# Spectrum Comparison between GFDM, OFDM and GFDM Behavior in a Noise and Fading Channel

Preliminary Communication

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**Abstract** – GFDM (Generalized Frequency Division Multiplexing) is a multi-carrier multiplexing scheme for next generations of cellular networks. In this paper, a simulation of the GFDM transmitter in a noisy channel will be conducted. The spectrum benefit of GFDM over OFDM (Orthogonal Frequency Division Multiplexing) is treated, which will be used in applications such as machine-to-machine communications or cognitive radio. The behavior of GFDM after passing on the AWGN (Additive White Gaussian Noise) channel for different values of SNR (Signal-to-Noise Ratio) will be observed, as well as the GFDM behavior after passing through the Rayleigh channel.

Keywords – AWGN, GFDM, Rayleigh, SNR, spectrum benefit.

# 1. INTRODUCTION

OFDM is an efficient technique widely used in wireless communications as well as in many modern communications. However, it has some disadvantages like sensitivity in ICI (Inter-Carrier Interference) and a high PAPR (Peak-to-Average Power Ratio) [1]. In addition, the CP (Cyclic Prefix) is not efficient in the spectrum and it fails when the distribution of delay in the channel is longer than the CP length, which will result in ISI (Inter-Symbol Interference) [2]. In an effort to improve these disadvantages, various techniques have been proposed. One of them is called Generalized FDM (GFDM) and it is characterized by a reduction of leakage outside the passband.

To achieve spectrum and energy efficiency, GFDM does the dynamic adoption of pulse shaping optimizing on time and frequency on fading channels. Despite OFDM, GFDM can benefit transmitting multi-symbols per multi-carrier in a two-dimensional block-structure (time and frequency) [3]. The block structure is achieved by circularly convolving each individual sub-carrier with a shaping pulse. Thanks to the pulse shaping filters, the transmit signal shows a good placement in the sub-carrier frequency which enables less radiation in the stopband. In addition, windowing schemes on time can be applied to a GFDM block to control better leakage in the stopband for small values of CPs. Variable pulse shaping filters remove orthogonality between the sub-carriers. As a result of this process, ICI and ISI are obtained [3]. Since GFDM introduces self-interference because of signal design, the same disadvantages are shown at multiple access scenarios at 4G. GFDM allows for the design of simplest transmitters, leaving aside synchronization procedures and reducing signaling load. These characteristics of GFDM, along with the idea of shifting processing at base stations as much as possible, can contribute to reducing terminals energy spending.

This cannot be done at OFDM because of sync-pulse leakage and accurate synchronizing requirements to keep orthogonality between the sub-carriers [4]. As a generalization of OFDM, GFDM fits with it when the number of symbols for sub-carriers is one.

#### 2. GFDM SYSTEM DESCRIPTION

## 2.1. TRANSMITTER

Let us consider the baseband system in Figure 1 that spreads complex data symbols d(k,m) through K carriers and M symbols. Each sub-carrier is shaped with a transmit pulse  $g_{Tx}(n)$  and modulated with the center frequency of sub-carrier  $e^{j2\pi k \cdot kn/N}$ . In order to carry out

Nyquist criteria, every symbol is sampled *N* times, that gives *MN* samples per sub-carrier [5]. Filter  $g_{Tx}(n)$  is chosen to have the *MN* periodicity. After placement of data symbols in a d(k,*m*) column, arrow d and performing operations of upsampling, circular convolution with pulse shaping, upconversion and superposition, this equation can be written as:

$$x = Ad, \tag{1}$$

where x contains transmitted samples in time x(n) and A is an  $MN \times MN$  modulation matrix.

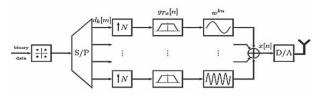


Fig. 1. GFDM baseband transmitter [5]

Let d(k,m) be a complex symbol of information,

$$d = \begin{bmatrix} d(0,0) & \cdots & d(0,M-1) \\ \vdots & \ddots & \vdots \\ d(K-1) & \cdots & d(K-1,M-1) \end{bmatrix}.$$
 (2)

Matrix  $K \ge M$  will be addressed as an information block. So, k = 0... K-1 represents a sub-carrier, where m = 0... M-1 represents the time slot. In order for symbols to be distributed in time and frequency, the discrete time impulse response of pulse shaping transmit filter g(n) should be movable in these dimensions. The expression  $g(n-nm)e^{j2\pi k \cdot kn/N}$  is used for these shifts, whereas for sampling time  $T_s$  symbol length is  $T_d = NT_s$  and  $1/NT_s$  signifies sub-carriers frequency separation. The transmit signal is as follows:

$$x(n) = \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} d(k,m)g(n-mN)e^{j2\pi \frac{kn}{N}}, 0 \le n \le NM, (3)$$

which results in a block from the sum of all shifted impulse responses that are composed of information symbols d(k,m). In addition, in order to equalize the frequency domain in the receiver, CP is added to x(n), and  $\bar{x}(n)$  is obtained, which is going to be transmitted in the radio channel.

#### 2.2. RECEIVER

The received signal of a time-invariant multipath channel with impulse response h(n) is given by the expression

$$\overline{y} = \overline{x} * h(n) + n(n), \tag{4}$$

where  $\bar{x}$  is the transmitted signal with the *CP* and *n*(*n*) is a vector of Gaussian noise samples with zero mean and variance  $\sigma_n^2$ . The symbol \* denotes convolution in discrete time. After removing the *CP*, the obtained equation is:

$$y(n) = x(n) + n(n).$$
(5)

Supposing that h(n) is known at the receiver, the  $K \ge M$  block of information symbols is equalized by

$$\overline{y} = \text{IDFT}\left(\frac{\text{DFT}(y(n))}{\text{DFT}(h(n))}\right),$$
 (6)

where DFT( $\bullet$ ) indicates Discrete Fourier Transform and IDFT( $\bullet$ ) its inverse function.

There are three ways of receiving the signal from (1) [5]. The first way is to find a matrix  $A^{\#}$  such that  $A^{\#}A=I$ , where I is an identity matrix. Depending on the rank of A, it can be found that

$$A^{\#} = (A^{\#} A)^{-1} A^{H} \text{ or } A^{\#} = A^{H} (A A^{H})^{-1},$$
(7)

where  $\overline{d}_{ZF} = A^{\#}y$  is a ZF (zero-forcing) receiver.

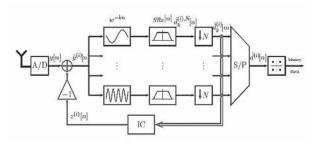
The second way of receiving the GFDM signal is to use the MF (matched filter) in the receiver according to Equation 8. The block diagram is shown in Figure 2.

$$\overline{d}_{MF} = A^{H} y. \tag{8}$$

The third way is the *MMSE* (minimum mean-square error) given by:

$$\overline{d}_{MMSE} = A^{a} y, \tag{9}$$

where,  $A^{\alpha} = (\sigma_n^2 / \sigma_d^2 \cdot I + A^H A)^{-l} A^H$ ,  $\sigma_n^2$  and  $\sigma_d^2$  are sample variances of noise and symbols.



**Fig. 2.** GFDM matched filter receiver model baseband with interference cancellation [5]

#### 3. GFDM IMPLEMENTATION

Simulation of the GFDM signal is conducted by using the Matlab software package. This simulation is conducted according to the scheme shown in Figure 3 and parameters in Table 1.

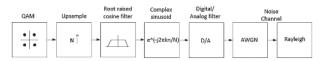


Fig. 3. GFDM transmitter and channel

Table 1. Input parameters of the system

Tg=224e-6	Period of GFDM,OFDM symbol
T=1.0938e-07	Elementary baseband period
B=9MH	Bandwidth
N=4096	Length of IFFT/FFT
Modulation	QAM
Waveform	OFDM, GFDM
Number of sub-carriers	OFDM: 1705   GFDM:114, M=15
Pulse	OFDM: Rect. GFDM:RRC, α=0.1

QAM (Quadrature Amplitude Modulation) symbols are obtained (Fig. 4.) and are upsampled.

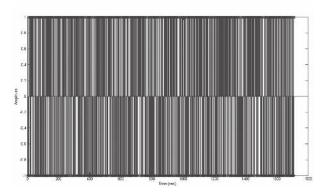


Fig. 4. QAM symbols

Circular convolution with the RRC (root raised cosine) pulse is performed on these information symbols, as displayed in Figure 5.

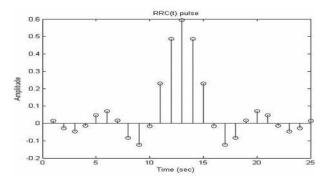


Fig. 5. Impulse response to the root raised cosine pulse

The result of this convolution is shown in Figure 6 (Circular convolution Inphase represents the real part, while Circular convolution Quadrature represents the imaginary part).

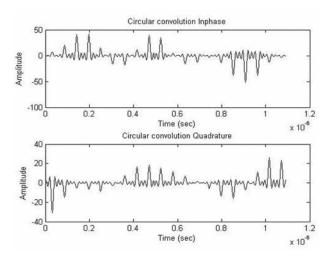


Fig. 6. Signal after circular convolution with RRC

This signal is converted into the baseband signal after filtering it with the LPF (Low Pass Filter) whose characteristic is shown in Figure 7.

The result of this filtering process is shown in Figure 8. The signal is modulated afterwards and the obtained signal is shown in Figure 9.

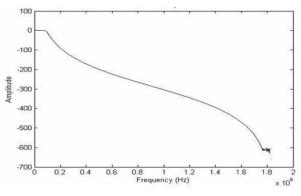


Fig. 7. Digital/Analog filter characteristic

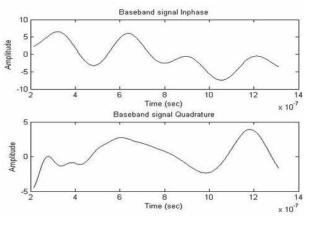


Fig. 8. Baseband signal after filtering

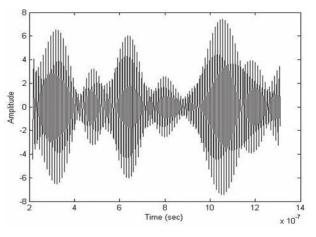


Fig. 9. Modulated signal with the complex sinusoid

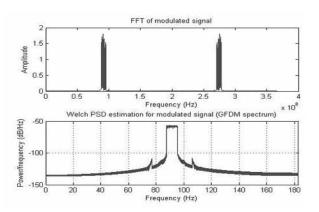


Fig. 10. FFT and GFDM signal spectrum

## 4. GFDM SYSTEM SIMULATION OF NOISY AND FADING CHANNELS

Multipath fading is a process that causes the presence of many copies of the main signal. These copies will attach to the main signal and make it very difficult to detect. In case of pure Rayleigh fading, there is no main signal. All components are reflected [6]. The Rayleigh channel is simulated using parameters in Table 2. AWGN is a real noise model that describes the effect of random processes [7].

Table 2. Rayleig	n channel parameters
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ChannelType	Rayleigh
InputSamplePeriod	1.0937e-07 (s)
DopplerSpectrum	[1x1 Doppler. Jakes] (s)
MaxDopplerShift	100 (Hz)
PathDelays	5.0000e-05 (s)
AvgPathGaindB	2 (dB)
NormalizePathGains	1 (0 or 1)
StoreHistory	0 (0 or 1)
StorePathGains	0 (0 or 1)
PathGains	0.5700 + 0.5425i
ChannelFilterDelay	457 (samples)
ResetBeforeFiltering	1 (0 or 1)
NumSamplesProcessed	81921

The power of the faded signal is shown in Figure 11.

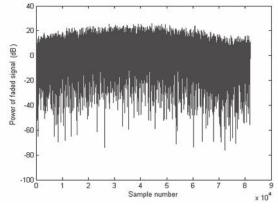


Fig. 11. Power of the faded signal

# 5. RESULTS

Figure 12 shows the signal spectrum gain of GFDM over OFDM 9-11 dB (on the left from -108 to -117 dB, and on the right from -110 to -121 dB). This spectrum gain is used in applications such as machine-to-machine communications or cognitive radio [8].

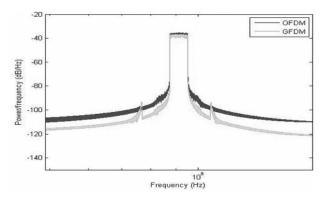
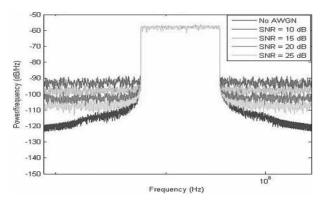


Fig. 12. Spectrum comparison between the GFDM and the OFDM signal

Figure 13 shows distortions the GFDM signal undergoes after passing through the AWGN channel with different SNRs (SNR = 10, 15, 20, 25 dB).



**Fig. 13.** Spectrum of the GFDM signal before and after passing through the AWGN channel for different SNRs

Figure 14 depicts distortions caused by the Rayleigh channel, in the case when the signal passes through AWGN with SNR = 10 dB. If the subtraction of the signal is done before and after passing through the Rayleigh channel, the curve shows that distortions made by Rayleigh in GFDM are similar to OFDM even though GFDM is a non-orthogonal technique.

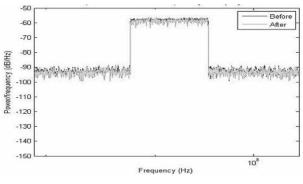


Fig. 14. GFDM signal after passing through the Rayleigh channel

## 6. CONCLUSION

From the presented results, it can be concluded that GFDM uses spectrum better than OFDM thanks to a RRC pulse that has been used and a spectrum gain is noticed compared to OFDM. In addition, channel losses (distortions) have been noticed that were caused by AWGN for different SNRs. Finally, a case when the signal passes through a Rayleigh channel has been shown. In the AWGN and Rayleigh channels, GFDM gives approximate results compared to OFDM.

## 7. REFERENCES

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