SPECTRAL THEORY AND DIFFERENTIAL OPERATORS

E. B. DAVIES
Department of Mathematics, King's College, Strand, London

CAMBRIDGE
UNIVERSITY PRESS

Contents

Preface		*page* vii
1	**The fundamental ideas**	1
1.1	Unbounded linear operators	1
1.2	Self-adjointness	7
1.3	Multiplication operators	15
1.4	Relatively bounded perturbations	17
	Exercises	19
2	**The spectral theorem**	22
2.1	Introduction	22
2.2	The Helffer–Sjöstrand formula	24
2.3	The first spectral theorem	32
2.4	Invariant and cyclic subspaces	33
2.5	The L^2 spectral representation	36
2.6	Norm resolvent convergence	41
	Exercises	43
3	**Translation invariant operators**	45
3.1	Introduction	45
3.2	Schwartz space	45
3.3	The Fourier transform	49
3.4	Distributions	52
3.5	Differential operators	57
3.6	Some L^p estimates	62
3.7	The Sobolev spaces $W^{n,2}(\mathbf{R}^N)$	67
	Exercises	70
4	**The variational method**	72
4.1	Classification of the spectrum	72

4.2	Compact operators	75
4.3	Positivity and fractional powers	78
4.4	Closed quadratic forms	81
4.5	The variational formulae	88
4.6	Lower bounds on eigenvalues	93
	Exercises	96
5	**Further spectral results**	99
5.1	The Poisson problem	99
5.2	The heat equation	101
5.3	The Hardy inequality	104
5.4	Singular elliptic operators	109
5.5	The biharmonic operator	113
	Exercises	116
6	**Dirichlet boundary conditions**	118
6.1	Dirichlet boundary conditions	118
6.2	The Dirichlet Laplacian	127
6.3	The general case	135
	Exercises	138
7	**Neumann boundary conditions**	140
7.1	Properties of the $W^{1,2}$ spaces	140
7.2	Neumann boundary conditions	146
7.3	Computation of eigenvalues	150
	Exercises	153
8	**Schrödinger operators**	155
8.1	Introduction	155
8.2	Definition of the operators	156
8.3	The positive spectrum	164
8.4	Compact perturbations	166
8.5	The negative spectrum	168
8.6	Double wells	171
	Exercises	177
Bibliography		178
Notation index		179
Index		180