Survey of Methods in Computer Graphics:

Spectral Surface Reconstruction From Noisy Point Clouds

Ravi Kolluri, Jonathan Shewchuk, James O'Brien. SGP 2004.

Article and presentation slides material from: <u>http://graphics.berkeley.edu/papers/Kolluri-SSR-2004-07/</u>

What is New ?

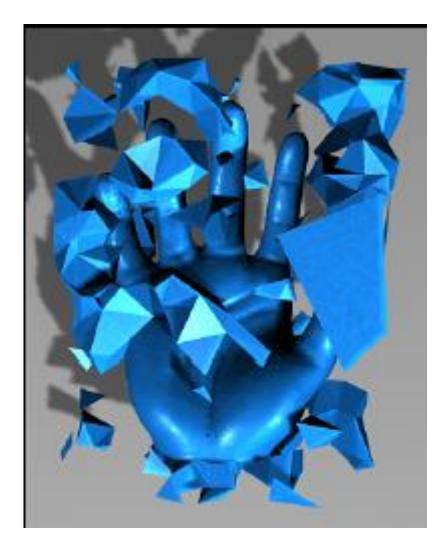
 Previous reconstruction methods based only in local decisions are weak to outliers, noise and undersampling.

 Spectral Surface Reconstruction propose an optimization framework that provides a global view of model. The obtained method is specially robust to outliers.

Spectral Reconstruction (Eigencrust)



Tight-Cocone



1200 Outliers

Method Pipeline

Input:

- Cloud of Points.
- Orientation **not** required.

Output:

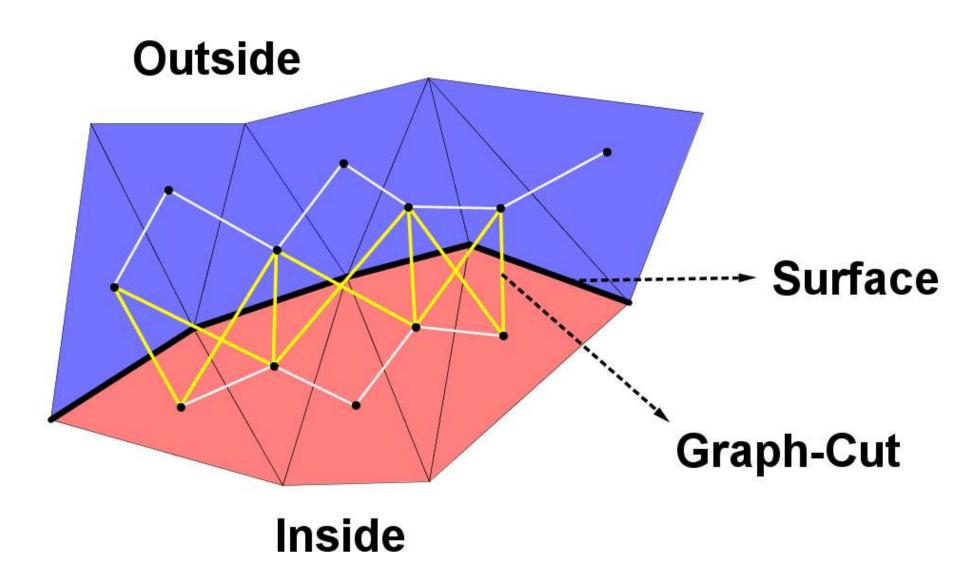
- Triangular Mesh.
- No boundary edges.

Guarantees:

- Always Watertight
- Manifold after post-processing.

Key Insight

Interpret the reconstruction problem as a segmentation of interior- exterior structures. Solve the segmentation problem using a graph-cut approach.

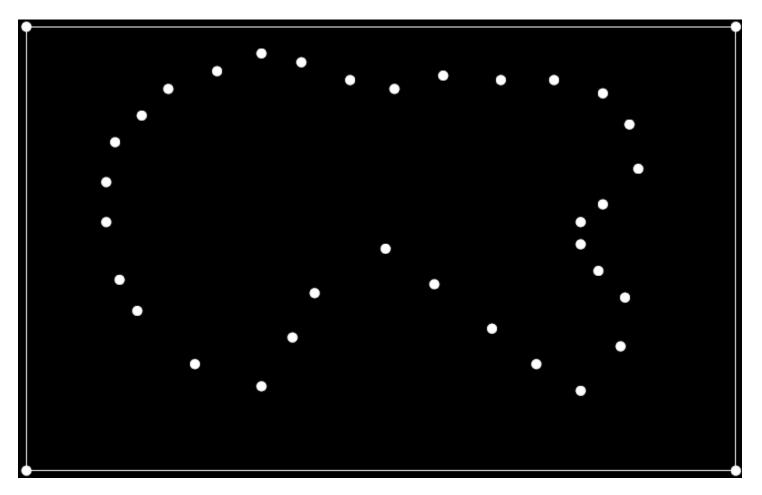


Method Sketch

- 1. Define the space partition structures. They will be classified as Interior or Exterior.
- Set a graph structure over these elements.
 Edge weights measures likelihood of belonging to a same component.
- 3. Find an approximate MIN CUT. This cut defines the structure set classification.
- 4. Post process the segmentation result to guarantee Manifold properties and reduce genus.

Space Partition

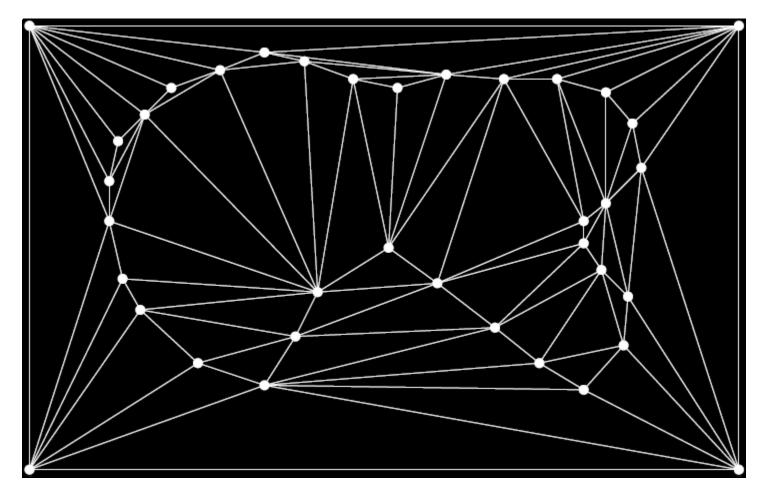
Delanauy Tetrahedralization



¹ Power Cells is another alternative to space partition.

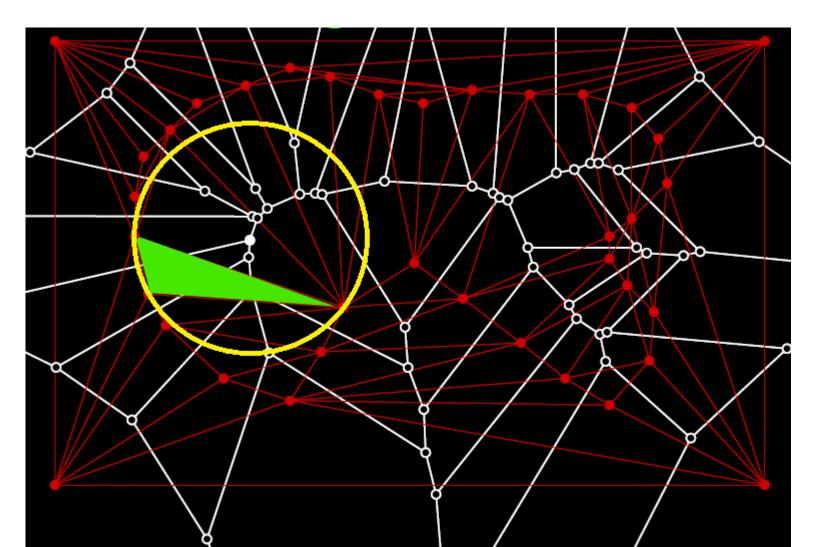
Space Partition

Delanauy Tetrahedralization



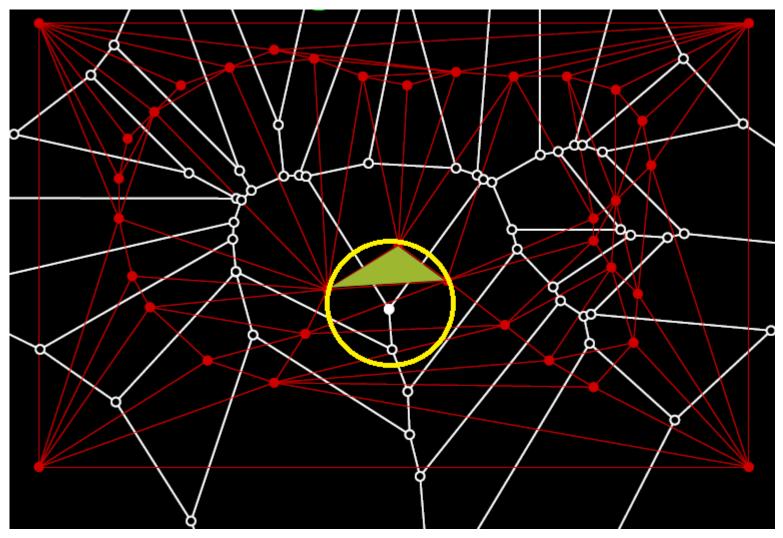
Identifying Reliable Structures

Circumcenters of reliable tetraheadra lay near medial axis



Identifying Reliable Structures

Circumenters of **ambiguous** tetrahedra lay near surface



Tetrahedra Classification Approach

• First: Classify reliable tetrahedra.



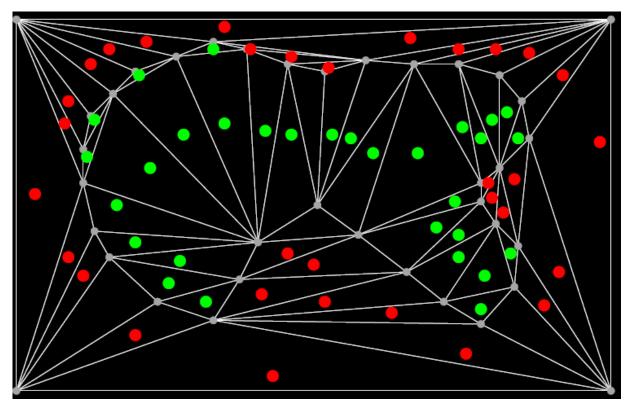
The Pole Graph G

• Second: Classify ambiguos tetrahedra.

Complement Graph H

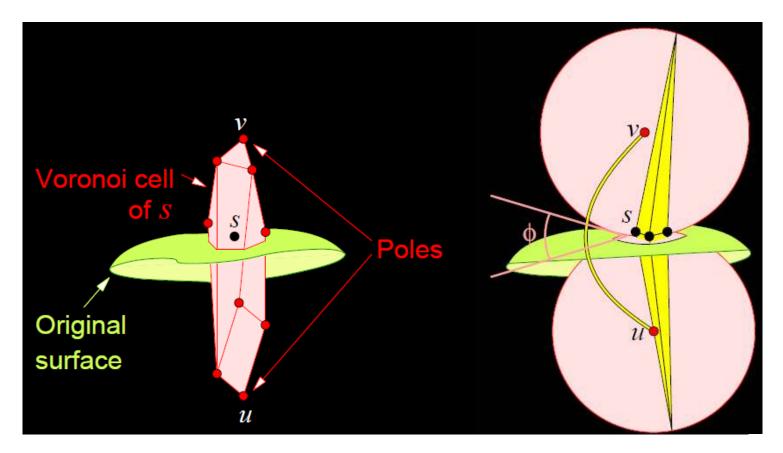
The Pole Graph: Vertices

Poles of Voronoi cells usually lays near medial axis. Therefore tetrahedra associated to poles are usually reliable.



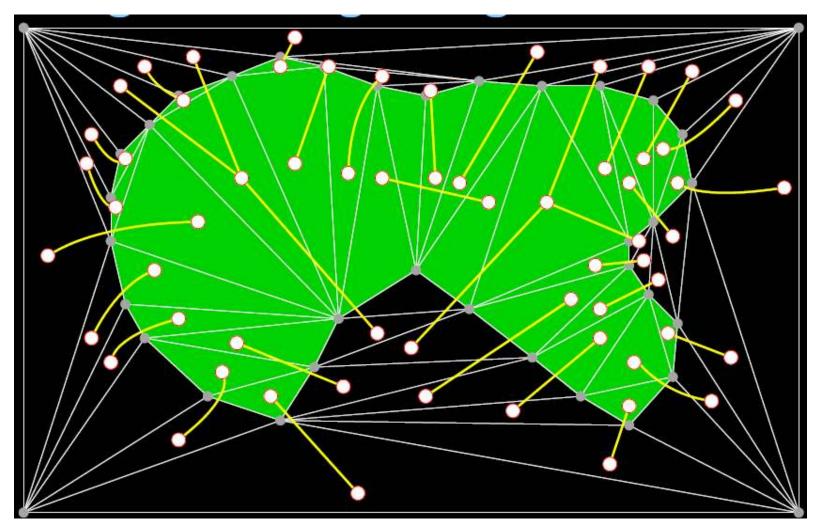
For each of such tetrahedra define a vertex.

Tipe I : Define egdes for tetrahedra associated to poles of a same Voronoi cell.

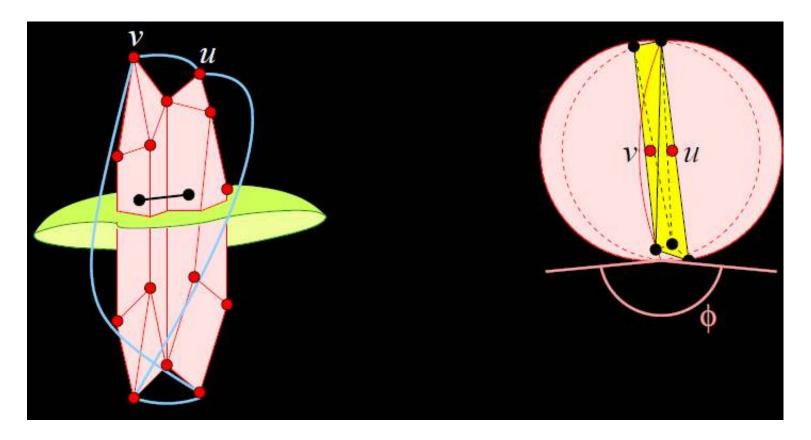


This kind of tetrahedra usually lay in opposite sides of the surface. Assign a negative (repelling) edge between the graph vertices of weight: $-e^{4+4cos\phi}$

Edges of **Type I** are shown in yellow. These are negative (repelling) edges.



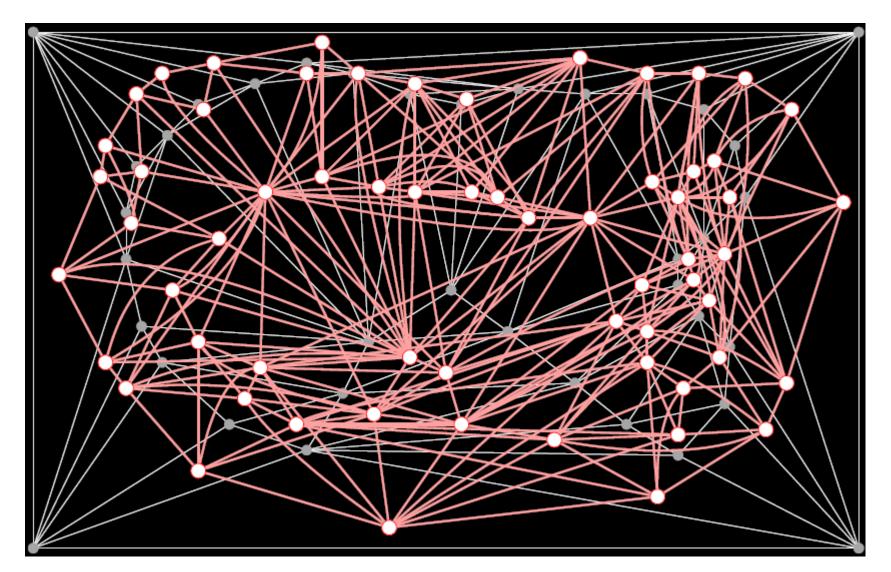
Tipe II : Define egdes for tetrahedra associated to poles of a adjacent Voronoi cells.



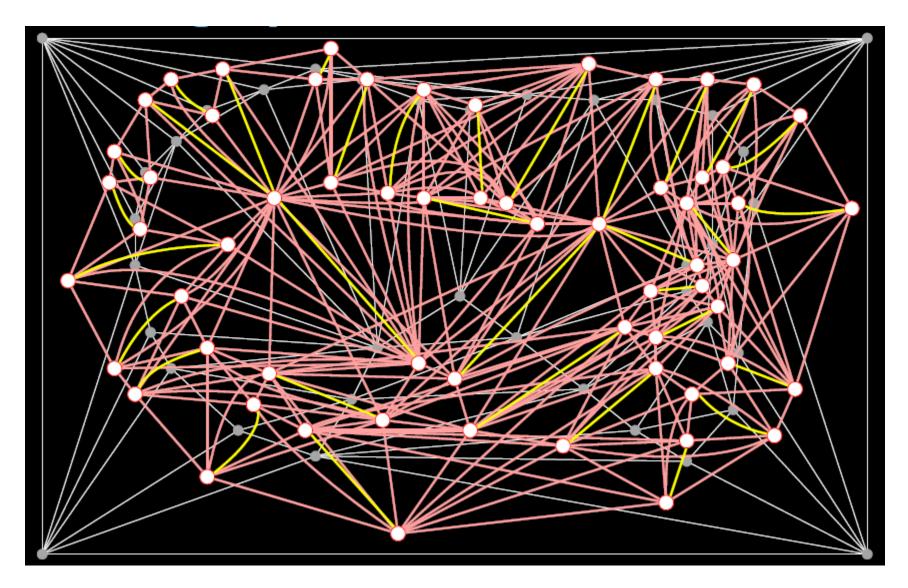
Pair of tetrahedra laying in identical sides usually have large circumsphere intersection. In this case we assign a positive (attractor) edge of weight: $e^{4-4cos\varphi}$

If the circumspheres do not intersect the edge is omitted.

Edges of **Type II** are shown in pink. These are positve (attractor) edges.

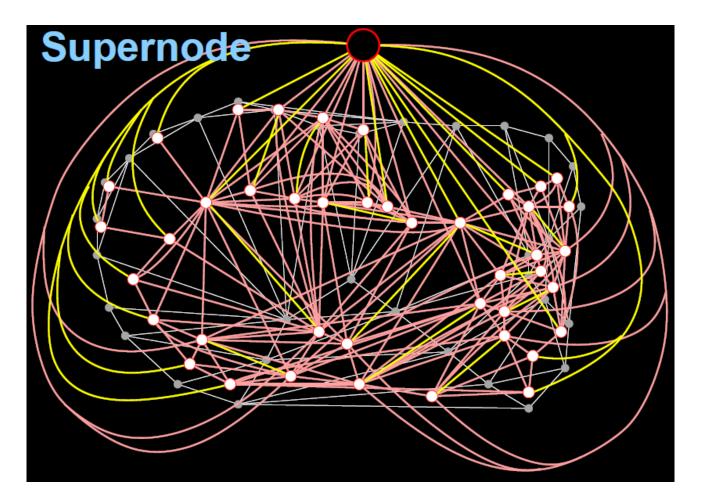


The Pole Graph



The Pole Graph

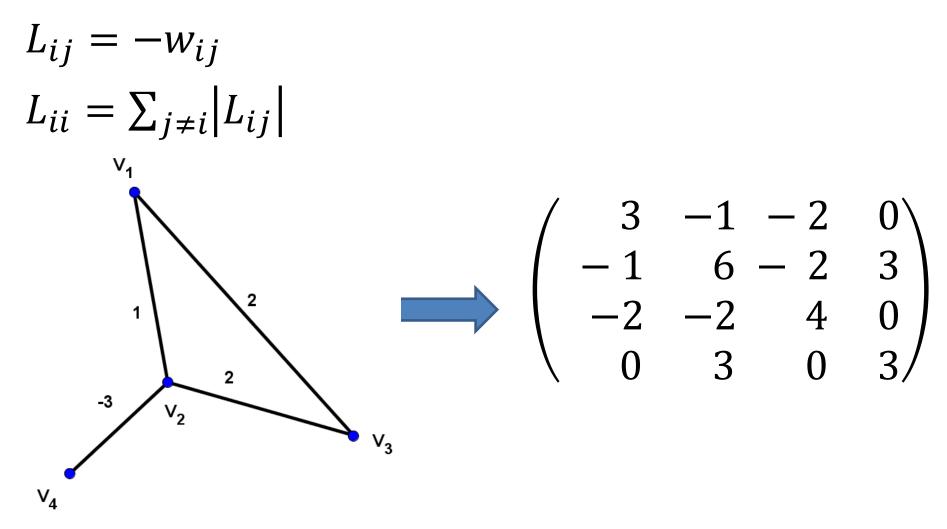
Tetrahedra with vertices at the bounding box are exterior. Collapse them in a single node.



Additional vertices can be collapsed to the exterior (or interior node) by manual seeding.

Graph Cut: Spectral Partitioning

Modified Laplacian Matrix

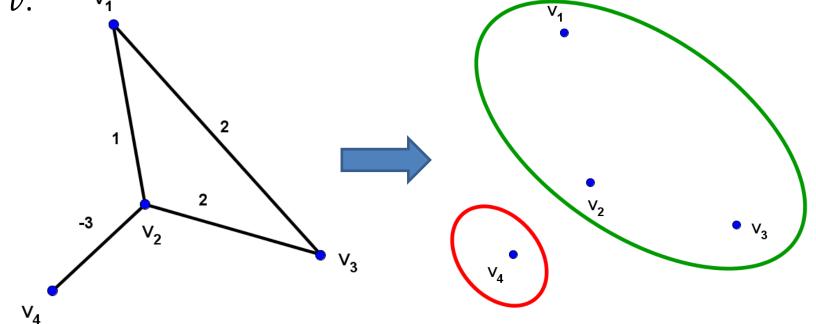


Graph Cut: Spectral Partitioning

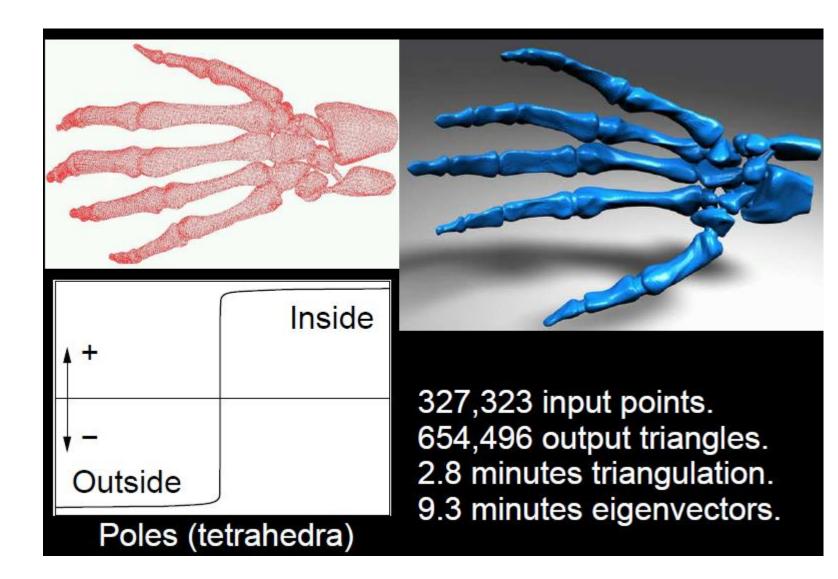
1) Compute an eigenvector \vec{v} of the smallest eigenvalue of L.

$$\vec{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

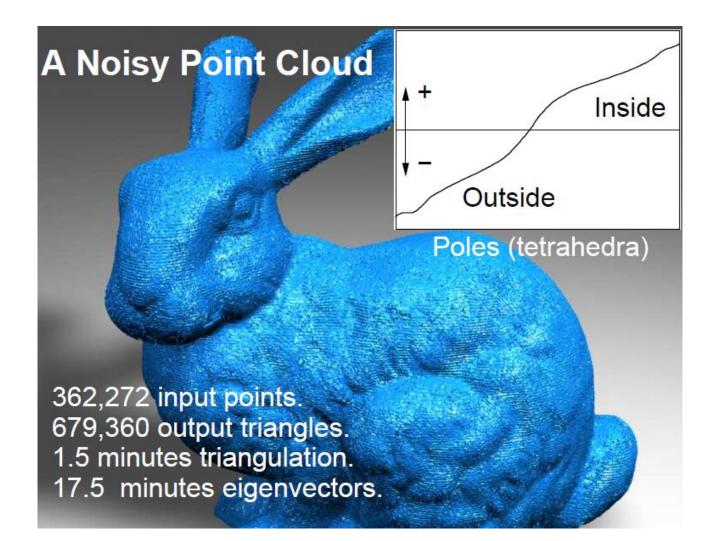
2) Partition the graph according to the sign of respective coordinates in \vec{v} . v_1



Noise Free Example



Noisy Example



Why Spectral Graph Partitioning works?

1) It resembles a system of spring and masses. At lowest frequency the "inside masses are usually found vibratring out of phase with the outside masses".¹

2) It can also be interpreted by transforming the CUT problem to a Total Weight problem.

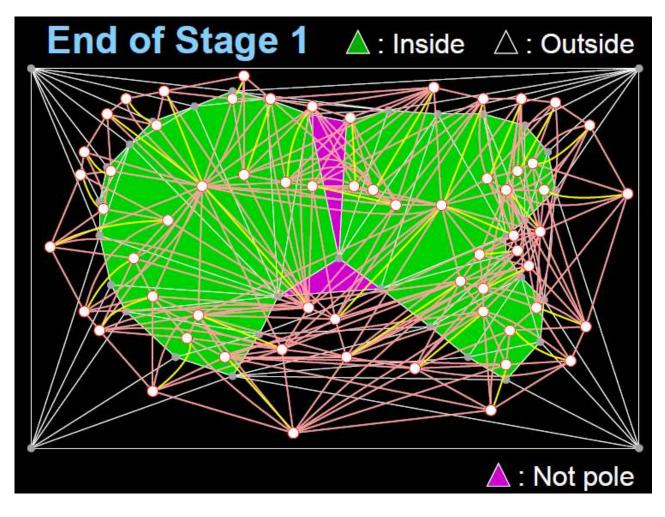
$$\min \sum_{i,j} w_{ij} (1 - v_i v_j) \iff \min \sum_{i,j} -w_{ij} v_i v_j$$

3) The computation of the objective eigenvector can be done using Lanczos iterative solver, which is $O(n\sqrt{n})$.

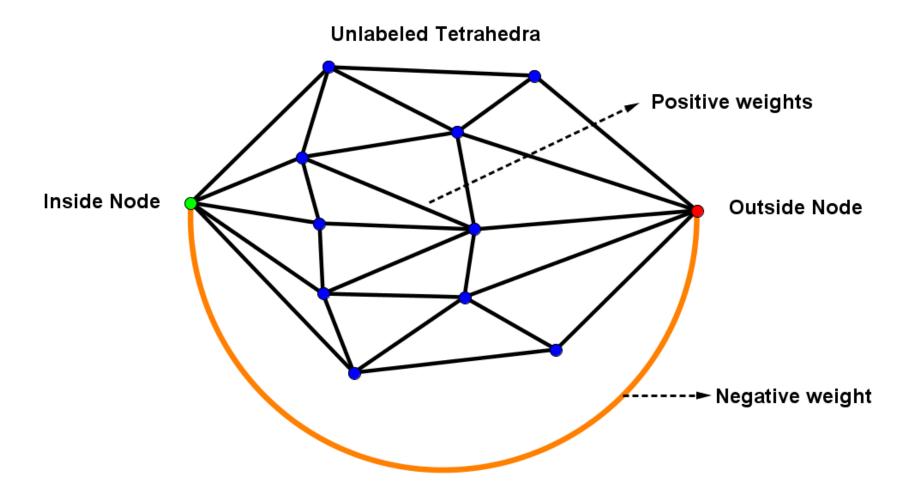
¹ See the link <u>www.cs.berkeley.edu/~jrs/papers/partnotes.pdf</u> for a deeper insight on this interpretation.

Complement Graph

The previous step just labeled tetrahedra associated to poles. Now we have to decide a label for the remaining tetrahedra.

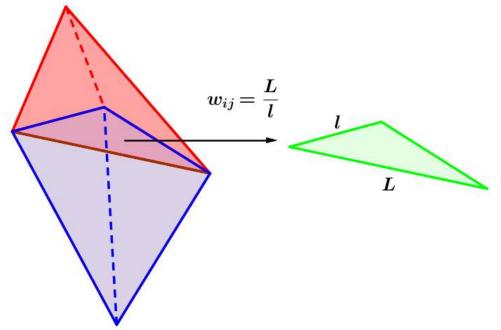


Complement Graph: Vertices and Edges



Cut the graph using spectral partitioning.

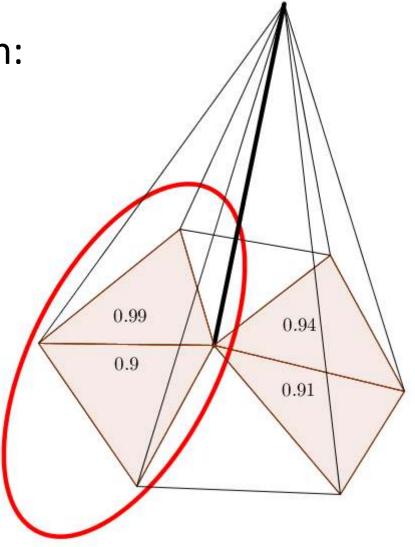
Complement Graph: Weights



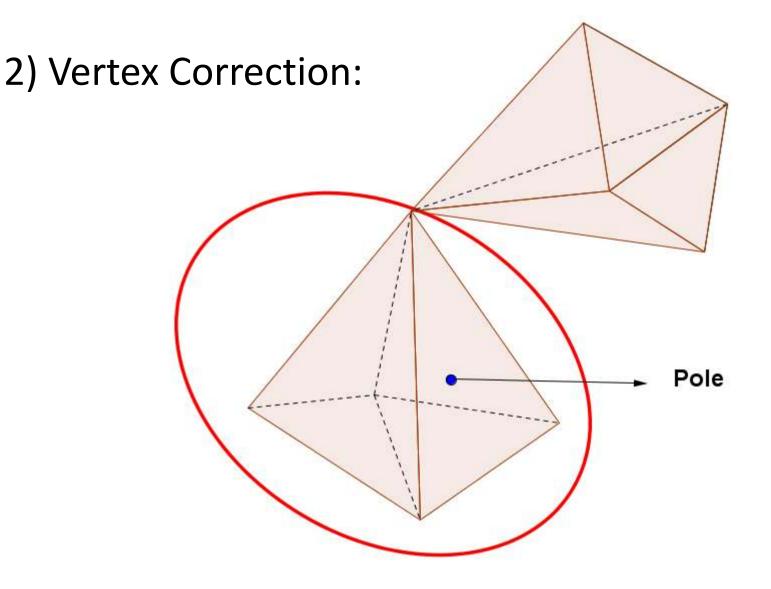
- 1) These kind of weights produced the best results in terms of low genus.
- 2) This also favorates a surface with regular shaped triangles.
- 3) The two supernodes are joined by a negative weighted edge that is the negative of the sum of all other edge weights.

Postprocessing: Make it Manifold

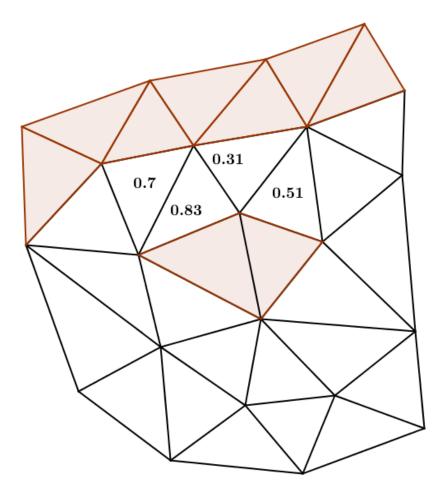
1) Edge Correction:

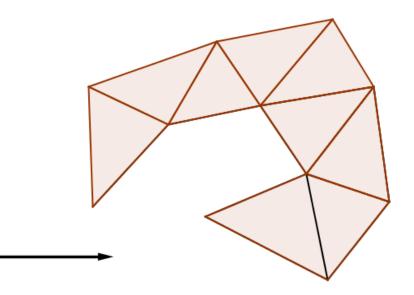


Postprocessing: Make it Manifold

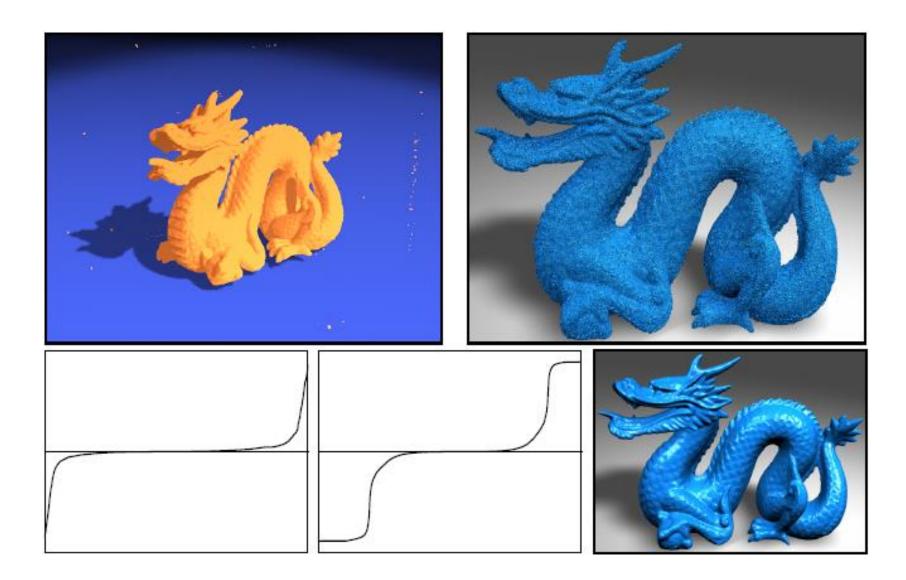


Postprocessing: Genus Reduction



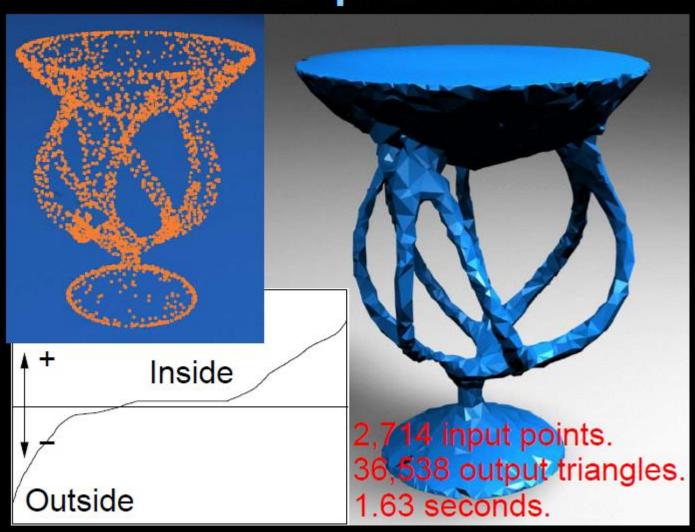


Results



Results

Undersampled Goblet



Authors Observations:

- 1) "It occasionally creates unwanted handles".
- 2) "Eigenvector computation is slow".
- 3) "Tetrahedra labeling algorithms do not reconstruct sharp cornes well".