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Spectrum Leasing to Femto Service Provider with Hybrid Access

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***Abstract*—The concept of femtocell that operates in licensed spectrum to provide home coverage has attracted interest in the wireless industry due to high spatial reuse, and extensive deployments of femtocells is expected in the future. In this paper, we consider the scenario that a femtocell service provider (FSP) expects to rent spectrum from the coexisting macrocell service provider (MSP) to serve its end users. In addition to the spectrum leasing payment, the FSP may allow hybrid access of macrocell users to improve the utilities of itself and MSP, which are defined as the sum of data traffic and payment/revenue. We propose the spectrum leasing framework taking hybrid access into consideration. The whole procedure is modeled as a three-stage Stackelberg game, where MSP and FSP determine the spectrum leasing ratio, spectrum leasing price and open access ratio sequentially to maximize their utilities, and the existence of the Nash Equilibrium of the sequential game is analyzed. We characterize the equilibrium, in terms of access price, spectrum acquisition of FSP, the open access ratio, and price of anarchy via simulation. Numerical results show that both MSP and FSP can benefit from spectrum leasing, and hybrid access of femtocell can further improve their utilities, which provide sufficient incentive for their cooperation.**

I. Introduction

Due to the scarcity of the spectrum and the increasing demand of mobile users, femtocell networks are in the process of being deployed to improve cellular capacity. Femtocells are low-power, low-cost, cellular Base Stations (BSs) with a typical coverage range of tens of meters, which is much smaller than that of macrocell (hundreds of meters). They operate in licensed spectrum as the macrocell does and utilize the end user's existing broadband internet access as backhaul. Since the femtocell has smaller coverage, femto users will experience superior signal reception and multiple femtocells can use the same channel simultaneously if they are far away from each other, which significantly increases the network capacity and the spectrum efficiency.

Moreover, according to the utilization of femtocell access opportunities, the access control mechanism for femtocells can be divided into three categories, closed access, open access, and hybrid access defined as follows [1]:

- Closed Access: only a subset of users, defined by the femtocell owner, can connect to the femtocell. This model is referred to as closed subscriber group (CSG) by the Third Generation Partnership Project (3GPP).
- Open Access: All customers of the operator have the right to make use of any femtocell.
- Hybrid Access: A limited amount of the femtocell resources are available to all users, while the rest are operated in a CSG manner.

Obviously, the close access and open access can be treated as the special cases of hybrid access with the definition of open access ratio, which could be the ratio of the open spectrum for sharing to all the spectrum obtained by the FSP, or the portion of femtocells in open access mode, or the probability a passing by macrocell user is served by the femtocell, etc. When the open access ratio is zero, all the femtocell access opportunities are reserved for the authorized femtocell users, the femtocell is in closed access mode. In this mode, the quality of service (QoS) of femtocell users is guaranteed at the expense of decreasing the spectral efficiency. In contrast, when the open access ratio is one, the femtocell access opportunities are shared with all the users passing by, the femtocell is in open access mode. The femtocell BSs can offload the burden of traffic from the macrocell BS within the same area to improve network capacity. Therefore, the selection of an open access ratio for femtocells has dramatic effects on the performance of the overall network, mainly due to the tradeoff between QoS of the femtocell users and the spectral efficiency of the whole network.

Note that the deployment of femtocells requires both the broadband internet access as backhaul and the wireless spectrum to serve the end user, the fixed-line service provider may not necessarily own spectrum, while the wireless service provider may not support fixed-line broadband internet access. Therefore, they need to cooperate with each other to expand their services with femtocell network.

In this paper, we consider the scenario that the macrocell service provider (MSP) coexists with a femtocell service provider (FSP). The MSP is a traditional wireless service provider which provides wireless access service to its end users, while the FSP is a fixed-line service provider which intends to provide wireless access service to some dedicate users with femto access points (FAPs). However, the FSP owns no spectrum and expects to lease some spectrum from the MSP. We investigate the spectrum leasing issue among MSP and FSP with the consideration of hybrid access control. The main challenges include: 1) how to motivate the MSP to lease spectrum to the FSP, and how to determine the spectrum leasing amount. 2) how to motivate the FSP open part of its obtained spectrum to serve macrocell users, and how to determine the open access ratio accordingly. To answer these questions, we propose a spectrum leasing framework based on game theory. The whole decision making process is modeled as a three-stage Stackelberg game, the FSP and MSP decide the spectrum leasing ratio, and the open access ratio jointly. In the first stage, the FSP decides the open access ratio, and in the second stage, the MSP announces the spectrum leasing price, and in the third stage, the FSP determines the spectrum it will rent from MSP. Using backward induction, we can obtain the best strategies of MSP and FSP in each stage. Simulation results show that both the MSP and FSP have the incentive to cooperate with each other, since the utilities of them increases dramatically.

The main contributions of this paper are three folds. First, we propose a framework to analyze the spectrum leasing problem between a MSP and FSP with the consideration of hybrid access of femtocell. To the best of our knowledge, this is the first work that considering hybrid access in spectrum leasing of such two-tier femtocell network. Second, we formulate the problem into a three-stage Stackelberg game, and achieve the Nash Equilibrium (NE) which gives the best strategies of MSP and FSP on spectrum leasing ratio and open access ratio determination. Third, numerical results show that under the proposed framework, both the MSP and FSP can achieve higher utility with spectrum leasing, as well as hybrid access of the femtocells.

The rest of this paper is organized as follows. In Section II, we introduce the system architecture, including the scenario, and system model in this paper, and formulate the spectrum leasing problem. In Section III, the spectrum leasing between MSP and FSP is modeled as a three-stage Stackelberg game, backward induction is adopted to analyze the best strategy of each player, and the Nash Equilibrium point is given. The advantage of the proposed framework is demonstrated with numerical results in Section IV. Related works are reviewed in Section V. Finally, Section VI concludes the whole paper.

II. System Model and Problem Formulation

In this section, we first introduce the scenario we consider in this work, and describe the channel model and long term throughput which are needed for the definition of the utility functions for the two service providers. Then, we formulate the utility maximization problem in the cooperative spectrum leasing.

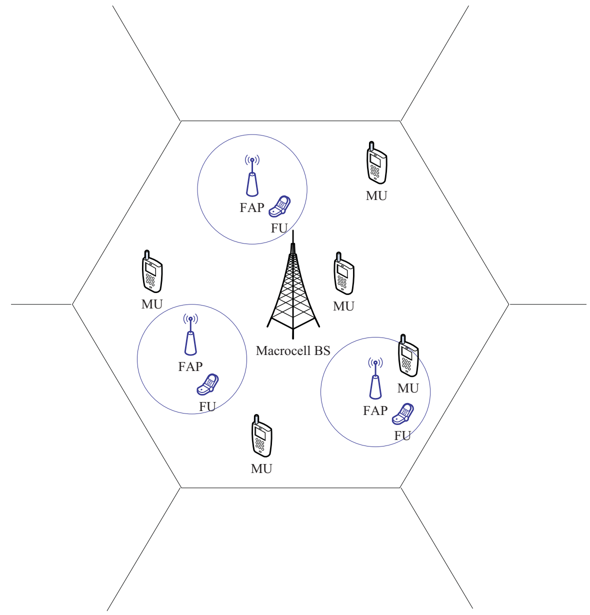

Fig. 1. The MSP and FSP coexistence Scenario.

A. Scenario

We consider the scenario that multiple femtocells and macrocells coexist with each other, which belong to a FSP and a MSP, respectively. The MSP is the spectrum licence holder who own spectrum, and the FSP owns no spectrum and want to rent spectrum from MSP according to the price charged by it. The FSP can also allow hybrid access of macrocell users in the expectation of obtaining more spectrum from the MSP with lower price. Assume similar distribution of femtocells in each macrocell, we can analyze the case of one macrocell for simplification without loss of generality.

The setup consists of a hexagonal region $\mathcal{H}$ of radius R_m with a central macro BS providing coverage $|\mathcal{H}| = \frac{3\sqrt{3}}{2}R_m^2$, which is surrounded by two rings of interfering macrocells [2]. The macrocell network is overlaid with multiple femtocell hotspots of radius R_f. The mean number of femtocells per macrocell is denoted as N_F. N_m macrocell users are assumed to be uniformly distributed inside each cell site. Each femtocell is assumed to provide hybrid access to N_f licensed indoor users and macrocell users who fall whin the radio range R_f.

The total available spectrum bandwidth is W Hz, which comprises F frequency subchannels each with bandwidth W/F Hz. We will determine the optimal allocation (F_c, F_f), where F_c subchannels are reserved for macrocell transmission, and $F_f = F - F_c$ subchannels will be leased to FSP for femtocells transmission. For the ease of analysis, we denote $\rho = F_f/F$ as the spectrum leasing ratio, which is the fraction of spectrum leased to FSP.

For the channel allocation among multiple femtocells, if a femtocell transmits over all its allotted subchannels, it may cause excessive interference to surrounding femtocells. Therefore, to avoid intercell interference with neighboring femtocells, the femtocell will access the spectrum in a frequency ALOHA fashion [2], i.e., each femtocell will transmit over only k subchannels among their allotted F_f subchannels. The portion of accessed spectrum per femtocell equals to $\rho \times \rho_f$, where $\rho_f = k/F_f$.

In the consideration of hybrid access, we denote the open access ratio of femtocells as ϵ, $0 \le \epsilon \le 1$. Note that $\epsilon = 0$ and $\epsilon = 1$ correspond to closed access and open access, respectively. The FSP will determine the optimal ϵ that maximizes its utility.

B. Channel Model

The channel between each BS and its users is composed of a fixed distance dependent path loss, a slowly varying component modeled by lognormal shadowing and Rayleigh fast fading with unit average power. For simplicity, thermal noise is neglected at the receiver since cellular systems are interference-limited.

Assume that there is no power control, and the BSs assign equal transmission powers to all subchannels. Each BS assigns rate adaptively based on the received signal-to-interference ratio (SIR) per user. Let G denote the gap between the Shannon capacity and the achievable rate with M-QAM transmission. Assume an instantaneous transmission rate of b_i bps/Hz if the instantaneous SIR lies in $[\Gamma_i, \Gamma_{i+1})$. Using adaptive modulation with L discrete rates, the instantaneous rate Wb in a W Hz wide subchannel is chosen as [3],

$$\begin{aligned} b &= b_i, \text{when SIR} \in [\Gamma_i, \Gamma_{i+1}), 1 \le i \le L \\ b_i &= \log_2\left(1 + \frac{\Gamma_i}{G}\right) \text{bps/Hz} \end{aligned} \tag{1}$$

Since the time duration of spectrum leasing is relatively long, we consider the throughput in a statistical manner, i.e, long term throughout, rather than the instantaneous one. Assuming identical statistics over all frequency subchannels, the long term expected throughput (in b/s/Hz) per macrcell/femtocell in each subchannel is given as

$$T_l = \sum_{l=1}^{L-1} l * Pr[\Gamma_l \le \text{SIR} < \Gamma_{l+1}] + L * Pr[\text{SIR} \ge \Gamma_L]. \tag{2}$$

The long term throughput of the macrocell/femtocell can be obtained via statistics or theoretical estimation. In [2], the long term throughput of macrocell/femtocell is derived theoretically and validated by simulation. In our analysis below, we will leverage some conclusions in the paper. The user scheduling method of macrocell and femtocell considered in our scenario is Round-Robin (RR). It is safe to mention that our analysis can also be extended to other scheduling methods. The long term throughput of macrocell and femtocell are denoted as T_c and T_f, respectively.

Since there are multiple users in each network, they will share the channels and the expected effective serving rate can be calculated as

$$r = \frac{N_c T_l}{N_u}. \tag{3}$$

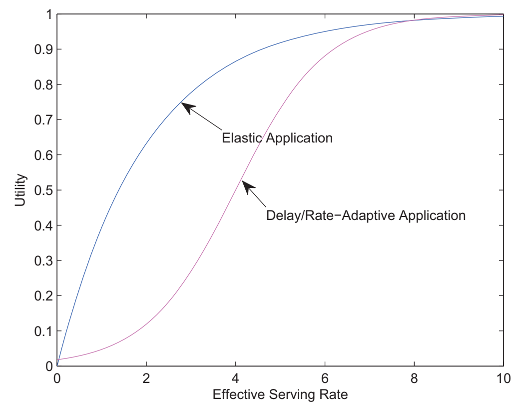

Fig. 2. Utility function illustrations.

where N_u is the number of users, and N_c is the number of channels shared by N_u users. Obviously, the network needs more channels to satisfy users' rate requirement when the number of users increases. In addition, since the macrocell and femtocell have different coverage range, the long term throughput of the two network will be different. With smaller coverage, the femtocell users are much closer to their APs, and thus they will have higher data rate as comparing with macrocell users. Moreover, the spatial reuse ratio of femtocell is much higher than the macrocell. Therefore, the spectrum can be better utilized if more spectrum are allocated to femtocell. However, the MSP also has to take into consideration the service quality of its own end users. Unless the femtocells are in open access or hybrid access mode, the spectrum obtained by the FSP will be very limited.

C. The utility functions

Obviously, the revenue of the both two service providers are comprised of two components, the revenue obtained from the users they served according to their service quality, and the payment of the spectrum leasing. We consider that the MSP determines the price per unit access spectrum, and the FSP will pay according to the spectrum it obtained from MSP. To quantify the service quality of each end user, we define the utility function, which related to the effective serving rate r to application performance, as $U(r)$. Assume that function $U(r)$ is the same for all users in each network.

Generally, the utility of each end user is a monotonically increasing function of effective serving rate. According to their characteristics, utility functions can be categorized into two types as demonstrated in Fig. 2. One is for elastic applications, where the curve is concave over the entire range and approach a fixed value as the rate approach infinite. The other type is for delay/rate-adaptive applications, which is convex for a portion of the curve, representing the fact that, once the rate is below a certain threshold, the user-perceived application performance drops sharply. Similar to elastic applications, the utility approach a certain value as channel rate goes to infinite, which is similar to the margin effect in economics [4].

In general, utility curves, which usually differ for different applications, can be constructed for any given network performance matrices, such as rate and delay. The best way to obtain utility curves is through sophisticated subjective surveys, in which users are asked to judge application performance under a wide range of network conditions. In [5], the authors use different utility functions to approach the best effect traffic, which include exponential, logarithmic, and power functions. In the rest of this paper, we will use exponential function as the start point of best effect traffic, and for rest more complicated functions for delay/rate-adaptive traffic, they can be analyzed in a similar way.

D. Problem Formulation

For the spectrum leasing to femtocell service provider with hybrid access, two fundamental questions need to be answered: first, how to motivate the MSP to lease spectrum to the FSP, and what is the optimal spectrum leasing ratio ρ; second, how to motivate the FSP open part of its obtained spectrum to serve macrocell users, and what is the open access ratio ϵ accordingly. To answer the questions, we formulate utility maximization problem for the two service providers.

1) FSP: The FSP aims to get most benefits at the least possible payment for the spectrum leasing, so the utility function of FSP consists of two components, the utility from femtocell users' throughput U_f, minus the payment to the MSP for the spectrum rented U_r,

$$U_F = \omega_f U_f(r_f) - U_r(\rho), \tag{4}$$

where ω_f is the weighting factor of the utility of end users on the total utility. With larger ω_f, the service provider weights more on the end users' performance than the spectrum leasing payment. U_f is defined as

$$U_f(T_f) = \sum_{N_F} \sum_{N_f} \left(1 - e^{-\alpha_f r_f}\right), \tag{5}$$

where r_f is the effective serving rate of femtocell users, which is given by (according to (3))

$$r_f = \frac{\rho_f T_f (1-(1-P_f)\epsilon)\rho W}{N_f}, \tag{6}$$

and P_f is the portion of femtocell throughput obtained by the femto users if femtocells are in open access mode,

$$P_f = \frac{N_f}{N_f + \frac{2\pi R_f^2}{\sqrt{3} R_m^2} N_m}. \tag{7}$$

Note that if the femtocell is operated in hybrid mode, the macro users will be able to obtain extra throughput from femtocells.

U_r is the payment from FSP to MSP for the spectrum access authority ρ, which is defined by

$$U_r(\rho) = c\rho W. \tag{8}$$

Then, the optimization problem for the FSP can be formulated as:

$$\max_{\{\rho,\epsilon\}} \; U_F = \omega_f U_f - U_r, \tag{9}$$

with the constraints

$$0 \le \rho \le 1, 0 \le \epsilon \le 1. \tag{10}$$

If with optimal ρ and ϵ, the resulting utility of FSP is such that $U_F^* \le 0$, then the FSP will quit from the market since it can not cover the spectrum leasing cost from the benefit collected from the end users.

2) MSP: The MSP aims to earn the payment not only from its own users but also gain as much extra profit as possible from spectrum leasing, so the utility function of MSP is comprised of two components, the throughput related utility U_m, plus the revenue obtained from spectrum leasing U_r,

$$U_M = \omega_m U_m(r_m) + U_r(\rho), \tag{11}$$

where ω_m is the weighting factor which reflects the preference of the two components (similar to ω_f). The throughput related utility is defined as

$$U_m(r_m) = \sum_{N_m} \left(1 - e^{-\alpha_m r_m}\right), \tag{12}$$

and r_m is the effective serving rate of macrocell user which is defined as (according to (3))

$$r_m = \frac{(1-\rho)T_c + N_F \rho_f (1-P_f)\epsilon\rho T_f}{N_m} W. \tag{13}$$

Note that with higher ϵ or ρ, the macro users will be able to obtain more throughput from femtocells. Therefore, the MSP should determine a suitable spectrum leasing price and motivate the FSP open more resource for open access.

Then, the optimization problem for MSP can be formulated as

$$\max_{c} U_M = \omega_m U_m + U_r, \; s.t. \; c > 0. \tag{14}$$

Therefore, the ultimate goal of the utilization maximization problem is to decide the optimal price c that maximizes the profit of the MSP U_M, and the optimal ρ and ϵ to maximize the profit of the FSP U_F. Notice that the only information required between the MSP and FSP are the long term throughput, the number of corresponding users, and the number of femtocells.

III. Cooperative Spectrum Leasing Framework

In this section, we model the spectrum leasing and hybrid access between the MSP and FSP as a sequential game, and analyze the best strategies of them based on their utility functions. The existence of the Nash equilibrium of the game is obtained.

Since the femtocell has much small coverage, the users accessing femto AP will experience superior signal reception and multiple femtocells can use the same channel simultaneously if the inter-cell interference is ignorable, which significantly increases the network capacity and the spectrum efficiency. Obviously, if the MSP leases part of the spectrum to FSP, they will be able to enhance the user experience if they can access femto APs in addition to extra revenue obtained from leasing. However, the macro users will occupy the precious access opportunity of femtocells if FSP allows macro users to

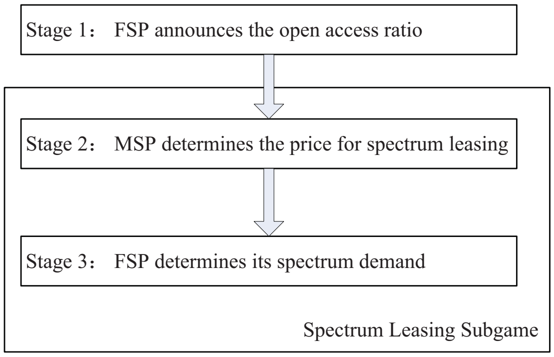

Fig. 3. A three-stage Stackelberg game.

access its network. Therefore, we should motivate the FSP to operate in open access or hybrid access mode.

We cast the competition and cooperation between MSP and FSP as a three-stage sequential game, as shown in Fig. 3, where MSP and FSP adapt their decisions dynamically to reach an equilibrium. The FSP first announces its open access ratio in Stage 1; and then the MSP determines the spectrum leasing price in Stage 2. Finally, the FSP decides the amount of spectrum it will rent from the FSP that maximizes its utility in Stage 3. The open access ratio of the femtocells should be determined in the first stage, because if the spectrum leasing price is announced first, the FSP will decide the spectrum bandwidth it expects to rent, while the best operation mode for the femtocell is closed access which maximizes its utility defined in (4). As a result, there will be no incentive for the FSP to operate in hybrid access mode, which will decrease the utility of both MSP and FSP due to low spectrum efficiency. Moreover, the spectrum leasing price could not be decided in the third stage, because the utility of MSP defined in (11) is a monotonic increasing function of price c, if the spectrum leasing ratio is already determined by the FSP, the MSP can set an infinite high price to maximize its utility which is not desirable in the market.

The whole game can be viewed as the combination of a spectrum leasing subgame and a hybrid access subgame. In the spectrum leasing subgame, the MSP plays as the leader, because it owns the spectrum, and thus it has such a priority and the market power. It first announces the spectrum leasing price per unit spectrum. The FSP plays as the follower, and determines the spectrum amount it expects to rent from MSP. In the hybrid access subgame, the FSP plays as the leader because it has the priority to decide the open access ratio to serve the passing by macrocell users, the MSP plays as the follower to lead the spectrum leasing subgame. Therefore, the MSP and FSP should take the open access ratio into consideration when they play the spectrum leasing subgame.

A. Backward Induction

The common method to analyze the equilibrium of the sequential game is backward induction [6], [7]. Therefore, we will first analyze the two-stage spectrum leasing subgame given the open access ratio, in which the best response function of the FSP is obtained given the spectrum leasing price, and then the optimal price is determined at MSP with the best response function of the FSP. At last, the best strategy of the FSP on open access ratio determination is obtained with aware of the equilibrium of the spectrum leasing subgame.

1) The best strategy of FSP on spectrum leasing ratio: First, we will analyze the best strategy of the FSP on the spectrum demand ρ in the spectrum leasing subgame. At this stage, the target of FSP is to maximize its utility given the spectrum leasing price determined by MSP. The best response function of FSP is defined and calculated as follows.

Definition 1: $\rho^*(c)$ is defined as the best response function of FSP if the utility of FSP is maximized at $\rho^*(c)$ given the spectrum leasing price c, i.e., $\forall c > 0$, we have $U_F(\rho^*(c), c) \geq U_F(\rho(c), c)$.

According to the Definition 1, we have the following theorem,

Theorem 1: The best response function of FSP is given by

$$\rho^*(c) = \frac{1}{B_f W} \ln \frac{\omega_f N_F N_f B_f}{c}, \tag{15}$$

where

$$B_f = \frac{\alpha_f \rho_f (1-(1-P_f)\epsilon) T_f}{N_f}. \tag{16}$$

Proof: It is easy to prove that, with the increase of ρ, U_F is first an increasing function of ρ and then a decreasing function. Therefore, a global maximum point for U_F is achieved when the first order derivative of U_F in respect with ρ equals to 0. Given the fist order derivation of (4) and set it to zero, we have

$$\frac{\partial U_F}{\partial \rho} = \omega_f N_F N_f B_f W e^{-B_f W \rho} - cW = 0. \tag{17}$$

By solving the above equation, we can have

$$c = \omega_f N_F N_f B_f e^{-B_f W \rho}, \tag{18}$$

with transformation, we have (15).

The proof is completed. ■

Note that the spectrum demand of the FSP will decrease as the price per unit spectrum bandwidth increases, which property will prevent the MSP to set the price at a unreasonable high level. Given the spectrum leasing price, the FSP decides the ρ according to (15), and the utility of the FSP is determined accordingly.

2) The best strategy of MSP on spectrum leasing price: Since the open access ratio is fixed at this stage, given the best response function of FSP, the MSP can determine its best strategy in the spectrum leasing subgame, i.e., the optimal price for spectrum leasing. According to (15), the price determination is equivalent to deciding the spectrum partition between them. i.e., ρ. The target of MSP is to maximize its utility defined by (11), i.e.,

$$\begin{aligned} \max\ \omega_m N_m \left(1 - \exp\left\{-\frac{\alpha_m T_c W}{N_m} - A_c W \rho\right\}\right) \\ + \omega_f N_F N_f B_f W \rho e^{-B_f W \rho} \end{aligned} \tag{19}$$

with the constraint

$$0 \leq \rho \leq 1. \tag{20}$$

where

$$A_c = \frac{\alpha_m(N_F\rho_f(1-P_f)\epsilon T_f - T_c)}{N_m}.$$

To solve the above problem, we have the following theorem,

Theorem 2: The best strategy of MSP on spectrum partition is given by

$$\rho^* = \max\{0, \min\{\rho_0, 1\}\}. \tag{21}$$

where ρ_0 is obtained as follows

$$\rho_0 = \frac{1}{B_f} + \frac{1}{A_c - B_f} \times lambertw(0, \frac{\omega_m N_m A_c(A_c - B_f)}{\omega_f N_F N_f B_f^2} \exp(-\frac{\alpha_m T_c W}{N_m} - \frac{A_c - B_f}{B_f})). \tag{22}$$

Note that the $w = lambertw(k, x)$ is the k-th branch of the Lambert W function at the elements of x, which solves the equation $we^w = x$. The proof of Theorem 2 is as follow.

Proof: By deriving the first order derivation of (19) about ρ, we obtain

$$\frac{\partial U_M}{\partial \rho} = \omega_m N_m A_c W \exp\left\{-\frac{\alpha_m T_c}{N_m} - A_c W \rho\right\} + \omega_f N_F N_f B_f W e^{-B_f \rho W} - \omega_f N_F N_f B_f^2 W^2 \rho e^{-B_f W \rho}, \tag{23}$$

and set it to zero. With equivalent transformation, we obtain the following equation,

$$\exp\left\{-\frac{\alpha_m T_c}{N_m} - (A_c - B_f)W\rho\right\} = \frac{\omega_f N_F N_f B_f}{\omega_m N_m A_c}(1 - B_f W \rho). \tag{24}$$

by solving (23), we obtain (22).

Note that there is only one zero point of $\partial U_M/\partial \rho$, and $\lim_{\rho \to -\infty} \frac{\partial U_M}{\partial \rho} > 0$, while $\lim_{\rho \to +\infty} \frac{\partial U_M}{\partial \rho} < 0$. Therefore, U_M is first an increasing function of ρ, then a decreasing one, and maximum U_M is achieved when $\frac{\partial U_M}{\partial \rho} = 0$. Considering the boundary condition of ρ, $0 \leq \rho \leq 1$, There are three cases for ρ_0:

- The case with $\rho_0 < 0$: Since $\frac{\partial U_M}{\partial \rho} < 0$, when $\rho > \rho_0$, U_M is monotonically decreasing at $[0, 1]$, and U_M is maximized at $\rho = 0$;
- The case with $0 \leq \rho_0 \leq 1$: Obviously, U_M is maximized at $\rho = \rho_0$;
- The case with $\rho_0 > 1$: Since $\frac{\partial U_M}{\partial \rho} > 0$, when $\rho < \rho_0$, U_M is monotonically increasing at $[0, 1]$, and U_M is maximized at $\rho = 1$;

In summary, the optimal spectrum partition is given by (21). The proof is completed. ■

With optimal spectrum leasing ratio ρ^*, the spectrum leasing price c^* can be obtained according to (18). Therefore, we have the following theorem,

Theorem 3: $\{\rho^*, c^*\}$ is the Nash Equilibrium of the spectrum leasing subgame.

3) the best strategy of FSP on open access ratio: In the spectrum leasing subgame, given the price determined by the MSP, the FSP can decide the spectrum bandwidth it expects to rent. Since the optimal price is the function of the open access ratio ϵ, the FSP should decide the optimal ϵ that maximize its utility.

As defined in (4), the utility function as a function of ϵ is given by

$$U_F(\epsilon) = \omega_f N_F N_f \left(1 - \exp\left\{-\frac{\alpha_f T_f \rho(\epsilon)(1-(1-P_f)\epsilon)}{N_f}\right\}\right) - c(\epsilon)\rho(\epsilon). \tag{25}$$

The target of the FSP in this hybrid access game is to maximize $U_F(\epsilon)$ with the constraint that

$$0 \leq \epsilon \leq 1. \tag{26}$$

To obtain the best strategy of FSP, we have the following theorem,

Theorem 4: There exists a ϵ^* in the interval $[0, 1]$ that maximizes the utility of FSP U_F in this sequential game.

Proof: Obviously, the utility of FSP U_F as a function of ϵ (25) is continuous in the closed and bounded interval $[0, 1]$, according to the Extreme Value Theorem [8], the U_F must attain its maximum value at once, i.e., there must be one ϵ^* in $[0, 1]$ that maximizes U_F.

The proof is completed. ■

Unfortunately, we are not able to obtain the close form solution for ϵ^* due to the complicated expression of ρ_*. In our evaluation, we obtain the optimal ϵ using bisection method, while other numerical methods like Newton method and gradient method can also be leveraged to obtain the ϵ^* that solve the utility maximize problem of FSP.

So finally we can get the optimal open access ratio to maximize the utility of the FSP in the first stage. In such a two-player static game, the convergence of the game is self-evident.

B. Properties of the Equilibrium

In this subsection, we discuss the efficiency of the equilibrium and the existence of the equilibrium of utility functions of other forms.

Theorem 5: The $\{\rho^*, c^*, \epsilon^*\}$ solved in the previous subsections is the equilibrium for this three-stage Stackelberg game.

Proof: When the FSP makes its decision on the open access ratio, according to Theorem 4, $U_F(\epsilon^*) \geq U_F(\epsilon)$, i.e., ϵ^* is the optimal response strategy for FSP. For the analysis in Theorem 2, if ϵ^* is selected by FSP, it can always find its optimal price c^*, i.e., at the optimal point $U_M(c^*) \geq U_M(c)$. Then, given the spectrum leasing price c^*, according to Theorem 1, FSP can determine the optimal spectrum leasing ratio ρ^*. Therefore, $U_F(\rho^*) \geq U_F(\rho)$. So the following equations holds

$$\begin{aligned} U_F(\rho^*, \epsilon^*) &= \sup_{\rho, \epsilon \in [0,1]} U_F(\rho, \epsilon), \\ U_M(c^*) &= \sup_{c \geq 0} U_M(c). \end{aligned} \tag{27}$$

meaning $\{\rho^*, c^*, \epsilon^*\}$ is the equilibrium of the sequential game.

The proof is completed. ■

1) Price of Anarchy: In non-cooperative game without centralized authorities, the interaction among rational but selfish players may lead to inefficient Nash Equilibrium (NE) point. The concept of "Price of Anarchy" is often used to describe the efficiency of the NE of non-cooperative game [9], which is defined as the ratio between the NE and the social optimum (SO) that can be achieved only when a central authority is available.

The aggregated utility of the two service providers at the NE point is defined $U_{NE} = U_{M,NE} + U_{F,NE}$, where $U_{M,NE}$ and $U_{F,NE}$ are the utilities of MSP and FSP at the NE, respectively. Meanwhile, the social optimum of the aggregated utility is obtained by solving the following optimization problem,

$$\begin{aligned} U_{SO} &= \max_{\rho,\epsilon}\ U_M + U_F \\ &= \max_{\rho,\epsilon}\ \omega_m N_m (1 - e^{-\alpha_m r_m}) + \omega_f N_F N_f (1 - e^{-\alpha_f r_f}). \end{aligned} \tag{28}$$

with the constraints,

$$0 \leq \rho \leq 1;\ 0 \leq \epsilon \leq 1.$$

It is easily proved that the objective function is concave, and U_{SO} can be obtained with Lagrange multiplier method. Therefore, the efficiency of the NE is defined as $POA = \frac{U_{NE}}{U_{SO}}$. The NE is more efficient with higher POA. In the next section, we will illustrate the efficiency of the NE in our framework with numerical results.

2) Other Utility Functions: In the previous analysis, we define the utility function of MSP and FSP as elastic one which is for best-effort traffic, we will also discuss the utility functions of other forms to show the robustness of the proposed framework.

In previous works, sigmoid function is commonly employed to represent the utility for rate adaptive application [6]. The cooperative spectrum leasing procedure based on the rate adaptive utility function can be analyzed in the similar way as in the previous subsection. It can be easily proved with Extreme Value Theorem that Nash Equilibrium also exists in the new sequential game for the two players with rate adaptive utility functions. Therefore, the effectiveness of the proposed spectrum leasing framework is not limited by the concrete expression of the utility functions.

IV. Numerical Results

In this subsection, we show the performance gain of the proposed framework, and the impact of some system parameters on the performance with simulation results. Some key system parameters are illustrated in Table I. The other parameters are preset as follow: $\alpha_m = 8$, $\alpha_f = 0.8$, $\Omega_m = 100$ and $\Omega_f = 100$. Unless explicitly otherwise stated, the number of femtocells per macrocell is $N_F = 50$, the number of macrocell users per cell $N_m = 50$, the number of femtocell user per femtocell is 2, and the available spectrum bandwidth is 5MHz. We compare the proposed framework with spectrum leasing without cooperation, i.e., the femtocells operate in closed access mode, as well as the case of no spectrum leasing.

TABLE I
System Paramters

Variables	Parameter Name	Value
R_c, R_f	Macrocell/Femtocell radius	288m, 40m
N_f	Femto user per femto-cell	1
β_f	Femtocell path loss exponent	3
β_m	Macrocell path loss exponent	4
W_{dB}	Wall penetration loss	5dB
ρ_f	Femtocell F-ALOHA spectrum reuse ratio	0.8

Impact of femtocell density

First, we illustrate the impact of the femtocell density on the performance of MSP and FSP. As illustrated in Fig. 4, with higher femtocell density, the spectrum leasing price will increase, while the spectrum obtained by FSP will increase if the femtocells operate in closed access mode, because the more femtocells, the inter-tier interference will be more severe, which results in lower long term throughput. Therefore, the femtocell needs more spectrum to meet the requirement of its users. In contrast, if the femtocells work in hybrid access mode, the spectrum obtained from MSP will decrease due to more access opportunities provided by more dense femto APs for macrocell users. In addition, the femtocell can reduce its open access ratio as shown in Fig. 5 to improve its own utility. Moreover, the efficiency of the NE decreases as the femtocell density increases due to the decrease of both spectrum acquisition and open access ratio of FSP. Fig. 6 shows the utilities of MSP and FSP versus the femtocell density. If there is no spectrum leasing, the utility of FSP is zero, because it will not be able to serve its end users. The utility of both MSP and FSP will increase as the number of femtocells increases because the MSP can obtain more revenue from spectrum leasing, while the FSP will benefit from the increasing number of served users. As shown in Fig. 6, both MSP and FSP will achieve higher utilities if the femtocells operate in hybrid access mode as comparing with femtocells with closed access. Therefore, the FSP has the incentive to open its access opportunity to macrocell users. Since the utility of FSP will increase as the femtocell density increases, the FSP also has the incentive to deploy more femtocells.

Impact of macrocell user density

Moreover, Fig. 7 - Fig. 9 demonstrate the impact of macrocell user density, i.e., the number of macrocell users per cell, on the utilities of MSP and FSP. As shown in Fig. 7, if femtocells operate in close access mode, the spectrum leasing price will increase and the spectrum obtained by FSP decreases, because the revenue obtained from the throughput increase of its users is more than the payment from FSP, the MSP will reserve more spectrum for its own users by setting relatively higher price. However, if the femtocells allow the opportunistic access of macrocell users, more spectrum will be leased to FSP with lower price, because the macrocell users

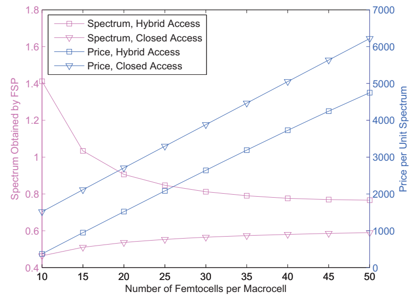

Fig. 4. Price and spectrum leasing amount versus femtocell density.

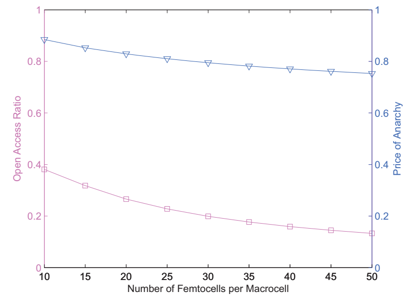

Fig. 5. Open access ratio of femtocell and POA versus femtocell density.

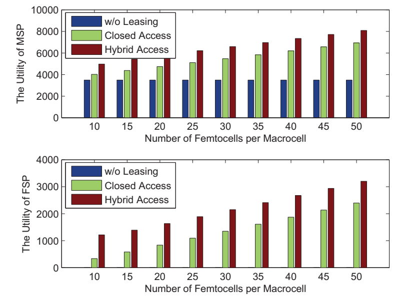

Fig. 6. Utilities of MSP and FSP versus femtocell density.

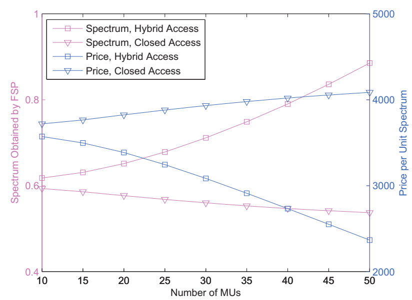

Fig. 7. Price and spectrum leasing amount versus macrocell user number.

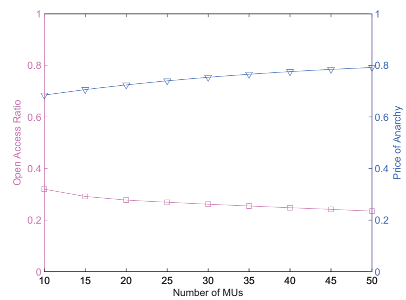

Fig. 8. Open access ratio of femtocell and POA versus macrocell user number.

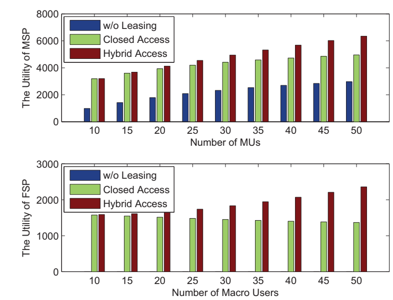

Fig. 9. Utilities of MSP and FSP versus macrocell user number.

are able to obtain more access opportunity from femtocells, although the open access ratio will slightly decrease as shown in Fig. 8. Meanwhile, the efficiency of the NE increases due to the cooperation between MSP and FSP. Fig. 9 shows the utility gain obtained from hybrid access increases for both MSP and FSP when there are more macrocell users, which is consistent with Fig. 7. Therefore, when the user load of the macrocells is high, the MSP tends to lease more spectrum to the FSP to improve the performance of macrocell users as well as the total utility.

Impact of spectrum bandwidth

Finally, Fig. 10 - Fig. 12 shows the effect of the total bandwidth on the utilities of MSP and FSP. With more available bandwidth, the spectrum obtained by close access femtocells will also increase, because the MSP will achieve higher revenue from spectrum leasing comparing with the utility obtained from its users if the spectrum reserved. In contrast, the femtocells with hybrid access will obtain less spectrum because the MSP has more spectrum to serve its own user, and access opportunity provided by femtocell is less necessary. An interesting observation in Fig. 11 is that both the open access ratio and POA stay nearly constant as the total bandwidth increases. As shown in Fig. 12, the utility gain obtained from hybrid access of FSP will decrease with the increase of total spectrum bandwidth. The reason is that when the spectrum is scarce, the throughput of macrocell users obtained from hybrid access of femtocells will be more beneficial according to the definition of the utility function of MSP (12). However, when the spectrum is sufficient, the macrocell users can reach relatively high throughput, and the benefit of extra throughput obtained from hybrid access is less than the revenue obtained from spectrum leasing if more spectrum is rented by the FSP.

V. Related Works

The problem we considered in this paper is related to two areas in wireless network, namely, the resource allocation in two-tier femtocell network, and the game theory based spectrum leasing. Therefore, we restrict our literature review to those papers that are the most relevant.

There has been a growing concern on the research on two-tier femtocell network [10]–[12]. In [11], the authors propose a location-based resource management solution for maximal spatial reuse from femto cells. A distributed utility-based SINR adaptation at femtocells is proposed in order to alleviate cross-tier interference at the macrocell from cochannel femtocells [12]. In [13], the resource allocation among femtocells is described as a non-cooperative game, and a decentralized power control algorithm is proposed. In [14], a Stackelberg game is formulated to study the joint utility maximization of the macrocell and the femtocells subject to a maximum tolerable interference power constraint at the macrocell base station. Our work differs from the aforementioned because we consider that the spectrum allocated between macrocell and femtocell is orthogonal in frequency domain, i.e., there is no cross-tier interference. Moreover, none of the aforementioned works considers the concrete impact of hybrid access.

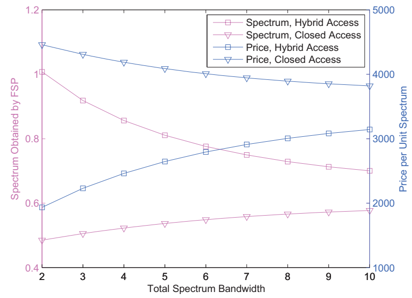

Fig. 10. Price and spectrum leasing amount versus spectrum bandwidth.

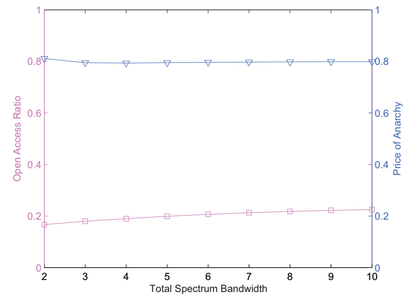

Fig. 11. Open access ratio of femtocell and POA versus spectrum bandwidth.

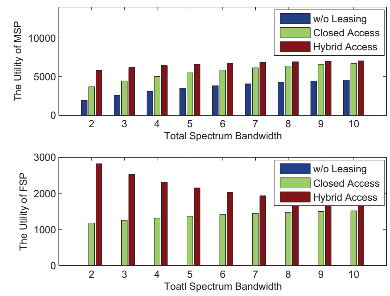

Fig. 12. Utilities of MSP and FSP versus spectrum bandwidth.

Game theory is widely used in the spectrum leasing context. Auction mechanism [15] and linear/non-linear pricing [16] are two main tools for spectrum trading. In [16], a Stackelberg game between three players: spectrum owner, primary users and secondary users is presented under the opportunistic spectrum access (OSA) model, where the secondary users share the channel with the probability of interference to the primary users below a tolerance threshold, and pay subscribe fee to the spectrum owner in proportion to that of the primary users. In [17], a pricing-based spectrum trading mechanism is proposed that enables secondary users to contend for channel usage by random access in a distributed manner. The first difference between the aforementioned works and our work is that we consider the spectrum leasing in a statistics manner, rather than a deterministic one. Another difference between those works and this one lines in the hybrid access we considered. To the best of our knowledge, this is the first work that takes hybrid access into the spectrum leasing.

VI. Conclusions

In this paper, we propose a spectrum leasing framework for coexisted MSP and FSP, which considers the hybrid access of femtocells. Based on three-stage Stackelberg game model, the optimal spectrum leasing price, spectrum leasing ratio, and open access ratio are obtained from the Nash Equilibrium, which maximizes the utilities of MSP and FSP. Numerical results show that the proposed framework can improve the utilities of MSP and FSP comparing with both no spectrum leasing and spectrum leasing to closed access femtocells. Therefore, the MSP has the incentive to share spectrum with FSP, and FSP is willing to open access opportunities to the macrocell users.

A natural next step is to explore the framework with multiple FSPs or MSPs, when they will compete with each other in this spectrum leasing market. Another potential direction lies in the behaviors of the end users on service selection from multiple service providers.

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