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Spectrum sharing on interference channels with a cognitive relay

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Abstract

In this paper, an interference channel with a cognitive relay (IFC-CR) is considered to achieve spectrum sharing between a licensed primary user and an unlicensed secondary user. The CR assists both users in relaying their messages to the respective receivers, under the constraint that the performance of the legacy primary user is not degraded. Without requiring any non-causal knowledge, the CR uses a successive interference cancellation to first decode the primary and secondary messages after a transmission phase. A power allocation is then performed to forward a linear weighted combination of the processed signals in the relaying phase. Closed-form expressions of the end-to-end outage probability are derived for both primary and secondary users under the proposed approach. Furthermore, by exploiting the decoded primary and secondary messages in the first phase, we propose the use of dirty paper coding (DPC) at CR to pre-cancel the interference seen at the secondary (or primary) receiver in the second phase, which results in a performance upper bound for the secondary (or primary) user without affecting the other user. Simulation results demonstrate that with a joint consideration of the power control at the secondary transmitter and the power allocation at CR, performance gains can be achieved for both primary and secondary users.

Keywords: Cognitive spectrum sharing; Interference channel with a cognitive relay; Successive interference cancellation; Dirty paper coding

1 Introduction

1.1 Background and related work

Consider a spectrum sharing system, as shown in Fig. 1, where two interfering users co-exist with a cognitive relay in the same frequency band. For both users to operate properly, the cognitive relay [1, 2] assists in forwarding the messages from both transmitters to their respective receivers [3–6] and at the same time coordinates the mutual interference. This constitutes an interference channel with a cognitive relay (IFC-CR), which has been intensively studied from an information-theoretic perspective [7–15].

A two-user symmetric Gaussian IFC-CR was first introduced in [7], where the CR was assumed to be full duplex and adopt a decode-and-forward (DF) processing. Through rate splitting [16] at both sources and joint decoding at each destination, an achievable rate region was obtained. Then, this achievable rate was improved
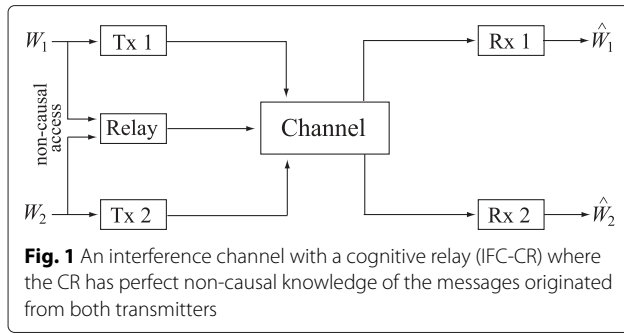
in [8] by performing sophisticated coding strategies that require non-causal information of both transmitters at the CR prior to information transmission. By combining the Han-Kobayashi coding scheme [16] for interference channels with dirty paper coding (DPC) [17], a generalization of the achievable rate region obtained in [8] was derived in [9]. In [10], an outer bound for the capacity of a general IFC-CR was first derived. New inner and outer bounds for the capacity of IFC-CR were derived later in [11–15], under various conditions.

A Gaussian interference channel with an out-of-band relay was investigated in [18, 19] where the relay was assumed to operate over orthogonal bands to the underlying interference channel. In [18], the entire system was characterized by two parallel channels, namely a Gaussian interference channel and a Gaussian relay channel. To characterize the capacity, relay operations were optimized with separable or nonseparable encoding between the interference channel and the out-of-band relay channel. In [19], the impact of the out-of-band relay channel and the corresponding signal interactions on the capacity were investigated under general channel conditions.

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Fig. 1 An interference channel with a cognitive relay (IFC-CR) where the CR has perfect non-causal knowledge of the messages originated from both transmitters

In the above works, the two interfering users are assumed to be part of the peer users in the same radio system. A question that arises is as follows: what if the two interfering users belong to different radio systems that are of different priorities. In view of the mutual interference between the two users and the inherent cognition and cooperation ability equipped at the CR, it is natural to evaluate IFC-CR under a cognitive spectrum sharing setup between, e.g., a licensed primary user and an unlicensed secondary user [20–23]. Under such circumstances, we assume that the CR belongs to the secondary system or a third-party agent. Then, instead of characterizing the capacity or sum rate of the entire system as in [7–15], it is more pragmatic to enhance the performance of the secondary user under the constraint that no harm is caused to the legacy primary user [24–26].
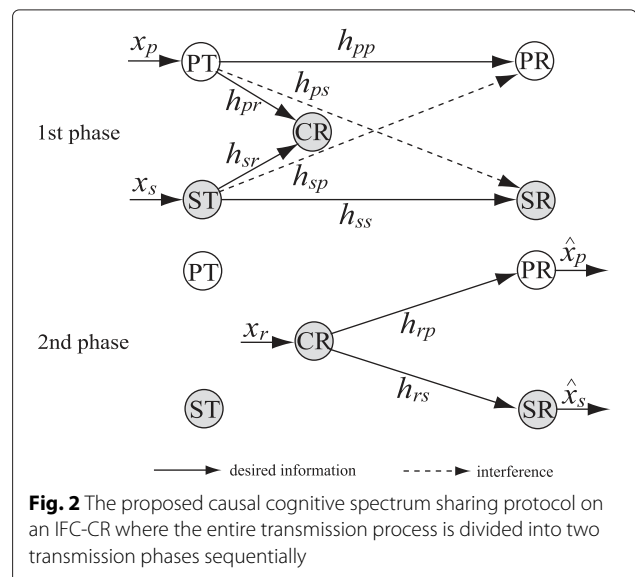
A spectrum sharing protocol was proposed on the interference channel in [26]. With the assumption that the secondary transmitter has non-causal knowledge of the codewords originated at the primary transmitter, achievable rates of the secondary user were characterized under the constraint that no rate degradation was created for the primary system. As a variant of [26], a spectrum sharing protocol was proposed between a primary and a secondary user on an IFC-CR in [27]. With the assumption that non-causal knowledge of the primary codewords is available at both the secondary transmitter and the CR, an enhanced throughput was achieved for the secondary user without degrading the throughput of the primary user. In [28], a spectrum sharing protocol was proposed on an IFC-CR where the CR helps both the primary and secondary transmissions. A DF relay protocol was considered where only when *both* primary and secondary messages are successfully decoded at CR, they are forwarded in the second phase with a certain power allocation. Conditioned on the decoding results at CR, the received SNR at each receiver was analyzed, through which an upper bound of the outage probability was derived. In [29], an opportunistic adaptive relaying protocol is proposed on IFC-CR, where CR is able to determine when to cooperate with the primary user, when to cooperate with the secondary user, and when to cooperate with both users

simultaneously. An upper bound of the secondary outage probability was derived under a primary outage probability threshold. In [30], an amplify-and-forward (AF) relay protocol was performed at CR to help relay the signals of both primary and secondary users over independent Nakagami-*m* fading channels. Assuming there are no cross links between primary and secondary users, end-to-end outage probabilities of the primary and secondary users were obtained. Simulation results demonstrated a performance gain for both primary and secondary users.

1.2 Our contributions

Motivated by the above works, we propose a causal cognitive spectrum sharing protocol on a fading IFC-CR [31]. As depicted in Fig. 2, both primary transmitter (PT) and secondary transmitter (ST) transmit simultaneously in the first phase. Without requiring non-causal knowledge of the messages from PT or ST, CR attempts to decode both messages using successive interference cancellation (SIC) [32]. To compensate the interference seen at the primary receiver (PR) that is caused by cross talk, we consider a power allocation scheme at CR. To be specific, a hybrid AF-DF relay protocol [33] is considered such that when at least one of the two messages is successfully decoded, a fraction α, $0 < \alpha < 1$, of the transmit power of CR is used to forward the primary signal, with the remaining power to forward the secondary signal, in the second phase. If however, neither of the messages is decoded, the CR simply stays silent and both PT and ST perform a retransmission simultaneously in the second phase.

At the end of the second phase, by exploiting the received signals in two phases, maximal-ratio combining (MRC) is employed at PR to decode the desired message. Without incurring additional overhead to the legacy

Fig. 2 The proposed causal cognitive spectrum sharing protocol on an IFC-CR where the entire transmission process is divided into two transmission phases sequentially

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primary system, we assume that PR is only aware of a relay terminal but oblivious of the secondary system [20–23], thus the component of the secondary signal is simply treated as noise at PR [24–26]. This provides a performance lower bound for the primary system compared to the cases with sophisticated decoding strategies. On the other hand, at the secondary receiver (SR), by exploiting the received signals in two phases, SIC is employed to decode the desired message. The contributions of this paper are summarized as follows.

- In decoding the mixed signals using SIC, we define an event to describe whether a specific message can be successfully recovered. In order to illustrate the correlation between the successive events in decoding mixed signals, we introduce a graphical representation by which each event can be represented by the corresponding region in a 2-dimensional (or 3-dimensional) graph. On this basis, by integrating over the respective regions of events, accurate closed-form expressions of the end-to-end outage probability can be derived for both primary and secondary users under the proposed protocol.

- Without requiring non-causal knowledge, CR attempts to decode both primary and secondary messages after a first transmission phase in the proposed protocol. For the case where both messages are successfully recovered at CR after the first transmission phase, in order to further mitigate the mutual interference, we propose using DPC at CR to pre-cancel the interference seen at PR or SR in the subsequent relaying phase. Numerical results demonstrate a performance upper bound for the primary (or secondary) user, without affecting the performance of the other user.

- To guarantee that no harm is caused to the primary system, besides the power allocation performed at CR to forward the primary and secondary messages respectively, we find that a power control at ST is also needed to facilitate the SIC decoding at CR as well as to limit the interference caused to PR. Numerical results demonstrate that with a proper design of the power allocation at CR and the transmit power at ST, the secondary user is allowed to access the licensed spectrum and at the same time performance gains can be achieved for both primary and secondary systems.

The rest of this paper is organized as follows. Section 2 describes the system model, where the two successive phases are discussed and the end-to-end outage probability of IFC-CR is defined. In Section 3, based on the possible decoding results at CR in the first transmission phase, the corresponding performance at PR and SR in the second phase is analyzed. By exploiting the decoded messages, in Section 4, we propose using DPC at CR, which provides a performance upper bound. Simulation results are presented in Section 5 where the effects of different parameters are evaluated. Finally, Section 6 concludes the paper.

2 System model and protocol description

As shown in Fig. 2, we consider an IFC-CR where a ST/SR pair co-exists with a PT/PR pair in the same frequency band with the assistance of a CR. It is assumed that all nodes operate in half-duplex mode and the channels experience independent block Rayleigh fading. For notational simplicity, we let h_{pp} and h_{ss} denote the coefficients of the direct channels from PT→PR and ST→SR, let h_{ps} and h_{sp} denote the coefficients of the cross interfering channels from PT→SR and ST→PR, and let h_{pr}, h_{sr}, h_{rp}, and h_{rs} denote the coefficients of the relay channels from PT→CR, ST→CR, CR→PR, and CR→SR, respectively. Then, we have the channel coefficient $h_{ij} \sim \mathcal{CN}\left(0, \delta_{ij}^{-1}\right)$, where $ij \in \{pp, ss, ps, sp, pr, sr, rp, rs\}$ and δ_{ij}^{-1} denotes the corresponding average channel power gain. By letting $\gamma_{ij} = |h_{ij}|^2$, γ_{ij} follows an exponential distribution that $\gamma_{ij} \sim \exp\left(\delta_{ij}\right)$ [34]. For ease of exposition, we assume that the additive white Gaussian noise (AWGN) n_0 is of zero mean and with unitary variance at each receiver. Any channel can be reduced to this normalized form. The transmit powers at PT, ST, and CR are denoted as P_P, P_S, and P_R, respectively. x_p and x_s denote the messages originated at PT and ST, with target rates R_{pt} and R_{st}, respectively. For easy reference, we summarize the abbreviations, notations, and symbols that appear in this paper in Table 1.

As shown in Fig. 2, in order to maintain the causality of the system, we divide the entire transmission process into two phases as discussed in the following.

2.1 First transmission phase

In the first phase, both PT and ST transmit their respective messages simultaneously. Then, the corresponding received signal at PR, SR, and CR is given as

$$y_j(1) = h_{pj}\sqrt{P_P}x_p + h_{sj}\sqrt{P_S}x_s + n_0, \quad j \in \{p, s, r\}. \quad (1)$$

In order to recover x_p and x_s that are mixed together, SIC is performed at CR. Then, for the decoding results of x_p and x_s, we define the following possible events that are mutually exclusive:

1. $\mathcal{E}^{(1)} = \{$Both x_p and x_s are successfully decoded at CR$\}$;
2. $\mathcal{E}^{(2)} = \{$Only x_p is successfully decoded at CR$\}$;
3. $\mathcal{E}^{(3)} = \{$Only x_s is successfully decoded at CR$\}$;
4. $\mathcal{E}^{(4)} = \{$Neither of x_p and x_s is successfully decoded at CR$\}$.

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Table 1 A summary of abbreviations, notations, and symbols

SIC	Successive interference cancellation
IFC-CR	Interference channel with a cognitive relay
DPC	Dirty paper coding
AF/DF	Amplify-and-forward/decode-and-forward
PT/PR	Primary transmitter/primary receiver
ST/SR	Secondary transmitter/secondary receiver
MRC	Maximal-ratio combining
AWGN	Additive white Gaussian noise
x_p	Signal transmitted from PT
x_s	Signal transmitted from ST
x_r	Signal transmitted from CR
y_p	Signal received at PR
y_s	Signal received at SR
y_r	Signal received at CR
n_0	AWGN that is with unitary variance
h_{ij}	Channel coefficient of link $i \rightarrow j$
$\gamma_{ij} = |h_{ij}|^2$	Channel power gain of link $i \rightarrow j$
$\exp(\delta)$	An exponential distribution with mean δ^{-1}
P_P	Transmit power at PT
P_S	Transmit power at ST
P_R	Transmit power at CR
R_{pt}	Target rate at PT
R_{st}	Target rate at ST
$\mathcal{E}, \overline{\mathcal{E}}$	An event and its complementary event
$\Pr\{\mathcal{E}\}$	Probability of event $\mathcal{E}$
O_P	End-to-end outage probability of the primary system
O_S	End-to-end outage probability of the secondary system
α	Power allocation factor at CR
θ	Ratio between P_S and P_P
τ	Ratio between δ_{sr}^{-1} and δ_{pr}^{-1}
φ	Ratio between δ_{sp}^{-1} and δ_{pp}^{-1}

The corresponding probabilities are defined as $\Pr\left\{\mathcal{E}^{(1)}\right\}$, $\Pr\left\{\mathcal{E}^{(2)}\right\}$, $\Pr\left\{\mathcal{E}^{(3)}\right\}$, and $\Pr\left\{\mathcal{E}^{(4)}\right\}$ that will be derived in Section 3.1.

2.2 Second transmission phase

When at least one of x_p and x_s is successfully decoded by CR, a power allocation is performed to forward a linear weighted combination of x_p and x_s in the second phase. Otherwise, CR simply stays silent and both PT and ST perform retransmissions simultaneously.

2.2.1 Conditioned on event $\mathcal{E}^{(1)}$

In the second phase, CR broadcasts a composite message

$$x_r = \sqrt{\alpha P_R}x_p + \sqrt{(1-\alpha)P_R}x_s, \qquad (2)$$

where $\alpha \in (0, 1)$ is the power allocation factor for relaying the primary message x_p. Then, the corresponding received signal at PR and SR is given as

$$y_j(2) = h_{rj}\sqrt{\alpha P_R}x_p + h_{rj}\sqrt{(1-\alpha)P_R}x_s + n_0, \quad j \in \{p, s\}. \qquad (3)$$

2.2.2 Conditioned on event $\mathcal{E}^{(2)}$

In the second phase, CR broadcasts a composite message

$$x_r = \sqrt{\alpha P_R}x_p + \beta\left(h_{sr}\sqrt{P_S}x_s + n_0\right), \quad \text{where}$$
$$\beta = \sqrt{\frac{(1-\alpha)P_R}{\gamma_{sr}P_S + 1}}. \qquad (4)$$

Then, the corresponding received signal at PR and SR is given as

$$y_j(2) = h_{rj}\sqrt{\alpha P_R}x_p + h_{rj}\beta\left(h_{sr}\sqrt{P_S}x_s + n_0\right) + n_0, \quad j \in \{p, s\}. \qquad (5)$$

2.2.3 Conditioned on event $\mathcal{E}^{(3)}$

In the second phase, CR broadcasts a composite message

$$x_r = \beta\left(h_{pr}\sqrt{P_P}x_p + n_0\right) + \sqrt{(1-\alpha)P_R}x_s, \quad \text{where}$$
$$\beta = \sqrt{\frac{\alpha P_R}{\gamma_{pr}P_P + 1}}. \qquad (6)$$

Then, the corresponding received signal at PR and SR is given as

$$y_j(2) = h_{rj}\beta\left(h_{pr}\sqrt{P_P}x_p + n_0\right) + h_{rj}\sqrt{(1-\alpha)P_R}x_s + n_0, \quad j \in \{p, s\}. \qquad (7)$$

2.2.4 Conditioned on event $\mathcal{E}^{(4)}$

CR simply stays silent and PT and ST retransmit x_p and x_s respectively in the second phase. Then, the corresponding received signal at PR and SR is given as

$$y_j(2) = h_{pj}\sqrt{P_P}x_p + h_{sj}\sqrt{P_S}x_s + n_0, \quad j \in \{p, s\}. \qquad (8)$$

2.3 End-to-end performance

For the decoding at PR at the end of the second phase, MRC is performed to decode x_p by utilizing the received signals in two successive phases, i.e., $y_p(1)$ and $y_p(2)$, while treating the secondary component of x_s simply as noise. Then, depending on the decoding results $\mathcal{E}^{(1)}$, $\mathcal{E}^{(2)}$, $\mathcal{E}^{(3)}$, and $\mathcal{E}^{(4)}$, at CR at the end of the first phase, we define $O_P^{(1)}$, $O_P^{(2)}$, $O_P^{(3)}$, and $O_P^{(4)}$ as the corresponding outage probabilities at PR at the end of the second phase. On the other hand, SIC is performed at SR to decode the desired message x_s by utilizing both received signals $y_s(1)$ and $y_s(2)$.

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Similarly, we define $O_S^{(1)}$, $O_S^{(2)}$, $O_S^{(3)}$, and $O_S^{(4)}$ as the corresponding outage probabilities at SR at the end of the second phase.

Theorem 1. *In the proposed spectrum sharing protocol on IFC-CR, taking into account all possible decoding results at CR at the end of the first phase, the overall end-to-end outage probabilities of the primary and secondary systems can be respectively derived as*

$$O_P = \Pr\left\{\mathcal{E}^{(1)}\right\}O_P^{(1)} + \Pr\left\{\mathcal{E}^{(2)}\right\}O_P^{(2)} + \Pr\left\{\mathcal{E}^{(3)}\right\}O_P^{(3)}$$

$$+ \Pr\left\{\mathcal{E}^{(4)}\right\}O_P^{(4)}, \tag{9}$$

$$O_S = \Pr\left\{\mathcal{E}^{(1)}\right\}O_S^{(1)} + \Pr\left\{\mathcal{E}^{(2)}\right\}O_S^{(2)} + \Pr\left\{\mathcal{E}^{(3)}\right\}O_S^{(3)}$$

$$+ \Pr\left\{\mathcal{E}^{(4)}\right\}O_S^{(4)}. \tag{10}$$

Next, we proceed to analyze the decoding performance at CR as well as PR and SR in two successive phases, respectively.

3 Numerical analysis

3.1 Decoding performance at CR in the first phase

At the end of the first phase, CR attempts to decode both x_p and x_s from $y_r(1)$ using SIC. From (1), if the power level of x_p is higher than that of x_s, then CR attempts to decode x_p first by considering the component of x_s as noise. Defining $C(\Lambda) = \frac{1}{2}\log_2(1 + \Lambda)$, the achievable rate of x_p is given as

$$R_{1,p} = C\left(\frac{P_P\gamma_{pr}}{P_S\gamma_{sr} + 1}\right). \tag{11}$$

Thus, x_p can be first decoded if event

$$\mathcal{E}_{1,p} = \{R_{1,p} \geq R_{pt}\} = \left\{\gamma_{pr} \geq \frac{R'_{pt}P_S}{P_P}\gamma_{sr} + \frac{R'_{pt}}{P_P}\right\} \tag{12}$$

occurs, where $R'_{pt} = 2^{2R_{pt}} - 1$. By reconstructing and removing the component of x_p from $y_r(1)$, the achievable rate of the remaining x_s is given as

$$R_{2,s} = C\left(P_S\gamma_{sr}\right), \tag{13}$$

which can be decoded successively if event

$$\mathcal{E}_{2,s} = \{R_{2,s} \geq R_{st}\} = \left\{\gamma_{sr} \geq \frac{R'_{st}}{P_S}\right\} \tag{14}$$

occurs, where $R'_{st} = 2^{2R_{st}} - 1$.

Conversely, if the power level of x_s is higher than that of x_p, then CR attempts to decode x_s first by considering the

component of x_p as noise. Then, the achievable rate of x_s that is subject to the interference from x_p is given as

$$R_{1,s} = C\left(\frac{P_S\gamma_{sr}}{P_P\gamma_{pr} + 1}\right), \tag{15}$$

which can be first decoded if event

$$\mathcal{E}_{1,s} = \{R_{1,s} \geq R_{st}\} = \left\{\gamma_{pr} \leq \frac{P_S}{R'_{st}P_P}\gamma_{sr} - \frac{1}{P_P}\right\} \tag{16}$$

occurs. Similarly, by reconstructing and removing the component of x_s from $y_r(1)$, the achievable rate of the remaining x_p is given as

$$R_{2,p} = C\left(P_P\gamma_{pr}\right), \tag{17}$$

which can be decoded successively if event

$$\mathcal{E}_{2,p} = \{R_{2,p} \geq R_{pt}\} = \left\{\gamma_{pr} \geq \frac{R'_{pt}}{P_P}\right\} \tag{18}$$

occurs.

Thus, from (12), (14), (16), and (18), all possible decoding results at CR using SIC can be expressed as

$$\mathcal{E}^{(1)} = \left\{\left(\mathcal{E}_{1,p} \cap \mathcal{E}_{2,s}\right) \cup \left(\mathcal{E}_{1,s} \cap \mathcal{E}_{2,p}\right)\right\}, \tag{19a}$$

$$\mathcal{E}^{(2)} = \left\{\mathcal{E}_{1,p} \cap \bar{\mathcal{E}}_{2,s}\right\}, \tag{19b}$$

$$\mathcal{E}^{(3)} = \left\{\mathcal{E}_{1,s} \cap \bar{\mathcal{E}}_{2,p}\right\}, \tag{19c}$$

$$\mathcal{E}^{(4)} = \left\{\bar{\mathcal{E}}_{1,p} \cap \bar{\mathcal{E}}_{1,s}\right\}. \tag{19d}$$

Take a close look at (12), (14), (16), and (18); since the multiple-access channels h_{sr} and h_{pr} are independent with each other, we can draw a 2-dimensional graph of γ_{sr} and γ_{pr} where the events defined in (19a)–(19d) are represented by their respective regions, as shown in Fig. 3. Here, we assume that $R_{pt}, R_{st} \geq 1$ such that there is no intersection between the regions of events $\mathcal{E}_{1,p}$ and $\mathcal{E}_{1,s}$. With the respective regions defined by $\{\gamma_{sr}, \gamma_{pr}\}$, the corresponding probability of each event can thus be obtained by

$$\int\int_{\{\gamma_{sr},\gamma_{pr}\}} f(\gamma_{sr}, \gamma_{pr})d\gamma_{pr}d\gamma_{sr}$$

$$= \int_{\{\gamma_{sr}\}} f(\gamma_{sr})\left[\int_{\{\gamma_{pr}\}} f(\gamma_{pr})d\gamma_{pr}\right]d\gamma_{sr}, \tag{20}$$

where $f(\gamma_{sr}) = \delta_{sr}e^{-\delta_{sr}\gamma_{sr}}$ and $f(\gamma_{pr}) = \delta_{pr}e^{-\delta_{pr}\gamma_{pr}}$ denote the respective probability density functions (PDF) of γ_{sr} and γ_{pr}, and $f(\gamma_{sr}, \gamma_{pr}) = f(\gamma_{sr})f(\gamma_{pr})$ denotes the joint PDF [34].

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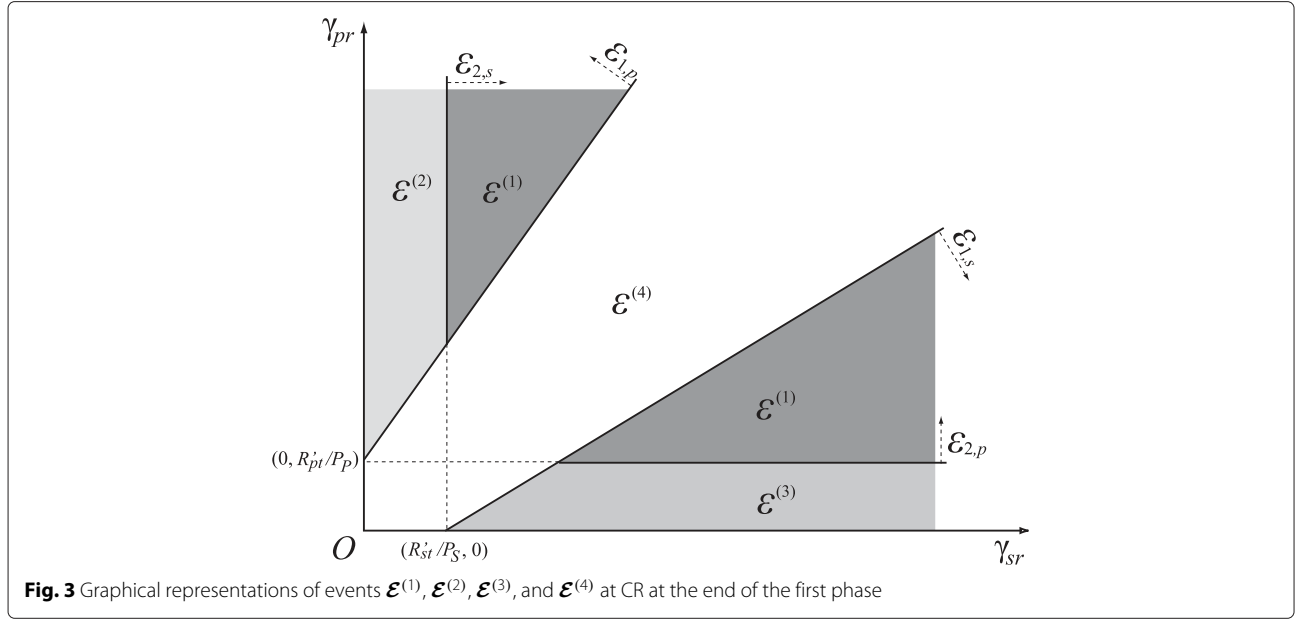
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Fig. 3 Graphical representations of events $\mathcal{E}^{(1)}$, $\mathcal{E}^{(2)}$, $\mathcal{E}^{(3)}$, and $\mathcal{E}^{(4)}$ at CR at the end of the first phase

Lemma 1. *By employing SIC at CR to decode both x_p and x_s, the respective probabilities of the events defined in (19a)–(19d) can be obtained as*

$$\Pr\left\{\mathcal{E}^{(1)}\right\} = \frac{\delta_{sr} e^{-\left(\frac{\delta_{sr}R'_{st}}{P_S} + \frac{\delta_{pr}R'_{pt}(1+R'_{st})}{P_P}\right)}}{\delta_{sr} + \frac{\delta_{pr}R'_{pt}P_S}{P_P}}$$

$$+ \frac{\delta_{pr} e^{-\left(\frac{\delta_{pr}R'_{pt}}{P_P} + \frac{\delta_{sr}R'_{st}(1+R'_{pt})}{P_S}\right)}}{\delta_{pr} + \frac{\delta_{sr}R'_{st}P_P}{P_S}}, \qquad (21a)$$

$$\Pr\left\{\mathcal{E}^{(2)}\right\} = \frac{\delta_{sr}\left[e^{-\frac{\delta_{pr}R'_{pt}}{P_P}} - e^{-\left(\frac{\delta_{sr}R'_{st}}{P_S} + \frac{\delta_{pr}R'_{pt}(1+R'_{st})}{P_P}\right)} \right]}{\delta_{sr} + \frac{\delta_{pr}R'_{pt}P_S}{P_P}}, (21b)$$

$$\Pr\left\{\mathcal{E}^{(3)}\right\} = \frac{\delta_{pr}\left[e^{-\frac{\delta_{sr}R'_{st}}{P_S}} - e^{-\left(\frac{\delta_{pr}R'_{pt}}{P_P} + \frac{\delta_{sr}R'_{st}(1+R'_{pt})}{P_S}\right)} \right]}{\delta_{pr} + \frac{\delta_{sr}R'_{st}P_P}{P_S}}, (21c)$$

$$\Pr\left\{\mathcal{E}^{(4)}\right\} = 1 - \frac{\delta_{sr} e^{-\frac{\delta_{pr}R'_{pt}}{P_P}}}{\delta_{sr} + \frac{\delta_{pr}R'_{pt}P_S}{P_P}} - \frac{\delta_{pr} e^{-\frac{\delta_{sr}R'_{st}}{P_S}}}{\delta_{pr} + \frac{\delta_{sr}R'_{st}P_P}{P_S}}. \qquad (21d)$$

Please find in Appendix A for the detailed derivations.

3.2 Decoding performance at PR in the second phase
3.2.1 Conditioned on event $\mathcal{E}^{(1)}$

Together with (1) and (3), MRC is performed at PR to decode the desired message x_p. Then, the corresponding achievable rate is given as

$$R_p^{(1)} = C\left(\frac{P_P\gamma_{pp}}{P_S\gamma_{sp} + 1} + \frac{\alpha}{1 - \alpha + \frac{1}{P_R\gamma_{rp}}} \right)$$

$$\approx C\left(\frac{P_P\gamma_{pp}}{P_S\gamma_{sp} + 1} + \frac{\alpha}{1 - \alpha} \right), \qquad (22)$$

where the approximation is obtained assuming $P_R \gg 1$ [24, 25].

Lemma 2. *Conditioned on event $\mathcal{E}^{(1)}$ that both x_p and x_s are successfully decoded at CR at the end of the first phase, the corresponding outage probability at PR at the end of the second phase can be derived as*

$$O_P^{(1)} = \Pr\left\{ R_p^{(1)} < R_{pt} \right\}$$

$$\approx \begin{cases} 0, & \alpha \geq \frac{R'_{pt}}{R'_{pt}+1} \\ 1 - \frac{\delta_{sp} e^{-\frac{\delta_{pp}\left(R'_{pt} - \frac{\alpha}{1-\alpha}\right)}{P_P}}}{\delta_{sp} + \frac{\left(R'_{pt} - \frac{\alpha}{1-\alpha}\right)P_S\delta_{pp}}{P_P}}, & \alpha < \frac{R'_{pt}}{R'_{pt}+1} \end{cases}. \quad (23)$$

Please find in Appendix B for the detailed derivations.

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3.2.2 Conditioned on event $\mathcal{E}^{(2)}$

Together with (1) and (5), by employing MRC at PR, the achievable rate of x_p is given as

$$R_p^{(2)} = C\left(\frac{P_P\gamma_{pp}}{P_S\gamma_{sp}+1} + \frac{\alpha P_R\gamma_{rp}}{\gamma_{rp}\beta^2(P_S\gamma_{sr}+1)+1}\right)$$

$$= C\left(\frac{P_P\gamma_{pp}}{P_S\gamma_{sp}+1} + \frac{\alpha}{(1-\alpha)+\frac{1}{P_R\gamma_{rp}}}\right), \quad (24)$$

which is exactly the same (22). This is because in both cases, x_p is successfully decoded and forwarded by CR with power αP_R. Thus, the corresponding outage probability at PR at the end of the second phase is

$$O_P^{(2)} = O_P^{(1)}. \quad (25)$$

3.2.3 Conditioned on event $\mathcal{E}^{(3)}$

Together with (1) and (7), by employing MRC at PR, the achievable rate of x_p is given as

$$R_p^{(3)} = C\left(\frac{P_P\gamma_{pp}}{P_S\gamma_{sp}+1} + \frac{\gamma_{rp}\beta^2 P_P\gamma_{pr}}{(1-\alpha)P_R\gamma_{rp} + \gamma_{rp}\beta^2 + 1}\right)$$

$$= C\left(\frac{P_P\gamma_{pp}}{P_S\gamma_{sp}+1}\right.$$

$$\left. + \frac{\alpha}{(1-\alpha)+\frac{1}{P_P\gamma_{pr}}+\frac{1}{P_R\gamma_{rp}}+\frac{1}{P_P\gamma_{pr}P_R\gamma_{rp}}}\right)$$

$$\approx C\left(\frac{P_P\gamma_{pp}}{P_S\gamma_{sp}+1} + \frac{\alpha}{1-\alpha}\right), \quad (26)$$

where the approximation in (26) is obtained assuming $P_P, P_R \gg 1$ [24, 25]. Since (26) is of the same form as (22), the corresponding outage probability at PR at the end of the second phase is

$$O_P^{(3)} \approx O_P^{(1)}. \quad (27)$$

3.2.4 Conditioned on event $\mathcal{E}^{(4)}$

Together with (1) and (8), by employing MRC at PR, the achievable rate of x_p is given as

$$R_p^{(4)} = C\left(\frac{2P_P\gamma_{pp}}{P_S\gamma_{sp}+1}\right). \quad (28)$$

Lemma 3. *Conditioned on event $\mathcal{E}^{(4)}$ that neither of x_p and x_s is successfully decoded at CR at the end of the first*

phase, the corresponding outage probability at PR at the end of the second phase can be derived as

$$O_P^{(4)} = \Pr\left\{R_p^{(4)} < R_{pt}\right\} = 1 - \frac{\delta_{sp}e^{-\frac{\delta_{pp}R'_{pt}}{2P_P}}}{\delta_{sp} + \frac{\delta_{pp}R'_{pt}P_S}{2P_P}}. \quad (29)$$

Please find in Appendix C for the detailed derivations.

Substituting (23), (25), (27), and (29) into (9), we can thus obtain the end-to-end outage probability O_P of the primary system.

Theorem 2. *Take a close look at (23), (25), and (27), all $O_P^{(1)}$, $O_P^{(2)}$, and $O_P^{(3)}$ approach 0 when $\alpha \geq \frac{R'_{pt}}{R'_{pt}+1}$. Then, from (9), the term $\Pr\{\mathcal{E}^{(4)}\}O_P^{(4)}$ dominates the end-to-end outage probability O_P. In other words, the component of $\Pr\{\mathcal{E}^{(4)}\}O_P^{(4)}$ brings a lower bound to O_P. Defining $\alpha^* = \frac{R'_{pt}}{R'_{pt}+1}$, the end-to-end outage performance of the primary system is thus optimized when $\alpha \geq \alpha^*$, i.e.,*

$$\arg\min_{\alpha}\{O_P\} = \left\{\alpha|\alpha \geq \alpha^*\right\}, \text{ and}$$

$$\min_{\alpha}\{O_P\} \geq \Pr\left\{\mathcal{E}^{(4)}\right\}O_P^{(4)}. \quad (30)$$

Remark 1. *From Theorem 2, CR may simply select a power allocation factor $\alpha = \alpha^* = \frac{R'_{pt}}{R'_{pt}+1}$ such that a reasonably good performance is achieved for the primary system, without requiring the CSI or other relevant information.*

Remark 2. *For comparison purposes, we consider a benchmark case for the primary system without spectrum sharing, where PT transmits to PR directly without a relay. Then, with target rate R_{pt}, the corresponding outage probability is*

$$O'_P = 1 - e^{-\frac{\delta_{pp}\left(2^{R_{pt}}-1\right)}{P_P}}. \quad (31)$$

The details are omitted here for the sake of brevity. Together with (9) and (31), in order to provide the primary system an incentive to participate in the spectrum sharing, the following condition has to be satisfied

$$O_P \leq O'_P. \quad (32)$$

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Remark 3. *From (30) and (31), if* $\Pr\left\{\mathcal{E}^{(4)}\right\}O_P^{(4)} \leq O'_P$, *then it is possible to find a suitable power allocation factor* α, *e.g.,* $\alpha \geq \alpha^*$, *such that the condition in (32) is satisfied. If however,* $\Pr\left\{\mathcal{E}^{(4)}\right\}O_P^{(4)} > O'_P$, *then the primary system experiences a performance loss compared to the benchmark case even when* $\alpha \to 1$. *Thus, in order to guarantee the performance of the legacy primary system, apart from selecting a proper power allocation factor* $\alpha \geq \alpha^*$ *at CR, the transmit power* P_S *also needs to be properly designed.*

For the power control of P_S, it is assumed that the statistical CSI of h_{sp} and h_{pp} is available at ST, which is a common assumption made in existing works [20–23]. Assuming that the channels are reciprocal, the CSI can be acquired at ST through a feedback channel from PR [35–37]. In addition, other relevant information is also required, i.e., P_P and R_{pt}, which is usually inserted in the header of a packet that can be overheard by ST. With these information, both $\Pr\left\{\mathcal{E}^{(4)}\right\}O_P^{(4)} \leq O'_P$ and O'_P can be estimated at ST. Furthermore, we assume that the probability $\Pr\left\{\mathcal{E}^{(4)}\right\}$ is available at ST through a feedback channel from CR. Thus, although it is intractable to analytically derive P_S such that $O_P \leq O'_P$, as long as a suitable P_S is found to make sure that $\Pr\left\{\mathcal{E}^{(4)}\right\}O_P^{(4)} \leq O'_P$, it is possible to achieve cognitive spectrum sharing while proving a performance gain to the primary system.

3.3 Decoding performance at SR in the second phase
3.3.1 Conditioned on event $\mathcal{E}^{(1)}$
Together with (1) and (3), SR attempts to decode the desired message x_s from the the mixed signals of x_s and x_p using SIC. Similar to (11), (13), and (15), we have the following achievable rates in decoding x_p and x_s successively

$$R_{1,p}^{(1)} = C\left(\frac{P_P\gamma_{ps}}{P_S\gamma_{ss}+1} + \frac{\alpha}{1-\alpha+\frac{1}{P_R\gamma_{rs}}}\right)$$

$$\approx C\left(\frac{P_P\gamma_{ps}}{P_S\gamma_{ss}+1} + \frac{\alpha}{1-\alpha}\right), \tag{33a}$$

$$R_{2,s}^{(1)} = C\left(P_S\gamma_{ss} + (1-\alpha)P_R\gamma_{rs}\right), \tag{33b}$$

$$R_{1,s}^{(1)} = C\left(\frac{P_S\gamma_{ss}}{P_P\gamma_{ps}+1} + \frac{1-\alpha}{\alpha+\frac{1}{P_R\gamma_{rs}}}\right)$$

$$\approx C\left(\frac{P_S\gamma_{ss}}{P_P\gamma_{ps}+1} + \frac{1-\alpha}{\alpha}\right), \tag{33c}$$

where the approximations in (33a) and (33c) are obtained assuming $P_R \gg 1$ [24, 25]. Here, $R_{1,p}^{(1)}$ denotes the achievable rate of decoding x_p directly by considering the component of x_s simply as noise. Upon successfully decoding and removing x_p, $R_{2,s}^{(1)}$ denotes the achievable rate of decoding the remaining x_s successively. Conversely, $R_{1,s}^{(1)}$ denotes the achievable rate of decoding x_s directly that is subject to the interference from x_p. Then, similar to (12), (14), and (16), we have the corresponding events in decoding the mixed signals of x_p and x_s by using SIC

$$\mathcal{E}_{1,p}^{(1)} = \left\{R_{1,p}^{(1)} \geq R_{pt}\right\} \tag{34a}$$

$$\approx \begin{cases} \gamma_{ps} \geq \frac{\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}\gamma_{ss} + \frac{R'_{pt}-\frac{\alpha}{1-\alpha}}{P_P}, & \alpha < \frac{R'_{pt}}{R'_{pt}+1} \\ \\ \text{certain event,} & \alpha \geq \frac{R'_{pt}}{R'_{pt}+1} \end{cases}$$

$$\mathcal{E}_{2,s}^{(1)} = \left\{R_{2,s}^{(1)} \geq R_{st}\right\}$$

$$= \left\{\gamma_{rs} \geq \frac{R'_{st}}{(1-\alpha)P_R} - \frac{P_S}{(1-\alpha)P_R}\gamma_{ss}\right\}, \tag{34b}$$

$$\mathcal{E}_{1,s}^{(1)} = \left\{R_{1,s}^{(1)} \geq R_{st}\right\}$$

$$\approx \begin{cases} \gamma_{ps} \leq \frac{P_S}{\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)P_P}\gamma_{ss} - \frac{1}{P_P}, & \alpha > \frac{1}{R'_{st}+1} \\ \\ \text{certain event,} & \alpha \leq \frac{1}{R'_{st}+1} \end{cases} \tag{34c}$$

From (34), the event of successfully decoding x_s by using SIC can thus be expressed as

$$\mathcal{E}_{1,s}^{(1)} \cup \left(\bar{\mathcal{E}}_{1,s}^{(1)} \cap \mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{2,s}^{(1)}\right) = \mathcal{E}_{1,s}^{(1)} \cup \left(\mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{2,s}^{(1)}\right). \tag{35}$$

Lemma 4. *Conditioned on event* $\mathcal{E}^{(1)}$ *that both* x_p *and* x_s *are successfully decoded at CR at the end of the first phase, the corresponding outage probability at SR at the end of the second phase can be derived as*

$$O_S^{(1)} = 1 - \Pr\left\{\mathcal{E}_{1,s}^{(1)}\right\} - \Pr\left\{\mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{2,s}^{(1)}\right\}$$

$$\hspace{3em} + \Pr\left\{\mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{1,s}^{(1)}\right\}, \tag{36}$$

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$$\Pr\left\{\mathcal{E}_{1,s}^{(1)}\right\} \approx \begin{cases} \dfrac{\delta_{ps}e^{-\frac{\delta_{ss}\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)}{P_S}}}{\delta_{ps}+\frac{\delta_{ss}\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)P_P}{P_S}}, & \alpha > \frac{1}{R'_{st}+1} \\ 1, & \alpha \le \frac{1}{R'_{st}+1} \end{cases} \tag{37a}$$

$$\Pr\left\{\mathcal{E}_{1,p}^{(1)}\cap\mathcal{E}_{2,s}^{(1)}\right\} \approx \begin{cases} \dfrac{\delta_{ss}e^{-\frac{\delta_{rs}R'_{st}}{(1-\alpha)P_R}}-\frac{\delta_{rs}P_S}{(1-\alpha)P_R}e^{-\frac{\delta_{ss}R'_{st}}{P_S}}}{\delta_{ss}-\frac{\delta_{rs}P_S}{(1-\alpha)P_R}}, & \alpha \ge \frac{R'_{pt}}{R'_{pt}+1} \\[4mm] \dfrac{\delta_{ss}e^{-\left(\frac{\delta_{sr}R'_{st}}{(1-\alpha)P_R}+\frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)}{P_P}\right)}-\delta_{ss}e^{-\left(\frac{\delta_{ss}R'_{st}}{P_S}+\frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)\left(1+R'_{st}\right)}{P_P}\right)}}{\delta_{ss}+\frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}-\frac{\delta_{rs}P_S}{(1-\alpha)P_R}} \\[6mm] \quad +\dfrac{\delta_{ss}e^{-\left(\frac{\delta_{ss}R'_{st}}{P_S}+\frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)\left(1+R'_{st}\right)}{P_P}\right)}}{\delta_{ss}+\frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}}, & \alpha < \frac{R'_{pt}}{R'_{pt}+1} \end{cases} \tag{37b}$$

$$\Pr\left\{\mathcal{E}_{1,p}^{(1)}\cap\mathcal{E}_{1,s}^{(1)}\right\} \approx \begin{cases} 0, \\ \quad \text{when } \left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)\left(R'_{st}-\frac{1-\alpha}{\alpha}\right) \ge 1; \\[3mm] \dfrac{\delta_{ss}e^{-\frac{\delta_{ss}\left(R'_{pt}-\frac{\alpha}{1-\alpha}+1\right)\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)}{P_S\left[1-\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)\right]}}}{\delta_{ss}-\frac{\delta_{ps}P_S\left[1-\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)\right]}{\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)P_P}}, & \frac{1}{R'_{st}+1} < \alpha < \frac{R'_{pt}}{R'_{pt}+1} \\[3mm] \quad \text{when } \left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)\left(R'_{st}-\frac{1-\alpha}{\alpha}\right) < 1. \\[3mm] 1, & \frac{R'_{pt}}{R'_{pt}+1} \le \alpha \le \frac{1}{R'_{st}+1} \\[3mm] \dfrac{\delta_{ss}e^{-\frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)}{P_P}}}{\delta_{ss}+\frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}}, & \alpha < \min\left(\frac{R'_{pt}}{R'_{pt}+1}, \frac{1}{R'_{st}+1}\right) \\[3mm] \dfrac{\delta_{ps}e^{-\frac{\delta_{ss}\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)}{P_S}}}{\delta_{ps}+\frac{\delta_{ss}\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)P_P}{P_S}}, & \alpha > \max\left(\frac{R'_{pt}}{R'_{pt}+1}, \frac{1}{R'_{st}+1}\right) \end{cases} \tag{37c}$$

Please find in Appendix D for the detailed derivations. where

3.3.2 Conditioned on event $\mathcal{E}^{(2)}$

Together with (1) and (5), SR attempts to decode the desired message x_s using SIC. Similarly, we have the following achievable rates in decoding x_p and x_s successively

$$R_{1,p}^{(2)} = C\left(\frac{P_P\gamma_{ps}}{P_S\gamma_{ss}+1} + \frac{\alpha}{1-\alpha+\frac{1}{P_R\gamma_{rs}}}\right)$$
$$\approx C\left(\frac{P_P\gamma_{ps}}{P_S\gamma_{ss}+1} + \frac{\alpha}{1-\alpha}\right), \tag{38a}$$

$$R_{2,s}^{(2)} = C\left(P_S\gamma_{ss} + \frac{(1-\alpha)P_R\gamma_{rs}P_S\gamma_{sr}}{(1-\alpha)P_R\gamma_{rs}+P_S\gamma_{sr}+1}\right)$$
$$\approx C\left(P_S\gamma_{ss} + P_S\gamma_{sr}\right), \tag{38b}$$

$$R_{1,s}^{(2)} = C\left(\frac{P_S\gamma_{ss}}{P_P\gamma_{ps}+1} + \frac{1-\alpha}{\alpha+\frac{1}{P_S\gamma_{sr}}+\frac{1}{P_R\gamma_{rs}}+\frac{1}{P_S\gamma_{sr}P_R\gamma_{rs}}}\right)$$
$$\approx C\left(\frac{P_S\gamma_{ss}}{P_P\gamma_{ps}+1} + \frac{1-\alpha}{\alpha}\right), \tag{38c}$$

where the approximation in (38a) is obtained assuming $P_R \gg 1$, the approximation in (38b) is obtained assuming $P_R\gamma_{rs} \gg P_S\gamma_{sr}$, and the approximation in (38c) is obtained assuming $P_S, P_R \gg 1$ [24, 25], respectively. Then, we have the corresponding events in decoding x_p and x_s successively

$$\mathcal{E}_{1,p}^{(2)} = \left\{R_{1,p}^{(2)} \ge R_{pt}\right\} \tag{39a}$$
$$\approx \begin{cases} \gamma_{ps} \ge \frac{\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}\gamma_{ss}+\frac{R'_{pt}-\frac{\alpha}{1-\alpha}}{P_P}, & \alpha < \frac{R'_{pt}}{R'_{pt}+1} \\ \text{certain event}, & \alpha \ge \frac{R'_{pt}}{R'_{pt}+1} \end{cases},$$

$$\mathcal{E}_{2,s}^{(2)} = \left\{R_{2,s}^{(2)} \ge R_{st}\right\} = \left\{\gamma_{sr} \ge \frac{R'_{st}}{P_S}-\gamma_{ss}\right\}, \tag{39b}$$

$$\mathcal{E}_{1,s}^{(2)} = \left\{R_{1,s}^{(2)} \ge R_{st}\right\}$$
$$\approx \begin{cases} \gamma_{ps} \le \frac{P_S}{\left(R'_{st}-\frac{1-\alpha}{\alpha}\right)P_P}\gamma_{ss}-\frac{1}{P_P}, & \alpha > \frac{1}{R'_{st}+1} \\ \text{certain event}, & \alpha \le \frac{1}{R'_{st}+1} \end{cases}. \tag{39c}$$

Similarly, the outage probability at SR at the end of the second phase can be derived as

$$O_S^{(2)} = 1 - \Pr\left\{\mathcal{E}_{1,s}^{(2)}\right\} - \Pr\left\{\mathcal{E}_{1,p}^{(2)} \cap \mathcal{E}_{2,s}^{(2)}\right\}$$
$$+ \Pr\left\{\mathcal{E}_{1,p}^{(2)} \cap \mathcal{E}_{1,s}^{(2)}\right\}. \tag{40}$$

From (34) and (39), since $\mathcal{E}_{1,p}^{(2)}$ and $\mathcal{E}_{1,s}^{(2)}$ are of the same form as $\mathcal{E}_{1,p}^{(1)}$ and $\mathcal{E}_{1,s}^{(1)}$ respectively, we have

$$\Pr\left\{\mathcal{E}_{1,s}^{(2)}\right\} = \Pr\left\{\mathcal{E}_{1,s}^{(1)}\right\}, \tag{41}$$

$$\Pr\left\{\mathcal{E}_{1,p}^{(2)} \cap \mathcal{E}_{1,s}^{(2)}\right\} = \Pr\left\{\mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{1,s}^{(1)}\right\}. \tag{42}$$

Then, by integrating over the corresponding region of event $\left\{\mathcal{E}_{1,p}^{(2)} \cap \mathcal{E}_{2,s}^{(2)}\right\}$, we can obtain

$$\Pr\left\{\mathcal{E}_{1,p}^{(2)} \cap \mathcal{E}_{2,s}^{(2)}\right\} \tag{43}$$

$$\approx \begin{cases} \dfrac{\delta_{ss}e^{-\frac{\delta_{sr}R'_{st}}{P_S}} - \delta_{sr}e^{-\frac{\delta_{ss}R'_{st}}{P_S}}}{\delta_{ss} - \delta_{sr}}, & \alpha \geq \dfrac{R'_{pt}}{R'_{pt}+1} \\[4mm] \dfrac{\delta_{ss}e^{-\left(\frac{\delta_{sr}R'_{st}}{P_S} + \frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)}{P_P}\right)} - \delta_{ss}e^{-\left(\frac{\delta_{ss}R'_{st}}{P_S} + \frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)\left(1+R'_{st}\right)}{P_P}\right)}}{\delta_{ss} + \frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P} - \delta_{sr}} \\[4mm] + \dfrac{\delta_{ss}e^{-\left(\frac{\delta_{ss}R'_{st}}{P_S} + \frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)\left(1+R'_{st}\right)}{P_P}\right)}}{\delta_{ss} + \frac{\delta_{ps}\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}}, & \alpha < \dfrac{R'_{pt}}{R'_{pt}+1} \end{cases}.$$

3.3.3 Conditioned on event $\mathcal{E}^{(3)}$

Together with (1) and (7), SR attempts to decode the desired message x_s using SIC. Similarly, we have the following achievable rates in decoding x_p and x_s successively

$$R_{1,p}^{(3)} = C\left(\dfrac{P_P\gamma_{ps}}{P_S\gamma_{ss} + 1}\right.$$
$$\left. + \dfrac{\alpha}{1-\alpha + \frac{1}{P_P\gamma_{pr}} + \frac{1}{P_R\gamma_{rs}} + \frac{1}{P_P\gamma_{pr}P_R\gamma_{rs}}}\right)$$
$$\approx C\left(\dfrac{P_P\gamma_{ps}}{P_S\gamma_{ss} + 1} + \dfrac{\alpha}{1-\alpha}\right), \tag{44a}$$

$$R_{2,s}^{(3)} = C\left(P_S\gamma_{ss} + (1-\alpha)P_R\gamma_{rs}\right), \tag{44b}$$

$$R_{1,s}^{(3)} = C\left(\dfrac{P_S\gamma_{ss}}{P_P\gamma_{ps} + 1} + \dfrac{1-\alpha}{\alpha + \frac{1}{P_R\gamma_{rs}}}\right)$$
$$\approx C\left(\dfrac{P_S\gamma_{ss}}{P_P\gamma_{ps} + 1} + \dfrac{1-\alpha}{\alpha}\right), \tag{44c}$$

where the approximation in (44a) is obtained assuming $P_P, P_R \gg 1$ and the approximation in (44c) is obtained assuming $P_R \gg 1$ [24, 25], respectively.

From (44) and (33), since in both cases x_s is successfully decoded and forwarded by CR with power $(1 - \alpha)P_R$, we have

$$O_S^{(3)} = O_S^{(1)}. \tag{45}$$

3.3.4 Conditioned on event $\mathcal{E}^{(4)}$

Again, SIC is performed at SR to decode the desired message x_s by exploiting the received signals in (1) and (8). Then, we have the following achievable rates in decoding x_p and x_s successively

$$R_{1,p}^{(4)} = C\left(\dfrac{2P_P\gamma_{ps}}{P_S\gamma_{ss} + 1}\right), \tag{46a}$$

$$R_{2,s}^{(4)} = C\left(2P_S\gamma_{ss}\right), \tag{46b}$$

$$R_{1,s}^{(4)} = C\left(\dfrac{2P_S\gamma_{ss}}{P_P\gamma_{ps} + 1}\right). \tag{46c}$$

Correspondingly, we have the following events in decoding x_p and x_s successively

$$\mathcal{E}_{1,p}^{(4)} = \left\{R_{1,p}^{(4)} \geq R_{pt}\right\} = \left\{\gamma_{ps} \geq \dfrac{R'_{pt}P_S}{2P_P}\gamma_{ss} + \dfrac{R'_{pt}}{2P_P}\right\}, \tag{47a}$$

$$\mathcal{E}_{2,s}^{(4)} = \left\{R_{2,s}^{(4)} \geq R_{st}\right\} = \left\{\gamma_{ss} \geq \dfrac{R'_{st}}{2P_S}\right\}, \tag{47b}$$

$$\mathcal{E}_{1,s}^{(4)} = \left\{R_{1,s}^{(4)} \geq R_{st}\right\} = \left\{\gamma_{ps} \leq \dfrac{2P_S}{R'_{st}P_P}\gamma_{ss} - \dfrac{1}{P_P}\right\}. \tag{47c}$$

For the considered scenario where $R_{pt}, R_{st} \geq 1$, there is no intersection between events $\mathcal{E}_{1,p}^{(4)}$ and $\mathcal{E}_{1,s}^{(4)}$. Thus, from Lemma 4, the corresponding outage probability at SR can be derived as

$$O_S^{(4)} = 1 - \Pr\left\{\mathcal{E}_{1,s}^{(4)}\right\} - \Pr\left\{\mathcal{E}_{1,p}^{(4)} \cap \mathcal{E}_{2,s}^{(4)}\right\}$$
$$= 1 - \int_{\frac{R'_{st}}{2P_S}}^{\infty} \delta_{ss}e^{-\delta_{ss}\gamma_{ss}}\left[\int_{\frac{R'_{pt}P_S}{2P_P}\gamma_{ss}+\frac{R'_{pt}}{2P_P}}^{\infty} \delta_{ps}e^{-\delta_{ps}\gamma_{ps}}d\gamma_{ps}\right]d\gamma_{ss}$$
$$- \int_{\frac{R'_{st}}{2P_S}}^{\infty} \delta_{ss}e^{-\delta_{ss}\gamma_{ss}}\left[\int_{0}^{\frac{2P_S}{R'_{st}P_P}\gamma_{ss}-\frac{1}{P_P}} \delta_{ps}e^{-\delta_{ps}\gamma_{ps}}d\gamma_{ps}\right]d\gamma_{ss}$$
$$= 1 - \dfrac{\frac{2\delta_{ps}P_S}{R'_{st}P_P}e^{-\frac{\delta_{ss}R'_{st}}{2P_S}}}{\delta_{ss} + \frac{2\delta_{ps}P_S}{R'_{st}P_P}} - \dfrac{\delta_{ss}e^{-\left(\frac{\delta_{ps}R'_{pt}}{2P_P} + \frac{\delta_{ss}R'_{st}}{2P_S} + \frac{\delta_{ps}R'_{pt}R'_{st}}{4P_P}\right)}}{\delta_{ss} + \frac{\delta_{ps}R'_{pt}P_S}{2P_P}}, \tag{48}$$

where $\delta_{ss}e^{-\delta_{ss}\gamma_{ss}}$ and $\delta_{ps}e^{-\delta_{ps}\gamma_{ps}}$ denote the respective PDFs of γ_{ss} and γ_{ps}.

Substituting $O_S^{(1)}$, $O_S^{(2)}$, $O_S^{(3)}$, and $O_S^{(4)}$ into (10), we can thus obtain the overall end-to-end outage probability of the secondary system.

4 A performance upper bound

Conditioned on event $\mathcal{E}^{(1)}$ that both x_p and x_s are successfully decoded at CR at the end of the first phase, we propose using DPC at CR to pre-cancel the interference seen at SR (or PR) in the second phase.

4.1 A performance upper bound for secondary user

We propose using DPC [17] at CR that transmits a composite message

$$x'_r = \sqrt{\alpha P_R} x_p + x'_s \qquad (49)$$

in the second phase. Employing a similar coding scheme as in [26, Eq. (15)], x'_s is encoded using DPC by treating the component of $\sqrt{\alpha P_R} x_p$ as the known interference that will corrupt the reception at SR in the second phase. Thus, after dirty paper decoding, the effectively received signal at SR in the second phase is given as

$$y'_s(2) = h_{rs}\sqrt{(1-\alpha)P_R} x_s + n_0. \qquad (50)$$

That is, SR sees no interference in the second phase. Then, together with (1) and (50), SR attempts to decode x_s using SIC. Following the same steps as in Lemma 4, the corresponding outage probability at SR at the end of the second phase can be similarly derived. The details are omitted here for the sake of brevity.

On the other hand, with the same power αP_R allocated to forward x_p at CR, the outage performance of the primary user is the same as that in (23).

4.2 A performance upper bound for primary user

The interference seen at PR can be also pre-cancelled by performing DPC at SR, where a composite message

$$x'_r = x'_p + \sqrt{(1-\alpha)P_R} x_s \qquad (51)$$

is transmitted in the second phase. Similarly, x'_p is encoded using DPC by treating the component of $\sqrt{(1-\alpha)P_R} x_s$ as the known interference that will corrupt the reception at PR. Thus, after dirty paper decoding, the effectively received signal at PR in the second phase is given as

$$y'_p(2) = h_{rp}\sqrt{\alpha P_R} x_p + n_0. \qquad (52)$$

That is, PR sees no interference in the second phase. Again, together with (1) and (52), MRC is employed at PR to decode x_p. The details are omitted here for the sake of brevity.

On the other hand, with the same power $(1-\alpha)P_R$ allocated to forward x_s at CR, the outage performance of the secondary user is the same as that in (36).

Remark 4. *From the above analysis, with a DPC performed at CR to exploit the successfully decoded messages received in the first phase, the interference seen at SR (or PR) in the second phase can be pre-cancelled, thus obtaining a performance upper bound for the secondary (or primary) user without affecting the performance of the other user.*

5 Simulation results

In this section, we illustrate the outage performance of both primary and secondary users in the proposed spectrum sharing protocol. In order to limit the interference caused to the primary user, we consider a power control at ST where $P_S = \theta P_P$. To evaluate the SIC decoding at CR at the end of the first phase, we let $\delta_{sr}^{-1} = \tau\delta_{pr}^{-1}$ for the multiple-access channels h_{sr} and h_{pr} at CR. To evaluate the interference seen at PR due to cross talk, we let $\delta_{sp}^{-1} = \varphi\delta_{pp}^{-1}$ for the multiple-access channels h_{sp} and h_{pp} at PR. Unless otherwise specified, we let $P_P = 30$ dB, $P_S = 10$ dB, $P_R = 40$ dB, and $R_{st} = R_{pt} = 1$. In order to reflect the geometric structure of the considered network shown in Fig. 2, we let $\delta_{pp}^{-1} = \delta_{ss}^{-1} = 0$ dB for the direct links, $\delta_{sp}^{-1} = \delta_{ps}^{-1} = -10$ dB for the cross links, and $\delta_{pr}^{-1} = \delta_{sr}^{-1} = \delta_{rp}^{-1} = \delta_{rs}^{-1} = 10$ dB for the relay links, respectively. Simulation results are presented in Figs. 4, 5, 6, 7, 8, 9, 10, and 11 where lines denote the analytical results obtained in this paper and markers denote the results of Monte Carlo simulations.

5.1 SIC decoding at CR

From Theorem 2, in order to fully exploit the relay transmissions, event $\mathcal{E}^{(4)}$ that neither of x_p and x_s is decoded at CR should be avoided as much as possible. Figure 4 displays the probability of $\Pr\{\mathcal{E}^{(4)}\}$ with respect to θ where $P_S = \theta P_P$. Various channel conditions are considered where $\delta_{pr}^{-1} = 10$ dB and $\delta_{sr}^{-1} = \tau\delta_{pr}^{-1}$. As can be seen from Fig. 4, with an increase in θ, the probability of $\Pr\{\mathcal{E}^{(4)}\}$ first increases and then decreases. This is reasonable as when there is a significant difference between the power levels of x_p and x_s received at CR, e.g., $\tau\theta \ll 1$ or $\tau\theta \gg 1$,
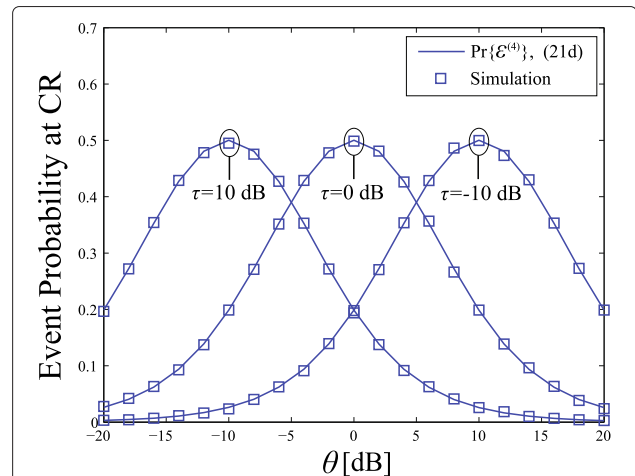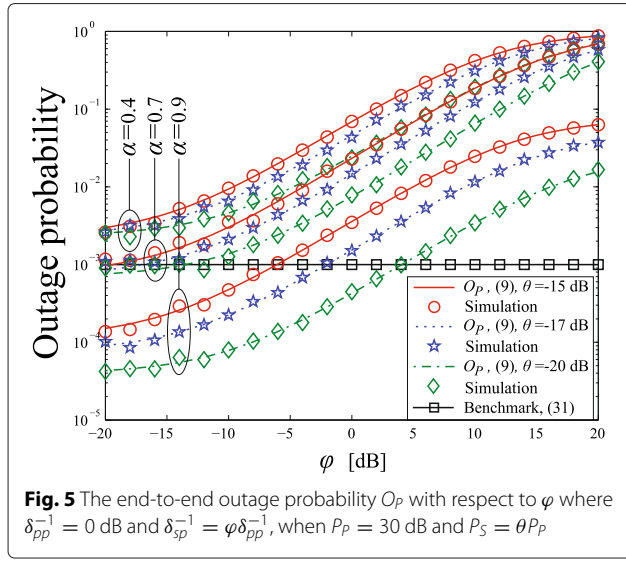

Fig. 4 The probability $\Pr\{\mathcal{E}^{(4)}\}$ with respect to θ where $P_P = 30$ dB and $P_S = \theta P_P$, when $\delta_{pr}^{-1} = 10$ dB and $\delta_{sr}^{-1} = \tau\delta_{pr}^{-1}$

Fig. 5 The end-to-end outage probability O_P with respect to φ where $\delta_{pp}^{-1} = 0$ dB and $\delta_{sp}^{-1} = \varphi\delta_{pp}^{-1}$, when $P_P = 30$ dB and $P_S = \theta P_P$

SIC is facilitated and it would be easy to decode x_p and x_s successively. In contrast, if the components of x_p and x_s are of comparable power levels, e.g., $\tau\theta \approx 1$, then the SIC decoding is limited by the mutual interference and it would be difficult to decode either of x_p and x_s. This can be observed in Fig. 4 where $\Pr\{\mathcal{E}^{(4)}\}$ takes peak values at $\theta = -10, 0, 10$ dB for $\tau = 10, 0, -10$ dB, respectively.

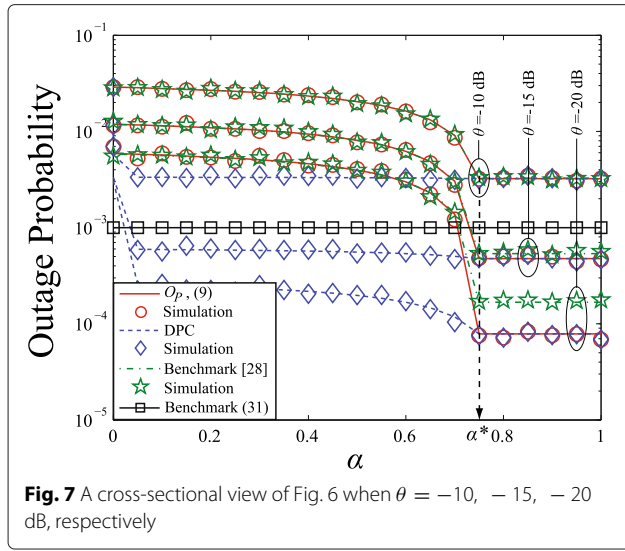### 5.2 End-to-end outage performance of the primary user

Firstly, we evaluate how the interference due to cross talk affects the end-to-end performance of the primary system. Let $\delta_{sp}^{-1} = \varphi\delta_{pp}^{-1}$, O_P is plotted with respect to φ in Fig. 5. The outage probability O_P' of the benchmark case considered in Remark 1 is also demonstrated. With an increase in φ, since the interference link ST→PR becomes

stronger, the corresponding performance of the primary system is impaired. On the other hand, with an increase in θ, ST transmits at a higher power that impedes the SIC decoding at CR as well as cause more interference to PR, thus similarly impairing the performance of the primary system. In addition, it is observed that a performance improvement is achieved for the primary system with a higher power allocation factor α. When $\alpha = 0.4$, even though ST transmits at a low power, e.g., $\theta = -20$ dB, and the interference link ST→PR is very weak, e.g., $\varphi = -20$ dB, the primary user experiences a performance loss, i.e., $O_P > O_P'$. Whereas when $\alpha = 0.9$, with all other parameters being the same, a significant performance improvement is achieved for the primary system. This means that besides the power control at ST, the power allocation at CR also needs to be designed to compensate the interference caused to the primary system due to secondary transmissions.

To further illustrate the effects of P_S and power allocation factor α, O_P is plotted with respect to θ and α in Fig. 6. The outage probability O_P' of the benchmark case (31) is also illustrated. It is observed that when θ takes values smaller than 0.05 AND when α takes values greater than 0.75, even subject to the interference from the secondary transmissions, the condition in (32) is always satisfied and a performance gain is achieved for the primary system. Otherwise, if $\theta > 0.05$, then we have $O_P > O_P'$ even when $\alpha \to 1$.

This can be better observed in Fig. 7 where a cross-sectional view of Fig. 6 is demonstrated. With an increase in θ, since the SIC at CR is impeded meanwhile more interference is caused to PR, the corresponding outage performance of the primary user is degraded. When $\theta = -20, -15$ dB, it is observed that we can always find

Fig. 6 The end-to-end outage probability O_P with respect to α and θ where $P_P = 30$ dB and $P_S = \theta P_P$

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Fig. 7 A cross-sectional view of Fig. 6 when $\theta = -10, -15, -20$ dB, respectively

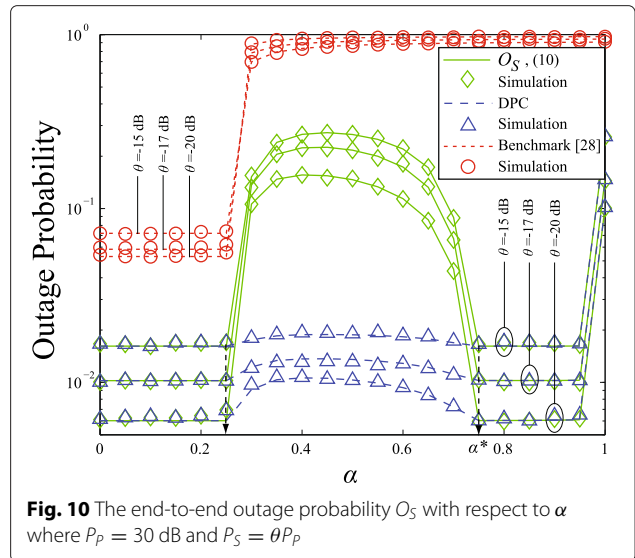**Fig. 9** The end-to-end outage probability O_P with respect to P_P where $\alpha = \alpha^*$
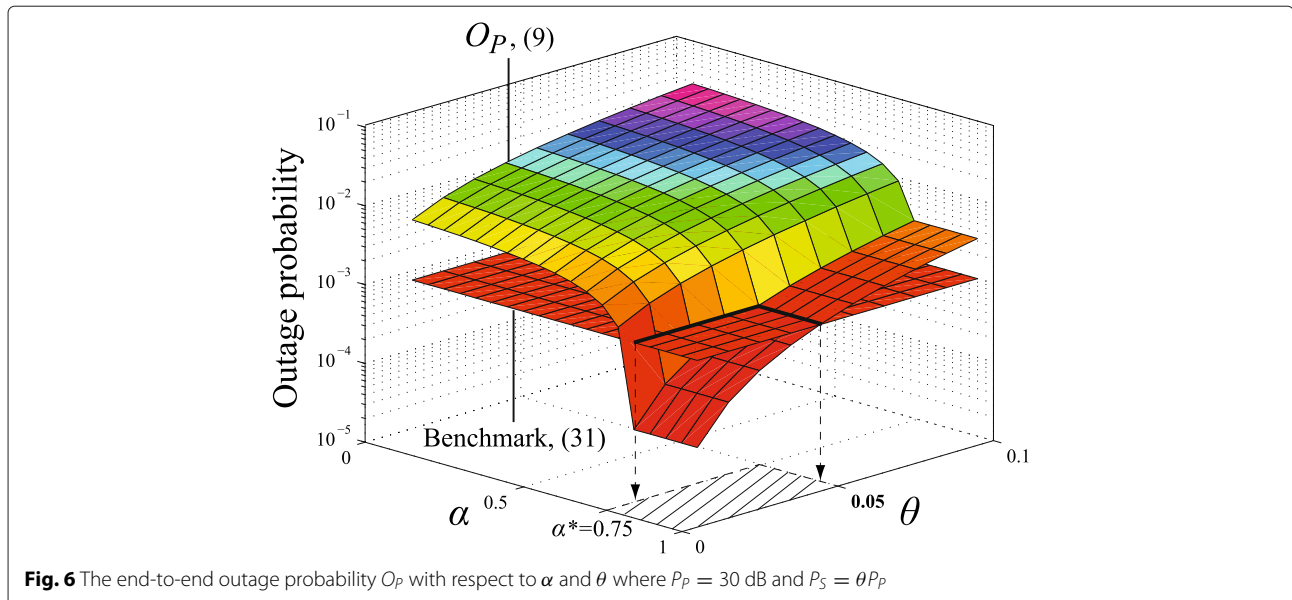
a suitable power allocation factor $\alpha \geq \alpha^* = \frac{R'_{pt}}{R'_{pt}+1} = 0.75$ such that a performance gain is achieved for the primary system. However, when θ is increased to -10 dB, it is observed that $O_P > O'_P$ even when $\alpha \to 1$. Similar phenomena can be observed for a benchmark case considered in [28], where a linear weighted combination of primary and secondary messages is forwarded by CR only when both messages are successfully decoded. Otherwise, CR simply stays silent and both PT and ST perform a retransmission in the second phase. It is observed that with the same system parameters, a better performance is achieved by the proposed approach compared to that in [28]. This is reasonable as in the proposed approach, CR
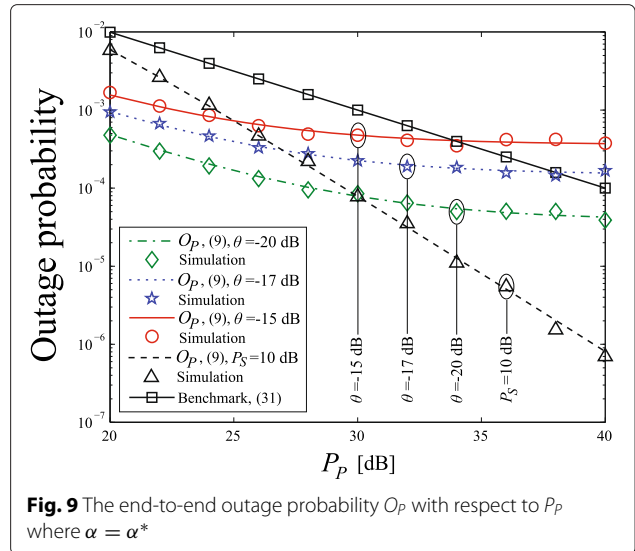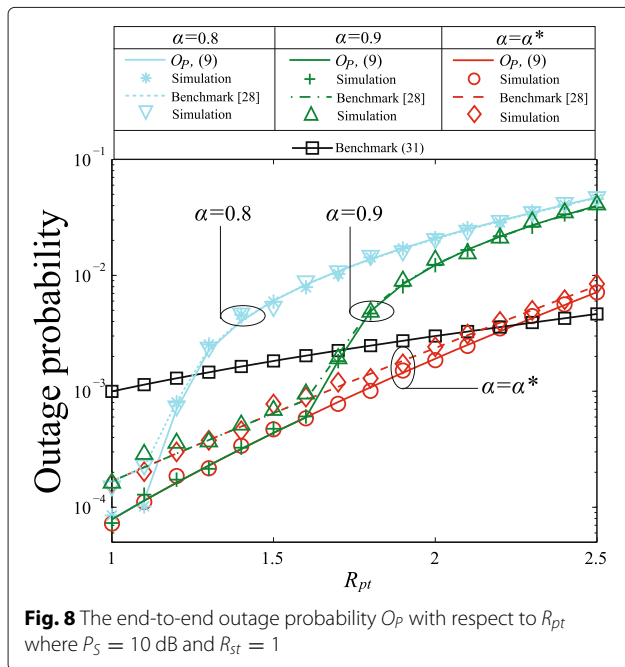
is able to help forward the received messages more frequently, which is more beneficial compared to a retransmission by PT and ST simultaneously that will cause severe interference to one another.

Furthermore, from Figs. 6 and 7, it is observed that when α takes values greater than $\alpha^* = \frac{R'_{pt}}{R'_{pt}+1} = 0.75$, O_P experiences a floor. This validates Theorem 2 that when $\alpha \geq \alpha^*$, $O_P^{(1)}$, $O_P^{(2)}$, and $O_P^{(3)}$ all approach 0 and thus $\Pr\{\mathcal{E}^{(4)}\} O_P^{(4)}$, which is irrelevant to α, dominates the overall outage performance. For a better illustration, the outage performance using DPC at CR is also presented in Fig. 7. Conditioned on event $\mathcal{E}^{(1)}$ that both x_p and x_s are successfully decoded at CR, by using DPC to pre-cancel the interference seen at PR, it is observed that a performance upper bound is achieved for the primary system.
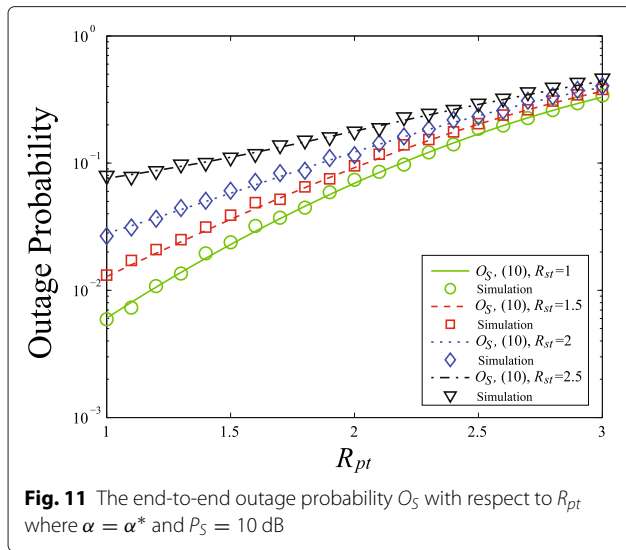

Fig. 8 The end-to-end outage probability O_P with respect to R_{pt} where $P_S = 10$ dB and $R_{st} = 1$

Fig. 10 The end-to-end outage probability O_S with respect to α where $P_P = 30$ dB and $P_S = \theta P_P$

Fig. 11 The end-to-end outage probability O_S with respect to R_{pt} where $\alpha = \alpha^*$ and $P_S = 10$ dB

For a better illustration of Theorem 2, O_P is plotted with respect to R_{pt} in Fig. 8. With an increase in R_{pt}, the corresponding outage performance of the primary system is degraded. Whereas with an increase in α, a better outage performance is achieved. When $\alpha = \alpha^* = \frac{R'_{pt}}{R'_{pt}+1}$, the corresponding performance outperforms that with fixed power allocation factor, e.g., $\alpha = 0.8, 0.9$, thus validating Theorem 2 that the end-to-end outage performance of the primary system is optimized with respect to α when $\alpha \geq \alpha^*$. Similar results can be observed for the benchmark case [28]. Again, a better performance is achieved by the proposed approach compared to that in [28]. Furthermore, in modest rate region where $R_{pt} < 2.2$, the condition in (32) is satisfied and a performance gain is achieved for the primary system compared to the benchmark case. Whereas in the high rate region, since both the SIC decoding at CR and the decoding at PR become more difficult, the primary user experiences a performance loss.

Figure 9 displays O_P with respect to P_P where a power allocation factor of $\alpha = \alpha^*$ is adopted at CR. When $P_S = 10$ dB, it is observed that a diversity order of 2 is achieved for the primary system when $P_P \rightarrow \infty$. This is reasonable as in the proposed protocol, two independent copies of x_p are received at PR from PT$\rightarrow$PR and CR$\rightarrow$PR respectively in two successive phases. However, when there is a fixed power ratio between P_P and P_S, i.e., $P_S = \theta P_P$, it is observed that the performance of the primary system is limited by the interference from secondary transmissions and no diversity gain is achieved.

5.3 End-to-end outage performance of the secondary user
In Fig. 10, the end-to-end outage probability of the secondary user O_S is plotted with respect to α where $P_P = 30$ dB and $P_S = \theta P_P$. It is observed that O_S experiences a plateau when α takes the values between 0.3 and 0.7. And

when α is less than 0.25 or greater than 0.75, a reasonably good outage performance is achieved for the secondary user. This is because of the employment of SIC at SR. In the regions where $\alpha \leq 0.25$ and $\alpha \geq 0.75$, there is a significant difference between the power levels of x_p and x_s received at SR in the second phase, thus x_s can either be first decoded or successively decoded using SIC with a high probability. Conversely, when α takes values between 0.3 and 0.7, SIC is limited by the comparable interference between x_p and x_s, which makes it difficult to decode either of them. For comparison purposes, the outage performance of the secondary user in the benchmark case [28] is also presented, where SR attempts to recover the desired signal x_s by using MRC of the received signals in two phases and considers the primary component simply as noise. As illustrated in Fig. 10, with an increase in α, since more power is allocated to forward the primary signal meanwhile higher interference is seen at SR, the outage performance of the secondary user is severely degraded.

Furthermore, it is observed from Fig. 10 that the performance of the secondary system is degraded with an increase in θ. This is because with a higher θ, e.g., θ is increased from -20 to -15 dB, from (34a), it becomes difficult to first decode and remove x_p and then decode x_s successively using SIC. In other words, it is not always beneficial to adopt a high transmit power at ST, even if the condition in (32) is met. Again, the outage performance using DPC at CR is illustrated in Fig. 10. Conditioned on event $\mathcal{E}^{(1)}$, by using DPC to pre-cancel the interference seen at SR, it is observed that a performance upper bound is achieved for the secondary user.

Together with Figs. 7 and 10, when $\alpha \geq \alpha^*$, it is possible to bring a performance gain for the primary system and at the same time achieve a reasonably good outage performance around 10^{-2} for the secondary system by the proposed approach. Whereas for the benchmark case considered in [28], although it is also possible to provide a performance gain to the primary user when $\alpha \geq \alpha^*$, the secondary message x_s can be hardly delivered from ST to SR.

In Fig. 11, O_S is plotted with respect to R_{pt} when $\alpha = \alpha^*$. Similarly, with an increase in R_{pt}, from (34a), it becomes difficult to decode x_p first and then decode x_s successively using SIC, thus the corresponding outage performance of the secondary system is degraded. On the other hand, with other parameters being the same, it is observed that a performance degradation is experienced by the secondary system with an increase in R_{st}.

From the above observations in Figs. 4, 5, 6, 7, 8, 9, 10, and 11, with properly designed parameters to facilitate the SIC decoding at CR as well as to limit the interference caused to PR due to the cross talk in the first phase, it is possible to find a suitable power allocation factor $\alpha \geq \alpha^*$ such that a performance gain is achieved for the primary

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user meanwhile the secondary user gains an opportunity to access the spectrum. Furthermore, with the same system parameters, performance gains are achieved for both primary and secondary systems by the proposed approach compared to a benchmark case in [28].

6 Conclusions

In this paper, the interference channel with a cognitive relay is exploited to achieve spectrum sharing between a licensed primary user and an unlicensed secondary user. A causal cognitive two-phase spectrum sharing protocol is proposed and closed-form expressions of the end-to-end outage probability are derived. In view of the inherent interference-limited property of the system, to guarantee the performance of the primary system, we consider a power control at ST together with a power allocation at CR to forward the processed primary and secondary messages, respectively. Simulation results demonstrate that by designing both the power control at ST and power allocation at CR, spectrum sharing is achieved between the primary and secondary systems and performance gains can be achieved for both parties.

Appendix A: Derivations of $\Pr\left\{\mathcal{E}^{(1)}\right\}$, $\Pr\left\{\mathcal{E}^{(2)}\right\}$, $\Pr\left\{\mathcal{E}^{(3)}\right\}$, and $\Pr\left\{\mathcal{E}^{(4)}\right\}$

From Fig. 3, upon determining the region of event $\mathcal{E}^{(1)}$ that is defined in (19a), the corresponding probability can be derived by

$$
\begin{aligned}
\Pr\{\mathcal{E}^{(1)}\} &= \Pr\left\{\left(\mathcal{E}_{1,p} \cap \mathcal{E}_{2,s}\right) \cup \left(\mathcal{E}_{1,s} \cap \mathcal{E}_{2,p}\right)\right\} \\
&= \int_{\frac{R'_{st}}{P_S}}^{\infty} \delta_{sr} e^{-\delta_{sr}\gamma_{sr}} \left[\int_{\frac{R'_{pt}P_S}{P_P}\gamma_{sr}+\frac{R'_{pt}}{P_P}}^{\infty} \delta_{pr} e^{-\delta_{pr}\gamma_{pr}} d\gamma_{pr}\right] d\gamma_{sr} \\
&\quad + \int_{\frac{R'_{pt}}{P_P}}^{\infty} \delta_{pr} e^{-\delta_{pr}\gamma_{pr}} \left[\int_{\frac{R'_{st}P_P}{P_S}\gamma_{pr}+\frac{R'_{st}}{P_S}}^{\infty} \delta_{sr} e^{-\delta_{sr}\gamma_{sr}} d\gamma_{sr}\right] d\gamma_{pr},
\end{aligned}
$$

(53)

where $\delta_{sr} e^{-\delta_{sr}\gamma_{sr}}$ and $\delta_{pr} e^{-\delta_{pr}\gamma_{pr}}$ denote the respective PDFs of γ_{sr} and γ_{pr}.

For event $\mathcal{E}^{(2)}$ defined in (19b), the corresponding probability can be derived by

$$
\begin{aligned}
\Pr\{\mathcal{E}^{(2)}\} &= \Pr\left\{\mathcal{E}_{1,p} \cap \bar{\mathcal{E}}_{2,s}\right\} \\
&= \int_{0}^{\frac{R'_{st}}{P_S}} \delta_{sr} e^{-\delta_{sr}\gamma_{sr}} \left[\int_{\frac{R'_{pt}P_S}{P_P}\gamma_{sr}+\frac{R'_{pt}}{P_P}}^{\infty} \delta_{pr} e^{-\delta_{pr}\gamma_{pr}} d\gamma_{pr}\right] d\gamma_{sr}.
\end{aligned}
$$

(54)

For event $\mathcal{E}^{(3)}$ defined in (19c), the corresponding probability can be derived by

$$
\begin{aligned}
\Pr\{\mathcal{E}^{(3)}\} &= \Pr\left\{\mathcal{E}_{1,s} \cap \bar{\mathcal{E}}_{2,p}\right\} \\
&= \int_{0}^{\frac{R'_{pt}}{P_P}} \delta_{pr} e^{-\delta_{pr}\gamma_{pr}} \left[\int_{\frac{R'_{st}P_P}{P_S}\gamma_{pr}+\frac{R'_{st}}{P_S}}^{\infty} \delta_{sr} e^{-\delta_{sr}\gamma_{sr}} d\gamma_{sr}\right] d\gamma_{pr}.
\end{aligned}
$$

(55)

Then the integrations in (53)–(55) can be solved to obtain the results in (21a)–(21c). Since events $\mathcal{E}^{(1)}$, $\mathcal{E}^{(2)}$, $\mathcal{E}^{(3)}$, and $\mathcal{E}^{(4)}$ are mutually exclusive, the corresponding probability of event $\mathcal{E}^{(4)}$ can be readily obtained by

$$
\Pr\{\mathcal{E}^{(4)}\} = 1 - \Pr\left\{\mathcal{E}^{(1)}\right\} - \Pr\left\{\mathcal{E}^{(2)}\right\} - \Pr\left\{\mathcal{E}^{(3)}\right\}.
$$

(56)

Appendix B: Derivations of $O_P^{(1)}$

From (22), with target rate R_{pt}, we have the corresponding outage probability at PR

$$
\begin{aligned}
O_P^{(1)} &= \Pr\left\{R_P^{(1)} < R_{pt}\right\} \\
&\approx \Pr\left\{C\left(\frac{P_P\gamma_{pp}}{P_S\gamma_{sp}+1} + \frac{\alpha}{1-\alpha}\right) < R_{pt}\right\} \\
&= \begin{cases} 0, & \alpha \geq \frac{R'_{pt}}{R'_{pt}+1} \\ \Pr\left\{\gamma_{pp} < \frac{\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}\gamma_{sp} + \frac{R'_{pt}-\frac{\alpha}{1-\alpha}}{P_P}\right\}, & \alpha < \frac{R'_{pt}}{R'_{pt}+1} \end{cases}.
\end{aligned}
$$

(57)

When $\alpha < \frac{R'_{pt}}{R'_{pt}+1}$, similarly, we can derive $O_P^{(1)}$ by integrating over the corresponding region

$$
\begin{aligned}
&\Pr\left\{\gamma_{pp} < \frac{\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}\gamma_{sp} + \frac{R'_{pt}-\frac{\alpha}{1-\alpha}}{P_P}\right\} \\
&= \int_0^{\infty} \delta_{sp} e^{-\delta_{sp}\gamma_{sp}} \left[\int_0^{\frac{\left(R'_{pt}-\frac{\alpha}{1-\alpha}\right)P_S}{P_P}\gamma_{sp} + \frac{R'_{pt}-\frac{\alpha}{1-\alpha}}{P_P}} \delta_{pp} e^{-\delta_{pp}\gamma_{pp}} d\gamma_{pp}\right] d\gamma_{sp},
\end{aligned}
$$

(58)

where $\delta_{sp} e^{-\delta_{sp}\gamma_{sp}}$ and $\delta_{pp} e^{-\delta_{pp}\gamma_{pp}}$ denote the respective PDFs of γ_{sp} and γ_{pp}. Then, this integration can be solved to obtain the results in (23).

Appendix C: Derivations of $O_P^{(4)}$

From (28), with target rate R_{pt}, we have the corresponding outage probability at PR

$$
\begin{aligned}
O_P^{(4)} &= \Pr\{R_P^{(4)} < R_{pt}\} \\
&= \Pr\left\{\gamma_{pp} < \frac{R'_{pt}P_S}{2P_P}\gamma_{sp} + \frac{R'_{pt}}{2P_P}\right\}.
\end{aligned}
$$

(59)

By integrating over the corresponding region defined in (59), $O_P^{(4)}$ can be derived by

$$
O_P^{(4)} = \int_0^{\infty} \delta_{sp} e^{-\delta_{sp}\gamma_{sp}} \left[\int_0^{\frac{R'_{pt}P_S}{2P_P}\gamma_{sp} + \frac{R'_{pt}}{2P_P}} \delta_{pp} e^{-\delta_{pp}\gamma_{pp}} d\gamma_{pp}\right] d\gamma_{sp}.
$$

(60)

Then, this integration can be solved to obtain the result in (29).

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Appendix D: Derivations of $O_S^{(1)}$

From (35), the outage probability at SR can be expressed as [34]

$$
\begin{aligned}
O_S^{(1)} &= 1 - \Pr\left\{ \mathcal{E}_{1,s}^{(1)} \cup \left(\mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{2,s}^{(1)} \right) \right\} \\
&= 1 - \Pr\left\{ \mathcal{E}_{1,s}^{(1)} \right\} - \Pr\left\{ \mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{2,s}^{(1)} \right\} \\
&\quad + \Pr\left\{ \mathcal{E}_{1,s}^{(1)} \cap \mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{2,s}^{(1)} \right\}.
\end{aligned}
\tag{61}
$$

Since $R_{1,s}^{(1)} \le R_{2,s}^{(1)}$ always holds, we have $\mathcal{E}_{1,s}^{(1)} \subseteq \mathcal{E}_{2,s}^{(1)}$, thus (61) can be rewritten as

$$
O_S^{(1)} = 1 - \Pr\left\{ \mathcal{E}_{1,s}^{(1)} \right\} - \Pr\left\{ \mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{2,s}^{(1)} \right\} + \Pr\left\{ \mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{1,s}^{(1)} \right\}.
\tag{62}
$$

From (34), by integrating over the corresponding regions of the events in (62), we have

$$
\Pr\left\{ \mathcal{E}_{1,s}^{(1)} \right\} \approx \int_{\frac{R_{st}' - \frac{1-\alpha}{\alpha}}{P_S}}^{\infty} \delta_{ss} e^{-\delta_{ss}\gamma_{ss}} \left[\int_0^{\frac{P_S}{\left(R_{st}' - \frac{1-\alpha}{\alpha} \right) P_P}\gamma_{ss} - \frac{1}{P_P}} \delta_{ps} e^{-\delta_{ps}\gamma_{ps}} d\gamma_{ps} \right] d\gamma_{ss},
\tag{63}
$$

$$
\Pr\left\{ \mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{2,s}^{(1)} \right\} \approx
$$

$$
\begin{cases}
1 - \int_0^{\frac{R_{st}'}{P_S}} \delta_{ss} e^{-\delta_{ss}\gamma_{ss}} \left[\int_0^{\frac{R_{st}'}{(1-\alpha)P_R} - \frac{P_S}{(1-\alpha)P_R}\gamma_{ss}} \delta_{rs} e^{-\delta_{rs}\gamma_{rs}} d\gamma_{rs} \right] d\gamma_{ss}, & \alpha \ge \frac{R_{pt}'}{R_{pt}'+1} \\[3ex]
\int_0^{\frac{R_{st}'}{P_S}} \delta_{ss} e^{-\delta_{ss}\gamma_{ss}} \left[\int_{\frac{R_{st}'}{(1-\alpha)P_R} - \frac{P_S}{(1-\alpha)P_R}\gamma_{ss}}^{\infty} \delta_{rs} e^{-\delta_{rs}\gamma_{rs}} d\gamma_{rs} \right] \\
\quad \cdot \left[\int_{\frac{\left(R_{pt}' - \frac{\alpha}{1-\alpha} \right)P_S}{P_P}\gamma_{ss} + \frac{R_{pt}' - \frac{\alpha}{1-\alpha}}{P_P}}^{\infty} \delta_{ps} e^{-\delta_{ps}\gamma_{ps}} d\gamma_{ps} \right] d\gamma_{ss} + \\
\int_{\frac{R_{st}'}{P_S}}^{\infty} \delta_{ss} e^{-\delta_{ss}\gamma_{ss}} \left[\int_{\frac{\left(R_{pt}' - \frac{\alpha}{1-\alpha} \right)P_S}{P_P}\gamma_{ss} + \frac{R_{pt}' - \frac{\alpha}{1-\alpha}}{P_P}}^{\infty} \delta_{ps} e^{-\delta_{ps}\gamma_{ps}} d\gamma_{ps} \right] d\gamma_{ss}, & \alpha < \frac{R_{pt}'}{R_{pt}'+1}
\end{cases}
\tag{64}
$$

$$
\Pr\left\{ \mathcal{E}_{1,p}^{(1)} \cap \mathcal{E}_{1,s}^{(1)} \right\} \approx
$$

$$
\begin{cases}
\begin{cases}
0, \\
\quad \text{when } \left(R_{pt}' - \frac{\alpha}{1-\alpha} \right)\left(R_{st}' - \frac{1-\alpha}{\alpha} \right) \ge 1; \\
\int_{\frac{\left(R_{pt}' - \frac{\alpha}{1-\alpha} + 1 \right)\left(R_{st}' - \frac{1-\alpha}{\alpha} \right)}{P_S\left[1 - \left(R_{pt}' - \frac{\alpha}{1-\alpha} \right)\left(R_{st}' - \frac{1-\alpha}{\alpha} \right) \right]}}^{\infty} \delta_{ss} e^{-\delta_{ss}\gamma_{ss}} \\
\quad \cdot \left[\int_{\frac{\left(R_{pt}' - \frac{\alpha}{1-\alpha} \right)P_S}{P_P}\gamma_{ss} + \frac{R_{pt}' - \frac{\alpha}{1-\alpha}}{P_P}}^{\frac{P_S}{\left(R_{st}' - \frac{1-\alpha}{\alpha} \right)P_P}\gamma_{ss} - \frac{1}{P_P}} \delta_{ps} e^{-\delta_{ps}\gamma_{ps}} d\gamma_{pr} \right] d\gamma_{ss}, \\
\quad \text{when } \left(R_{pt}' - \frac{\alpha}{1-\alpha} \right)\left(R_{st}' - \frac{1-\alpha}{\alpha} \right) < 1.
\end{cases} & \frac{1}{R_{st}'+1} < \alpha < \frac{R_{pt}'}{R_{pt}'+1} \\[6ex]
\text{certain event,} & \frac{R_{pt}'}{R_{pt}'+1} \le \alpha \le \frac{1}{R_{st}'+1} \\[2ex]
\int_0^{\infty} \delta_{ss} e^{-\delta_{ss}\gamma_{ss}} \left[\int_{\frac{\left(R_{pt}' - \frac{\alpha}{1-\alpha} \right)P_S}{P_P}\gamma_{ss} + \frac{R_{pt}' - \frac{\alpha}{1-\alpha}}{P_P}}^{\infty} \delta_{ps} e^{-\delta_{ps}\gamma_{ps}} d\gamma_{ps} \right] d\gamma_{ss}, & \alpha < \min\left(\frac{R_{pt}'}{R_{pt}'+1}, \frac{1}{R_{st}'+1} \right) \\[4ex]
\int_{\frac{R_{st}' - \frac{1-\alpha}{\alpha}}{P_S}}^{\infty} \delta_{ss} e^{-\delta_{ss}\gamma_{ss}} \left[\int_0^{\frac{P_S}{\left(R_{st}' - \frac{1-\alpha}{\alpha} \right)P_P}\gamma_{ss} - \frac{1}{P_P}} \delta_{ps} e^{-\delta_{ps}\gamma_{ps}} d\gamma_{ps} \right] d\gamma_{ss}, & \alpha > \max\left(\frac{R_{pt}'}{R_{pt}'+1}, \frac{1}{R_{st}'+1} \right),
\end{cases}
\tag{65}
$$

respectively. Then, these integrations can be solved to obtain the results in (37a)–(37c).

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