# Spectral Problems Associated with Corner Singularities of Solutions to Elliptic Equations 

V. A. Kozlov<br>V. G. Maz'ya<br>J. Rossmann

## Contents

Introduction ..... 1
Part 1. Singularities of solutions to equations of mathematical physics ..... 7
Chapter 1. Prerequisites on operator pencils ..... 9
1.1. Operator pencils ..... 10
1.2. Operator pencils corresponding to sesquilinear forms ..... 15
1.3. A variational principle for operator pencils ..... 21
1.4. Elliptic boundary value problems in domains with conic points: some basic results ..... 26
1.5. Notes ..... 31
Chapter 2. Angle and conic singularities of harmonic functions ..... 35
2.1. Boundary value problems for the Laplace operator in an angle ..... 36
2.2. The Dirichlet problem for the Laplace operator in a cone ..... 40
2.3. The Neumann problem for the Laplace operator in a cone ..... 45
2.4. The problem with oblique derivative ..... 49
2.5. Further results ..... 52
2.6. Applications to boundary value problems for the Laplace equation ..... 54
2.7. Notes ..... 57
Chapter 3. The Dirichlet problem for the Lamé system ..... 61
3.1. The Dirichlet problem for the Lamé system in a plane angle ..... 64
3.2. The operator pencil generated by the Dirichlet problem in a cone ..... 74
3.3. Properties of real eigenvalues ..... 83
3.4. The set functions $\Gamma$ and $F_{\nu}$ ..... 88
3.5. A variational principle for real eigenvalues ..... 91
3.6. Estimates for the width of the energy strip ..... 93
3.7. Eigenvalues for circular cones ..... 97
3.8. Applications ..... 100
3.9. Notes ..... 105
Chapter 4. Other boundary value problems for the Lamé system ..... 107
4.1. A mixed boundary value problem for the Lamé system ..... 108
4.2. The Neumann problem for the Lamé system in a plane angle ..... 120
4.3. The Neumann problem for the Lamé system in a cone ..... 125
4.4. Angular crack in an anisotropic elastic space ..... 133
4.5. Notes ..... 138
Chapter 5. The Dirichlet problem for the Stokes system ..... 139
5.1. The Dirichlet problem for the Stokes system in an angle ..... 142
5.2. The operator pencil generated by the Dirichlet problem in a cone ..... 148
5.3. Properties of real eigenvalues ..... 155
5.4. The eigenvalues $\lambda=1$ and $\lambda=-2$ ..... 159
5.5. A variational principle for real eigenvalues ..... 168
5.6. Eigenvalues in the case of right circular cones ..... 175
5.7 . The Dirichlet problem for the Stokes system in a dihedron ..... 178
5.8. Stokes and Navier-Stokes systems in domains with piecewise smooth boundaries ..... 192
5.9. Notes ..... 196
Chapter 6. Other boundary value problems for the Stokes system in a cone ..... 199
6.1. A mixed boundary value problem for the Stokes system ..... 200
6.2. Real eigenvalues of the pencil to the mixed problem ..... 212
6.3. The Neumann problem for the Stokes system ..... 223
6.4. Notes ..... 225
Chapter 7. The Dirichlet problem for the biharmonic and polyharmonic equations ..... 227
7.1. The Dirichlet problem for the biharmonic equation in an angle ..... 229
7.2. The Dirichlet problem for the biharmonic equation in a cone ..... 233
7.3. The polyharmonic operator ..... 239
7.4. The Dirichlet problem for $\Delta^{2}$ in domains with piecewise smooth boundaries ..... 246
7.5. Notes ..... 248
Part 2. Singularities of solutions to general elliptic equations and systems ..... 251
Chapter 8. .The Dirichlet problem for elliptic equations and systems in an angle ..... 253
8.1. The operator pencil generated by the Dirichlet problem ..... 254
8.2. An asymptotic formula for the eigenvalue close to $m$ ..... 263
8.3. Asymptotic formulas for the eigenvalues close to $m-1 / 2$ ..... 265
8.4. The case of a convex angle ..... 272
8.5. The case of a nonconvex angle ..... 275
8.6. The Dirichlet problem for a second order system ..... 283
8.7. Applications ..... 286
8.8. Notes ..... 291
Chapter 9. Asymptotics of the spectrum of operator pencils generated by general boundary value problems in an angle ..... 293
9.1. The operator pencil generated by a regular boundary value problem ..... 293
9.2 . Distribution of the eigenvalues ..... 299
9.3. Notes ..... 305
Chapter 10. The Dirichlet problem for strongly elliptic systems in particular cones ..... 307
10.1. Basic properties of the operator pencil generated by the Dirichlet problem ..... 308
10.2. Elliptic systems in $\mathbb{R}^{n}$ ..... 313
10.3. The Dirichlet problem in the half-space ..... 319
10.4. The Sobolev problem in the exterior of a ray ..... 321
10.5. The Dirichlet problem in a dihedron ..... 332
10.6. Notes ..... 344
Chapter 11. The Dirichlet problem in a cone ..... 345
11.1. The case of a "smooth" cone ..... 346
11:2. The case of a nonsmooth cone ..... 350
11.3. Second order systems ..... 353
11.4. Second order systems in a polyhedral cone ..... 365
11.5. Exterior of a thin cone ..... 368
11.6. A cone close to the half-space ..... 376
11.7. Nonrealness of eigenvalues ..... 383
11.8. Further results ..... 384
11.9. The Dirichlet problem in domains with conic vertices ..... 386
11.10. Notes ..... 387
Chapter 12. The Neumann problem in a cone ..... 389
12.1. The operator pencil generated by the Neumann problem ..... 391
12.2. The energy line ..... 396
12.3. The energy strip ..... 398
12.4. Applications to the Neumann problem in a bounded domain ..... 411
12.5. The Neumann problem for anisotropic elasticity in an angle ..... 414
12.6. Notes ..... 415
Bibliography ..... 417
Index ..... 429
List of Symbols ..... 433

