

Spectral Narrowing in High-Gain Lasers

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Abstract—The dependence of spectral narrowing in lasers on the line-broadening mechanism is investigated including the effects of saturation and distributed loss. It is found that in the unsaturated regime the narrowing is essentially independent of the resonance broadening mechanism and the narrowed line approaches a Gaussian. The onset of saturation slows or reverses the narrowing process. Experimental results have been obtained using a 3.51- μ xenon laser.

I. INTRODUCTION

SPECTRAL narrowing refers to the fact that under some circumstances radiation incident on a laser amplifier or generated within an amplifier will emerge with a spectrum which is narrower than the one it started with [1]. In high-gain lasers spectral narrowing may be substantial and can provide a highly stable and monochromatic light source. The applications of such optical frequency standards in metrology are well known [2]. Comparison of the input and output spectra of a laser amplifier might provide a sensitive indirect measurement of the amplifier gain.

The purpose of this paper is to study in some detail the influence of an amplifying medium on a spectral continuum including the effects of saturation and distributed loss. Emphasis is placed on the important problem of superradiance, but narrowing in other amplifiers is also considered and limiting line widths are determined.

The starting point for this investigation is a general expression for the incremental gain of a monochromatic signal in a laser amplifier. From the derivation of Gordon *et al.* [3] the intensity of radiation at the frequency ν in a Doppler broadened medium is described by

$$\frac{dI_\nu}{dz} = \frac{kI_\nu}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 y^2)}{[1 + (y - y_i)^2] \left[1 + \frac{sI_\nu}{1 + (y - y_i)^2} \right]} dy \quad (1)$$

where $y(\nu) = 2(\nu - \nu_0)/\Delta\nu_h$ is a normalized frequency, k is a pumping constant, s is a saturation parameter, and the natural damping ratio $\epsilon = (\Delta\nu_h/\Delta\nu_D)\sqrt{\ln 2}$ measures the relative importance of homogeneous and

Doppler broadening. The homogeneous and Doppler line widths are given respectively by $\Delta\nu_h$ and $\Delta\nu_D$. The derivation can be generalized in a straightforward fashion to the interaction with an optical continuum and the result is

$$\frac{dI(y_i)}{dz} = \frac{kI(y_i)}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 y^2)}{[1 + (y - y_i)^2] \left[1 + s \int_{-\infty}^{\infty} \frac{I(y_n) dy_n}{1 + (y - y_n)^2} \right]} dy \quad (2)$$

where $I(y_i)$ is the spectral density at the frequency y_i . This result neglects any coherent interaction between the frequency components of the spectrum. However, it is found in practice that even with monochromatic fields coherence effects are usually unimportant [4] and the neglect of coherence makes possible a number of useful analytic solutions. Also, the extreme saturation that is necessary for coherence effects is generally found to impede the narrowing process, so that in any practical situation strong saturation would be avoided.

II. UNSATURATED AMPLIFIERS

In an unsaturated amplifier the growth of an intensity continuum is governed by

$$\frac{dI(y_i, z)}{dz} = g(y_i)I(y_i, z) - \alpha I(y_i, z) + \eta g(y_i). \quad (3)$$

The unsaturated incremental gain spectrum $g(y_i)$ is, for simplicity, assumed to be independent of the distance z . The second term on the right side of (3) represents distributed losses. The last term is the spontaneous emission, which has the same frequency dependence $g(y_i)$ as the incremental gain. This one-dimensional model is approximately valid in narrow bore amplifiers having a length much greater than the diameter. The coefficient η is then proportional to the spontaneous emission rate and to a geometrical factor that depends on the amplifier dimensions. A more rigorous three-dimensional treatment would have to account for the spatial distribution of the emitting atoms [5], the strong focusing effects that are common in high-gain lasers [6], and reflections at the boundaries of the amplifying medium. Solving for an amplifier of length z yields

$$I(y_i, z) = I(y_i, 0) \exp [(g(y_i) - \alpha)z] + \frac{\eta g(y_i)}{g(y_i) - \alpha} (\exp [(g(y_i) - \alpha)z] - 1). \quad (4)$$

Superradiance will be considered first. In a superradiant source there is no input and if losses are negli-

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gible (4) simplifies to

$$I(y_i, z) = \eta(\exp [g(y_i)z] - 1). \quad (5)$$

Defining $f(y_i)$ as the fraction of the line center intensity at the frequency y_i yields

$$f(y_i, z) = \frac{I(y_i, z)}{I(0, z)} = \frac{\exp [g(y_i)z] - 1}{\exp [g(0)z] - 1}. \quad (6)$$

Most lasers can be classed as either homogeneously or inhomogeneously broadened and only these cases will be considered here.

For homogeneous broadening ($\epsilon \gg 1$) and no saturation the gain from (2) is simply the Lorentzian

$$g(y_i)_{\text{hom}} = \frac{k}{\sqrt{\pi} \epsilon} \frac{1}{1 + y_i^2}. \quad (7)$$

The spectral width, defined as the separation between the two frequencies at which the spectral intensity is down to $\frac{1}{2}$ its peak value, is obtained by combining (7) with (6) for $f = \frac{1}{2}$. The result is

$$\Delta\nu_{\text{hom}} = \Delta\nu_h \sqrt{\frac{kz}{\sqrt{\pi} \epsilon} \frac{1}{\ln \frac{1}{2}(\exp [kz/\sqrt{\pi} \epsilon] + 1)} - 1}. \quad (8)$$

For short distances ($kz/\sqrt{\pi} \epsilon \ll 1$) the line width given by (8) is just the homogeneous line width $\Delta\nu_h$. For long distances ($kz/\sqrt{\pi} \epsilon \gg 1$) (8) simplifies to

$$\Delta\nu_{\text{hom}} \simeq \Delta\nu_h \sqrt{\frac{\sqrt{\pi} \epsilon}{kz} \ln 2} = \Delta\nu_h \sqrt{\frac{\ln 2}{g(0)_{\text{hom}} z}}. \quad (9)$$

Thus the narrowing effect becomes important when the product $g(0)_{\text{hom}} z$ becomes comparable to unity. The approximate result given by (9) becomes valid after the width is narrowed to about one half of its initial value.

For an unsaturated inhomogeneously broadened amplifier ($\epsilon \ll 1$) and for radiation not too far in the wings of the line ($y_i \ll 1/2 \epsilon^2$) the gain from (2) is the Gaussian

$$g(y_i)_{\text{inhom}} = k \exp (-\epsilon^2 y_i^2). \quad (10)$$

Combining this with (6) for $f = \frac{1}{2}$ yields

$$\Delta\nu_{\text{inhom}} = \Delta\nu_D \sqrt{\frac{\ln kz - \ln \ln \frac{1}{2}[\exp (kz) + 1]}{\ln 2}}. \quad (11)$$

For short distance ($kz \ll 1$) the width of the emission is equal to the Doppler width $\Delta\nu_D$. For long distances ($kz \gg 1$) one finds

$$\Delta\nu_{\text{inhom}} \simeq \Delta\nu_D \frac{1}{\sqrt{kz}} = \frac{\Delta\nu_D}{\sqrt{g(0)_{\text{inhom}} z}}. \quad (12)$$

These results suggest that the narrowing proceeds in about the same fashion independent of the line broadening mechanisms. To verify this conclusion, one may consider a completely general incremental gain function $g(y_i)$ with a maximum at the frequency $y_0 = 0$. Assuming that $g(y_i)$ is differentiable in the neighborhood of

zero, it may be expanded as

$$g(y_i) = g_0 - g_2 y_i^2 + g_3 y_i^3 + g_4 y_i^4 + \dots \quad (13)$$

where g_0 and g_2 are positive. The spectrum of the superradiance is given by (6) as

$$\begin{aligned} f(y_i) &= \frac{\exp [(g_0 - g_2 y_i^2 + g_3 y_i^3 + \dots)z] - 1}{\exp [g_0 z] - 1} \\ &= \frac{\exp [g_0 z] \exp \left[-\left(\frac{y_i}{\Delta_2}\right)^2 \right] \exp \left[\left(\frac{y_i}{\Delta_3}\right)^3 \right] \exp [(\dots)^4] \dots - 1}{\exp [g_0 z] - 1} \end{aligned} \quad (14)$$

where $\Delta_n = (z g_n)^{-1/n}$.

At large distances ($g_0 z \gg 1$) the zero-order terms cancel, leaving

$$f(y_i) \simeq \exp \left[-\left(\frac{y_i}{\Delta_2}\right)^2 \right] \exp \left[\left(\frac{y_i}{\Delta_3}\right)^3 \right] \exp [(\dots)^4] \dots \quad (15)$$

Also, at very large distances one finds that $\Delta_n \ll \Delta_{n+1}$ so that the Gaussian factor is much narrower than the others. Consequently, all of the factors but the first may be replaced with their value at line center. Therefore,

$$f(y_i) \simeq \exp \left[-\left(\frac{y_i}{\Delta_2}\right)^2 \right], \quad \Delta_2 = \frac{1}{\sqrt{g_2 z}}. \quad (16)$$

This is in agreement with the previous results for homogeneous and inhomogeneous broadening as may be verified by expanding $g(y_i)$ of (7) and (10) in power series in y_i to obtain g_2 . More generally if the incremental gain has several maxima, the emission will eventually resolve itself into narrowing Gaussian lines centered on the gain maxima. This resolving effect has been observed by Parks *et al.* [7].

So far only superradiance has been considered. If an amplifier has an input $I(y_i, 0)$ and negligible spontaneous emission, (6) for the output light spectrum is replaced by

$$F(y_i, z) = \frac{I(y_i, 0) \exp [g(y_i)z]}{I(0, 0) \exp [g(0)z]}. \quad (17)$$

At large distances this becomes

$$F(y_i, z) \simeq \frac{I(y_i, 0)}{I(0, 0)} \exp \left[-\left(\frac{y_i}{\Delta_2}\right)^2 \right], \quad \Delta_2 = \frac{1}{\sqrt{g_2 z}}. \quad (18)$$

If the input is reasonably smooth the output spectrum is simply

$$F(y_i, z) \simeq \exp \left[-\left(\frac{y_i}{\Delta_2}\right)^2 \right]. \quad (19)$$

If the input is a narrow Gaussian of width Δ_{in} , the output will be a Gaussian of width Δ_{out} such that

$$\left(\frac{1}{\Delta_{\text{out}}}\right)^2 = \left(\frac{1}{\Delta_{\text{in}}}\right)^2 + \left(\frac{1}{\Delta_2}\right)^2. \quad (20)$$

The concept of gain narrowing is sometimes useful instead of spectral narrowing. The two are obviously

closely related. If one assumes that the input spectrum in (17) is white, then the intensity factors cancel and $F(y_i)$ is a general expression for the gain spectrum of an amplifier normalized to unity on line center. Expressions for the gain line width, for example, are then obtained by setting F equal to one half and solving (17) for $y_{1/2}$. For an inhomogeneously broadened amplifier the results may be made to conform with those of Hotz [8].

In summary, one may conclude that in an unsaturated amplifier the width of the amplified spectrum decreases with distance. The shape of the spectrum approaches a Gaussian as the effective part of the gain approaches a quadratic. As an example, in a helium-xenon discharge the gain may be 400 dB/m [5] or nearly $k = 100$ so that the spectrum of a superradiant source 1 m long would be narrowed according to (12) by a factor of ten provided saturation did not occur. The Doppler width of xenon at room temperature is about $\Delta\nu_D = 100$ MHz, so the narrowed radiation would have a width of about 10 MHz. Since the frequency of the 3.51- μ transition is about 10^8 MHz, the light would be monochromatic to a part in 10^7 . Collimation in a long high-gain gas laser is taken care of by the gain profile of the medium itself [6] and there should not be much difficulty in constructing a gas discharge amplifier of arbitrary length. However, care is necessary to prevent saturation as will be shown in the next section.

III. SATURATION EFFECTS

The intensity of amplified spontaneous emission can easily reach the saturation level. When this happens the behavior of the spectrum becomes considerably more complicated and for a completely general treatment computer solutions are required. We present here analytic solutions for certain important limiting situations and also some general computer results.

For a homogeneously broadened amplifier all of the factors but the Gaussian may be removed from the integral in (2). The result of the integration is

$$\frac{dI(y_i)}{dz} = \frac{kI(y_i)}{\sqrt{\pi} \epsilon h(z)} \frac{1}{1 + y_i^2} \quad (21)$$

where

$$h(z) = 1 + s \int_{-\infty}^{\infty} \frac{I(y_n) dy_n}{1 + y_n^2} \quad (22)$$

and distributed losses are ignored. Thus the gain profile remains Lorentzian even with saturation, although its amplitude decreases. The quadratic term in the frequency expansion of the gain decreases in magnitude and hence a narrow Gaussian beam will continue to narrow, but at a reduced rate as saturation becomes important. Solving as before yields for large distances

$$\Delta\nu_{\text{hom}} = \Delta\nu_h \sqrt{\sqrt{\pi} \epsilon \ln 2 / k \int \frac{dz}{h(z)}} \quad (23)$$

To proceed, expressions for $h(z)$ must be obtained.

For an intensity spectrum that is narrow compared to $\Delta\nu_h$ (22) becomes

$$h(z) \simeq 1 + s \int_{-\infty}^{\infty} I(y_n) dy_n = 1 + sI_t \quad (24)$$

where I_t is the total intensity. Such a spectrum would be obtained, for example, at large distances ($g(0)_{\text{hom}} z \gg 1$) in a superradiant amplifier. To evaluate $h(z)$, it is first necessary to find expressions for I_t . In this approximation (21) may be integrated over frequency yielding

$$\frac{dI_t}{dz} = \frac{kI_t}{\sqrt{\pi} \epsilon (1 + sI_t)} \quad (25)$$

Thus in an unsaturated amplifier the intensity grows according to

$$I_t = I_{t0} \exp [kz / \sqrt{\pi} \epsilon] \quad (26)$$

while in a highly saturated amplifier the intensity is governed by

$$I_t = I_{t0} + \frac{kz}{\sqrt{\pi} \epsilon s} \quad (27)$$

The intensity in the saturated amplifier eventually reaches the point at which the loss term $-\alpha I_t$, which was neglected initially, is comparable to the saturated gain term. If losses are included, the intensity is governed by

$$\frac{dI_t}{dz} = \frac{kI_t}{\sqrt{\pi} \epsilon (1 + sI_t)} - \alpha I_t = 0, \quad (28)$$

which for $sI_t \gg 1$ has the steady-state solution

$$I_t = \frac{k}{\sqrt{\pi} \epsilon \alpha s} \quad (29)$$

These results may be collected as

$$I_t = \begin{cases} I_{t0} \exp \left[\frac{kz}{\sqrt{\pi} \epsilon} \right] & \text{lossless} \\ & \text{unsaturated} \\ & \text{regime} \\ I_{t0} + \frac{kz}{\sqrt{\pi} \epsilon s} & \text{lossless} \\ & \text{saturated} \\ & \text{regime} \\ \frac{k}{\sqrt{\pi} \epsilon \alpha s} & \text{loss-limited} \\ & \text{saturated regime.} \end{cases} \quad (30)$$

They are similar to Rigrod's solutions [9] for monochromatic radiation at gain center.

Using (24) and (30), (23) may be written for large distances as

$$\frac{\Delta\nu_{\text{hom}}}{\Delta\nu_h} = \begin{cases} \sqrt{\frac{\sqrt{\pi}\epsilon}{kz} \ln 2} & \text{lossless} \\ & \text{unsaturated} \\ & \text{regime} \\ \sqrt{\frac{\ln 2}{\ln \frac{kz}{\sqrt{\pi}\epsilon}}} & \text{lossless} \\ & \text{saturated} \\ & \text{regime} \\ \sqrt{\frac{\ln 2}{\alpha z}} & \text{loss-limited} \\ & \text{saturated regime.} \end{cases} \quad (31)$$

These results are valid provided that the spectrum is much narrower than the homogeneous line width before saturation sets in. As the gain is pulled down by saturation, the narrowing rate is slowed. When losses become important, the gain curve is clamped and the narrowing speeds up again. In a homogeneously broadened amplifier neither saturation nor losses stop the narrowing process. In Fig. 1 are some numerical solutions for the line width of the emission from a homogeneously broadened super-radiant laser amplifier as a function of the normalized distance $Z_{\text{hom}} = kz/(\sqrt{\pi}\epsilon \ln 2)$. For simplicity losses are assumed to be negligible. The parameter in these plots is the product $\xi = s\eta$.

For an inhomogeneously broadened amplifier ($\epsilon \ll 1$) it will be assumed that the intensity is nearly uniform over a natural line width. Then the intensity spectrum may be removed from the denominator integral in (2) and the result simplifies to

$$\frac{1}{I(y_i)} \frac{dI(y_i)}{dz} = \frac{k \exp[-\epsilon^2 y_i^2]}{1 + \pi s I(y_i)}. \quad (32)$$

The spectral density $I(y_i)$ is found in the various regions from (32) in a manner essentially identical to the homogeneous case. The results are

$$I(y_i, z) = \begin{cases} I(y_i, 0) \exp[kz \exp[-\epsilon^2 y_i^2]] & \text{lossless} \\ & \text{unsaturated} \\ & \text{regime} \\ I(y_i, 0) + \frac{kz}{\pi s} \exp[-\epsilon^2 y_i^2] & \text{lossless} \\ & \text{saturated} \\ & \text{regime} \\ \frac{k}{\pi \alpha s} \exp[-\epsilon^2 y_i^2] & \text{loss-limited} \\ & \text{saturated} \\ & \text{regime} \end{cases} \quad (33)$$

and consequently the spectral width for large distances is

$$\frac{\Delta\nu_{\text{inhom}}}{\Delta\nu_D} = \begin{cases} \frac{1}{\sqrt{kz}} & \text{lossless} \\ & \text{unsaturated} \\ & \text{regime} \\ 1 & \text{lossless} \\ & \text{saturated} \\ & \text{regime} \\ 1 & \text{loss-limited} \\ & \text{saturated regime.} \end{cases} \quad (34)$$

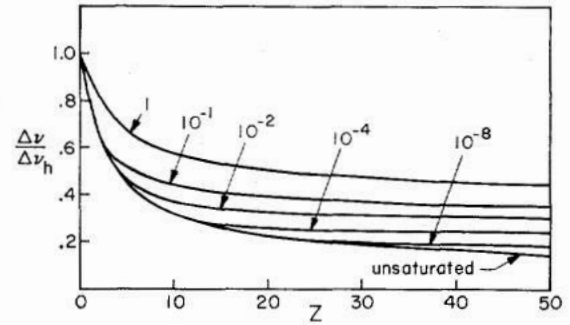


Fig. 1. Superradiant narrowing in a homogeneously broadened amplifier for various values of $\xi = s\eta$.

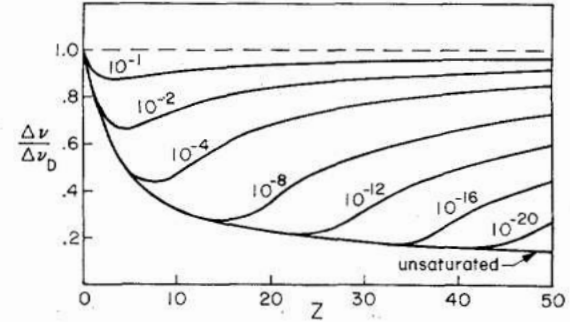


Fig. 2. Superradiant narrowing in a Doppler broadened amplifier for various values of $\xi = s\eta$.

It is evident that the effects of saturation on narrowing in an inhomogeneously broadened amplifier are significantly different from the effects in a homogeneously broadened amplifier. In the inhomogeneous case the onset of saturation reverses the narrowing process and restores the radiation to its Doppler line shape. This occurs because the center of the line saturates first, while the wings continue to grow exponentially. This analysis is valid provided that the intensity spectrum remains broad compared to the homogeneous line width. If the spectrum becomes narrow compared to $\Delta\nu_h$, the rebroadening would be expected to occur more slowly. Some numerical results for superradiant narrowing in an inhomogeneously broadened laser are shown in Fig. 2.

The minimum line width for a simple inhomogeneous superradiant source may be readily calculated. From (32) it is evident that saturation becomes important when $I(y_0) = 1/\pi s$. Then, using (5), one finds that saturation occurs at a distance z_{sat} given approximately by

$$g(0)z_{\text{sat}} = kz_{\text{sat}} = \ln\left(1 + \frac{1}{\pi s\eta}\right) \approx -\ln \pi s\eta. \quad (35)$$

Use of this expression in (12) yields the minimum line width

$$\Delta\nu_{\text{inhom min}} = \frac{\Delta\nu_D}{\sqrt{-\ln \pi s\eta}}. \quad (36)$$

Thus, if, for example, $\eta \simeq 5 \times 10^{-6}$ W/m² and $s \simeq 6 \times 10^{-9}$ m²/W in a xenon laser, then the Doppler line can be narrowed by at most a factor of about 5.5 before saturation becomes important at a distance (for $k \simeq 15$) of about 2.0 m. These values for η and s are judged to be reasonably valid for a simple xenon laser used in some of our experiments based on the data of Clark [10]. This narrowing is not too impressive, and decreasing η by orders of magnitude does not help much, since the dependence on η involves a logarithm and a square root.

A possible scheme for reducing the ultimate line width is to place attenuators between sections of the amplifying medium [11]. These would cut down the intensity to prevent saturation without affecting the narrowing process. Even in such a system, however, the spectral line width could never approach zero because there is always broadband background noise being added to the beam by spontaneous emission. The result of the background is that the spectrum must eventually approach a narrow limiting line shape.

The narrowest possible line would be obtained in a long amplifier with distributed losses which are just sufficient to keep the line center intensity somewhat below the saturation intensity $1/\pi s$. To get an estimate of this limiting line shape one can write (3) for steady state with $I(y_0) = 1/\pi s$

$$0 = \frac{g_0}{\pi s} - \frac{\alpha}{\pi s} + \eta g_0. \quad (37)$$

Thus, the appropriate value for the loss constant α is

$$\alpha = g_0(1 + \pi s \eta). \quad (38)$$

Using this result, (3) away from line center can be written at steady state as

$$0 = g(y_i)I(y_i) - g_0(1 + \pi s \eta)I(y_i) + \eta g(y_i) \quad (39)$$

with the solution

$$I(y_i) = \frac{\eta g(y_i)}{g_0(1 + \pi s \eta) - g(y_i)} \simeq \frac{\eta}{1 + \pi s \eta - \frac{g(y_i)}{g_0}}. \quad (40)$$

Keeping the second-order term in the power-series expansion of $g(y_i)$ leads finally to the intensity spectrum

$$I(y_i) = \frac{1/\pi s}{1 + (g_2/\pi s \eta g_0)y_i^2}. \quad (41)$$

Therefore, the narrowest possible line is a Lorentzian of width

$$\Delta\nu_{\min} = \Delta\nu_h \sqrt{\frac{\pi s \eta g_0}{g_2}}. \quad (42)$$

If the gain profile is the Gaussian given by (10) then $g_2 = g_0 \epsilon^2$ and the line width is simply

$$\Delta\nu_{\min} = \frac{\Delta\nu_h}{\epsilon} \sqrt{\pi s \eta} = \Delta\nu_D \sqrt{\frac{\pi s \eta}{\ln 2}}. \quad (43)$$

Using the approximate numbers given previously for s and η , one finds that the Doppler line would be narrowed by a factor of about 4×10^{-7} . A Doppler width of $\Delta\nu_D \simeq 10^8$ Hz could yield an intensity spectrum of about 40-Hz width. If the oscillation frequency were about 10^{14} Hz as in xenon, the output could be used as an absolute frequency standard with a stability of about four parts in 10^{13} . Similar calculations can be carried out for the limiting line shape in a laser incorporating discrete rather than continuous losses.

The preceding discussion suggests that superradiant lasers could be useful as extremely stable frequency standards. Some practical limitations on such a system should be emphasized. The intensity only approaches its limiting form at a rate given by (12). Thus, to obtain a line width of 40 Hz for a gain constant of $k \simeq 100$ m⁻¹ the overall length of the laser would have to be greater than 10^5 m. However, higher gain media are available and for some applications superradiant lasers should be useful as absolute frequency standards.

IV. EXPERIMENT

We describe here an experiment that has been performed in an effort to verify some of the conclusions of the previous sections regarding spectral narrowing. The apparatus used for this study consisted of a dc xenon discharge having an active region of 5.5 mm diameter and 1.1-m length. The xenon pressure was maintained at about 5 μ by means of a liquid nitrogen trap [12]. The laser was operated as a single mirror superradiant source.

To measure the width of the chopped superradiant output, we allowed it to impinge on an InAs junction detector. The resulting current was then fed into a conventional RF spectrum analyzer followed by a lock-in amplifier. It can be shown that under these conditions a Gaussian optical intensity spectrum $I(\xi)$ will give rise to a low frequency spectrum [13].

$$s(\nu) \propto \int_0^\infty I(\xi)I(\xi + \nu) d\xi, \quad (44)$$

which is also Gaussian and whose width is larger by $\sqrt{2}$ than that of $I(\xi)$.

A typical spectrum is shown in Fig. 3 for a current of 80 mA. This current corresponds to a gain of about $k \simeq 12$ m⁻¹. The spectrum is approximately Gaussian in shape with a width of about 20 MHz. Therefore, the width of the original intensity spectrum is about 14 MHz.

The double-pass length of the laser amplifying medium is 2.2 m. Using (12), the intensity spectrum should be narrowed from the Doppler width by the factor $\sqrt{kz} = \sqrt{(12)(2.2)} = 5.14$. If the Doppler width were $\Delta\nu_D = 100$ MHz, then the intensity spectral width should be 19.5 MHz which is in satisfactory agreement with the measured width of 14 MHz. The discrepancy, if significant, could be due to a small amount of nonresonant feedback scattered back into the laser. Care was also necessary to

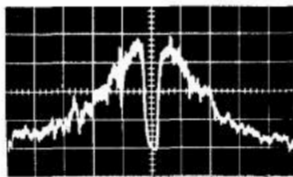


Fig. 3. Superradiant output spectrum for an 80-mA discharge current with 5-MHz/div dispersion.

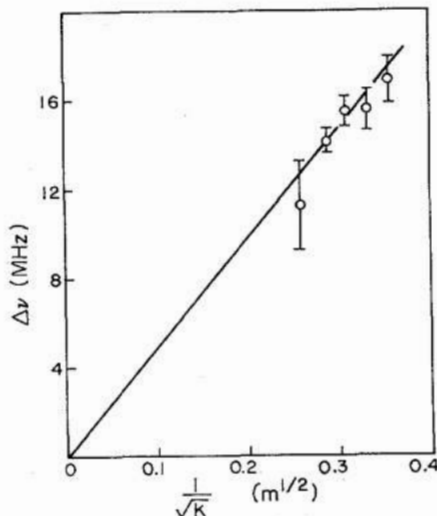


Fig. 4. Spectral width versus $k^{-1/2}$.

prevent resonant feedback, which could result, for example, from reflections off the detector. Resonant feedback led to longitudinal modes, which could be detected as periodic intensity fluctuations as the single mirror was scanned longitudinally.

Some experimental data are collected in Fig. 4. According to (12) the superradiant line width should be inversely proportional to the square root of the gain constant, which is in satisfactory agreement with the data. Line width data have not been obtained for lower values of gain because of the limited sensitivity of our detection system. At higher levels of gain the laser amplifier inevitably began to oscillate so that again meaningful data could not be obtained. These are only preliminary results and more refined experiments are in progress.

V. CONCLUSION

It has been shown in this paper that a gain profile which is quadratic in frequency near its maximum can support a narrowing Gaussian radiation spectrum. The

spectral width varies inversely as the square root of distance at long distances. In a homogeneously broadened amplifier saturation slows the narrowing process, while in an inhomogeneously broadened amplifier saturation restores the line to its original inhomogeneous line shape. Narrowed amplifiers are useful in spectroscopy, gain measurements, and anywhere a stable absolute frequency standard is needed.

Analogous results are obtained for laser oscillators if the coordinate z is replaced by the time coordinate ct/n_0 . One finds that the spectrum of an abruptly started laser oscillator should be a narrowing Gaussian that approaches ultimately a narrow limiting Lorentzian line because of spontaneous emission. The narrowing in this case results from the Fabry-Perot transmission resonances of the oscillator. Nonresonant oscillators operated near threshold may also be useful as frequency standards. Nonresonance can be obtained by making the cavity long so that the modes are closely spaced or by replacing one of the cavity mirrors with a scatterer [14].

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