\text{tr}}$	$\alpha^{\text{tr}}$ at mideclipse; maximum $\alpha$
$\beta$	instantaneous equatorial horizontal parallax of moon
$\beta_m$	mean equatorial horizontal parallax of moon; $\beta_m = 57 \text{ min } 3 \text{ sec}$
$\gamma$	angle at the center of the sun between the radius and the line of sight
$\delta$	angle between the centers of the moon and sun at any instant
$\theta$	longitude in orbit (from conjunction)
$\tau$	ratio of the light received from the solar disk at internal tangency (second or third contact) during an annular eclipse, to total light of that disk

## SPECTRAL IRRADIANCE CURVE CALCULATIONS FOR ANY TYPE OF SOLAR ECLIPSE

### I. INTRODUCTION

The spectral irradiance curve (SIC) gives the variation of the sun's radiation of a given wavelength reaching a distant observer for various positions of the moon across the sun. It is simply the light curve for a given wavelength ( $\lambda$ ). The SIC is defined by the difference ( $1-\alpha$ ) of the eclipse function (EF),  $\alpha$ , from unity. The EF gives the variation of the loss of solar radiation reaching the distant observer during the various phases of the eclipse. It is also referred to as the Russell Function in the theory of light curves for eclipsing binaries [1-3] and is defined in the List of Symbols.

According to von Oppolzer's Canon of Eclipses [4], an average of approximately 237 solar eclipses occur per century, of which 66 become total, 77 become annular, and 10 are annular-and-total, and about 84 are partial at maximum.

This report contains a simple procedure for calculating the eclipse function, and hence the SIC, for any type of solar eclipse and for radiation of any wavelength for which the solar limb-darkening obeys the limb-darkening cosine law:

$$J = J_c (1 - x + x \cos \gamma) \quad , \quad (1)$$

where the symbols are defined in the List of Symbols. The case of occultation (partial/total) eclipses is presented in Section II and that of transit (partial annular) eclipses is discussed in Section III.

The computation of the eclipse function or the SIC would be trivial but for the presence of the solar limb-darkening effect. The treatment here differs from that of Beard [5] which is based on the Method of Julius [6] and consists

essentially of dividing the sun into 12 somewhat arbitrary concentric zones and assigning to each an average brightness value. The treatment in this report is based on the theory for the light curves of eclipsing binaries [1-3], where it is assumed that the orbit is circular and that the stars are spherical and appear darkened at the limb according to the cosine law, equation (1). In the procedure here the binary system is replaced by the sun and the moon which are assumed to be spherical. Because the speed of the earth-moon system in its orbit around the sun is more than 30 times as great as the speed of the moon in its orbit around the earth, the relative orbit of the moon around the sun is always concave toward the sun and can be regarded as circular with respect to the sun throughout the entire interval of the eclipse.

Also, the authors of this report consider only those solar radiations for which the limb-darkening law of equation (1) holds for the sun. One can obtain the values of the limb-darkening coefficient ( $x$ ) for these wavelengths from Reference 6. Then, the pertinent portions of Merrill's Tables [1-3] (hereinafter called "Princeton 23") can easily be applied to the case of the solar eclipses.

As an example of the computations involved, calculations have been made for the March 7, 1970, eclipse. One can use the procedure in a similar manner to compute the eclipse function and the SIC for any other occultation or transit eclipse. The March 7, 1970, eclipse is an example of an occultation eclipse which is total for a portion of the time, as seen from within a certain narrow track.

Before proceeding to the description of the method, a brief explanation of the concepts of eclipsing binaries as applicable to the case of solar eclipses is given.

## A. Concepts and Relations Relevant to Solar Eclipses

If  $\theta$  is the longitude in orbit from conjunction and  $i$  is the inclination of the orbit, then the apparent distance  $\delta$  between the centers of the two disks (of the sun and the moon) is given by (Fig. 1)

$$\left(\frac{\delta}{f}\right)^2 = \cos^2 i + \sin^2 i \sin^2 \theta \quad (2)$$



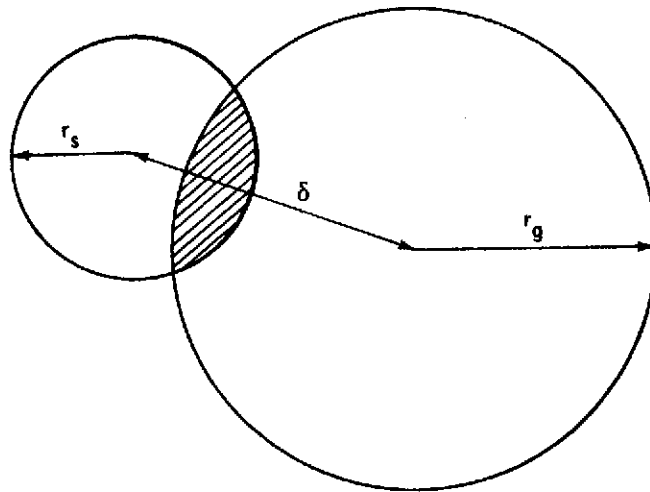


Figure 1. Apparent geometry of eclipses. [The eclipse is an occultation if the eclipsed area (shaded) is on the smaller disk. The eclipse is transit if the eclipsed area is on the larger disk.]

where  $f$  is simply a factor relating the distance units involved in the two systems. The geometrical depth of the eclipse is then defined by

$$p = (\delta - r_g) / r_s \quad , \quad (3)$$

where  $r_g$  and  $r_s$  are the respective angular radii of the larger and the smaller disks as seen by the observer. Depending on the distance of the moon from the observer (which varies about a mean distance,  $D_m$ ), the moon's disk may have its radius larger or smaller than that of the sun. When for any fixed observer the moon's semidiameter ( $r_m$ ) is greater than that of the sun ( $r_{sn}$ ), the eclipse is herein called occultation; if and while  $p \leq -1$ , the eclipse is complete and is usually called total. When for any fixed observer the moon's semidiameter ( $r_m$ ) is less than the sun's ( $r_{sn}$ ), the eclipse is herein called transit; if and while  $p \leq -1$ , the eclipse is complete and usually called annular. When  $p > -1$ , whether during occultation or transit, the eclipse is usually called partial.

The definition of  $p$  applies equally, whether the larger or the smaller disk is in front: The eclipse will be absent, partial, or complete, according to whether  $p \geq 1$ ,  $1 > p \geq -1$ , or  $p < -1$ . Because the amount of light obscured during an annular phase (if there is one) is least at internal tangency ( $p = -1$ ) and increases steadily with diminishing  $p$  to a maximum (loss) at concentric eclipse ( $p = -1/k$ ), if that occurs, it is more efficient to treat occultation eclipses as a whole and transit eclipses as a whole, by slightly different techniques.

The other important quantity is

$$k = r_s / r_g \quad , \quad (4)$$

which is always less than unity unless  $r_s = r_g$ . From equations (3) and (4), one obtains [1-3]

$$\delta = r_g (1 + kp) \quad . \quad (5)$$

Only for a concentric eclipse (annular or total),  $p_c = -1/k$  (where  $k$  and  $p$  are dimensionless quantities), do whole values completely define the geometrical circumstances of a given phase. With the aid of geometrical parameters of the solar eclipse, the EF or the SIC is derived for the occultation eclipse in the following sections.

## B. The Eclipse Function and the Spectral Irradiance Curve Calculations

The eclipse function,  $\alpha$ , referred to as the Russell Function in the nomenclature of eclipsing binaries, is defined as the ratio of the light loss at any instant during the eclipse to the light loss at internal tangency.  $\alpha$  is a function of the limb-darkening coefficient,  $x$ , and the eclipse parameters  $k$  and  $p$  defined previously, i.e.,

$$\alpha = \alpha(x, k, p) \quad . \quad (6)$$

One can invert relation (6) and obtain  $p$  as a function of  $x$ ,  $k$  and  $\alpha$ , i.e.,

$$p = p(x, k, \alpha) \quad (7)$$

during all partial and annular phases but not, of course, during actual total phases since  $\alpha$  is constant there.

If  $\alpha_0$  is the value of  $\alpha$  at mideclipse, then one can define the light loss at any time during the eclipses relative to that at mideclipse by

$$n = \alpha/\alpha_0 \quad . \quad (8)$$

Then by definition,

$$p_0 = p(x, k, \alpha_0) \quad . \quad (9)$$

From equations (2) and (5)

$$f(\cos^2 i + \sin^2 i \sin^2 \theta) = r_g^2 (1 + kp)^2 \quad . \quad (10)$$

For mideclipse,  $\theta = 0$  so that

$$f^2 \cos^2 i = r_g^2 (1 + kp_0)^2 \quad . \quad (11)$$

Furthermore, if  $p_n = p(x, k, n\alpha_0)$  and  $\theta(n)$  refers to the value of  $\theta$  when the radiation loss from the sun's disk is  $\alpha = n\alpha_0$ , one can obtain the following important relation [1-3]:

$$\frac{\sin^2 \theta (n)}{\sin^2 \theta \left(\frac{1}{2}\right)} = \frac{(1 + kp_n)^2 - (1 + kp_0)^2}{(1 + kp_{1/2})^2 - (1 + kp_0)^2} = \chi(x, k, \alpha_0, n) \quad (12)$$

or

$$\sin^2 \theta (n) = \sin^2 \theta (1/2) \chi(x, k, \alpha_0, n) \quad (13)$$

These equations apply to all eclipses. It should be noted that  $n$  becomes unity at second contact (internal tangency) and that during totality the independent variable in  $\chi$  is  $p$  rather than  $n$ .

The SIC, the light curve for a given  $\lambda$ , is defined by the relation between  $\theta$  and  $(1 - n\alpha_0)$  for any eclipse. The SIC can be easily obtained by using Princeton 23 [1-3] along with equation (13), as described in Sections II and III.

## II. THE OCCULTATION ECLIPSE

### A. The Total Eclipse

For all occultation-type eclipses, for all observers,  $k = r_{sn}/r_m \leq 1$ .

For an observer on the centerline of the path of totality,  $p_0 = -1/k$ ; for one on either edge of that path,  $p_0 = -1$ ; for any observer elsewhere within that path,  $-1/k < p_0 < -1$ ; and for one outside the path of totality but within the (very large) region where the eclipse is partial at maximum,  $-1 < p_0 < +1$ . For any observer within the path of totality  $\alpha_0^{oc} = 1$  and, in fact,  $\alpha^{oc} = \alpha_0^{oc} = 1$  throughout the time interval from second to third contact; for one outside that path but inside the large region mentioned,  $\alpha_0^{oc} < 1$ . The parameter  $p$  takes on the instantaneous values  $+1, -1, -1, +1$ , respectively, at first, second, third, and fourth contacts for an observer within the path of totality; second and third contacts are, of course, not observed outside that path, but the name "fourth contact" is by convention retained for the end of the phenomenon (Fig. 1).

Princeton 23 provides among other things  $\chi^{oc}$  as a function of  $k$  and  $\alpha_0^{oc}$  for a large number of selected values of  $\chi$  and  $n$ , and simple interpolating forms for finer screening in any one of the four variables  $k$ ,  $\alpha_0^{oc}$ ,  $\chi$ ,  $n$  where warranted.

An example of computation of  $\theta(n)$  values is given below for the March 7, 1970, centerline of totality for which  $p_0 = -1/k$ . From the Naval Observatory circular [7], one obtains for an observer at the centerline of totality the value of  $p_0 = -1.040$ , so that

$$k^{oc} = -1/p_0^{oc} = 0.962 \quad . \quad (14)$$

Then if the sun's semidiameter [7] with respect to the earth's surface is  $r_{sn} = 0.004688$  radians, that of the moon is

$$r_m = r_{sn}/k = 0.0048773 \text{ radians} \quad , \quad (15)$$

so that at first contact ( $p = +1$ ,  $n = 0.0$ )

$$\theta(n = 0.0) = r_{sn} + r_m = 0.00956451 \text{ radians} \quad . \quad (16)$$

From the  $x = 0.6$  section of Princeton 23 [1-3], one can obtain  ${}^6\chi^{oc}(n)$  to use in equation (13) or, in a form more convenient for our present purpose, such as

$$\sin \theta(n) = \sin \theta(0.5) \left[ {}^6\chi(n) \right]^{1/2} \quad (17)$$

to obtain  $\theta(n)$ . The value of  $\theta(0.5)$  is derived by substituting in equation (17) the value of  ${}^6\chi^{oc}(n = 0.0, k = 0.962, \alpha_0 = 1, n = 0.0)$  obtained from the next to the bottom row ( $\chi^c$ ) on page 180 of Princeton 23, and that of  $\theta(0.0)$  from equation (16):

$$\begin{aligned}
\sin\theta(0.5) &= \sin[\theta(0.0)] / \left[ 0.6 \chi(n) \right]^{1/2} \\
&= \sin(0.0095641) / 2.30954 \\
&= 0.0041413 \quad . \quad (18)
\end{aligned}$$

Therefore,  $\theta(0.5) = 0.0041413$ , for  $n = 0.5$  and  $(1 - n\alpha_0) = 0.5$ . Similarly, for  $n = 0.1$  and  $(1 - n\alpha_0) = 0.9$ :

$$\begin{aligned}
\sin\theta(0.1) &= \sin\theta(0.5) \left[ 6 \chi(0.1) \right]^{1/2} \\
&= 0.0041413 (1.83078) = 0.0075818
\end{aligned}$$

$$\theta(0.1) = 0.0075818$$

and so on. The results of the calculations for  $\theta(n)$  and  $\theta(1-n\alpha_0)$  are given in Table 1 and the SIC is plotted in Figure 2. The calculations have been made for Walterboro, S. C. (32 deg 54 min north, 80 deg 40 min west), which is on the centerline of totality for the March 7, 1970, eclipse (see map in Reference 7). The universal times [7] for the first and second contacts are 17 hr 4 min 56 sec and 18 hr 24 min 67 sec, respectively.

The time interval,  $\Delta t$ , elapsed after first contact can easily be obtained from  $\theta$ 's because of the uniform (circular) motion of the moon around the sun, over that interval. If the time interval between the first and second contacts is  $T_{12}$ , and  $\theta_1 = r_{sn} + r_m$  [as in equation (17)],

$$\Delta t(n) = T_{12} \left[ \frac{\theta_1 - \theta(n)}{\theta_1 - \theta_2} \right] \quad , \quad (19)$$

where  $\theta_2$  is the value at second contact.

TABLE 1. CALCULATIONS FOR  $\theta(n)$  AND  $\theta(1 - n\alpha_0)$  [For Walterboro, South Carolina (32 deg 54 min north, 80 deg 40 min west) on the Centerline of Totality and  $x = 0.6$  Corresponding to  $\lambda = 550 \text{ nm}$ ]

Eclipse Phase	n	1-n	$\theta(n)$ (radians)	$\Delta t(n)$ (min)
1st Contact	0.0	1.00	0.009561	0.0
	0.1	0.90	0.007579	16.780
	0.2	0.80	0.006536	25.617
	0.4	0.60	0.004883	39.621
	0.5	0.5	0.004144	45.880
	0.6	0.4	0.003427	51.953
	0.8	0.2	0.001987	64.149
	0.9	0.1	0.001210	70.730
	0.95	0.05	0.0007794	74.377
2nd Contact	1.00	0.0	0.0002125	79.1784

Note that, since  $\theta(n)$  is practically always so small during any solar eclipse (seen from a single point on the earth's surface),  $\theta(\text{radians})$  is always equal to  $\sin \theta(n)$  to the accuracy of the data. Its obvious implication would be that a permanent set of tables of  $[\chi]^{1/2}$  for relevant values of  $x, k, n$  could very easily be constructed from the appropriate  $\chi$ 's of Princeton 23.

If  $H_0(\lambda, 0)$  represents the spectral irradiance ( $\text{watts/cm}^2\text{-}\lambda$ ) of the sun at instant  $t = 0$  just prior to the first contact and  $H_0(\lambda, t)$  is that at time  $t$  after contact, the relative spectral irradiance is given by the ratio  $[H_0(\lambda, t)/H_0(\lambda, 0)]$  and is equal to  $(1 - n\alpha_0)$ . Hence the name SIC for the relation  $(1 - n\alpha_0)$  versus  $\theta$  or  $t$ .

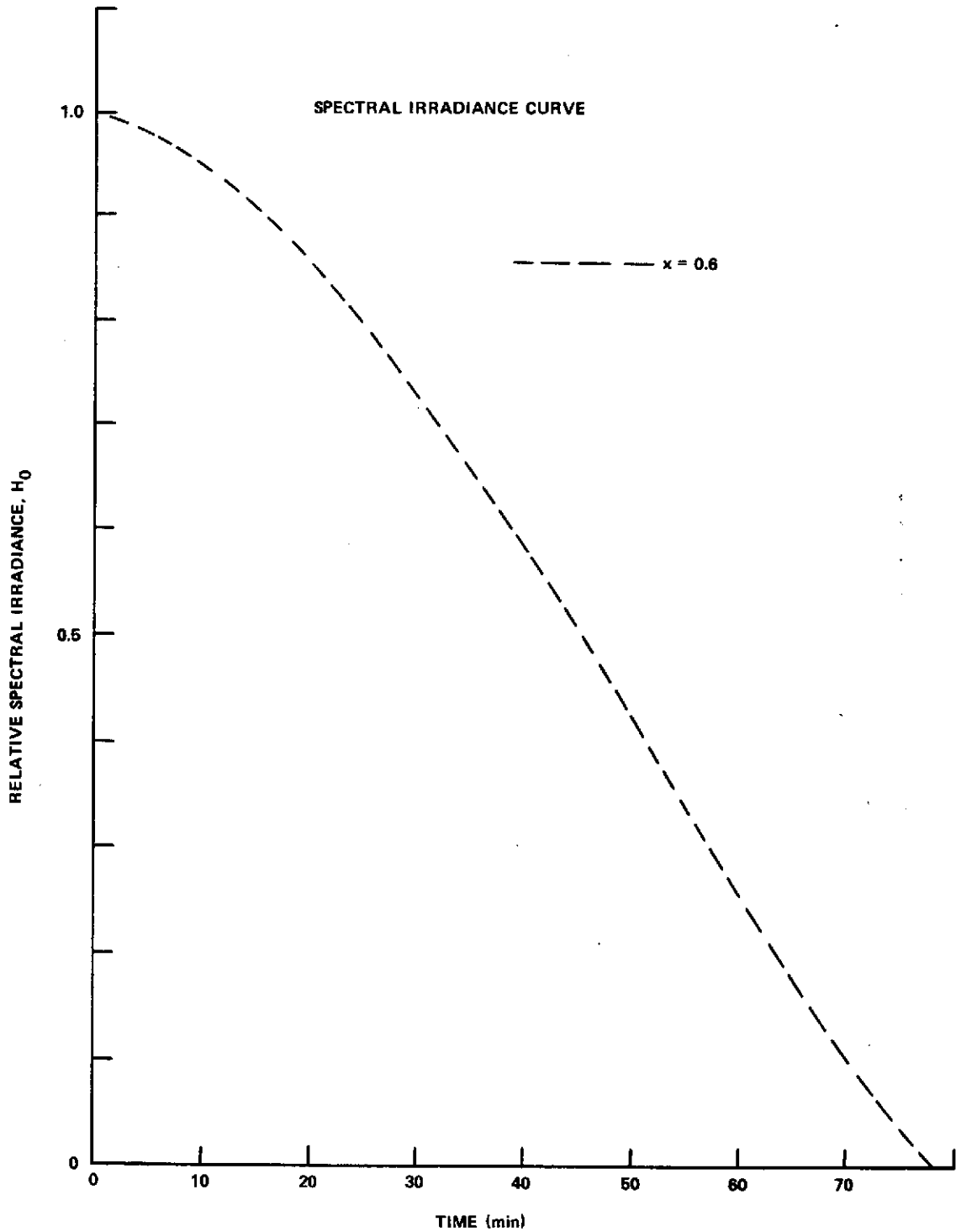


Figure 2. Computed spectral irradiance curve for the limb-darkening coefficient,  $x = 0.6$ .



## B. Occultation Eclipses Not Central at Maximum

For an observer situated away from the centerline of totality, the relation  $p_0 = -1/k$  does not hold and one has to obtain the values of  $k$  and  $p_0$  either from Duncombe's circular [7] (if listed there directly) or by simple geometry. If the values are listed, then one proceeds as before; otherwise, one may proceed as follows.

1. To obtain  $k$  from geometry (March 7, 1970, eclipse) — For an observer at P (Fig. 3), let the moon be at a zenith distance,  $Z = 36$  deg. The semidiameters of the moon and the sun given in the circular [7] are measured geocentrically and have the following values:  $R_s = 16$  min 6.8 sec,  $R_m = 16$  min 32 sec, respectively. From the earth's surface, the semidiameter of the moon is then given by the relation  $r_m = R_m D/d$  where  $D$  is the instantaneous distance between the centers of the moon and the earth and is given by the relation  $D = D_m (\beta_m / \beta)$ , and  $d = D \sin C / \sin Z$ :

$$r_m = R_m \sin Z / \sin (Z - M) \quad (20)$$

$$k = r_s / r_m = R_s \sin (Z - M) / (R_m \sin M) \quad (21)$$

where

$$\sin M = R_E \sin Z / D \quad (22)$$

For the March 7, 1970, eclipse,  $\beta = 60$  min 39.4 sec and, therefore,  $D = 359\,464$  km (224 665 mi) so that  $k = 0.961$ . Thus, knowing the zenith distance,  $Z$ , of the moon at the time of the eclipse and knowing  $D$ ,  $R_s$ ,  $R_m$  and  $R_E$ , one can determine  $k$  at any location.

2. To obtain the value of  $p_0$  (March 7, 1970, eclipse) — In the eclipsing binary notation,  $p_0$  is the mideclipse value of  $p$  at the centerline. For observers on the northern and southern limits of the track of totality (Fig. 4),

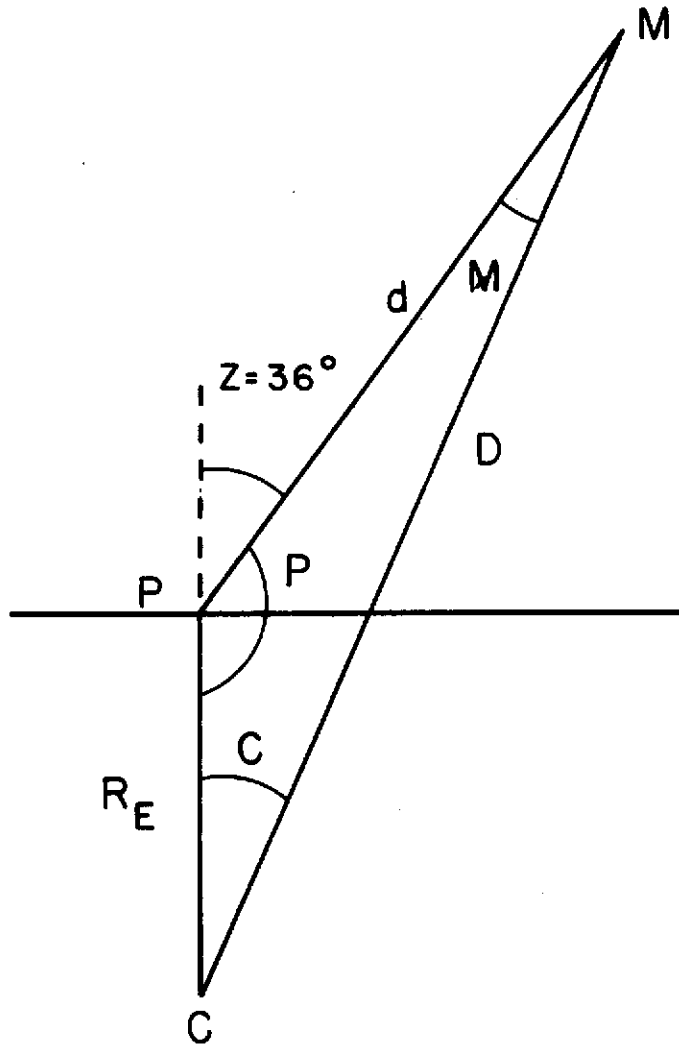


Figure 3. Geometry for deriving the instantaneous apparent diameter of the moon at a given place.

$p_0 = -1.000$ . The magnitude of eclipse values ( $m_E$ ) given by Duncombe [7] for points outside the path of totality, i.e., for the cases of occultation eclipse partial at maximum, are related to the values of the eclipsing binary  $p_0$  by the formula

$$p_0 = (1 - 2 m_E) \quad . \quad (23)$$

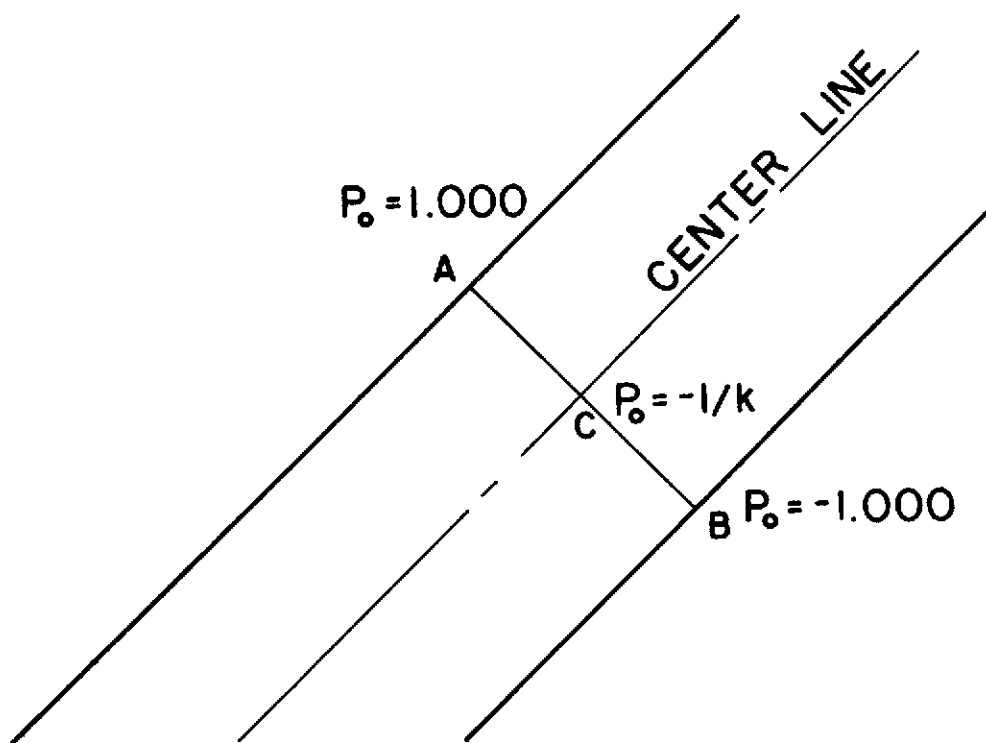


Figure 4. Track of totality (or annularity) for the occultation (or transit) solar eclipse.

If, however, the magnitude  $m_E$  for a place is not given, one can obtain the value of  $p_0$  by interpolating linearly with a distance from  $-1.040$  (at centerline) towards  $-1.000$  (at the edge approximately 44 km away) for a place  $P$  inside the path, or extrapolating beyond  $-1.000$  for a place not too far outside. The distances are measured perpendicular to the track. For places not too far away from the tracks, one can measure off perpendicular distances on the maps (in Duncombe's circular [7]) themselves. For example, consider the case of an observer at Tampa, Florida. Measuring the distances perpendicular to the track one obtains for Tampa: centerline (C.L.) to the edge of the track,  $d_t = 23$  mm, and centerline to Tampa,  $d_P = 77$  mm, so that for Tampa

$$\begin{aligned}
 [p_0]_{\text{Tampa}} &= -[p_0]_{\text{C.L.}} + \frac{d_P}{d_t} ([p_0]_{\text{C.L.}} - 1.00) \\
 &= -1.040 + \frac{77}{22} (0.040) = -0.906 \quad . \quad (24)
 \end{aligned}$$

On the other hand, the magnitude of eclipse at Tampa, as given in the circular [7], is  $m_E = 0.953$  so that

$$[p_0]_{\text{Tampa}} = 1 - 2(0.953) = -0.906, \quad (25)$$

which is in agreement with the result in equation (24).

3. To obtain the value of  $\alpha_0^{\text{oc}}$  needed — If P lies inside the path of totality,  $\alpha_0^{\text{oc}} = 1$  whatever the particular values of  $x$ ,  $k$  and  $p_0$ . If P lies in the large outside region where the eclipse is partial at maximum, an assumption must be made as to the degree of darkening  $\chi^{\text{oc}}$ ; then the needed  $\alpha_0^{\text{oc}}$  is located in the appropriate  $\alpha^{\text{oc}}(k, p)$  table of Princeton 23.

4. If the eclipse does not become total at any point of the earth's surface, then one resorts to the American Ephemeris and the Explanatory Supplement to the Ephemeris, or to local circumstances supplied directly on request by the Naval Observatory.

5. The SIC — Given the angular radii of the sun and moon, the  $x$ ,  $k$ ,  $\alpha_0^{\text{oc}}$  and  $p_0^{\text{oc}}$ , the determination of points ( $n$ ,  $\theta(n)$  or  $\Delta t$ ), and therefore points on the SIC, proceeds as before.

### III. THE TRANSIT ECLIPSE

In transit eclipses, the angular radius of the moon is less than that of the sun. Equations (1) through (13) are still valid but one uses the  $\text{tr}$  tables of Princeton 23, instead of their  $\text{occ}$  counterparts, in the calculations, with  $k^{\text{tr}} = r_m / r_{\text{sn}}$ ; while for occultation eclipses,  $\alpha_0^{\text{oc}} = 1$  at deepest phase for an observer anywhere within the path of totality, in the transit case  $\alpha_0^{\text{tr}} > 1$  for an observer within the path of annularity.

At centrality of the annular case  $k^{tr} = m_E$  and  $p_0^{tr} = -1/m_E$ , where  $m_E$  is the magnitude of eclipse. In the large region of the earth for which the transit eclipse is partial at maximum,  $k$  is obtained by the geometry for the phase (as for occultation eclipses) but  $p_0^{tr}$  is obtained from either  $p_0^{tr} = 1 - 2 m_E/R$  or is read from a map as in the example of equation (25).

All the procedures are essentially formally parallel throughout. Some relations appear different because

$$\alpha^{tr} = \frac{\text{light lost at a given phase of transit eclipse}}{\tau},$$

where  $\tau$  is the light that is or would be lost at internal tangency;  $\tau(x, k)$  is the tabulated in extemo in Princeton 23.

During the annular phases of an eclipse observed from a point within the path of annularity, the amount of light lost at a given instant is strongly dependent on the degree of limb-darkening of the solar disk at the wavelength of observation. For this reason the small section of the light curve (or SIC) during annularity holds special possible interest. For more detailed study of this small section, Princeton 23 contains, in each  $\chi$  section a table headed  $x_n^{ann}$ . It is used by workers in the field of eclipsing binaries, as it would be here, in conjunction with the  $\chi^{tr}$  tables for the same  $\chi$ , so it is placed with those tables. Its mode of use is obvious from its structure.

#### IV. CONCLUSIONS

A fairly direct method for obtaining the spectral irradiance curves for various assumed degrees of limb-darkening for any type of solar eclipse has

been indicated. As mentioned earlier, a permanent set of tables of  $[\chi]^{1/2}$  for relevant values of  $x, k, n$  could very easily be constructed from the appropriate  $\chi$ 's of Princeton 23, and the values could be stored in a computer data bank or on tapes for future use.

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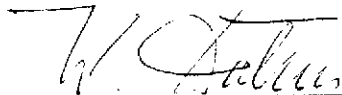
## APPROVAL

# SPECTRAL IRRADIANCE CURVE CALCULATIONS FOR ANY TYPE OF SOLAR ECLIPSE

By Adarsh Deepak and J. E. Merrill

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.



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W. K. DAHM  
Chief, Aerophysics Division



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J. A. LOVINGOOD  
Director, Aero-Astrodynamic Laboratory