Spectral Transformations for Two-Dimensional Filters via FFT

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***Abstract*—In this paper, a new fast algorithm for spectral transformations for two-dimensional digital filters is presented. The algorithm is based on the use of the fast Fourier transform. The computational complexity of this algorithm is evaluated. The simplicity and efficiency of the algorithm is illustrated by a numerical example.**

***Index Terms*—Discrete transforms, fast Fourier transforms, spectral transformation, two-dimensional filters.**

I. Introduction

Two-dimensional (2-D) digital filters have found numerous applications within the vast context of 2-D digital signal processing. In the area of real-time image processing, for example, they are used for the enhancement of X-ray images, blood cell analysis, thermography, ultrasound echography, computerized tomography, moving-objects recognition, underwater acoustics, remote-sensing, and robotics. Several implementation algorithms for 2-D digital filters have been widely reported [1]–[3]. Most of these algorithms are based on proper (2-D) rational (also called spectral) transformations. These transformations are useful for designing low-pass, high-pass, band-pass, and multiple passband filters. The bilinear transformation is a special case of the 1-D rational transformation. This transformation appears to have a long list of references in the recent literature [5]–[10].

It is well known that a linear shift-invariant causal single-input single-output (SISO) 2-D system can be described by the transfer function

$$G(s_1, s_2) = \frac{Q(s_1, s_2)}{F(s_1, s_2)} \tag{1}$$

where $Q(s_1, s_2)$ and $F(s_1, s_2)$ are coprime polynomials in the independent complex variables s_1 and s_2. As a unifying approach, the variables s_1 and s_2 are used for the continuous as well as for the discrete 2-D case. A transformation T is a mapping: $\mathbf{C}^2 \to \mathbf{C}^2$ with $s_1 = G_1(z_1, z_2), s_2 = G_2(z_1, z_2)$. Obviously the problem of transformation of the rational function (1) is reduced to transforming the polynomials $Q(s_1, s_2)$ and $F(s_1, s_2)$. Henceforth, we deal only with the 2-D rational transformation of the 2-D polynomial $F(s_1, s_2)$.

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A good many efficient techniques and fast algorithms for 1-D polynomial, and 1-D rational transformations of 1-D and m-D polynomials have been proposed lately. In [4]–[13], the bilinear and related transformations are carried out by using appropriate matrix multiplications. In [14], the same transformations are used in the multidimensional systems stability. In [15], a procedure for transforming a rational function to another rational function under an arbitrary rational transformation via appropriate matrix multiplication is outlined. In [16], the matrix multiplication technique for bilinear transformation of multivariable polynomials is developed as an extension of the corresponding 1-D technique. In [17], the same technique is applied for the discrete system stability. In [18], an efficient algorithm for bilinear transformation of multivariable polynomials is given. In [19], the bilinear transformation is extended to transform multivariable polynomials, using discrete convolution and the Kronecker product. In [20], some properties of the matrix associated with the bilinear transformation are given. In [21], properties of the same matrix in the multidimensional case are discussed. In [22], the structure of the transformation matrix for the general bilinear transformation is examined. A new transformation matrix technique for bilinear transformation is proposed in [23], while in [24] properties of the known transformation matrix are presented. In [25]–[30], in order to achieve the desired transformation the method of the synthetic division is used. In [31], a closed form relationship for the multiple bilinear transformation is used by Erfani, Ahmadi, and Ramachandran.

In [32]–[36], one can find very important applications of several rational transformations. In [37], a very efficient algorithm based on the well-known Horner's form is proposed by Waggener. This is a simple recursive algorithm first for bilinear transformation and secondly for other single-variable rational transformations. Heinen and Siddique extended this technique for any arbitrary 1-D rational transformation [38]. One should note that several authors use the term "polynomial transformation" instead of the term rational transformation. In [40], fast multivariable bilinear and Hadamard transforms are presented.

In [41], an algorithm for any (2-D) spectral transformation of 2-D filters [1]–[3], [39], which is based on the technique of [37] and [38], is presented.

In the present paper, a new algorithm for the same problem is proposed. The algorithm is based on the discrete Fourier transform (DFT). The exploitation of the DFT in similar problems of systems theory is already known. In [42], the DFT is used in order to determine the characteristic polynomial of a rectangular matrix and in [43] the same technique is used for the calculation of a determinantal polynomial. The extension of this technique in 2-D systems is given in [44]. The use of DFT for arbitrary transformations of one-variable polynomials and rational functions is known [45]. So, in the present paper the use of DFT is proposed in order to find a new efficient algorithm for spectral transformations for 2-D filters. The original polynomial under the transformation $T(s_1 = G_1(z_1, z_2), s_2 = G_2(z_1, z_2))$, G_1 and G_2 being rational functions in (z_1, z_2), is transformed to the rational function $(A(z_1, z_2)/B(z_1, z_2))$. In Section II, the Horner's formula for a 2-D polynomial is described. In Section III, the algorithm is fully presented. In Section IV, the computational complexity of the presented algorithms is evaluated. Finally, Section V contains some concluding remarks.

II. The Horner's Form for 2-D Polynomials

In what follows, it is necessary to describe the Horner's formula for 2-D Polynomials [41]. To this end, consider the polynomial $F(s_1, s_2) = \sum_{i_1=0}^{N_1} \sum_{i_2=0}^{N_2} f(i_1, i_2) s_1^{i_1} s_2^{i_2}$. This polynomial can be evaluated using the following 2-D Horner's form

$$F^{(k,l)}(s_1) = F^{(k-1,l)}(s_1) \cdot s_1 + f(N_1 - k, N_2 - l), \quad k = 1, \ldots, N_1 \tag{2}$$

with the initial condition $F^{(0,l)}(s_1) = f(N_1, N_2 - l)$ and

$$\hat{F}^{(l)}(s_1, s_2) = \hat{F}^{(l-1)}(s_1, s_2) \cdot s_2 + F^{(N_1,l)}(s_1, s_2), \quad l = 1, \ldots, N_2 \tag{3}$$

with the initial condition $\hat{F}^{(0)}(s_1, s_2) = F^{(N_1,0)}(s_1, s_2)$. In (2), k is the step of the iteration and l is simply a parameter $(l = 0, 1, \ldots, N_2)$ while in (3), l is the step of the iteration. Finally, $F(s_1, s_2) = \hat{F}^{(N_2)}(s_1, s_2)$.

III. Spectral Transformations for 2-D Filters

Consider the 2-D polynomial

$$F(s_1, s_2) = \sum_{i_1=0}^{N_1} \sum_{i_2=0}^{N_2} f(i_1, i_2) s_1^{i_1} s_2^{i_2} \tag{4}$$

Under the transformation

$$s_1 = G_1(z_1, z_2) = \frac{P_1(z_1, z_2)}{R_1(z_1, z_2)} \tag{5}$$

$$s_2 = G_2(z_1, z_2) = \frac{P_2(z_1, z_2)}{R_2(z_1, z_2)} \tag{6}$$

where

$$P_j(z_1, z_2) = \sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} p_j(i_1, i_2) z_1^{i_1} z_2^{i_2}, \qquad j = 1, 2 \tag{7}$$

and

$$R_j(z_1, z_2) = \sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} r_j(i_1, i_2) z_1^{i_1} z_2^{i_2}, \qquad j = 1, 2 \tag{8}$$

where $F = F(s_1, s_2)$ is transformed to the rational function $(A(z_1, z_2)/B(z_1, z_2))$. Suppose that

$$A(z_1, z_2) = \sum_{i_1=0}^{(N_1+N_2)M_1} \sum_{i_2=0}^{(N_1+N_2)M_2} a(i_1, i_2) z_1^{i_1} z_2^{i_2} \tag{9}$$

$$B(z_1, z_2) = \sum_{i_1=0}^{(N_1+N_2)M_1} \sum_{i_2=0}^{(N_1+N_2)M_2} b(i_1, i_2) z_1^{i_1} z_2^{i_2}. \tag{10}$$

It is easy to verify that $B(z_1, z_2) = R_1^{N_1}(z_1, z_2) R_2^{N_2}(z_1, z_2)$.

In order to compute $A(z_1, z_2), B(z_1, z_2)$, we consider a 2-D discrete Fourier transform at $(r_1 + 1) \times (r_2 + 1)$ equispaced points on the unit 2-D disk where $r_1 = (N_1 + N_2)M_1, r_2 = (N_1 + N_2)M_2$. So, we define

$$w_1 = e^{j2\pi/(r_1+1)} \tag{11}$$

and

$$w_2 = e^{j2\pi/(r_2+1)}. \tag{12}$$

The 2-D DFT for the double sequences of the coefficients $a(i_1, i_2)$ and $b(i_1, i_2)$, in (9) and (10), where $i_1 = 0, \ldots, r_1, i_2 = 0, \ldots, r_2$ is defined as follows:

$$\tilde{b}(k_1, k_2) = \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} b(i_1, i_2) w_1^{i_1 k_1} w_2^{i_2 k_2} \tag{13}$$

$$\tilde{a}(k_1, k_2) = \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} a(i_1, i_2) w_1^{i_1 k_1} w_2^{i_2 k_2}. \tag{14}$$

So, $\tilde{b}(k_1, k_2), \tilde{a}(k_1, k_2)$ can be considered as the polynomial values of $B(z_1, z_2)$ and $A(z_1, z_2)$ at the equally spaced points $z_1 = w_1^{k_1}$, $z_2 = w_2^{k_2}$ on the unit 2-D disc, where $k_1 = 0, \ldots, r_1, k_2 = 0, \ldots, r_2$. So, the polynomial values $\tilde{b}(k_1, k_2)$ are found as follows:

$$\tilde{b}(k_1, k_2) = R_1^{N_1}(z_1, z_2) R_2^{N_2}(z_1, z_2)\Big|_{z_1 = w_1^{k_1}, z_2 = w_2^{k_2}} \tag{15.1}$$

while the coefficients $b(i_1, i_2)$ are evaluated via the inverse 2-D FFT as follows:

$$b(i_1, i_2) = \frac{1}{r_1 + 1} \cdot \frac{1}{r_2 + 1} \sum_{k_1=0}^{r_1} \sum_{k_2=0}^{r_2} \tilde{b}(k_1, k_2) w_1^{-i_1 k_1} w_2^{-i_2 k_2}. \tag{15.2}$$

Therefore, the polynomial $B(z_1, z_2)$ has been computed. Furthermore, the polynomial values $\tilde{a}(k_1, k_2)$ are found by the formula

$$\tilde{a}(k_1, k_2) = B(z_1, z_2)\big|_{z_1 = w_1^{k_1}, z_2 = w_2^{k_2}} \cdot \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} f(i_1, i_2) \times \left(\frac{P_1(z_1, z_2)}{R_1(z_1, z_2)}\right)^{i_1} \left(\frac{P_2(z_1, z_2)}{R_2(z_1, z_2)}\right)^{i_2} \Bigg|_{z_1 = w_1^{k_1}, z_2 = w_2^{k_2}}$$

or equivalently

$$\tilde{a}(k_1, k_2) = \tilde{b}(k_1, k_2) \cdot \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} f(i_1, i_2) \times \left(\frac{P_1(z_1, z_2)}{R_1(z_1, z_2)}\right)^{i_1} \left(\frac{P_2(z_1, z_2)}{R_2(z_1, z_2)}\right)^{i_2} \Bigg|_{z_1 = w_1^{k_1}, z_2 = w_2^{k_2}} \tag{16.1}$$

where Horner's formula for 2-D polynomials should be used.

Using, now, an inverse 2-D FFT, the coefficients $a(i_1, i_2)$ are evaluated as follows:

$$a(i_1, i_2) = \frac{1}{r_1 + 1} \cdot \frac{1}{r_2 + 1} \sum_{k_1=0}^{r_1} \sum_{k_2=0}^{r_2} \tilde{a}(k_1, k_2) w_1^{-i_1 k_1} w_2^{-i_2 k_2}. \tag{16.2}$$

Therefore, the polynomial $A(z_1, z_2)$ has been found.

Remark: In order to transform a given rational function $G(s_1, s_2) = (Q(s_1, s_2)/F(s_1, s_2))$ under the transformation of (5) and (6), the previously developed procedure is applied for $Q(s_1, s_2)$ and $F(s_1, s_2)$ separately. Both these polynomials are transformed using the above algorithm where we are not interested in the common resultant denominator $B(z_1, z_2)$, but only for the different resultant numerators $A_1(z_1, z_2)$ and $A_2(z_1, z_2)$.

Example: Consider the 2-D digital filter with transfer function [2], [39] and [41]:

$$H(Z_1, Z_2) = \frac{F_1(Z_1, Z_2)}{F_2(Z_1, Z_2)} = 0.599\,44 \frac{[1 \quad Z_1] \begin{bmatrix} 1 & -0.046\,68 \\ -0.046\,68 & -0.467\,61 \end{bmatrix} \begin{bmatrix} 1 \\ Z_2 \end{bmatrix}}{[1 \quad Z_1] \begin{bmatrix} 1 & 0.426\,02 \\ 0.426\,02 & -0.106\,92 \end{bmatrix} \begin{bmatrix} 1 \\ Z_2 \end{bmatrix}}.$$

Under the spectral (rational) transformation

$$Z_1 = \frac{-0.2612 + 0.2612 z_1 + 0.2612 z_2 + z_1 z_2}{1 + 0.2612 z_1 + 0.2612 z_2 - 0.2612 z_1 z_2}$$
$$Z_2 = \frac{0.7154 - 0.7154 z_1 - 0.7154 z_2 + z_1 z_2}{1 - 0.7154 z_1 - 0.7154 z_2 + 0.7154 z_1 z_2}$$

the above transfer function is transformed into another transfer function. First, we apply the whole procedure described above for the polynomial of the numerator of $H(Z_1, Z_2)$ for $z_1 = e^{j2\pi(n/3)}, z_2 = e^{j2\pi(m/3)}, n = 0, 1, 2$ and $m = 0, 1, 2$. So, we find the following values that form a double sequence

$$\{\{0.157\,584, 0.459\,614 + 0.116\,335j, 0.459\,614 - 0.116\,335j\}$$
$$\{0.459\,614 + 0.116\,335j, 3.290\,08 - 1.527\,51j, 0.509\,692\}$$
$$\{0.459\,614 - 0.116\,335j, 0.509\,692, 3.290\,08 + 1.527\,51j\}\}.$$

Applying (double) inverse DFT, we find the double sequence

$$\{\{1.066\,18, -0.625\,201, -0.082\,038\,2\}$$
$$\{-0.625\,201, 0.001\,820\,5, 0.589\,87\}$$
$$\{-0.082\,038\,2, 0.589\,87, -0.675\,675\}\}$$

which corresponds to the polynomial of the numerator of the new transfer function. We then apply the same procedure for the polynomial of the denominator of $H(Z_1, Z_2)$ for $z_1 = e^{j2\pi(n/3)}, z_2 = e^{j2\pi(m/3)}, n = 0, 1, 2$ and $m = 0, 1, 2$. So, we find again the following values:

$$\{\{0.626\,389, 0.225\,212 + 0.298\,092j, 0.225\,212 - 0.298\,092j\}$$
$$\{0.225\,212 + 0.298\,092j, 2.671\,03 - 0.8238j, 2.026\}$$
$$\{0.225\,212 - 0.298\,092j, 2.026, 2.671\,03 + 0.8238j\}\}.$$

Applying (double) inverse DFT we find the double sequence

$$\{\{1.213\,48, -0.528\,443, -0.326\,097\}$$
$$\{-0.528\,443, 0.446\,268, 0.388\,004\}$$
$$\{-0.326\,097, 0.388\,004, -0.100\,284\}\}$$

which corresponds to the polynomial of the denominator of the new transfer function.

So, the resultant digital filter is described by the transfer function shown at the bottom of the page. This is a typical transformation of a high-pass 2-D digital filter [2].

IV. Computational Complexity

A. For Finding $B(z_1, z_2)$

Using the 2-D Horner's formula, for finding $R_1(z_1, z_2), M_1M_2 + M_1 + M_2$ complex multiplications and $M_1M_2 + M_1 + M_2$ complex additions are needed. Also, for finding $R_2(z_1, z_2)$, we need $M_1M_2 + M_1 + M_2$ complex multiplications and complex additions. So, in (15.1) there exists a cost of $2(M_1M_2 + M_1 + M_2) + N_1 + N_2 - 1$ complex multiplications as well as a cost of $2(M_1M_2 + M_1 + M_2)$ complex additions. Since $(r_1 + 1)(r_2 + 1)$ number of equation (15.1) are required, i.e., $((N_1 + N_2)M_1 + 1)((N_1 + N_2)M_2 + 1)$, the total cost is finally $((N_1 + N_2)M_1 + 1)((N_1 + N_2)M_2 + 1)(2(M_1M_2 + M_1 + M_2) + N_1 + N_2 - 1)$ complex multiplications as well as $2((N_1 + N_2)M_1 + 1)((N_1 + N_2)M_2 + 1)(M_1M_2 + M_1 + M_2)$ complex additions.

Equation (15.2) requires $(1/4)(r_1 + 1)(r_2 + 1)\log_2(r_1 + 1)\log_2(r_2 + 1)$ complex multiplications, i.e., $(1/4)((N_1 + N_2)M_1 + 1)((N_1 + N_2)M_2 + 1)\log_2((N_1 + N_2)M_1 + 1)\log_2((N_1 + N_2)M_2 + 1)$.

B. For Finding $A(z_1, z_2)$

In (16.1), using the 2-D Horner's formula, for finding $P_1(z_1, z_2)$ we need $M_1M_2 + M_1 + M_2$ complex multiplications and $M_1M_2 + M_1 + M_2$ complex additions. Also, for finding $P_2(z_1, z_2)$ we need $M_1M_2 + M_1 + M_2$ complex multiplications and $M_1M_2 + M_1 + M_2$ complex additions. Note that $R_1(z_1, z_2)$ and $R_2(z_1, z_2)$ are already known from the computation in (15.1). Additionally in (16.1) we need 2 complex divisions. So, we need $2(M_1M_2 + M_1 + M_2) + 2$ CMADs (complex multiplications and divisions). Applying the usual 2-D Horner's formula, one finds $(N_1N_2 + N_1 + N_2) + 2(M_1M_2 + M_1 + M_2 + 1) + 1$ CMAD's and $(N_1N_2 + N_1 + N_2) + 2(M_1M_2 + M_1 + M_2)$ complex additions.

$$H(z_1, z_2) = 0.599\,44 \frac{[1 \quad z_1 \quad z_1^2] \begin{bmatrix} 1.066\,18 & -0.625\,201 & -0.082\,038\,2 \\ -0.625\,201 & 0.001\,820\,5 & 0.589\,87 \\ -0.082\,038\,2 & 0.589\,87 & -0.675\,675 \end{bmatrix} \begin{bmatrix} 1 \\ z_2 \\ z_2^2 \end{bmatrix}}{[1 \quad z_1 \quad z_1^2] \begin{bmatrix} 1.213\,48 & -0.528\,443 & -0.326\,097 \\ -0.528\,443 & 0.446\,268 & 0.388\,004 \\ -0.326\,097 & 0.388\,004 & -0.100\,284 \end{bmatrix} \begin{bmatrix} 1 \\ z_2 \\ z_2^2 \end{bmatrix}}$$

Since $(r_1 + 1)(r_2 + 1)$ (16.1) are required, i.e., $((N_1 + N_2)M_1 + 1)((N_1 + N_2)M_2 + 1)$, the total cost is finally $((N_1+N_2)M_1+1)((N_1+N_2)M_2+1)((N_1N_2+N_1+N_2)+2(M_1M_2+M_1+M_2+1)+1)$ CMAD's as well as $((N_1+N_2)M_1+1)((N_1+N_2)M_2+1)((N_1N_2+N_1+N_2)+2(M_1M_2+M_1+M_2))$ complex additions.

Equation (16.2) requires $(1/4)(r_1+1)(r_2+1)\log_2(r_1+1)\log_2(r_2+1)$ complex multiplications, i.e., $(1/4)((N_1+N_2)M_1+1)((N_1+N_2)M_2+1)\log_2((N_1+N_2)M_1+1)\log_2((N_1+N_2)M_2+1)$.

The conclusion is that our method requires $\mathbf{O}(2(N_1+N_2)^2M_1^2M_2^2)$ CMAD's with respect to M_1, M_2 (this is the evaluated complexity if N_1, N_2 are considered constant and M_1, M_2 variable) as well as $\mathbf{O}((N_1+N_2)^2N_1N_2M_1M_2)$ CMAD's with respect to N_1, N_2 (this is the evaluated complexity if M_1, M_2 are considered constant and N_1, N_2 variable). Also our method requires $\mathbf{O}((N_1+N_2)^2M_1^2M_2^2)$ complex additions with respect to M_1, M_2 as well as $\mathbf{O}((N_1+N_2)^2N_1N_2M_1M_2)$ complex additions with respect to N_1, N_2. Note that each complex multiplication is implemented by four real multiplications and two real additions while each complex addition is implemented by two real additions.

So, the present method seems to be better than that of [41], where we had $(1/10)M_1^2M_2^2N_2\mathbf{O}(N_1^5)$ MAD's (and equal number of additions) with respect to N_1 and $(1.5/10)M_1^2M_2^2\mathbf{O}(N_2^5)$ MAD's (and equal number of additions) with respect to N_2 as well as $\mathbf{O}\Big(M_1^2M_2^2[(N_1^5N_2/10)+(3N_1^4N_2/10)+(3N_1^3N_2^2/2)+(3N_1^2N_2^3/2)+(3N_1N_2^4/4)+(3N_2^5/20)+(N_1^4/4)+N_1^3+(3N_1^2N_2^2/2)+N_1N_2^3+(N_2^4/4)]\Big)$ with respect to M_1, M_2.

V. Conclusion

The algorithm presented in this paper speeds up the various 2-D rational transformations. These rational transformations are of great interest in the area of 2-D digital filters analysis and synthesis as well as in the area of 2-D signal processing. An important example is the double bilinear transformation via which a discrete 2-D system is transformed to a continuous one and vice-versa. The comparison of this method with a previously published one, [41], proves that the present method is better with respect to the computational complexity. Moreover, the whole formulation of the present method is simpler and more comprehensive than that of [41].

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