Spectrum Sharing Single-Carrier in the Presence of Multiple Licensed Receivers

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***Abstract*—In this paper, maximal-ratio combining (MRC) and selection combining (SC) are proposed in spectrum sharing single-carrier networks with multiple primary user receivers (PU-Rxs). Taking into account the peak interference power at the PU-Rx's and the maximum transmit power at the secondary user (SU), the impact of multiple PU-Rx's on the secondary network is characterized when the secondary user receiver (SU-Rx) is equipped with multiple antennas. In doing so, exact and asymptotic expressions are derived for the cumulative distribution function, taking into account two realistic scenarios: 1) non-identical frequency selective fading between the secondary user transmitter (SU-Tx) and the PUs, and 2) frequency selective fading between the SU-Tx and the SU-Rx. Based on these, exact and asymptotic expressions for the outage probability and average bit error rate are derived. Furthermore, an exact closed-form expression for the ergodic capacity is derived. It is shown that the asymptotic diversity gain depends only on the number of receive antennas and the number of multipath channels. It is further shown that the number of PU-Rx's and fading severities between the SU-Tx and the PU-Rx's have no impact on the asymptotic diversity gain.**

***Index Terms*—Diversity, frequency selective fading, single-carrier transmission, spectrum sharing.**

I. Introduction

COGNITIVE RADIO (CR) network with spectrum-sharing, where the secondary user (SU) is able to share the same radio medium licensed to the primary user receiver (PU-Rx), is a promising approach to alleviate the inefficient use of the frequency spectrum [1]. In this paradigm, the SU transmit power is controlled such that its interference on the PU-Rx does not exceed a predefined threshold, which is determined by the quality-of-service (QoS) at the PU-Rx. To boost the performance of the SU, several enhanced diversity techniques such as spatial diversity combining have been proposed. Specifically, maximal-ratio combining (MRC) has been adopted to enhance the SU performance in CR networks [2], [3]. In [2], exact and asymptotic expressions for the average symbol error rate and the ergodic capacity were derived for Rayleigh fading channels under a maximum allowable interference power and peak transmit power. In [2], it was shown that a full diversity order equal to the total number of cognitive receive antennas is achieved when the peak transmit power is much smaller than the maximum allowable interference power. By relaxing this assumption, a more accurate asymptotic result was presented in [3]. Comparing these two power allocation constraints, only the maximum allowable interference power is considered in spectrum sharing systems in [4]–[8]. Furthermore, the ergodic capacity of spectrum-sharing over Nakagami-m fading channels was addressed in [9] and [10]. It was shown in [10] that MRC diversity at the SU receiver (SU-Rx) can achieve capacity enhancement and reduce the effect of asymmetric fading among the CR links on the SU performance. It is important to note that all these previous works have considered a single PU-Rx. Moreover, the impact of frequency selectivity in fading channels has not been reflected in the analysis of the aforementioned works.

To combat the effects of frequency selectivity in fading channels, orthogonal frequency division multiplexing (OFDM) has been proposed and adopted in several emerging technologies such as wireless local area networks (e.g. IEEE 802.11n [11]) and wireless mobile broadband communication systems (e.g. IEEE 802.16e [12]). However, OFDM transmission has intrinsically high peak-to-average power ratios (PAPRs) and high power back off in proportion to the number of subcarriers [13], [14]. Thus, single-carrier transmission has been proposed in very high-speed wireless networks (e.g. IEEE 802.11ad [15] and 3GPP Long-Term Evolution [16]) to maximize the use of battery power. For these reasons, single-carrier transmission is of interest for the up-link transmission instead of OFDM transmission [16]. For single-carrier transmission, several techniques have been proposed to fit different problems and accommodate different constraints. Among them, space-time-block coding (STBC) was proposed in [17] and distributed space-frequency-block coding (SFBC) was proposed in [18]. Cyclic delay diversity was investigated in [19] to achieve transmit diversity with a less complex transmitter. Frequency domain equalization (FDE) has been widely adopted due to its low computational requirements [20], [21]. Best relay selection and best terminal selection were proposed to improve

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the throughput of cooperative non-spectrum sharing systems in [22] and [23]. In addition, channel estimation was considered in [13] and [24]. Recently, single-carrier transmission has been proposed for spectrum sharing systems [25], [26]. In [25], the impact of interference from the PU transmitter (PU-Tx) on the secondary network was considered. In [26], several relay selection and power allocation constraints were proposed. In [25] and [26], a single antenna and a single PU-Rx were considered. Moreover, the channel impulse response in [25] and [26] comprised of independent and identically distributed (i.i.d.) complex Gaussian random variables with zero means and unit variances due to the assumption of Rayleigh fading channels. More importantly, spatial diversity has not been addressed in the aforementioned works. Against this backdrop, the impact of spatial diversity on spectrum sharing single-carrier networks is not intuitively obvious and is the main focus of this paper.

In this paper, we introduce spectrum sharing single-carrier transmission with *multiple* receive antennas in the secondary network. We also address the more complete scenario of *multiple* PU-Rx's in the primary network. With this in mind, the interplay between the transmit power with multiple receive antennas in the secondary network and the interference temperature with multiple PU-Rx's in the primary network is not straightforward. Against the background, the preeminent objective is to characterize the joint impact of multiple receive antennas in the secondary network and multiple PU-Rx's in the primary network in the more general scenario of frequency selective fading between the secondary user transmitter (SU-Tx) and the SU-Rx. We summarize the main contributions of this paper as follows.

1) We propose a single transmit antenna at the SU-Tx and multiple receive antennas at the SU-Rx. In contrast to previous works [2], [3], [25], [26], we consider the co-existence of multiple PU-Rx's in the network. We incorporate two realistic scenarios: i) non-identical frequency selective fading between the SU-Tx and all the PU-Rx's due to different multipath fading between them, and ii) frequency selective fading between the SU-Tx and the SU-Rx. Based on these, we present a unified comparative analysis of two diversity combining protocols at the SU-Rx, namely MRC and selection combining (SC) [1].
2) We consider two interrelated power constraints: i) peak interference power at the PU-Rx's, and ii) maximum transmit power at the SU. Based on these, we derive new exact closed-form expressions for the outage probability, average bit error rate (ABER), and ergodic capacity. We also derive new asymptotic closed-form expressions for the outage probability and the ABER. Our asymptotic expressions reveal important design insights into the joint impact of key network parameters − number of PU-Rx's, number of receive antennas at SU-Rx, and number of multipath channels − on the behavior of spectrum sharing single-carrier transmission.
3) We confirm that the asymptotic diversity gain is solely determined by two network parameters: i) receiver diversity gain which corresponds to the number of receive antennas at the SU, and ii) multipath diversity gain which corresponds to the number of multipath channels between the SU-Tx and the SU-Rx. This result is consistent with non-spectrum sharing single-carrier systems [22] and [23]. We corroborate that the asymptotic diversity gain is entirely independent of the primary network. For each diversity combining protocol, the asymptotic diversity gain is the same, irrespective of the number of PU-Rx's and the fading severities which are proportional to the number of multipaths between the SU-Tx and all the PU-Rx's.

[1]It is well-known that MRC is superior to SC at the cost of higher power consumption and multiple radio-frequency (RF) chains. Nonetheless, a performance/implementation tradeoff between MRC and SC is important, and as such it is worth investigating and comparing the performance of these two diversity combining techniques [27].

Notation: The superscript $(\cdot)^H$ denotes complex conjugate transposition; $E\{\cdot\}$ denotes expectation; $\boldsymbol{I}_N$ is an $N \times N$ identity matrix; $\mathbf{0}$ denotes an all zeros matrix of appropriate dimensions; $\mathcal{CN}\left(\mu, \sigma^2\right)$ denotes the complex Gaussian distribution with mean μ and variance σ^2; $\mathbb{C}^{m\times n}$ denotes the vector space of all $m \times n$ complex matrices; $F_\varphi(\gamma)$ denotes the cumulative distribution function (CDF) of the random variable (RV) φ; The probability density function (PDF) of φ is denoted by $f_\varphi(x)$; The binomial coefficient is denoted by $\binom{n}{k} \triangleq \frac{n!}{(n-k)!k!}$.

II. System and Channel Model

We assume a secondary network as shown in Fig. 1 in which the SU-Tx is equipped with a single transmit antenna and the SU-Rx is equipped with Q receive antennas. All K PU-Rx's are coexistent in the same frequency band. Similar to [28]–[30], we have assumed that the PU-Txs are located far enough away from the SUs so as not to impinge any significant interference upon the received signals at the SU-Rx. In addition, as noted in [31], the interference at the SU-Rx can be further neglected by treating it as noise under the condition that the signals transmitted from the PU-Tx's are generated by random Gaussian codebooks. Thus, interference in the SU network [25], [32], [33] from PU-Tx's are neglected in the considered system. Binary phase shift keying (BPSK) modulation is employed such that the modulated block data symbol transmitted from the SU-Tx, denoted by $\boldsymbol{x} \in \mathbb{C}^{N\times 1}$, satisfies $E\{\boldsymbol{x}\} = \mathbf{0}$ and $E\{\boldsymbol{x}\boldsymbol{x}^H\} = \boldsymbol{I}_N$. A cyclic prefix (CP) of N_g symbols is prefixed to the front of $\boldsymbol{x}$ to prevent inter-block symbol interference (IBSI) and intersymbol interference (ISI) [13], [14].

An instantaneous set of impulse channel responses from the SU-Tx to the kth PU-Rx, $\boldsymbol{g}^k$, is assumed to be comprised of m_k multipath channels, that is, $\boldsymbol{g}^k \triangleq [g_0^k, \ldots, g_{m_k-1}^k]^T \in \mathbb{C}^{m_k\times 1}$. A pass loss component over the channel $\boldsymbol{g}^k$ is denoted by α_k. An instantaneous set of impulse channel responses from the SU-Tx to the qth receive antenna at the SU-Rx is denoted by $\boldsymbol{h}^q \triangleq [h_0^q, \ldots, h_{N_h-1}^q]^T \in \mathbb{C}^{N_h\times 1}$, with N_h being the multipath channel length for all channels in the SU network. Comparing with α_k, a path loss component over a channel $\boldsymbol{h}^q$ is normalized to 1. To suppress IBSI and ISI in single carrier-transmission, it is assumed that $N_h \leq N_g$ and $\{m_k\}_{k=1}^K \leq N_g$.

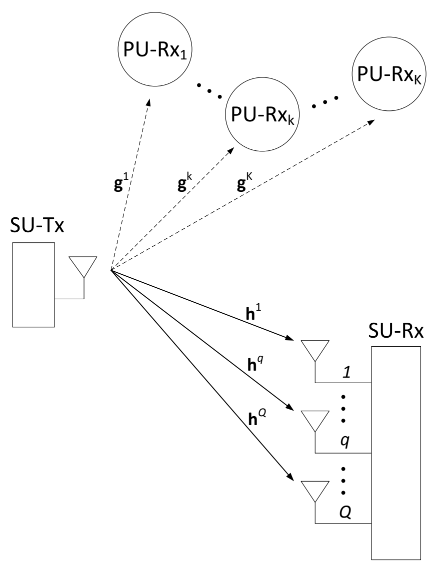

Fig. 1. Illustration of a spectrum sharing single-carrier network with multiple PU-Rx's and multiple receive antennas at SU-Rx.

The peak transmit power at the SU-Tx is denoted by P_T and the maximum allowable interference at all the PU-Rx's is denoted by I_p. Under a given peak transmit power and maximum allowable interference constraints, the transmit power allocation at the SU-Tx is defined as

$$P_s = \min\left(P_T, \frac{I_p}{\max_{k=1,\cdots,K}\{\alpha_k \|\boldsymbol{g}^k\|^2\}}\right). \quad (1)$$

After removing the signal associated with the CP, the received signal at the qth receive antenna can be written as

$$\boldsymbol{y}^q = \sqrt{P_s}\boldsymbol{H}^q\boldsymbol{x} + \boldsymbol{z}^q \quad (2)$$

where $\boldsymbol{z}^q \sim \mathcal{CN}(\boldsymbol{0}, \sigma_n^2\boldsymbol{I}_N)$. Recall that influential PU-Tx's are placed far away from the SU network, so that interference from the PU-Tx is neglected in the proposed system. As such, (2) corresponds to (1) in [25] without interference from the PU-Tx.

In single carrier transmission, the time varying right circulant channel matrix $\boldsymbol{H}^q \in \mathbb{C}^{N\times N}$ is determined by $\boldsymbol{h}^q \in \mathbb{C}^{N_h\times 1}$ [22], [34]. To construct the right circulant channel matrix, it is necessary to insert $(N-N_h)$ zero paddings which results in a size-N channel vector where $N > N_h$.

Definition 1: The instantaneous channel power of a channel matrix $\boldsymbol{A} \in \mathbb{C}^{N\times N}$ is defined by $\gamma_A \triangleq |\mathrm{Trace}(\boldsymbol{A})|^2 = \mathrm{Trace}(\boldsymbol{A})\mathrm{Trace}(\boldsymbol{A}^H)$. For a receive matrix $\boldsymbol{B}$, the Cauchy-Schwartz inequality for the instantaneous power of the channel after the receiving operation is given by

$$\begin{aligned}\gamma_{AB} &\triangleq |\mathrm{Trace}(\boldsymbol{A}^H\boldsymbol{B})|^2 \\ &\leq \mathrm{Trace}(\boldsymbol{A}\boldsymbol{A}^H)\mathrm{Trace}(\boldsymbol{B}\boldsymbol{B}^H) = \gamma_A\gamma_B \quad (3)\end{aligned}$$

with equality if and only if $\boldsymbol{B} = c\boldsymbol{A}$, $\forall c \neq 0$.

Definition 2: It was shown in [22] and [34] that when the channel impulse responses are composed of i.i.d. complex Gaussian random variables with zero means and unit variances, then the distribution of $\gamma_H = \frac{\mathrm{Trace}((\boldsymbol{H}^q)^H\boldsymbol{H}^q)}{N}$ follows a chi-squared distribution with $2N_h$ degrees of freedom for circulant matrices $\{\boldsymbol{H}^q, \forall q\}$. We express the distribution of γ_H as $\gamma_H \sim \chi^2(2N_h)$. The PDF and the CDF of γ_H are, respectively, given by

$$\begin{aligned} f_{\gamma_H}(x) &= \frac{1}{\Gamma(N_h)}x^{N_h-1}e^{-x}\mathrm{U}(x) \text{ and} \\ F_{\gamma_H}(x) &= \left(1 - e^{-x}\sum_{i=0}^{N_h-1}\frac{x^i}{i!}\right)\mathrm{U}(x) \quad (4)\end{aligned}$$

where $\mathrm{U}(\cdot)$ denotes the discrete unit step function and $\Gamma(N_h) \triangleq \int_0^\infty e^{-t}t^{N_h-1}dt$.

An RV $\tilde{\gamma}_H$ distributed by a modified chi-squared distribution with $2N_h$ degrees of freedom with a real-valued constant β_H is denoted by $\tilde{\gamma}_H \sim \chi^2(2N_h, \beta_H)$, whose PDF and CDF are, respectively, given by

$$\begin{aligned} f_{\tilde{\gamma}_H}(x) &= \frac{\beta_H^{N_h}}{\Gamma(N_h)}x^{N_h-1}e^{-\beta_H x}\mathrm{U}(x) \text{ and} \\ F_{\tilde{\gamma}_H}(x) &= \left(1 - e^{-\beta_H x}\sum_{i=0}^{N_h-1}\frac{(\beta_H x)^i}{i!}\right)\mathrm{U}(x). \quad (5)\end{aligned}$$

Note that $\beta_H \triangleq \frac{1}{\alpha_H}$, where α_H accounts for a path loss component over a particular channel. Based on (5), $\alpha_k\|\boldsymbol{g}^k\|^2 \sim \chi^2(2m_k, \beta_k)$ when $\boldsymbol{g}^k$ is composed of m_k i.i.d. complex Gaussian random variables with zero means and unit variances.

III. Distribution of Post-Processing SNR of SIMO Single-Carrier System

In this section, we first derive the distributions of post-processing SNRs of single-input multiple-output (SIMO) single-carrier systems employing either MRC or SC at the SU-Rx. To this end, instantaneous post-processing SNRs for each combining protocol are derived. Based on the instantaneous post-processing SNRs, corresponding CDFs are derived.

Assumption 1: Frequency selective fading channels between the SU-Tx and the K PU-Rx's follow independent modified chi-squared distributions with different degrees of freedom and path losses, whereas all the frequency selective fading channels between the SU-Tx and the Q receive antennas at the SU-Rx are comprised of N_h i.i.d. complex Gaussian random variables with zero means and unit variances.

A. MRC at SU-Rx

When MRC is employed at the SU-Rx, all the antennas are combined and the received signal is given by

$$\boldsymbol{y} = \sum_{q=1}^{Q}\sqrt{P_s}(\boldsymbol{G}^q)^H\boldsymbol{H}^q\boldsymbol{x} + \sum_{q=1}^{Q}(\boldsymbol{G}^q)^H\boldsymbol{z}^q \quad (6)$$

where $\boldsymbol{G}^q$ is the receive matrix for the qth receive antenna branch at the SU-Rx. Based on Definition 1, the instantaneous post-processing SNR at the SU-Rx is given by

$$\gamma_{\mathrm{MRC}} \leq \frac{P_s\left(\sum_{q=1}^{Q}\sqrt{\mathrm{Trace}((\boldsymbol{G}^q)^H\boldsymbol{G}^q)\mathrm{Trace}((\boldsymbol{H}^q)^H\boldsymbol{H}^q)}\right)^2}{\sigma_n^2\sum_{q=1}^{Q}\mathrm{Trace}((\boldsymbol{G}^q)^H\boldsymbol{G}^q)}. \quad (7)$$

When the receive matrix is $\boldsymbol{G}^q = \boldsymbol{H}^q$, the maximum achievable instantaneous post-processing SNR is given by

$$\gamma_{\mathrm{MRC}} = \frac{P_s \sum_{q=1}^{Q} \mathrm{Trace}((\boldsymbol{H}^q)^H \boldsymbol{H}^q)}{N\sigma_n^2}. \quad (8)$$

Upon applying the expressions for P_s defined in (1), we evaluate (8) as follows:

$$\gamma_{\mathrm{MRC}} = \min\left(P_T, \frac{I_p}{\max_{k=1,\cdots,K}\{\alpha_k \|\boldsymbol{g}^k\|^2\}}\right) \frac{\sum_{q=1}^{Q} \mathrm{Trace}((\boldsymbol{H}^q)^H \boldsymbol{H}^q)}{N\sigma_n^2}. \quad (9)$$

According to the properties of the right circulant matrix [22], [34], (9) becomes

$$\gamma_{\mathrm{MRC}} = \min\left(\tilde{P}_T, \tilde{I}_p/X\right) Y \quad (10)$$

where $\tilde{I}_p \triangleq \frac{I_p}{\sigma_n^2}$ and $\tilde{P}_T \triangleq \frac{P_T}{\sigma_n^2}$ are a normalized peak interference at all the PU-Rx's and the normalized transmit power at the SU-Tx, respectively. For notational purpose, we define $X \triangleq \max_{k=1,\cdots,K}\{\alpha_k \|\boldsymbol{g}^k\|^2\}$ and $Y \triangleq \sum_{q=1}^{Q} \sum_{l=0}^{N_h-1} |h_l^q|^2$.

To derive the CDF of γ_{MRC}, the exact knowledge of the distribution of X is necessary. The CDF $F_X(x)$ is derived in the following lemma.

Lemma 1: When $\alpha_k \|\boldsymbol{g}^k\|^2 \sim \chi^2(m_k, \beta_k)$, the CDF $F_X(x)$ is given by

$$F_X(x) = 1 + \sum_{k=1}^{K} \frac{(-1)^k}{k!} \underbrace{\sum_{n_1=1}^{K} \cdots \sum_{n_k=1}^{K}}_{|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k| = k} \sum_{l_1=0}^{m_{n_1}-1} \cdots \sum_{l_k=0}^{m_{n_k}-1} \prod_{t=1}^{k} \left(\frac{(\beta_{n_t})^{l_t}}{l_t!}\right) x^{\sum_{t=1}^{k} l_t} e^{-(\sum_{t=1}^{k} \beta_{n_t})x}$$
$$= 1 + \widetilde{\sum}\left[x^{\tilde{l}} e^{-\tilde{\beta}x}\right] \quad (11)$$

where $|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k|$ denotes the cardinality of the union of k indices. Also, $\widetilde{\sum}[\cdot] \triangleq \sum_{k=1}^{K} \frac{(-1)^k}{k!} \underbrace{\sum_{n_1=1}^{K} \cdots \sum_{n_k=1}^{K}}_{|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k| = k} \sum_{l_1=0}^{m_{n_1}-1} \cdots \sum_{l_k=0}^{m_{n_k}-1} \prod_{t=1}^{k} \left(\frac{(\beta_{n_t})^{l_t}}{l_t!}\right)[\cdot]$, $\tilde{l} \triangleq \sum_{t=1}^{k} l_t$, and $\tilde{\beta} \triangleq (\sum_{t=1}^{k} \beta_{n_t})$.

Proof: A proof of this lemma is provided in Appendix A. ∎

Using Assumption 1, Y follows a chi-squared distribution with $2N_hQ$ degrees of freedom. Thus, we denote the distribution of Y as $Y \sim \chi^2(2N_hQ)$. For this fading, the CDF of γ_{MRC} is given as

$$F_{\gamma_{\mathrm{MRC}}}(\gamma) = F_Y\left(\gamma\mu/\tilde{I}_p\right) - \frac{\left(\gamma/\tilde{I}_p\right)^{N_hQ}}{\Gamma(N_hQ)} \widetilde{\sum}\left[\left(\gamma/\tilde{I}_p + \tilde{\beta}\right)^{-(N_hQ+\tilde{l})} \Gamma\left(N_hQ + \tilde{l}, \mu\left(\gamma/\tilde{I}_p + \tilde{\beta}\right)\right)\right]$$
$$= 1 - \tilde{F}_{\gamma_{\mathrm{MRC}}}(\gamma) \quad (12)$$

where $\mu \triangleq \tilde{I}_p/\tilde{P}_T$ is the ratio of the maximum interference to the peak transmit power, and

$$\tilde{F}_{\gamma_{\mathrm{MRC}}}(\gamma) \triangleq \Gamma(N_hQ, \gamma\mu/\tilde{I}_p)/\Gamma(N_hQ) + \frac{\left(\gamma/\tilde{I}_p\right)^{N_hQ}}{\Gamma(N_hQ)} \widetilde{\sum}\left[\left(\gamma/\tilde{I}_p + \tilde{\beta}\right)^{-(\tilde{l}+N_hQ)} \Gamma\left(N_hQ + \tilde{l}, \mu\left(\gamma/\tilde{I}_p + \tilde{\beta}\right)\right)\right]. \quad (13)$$

A detailed derivation of (12) is provided in Appendix B.

B. SC at SU-Rx

When SC is employed at the SU-Rx, the strongest antenna is selected and the instantaneous post-processing SNR γ_{SC} is given by

$$\gamma_{\mathrm{SC}} = \min\left(\tilde{P}_T, \frac{\tilde{I}_p}{\max_{k\in[1,\cdots,K]}\{\alpha_k \|\boldsymbol{g}^k\|^2\}}\right) \max_{q\in[1,2,\cdots,Q]} \left(\sum_{l=0}^{N_h-1} |h_l^q|^2\right) = \min\left(\tilde{P}_T, \tilde{I}_p/X\right) Z \quad (14)$$

where $Z \triangleq \max_{q\in[1,2,\cdots,Q]} \left(\sum_{l=0}^{N_h-1} |h_l^q|^2\right)$. According to the derivations provided in [34], the PDF $f_Z(z)$ is given by

$$f_Z(z) = \frac{Q}{\Gamma(N_h)} \widehat{\sum}_{k'}\left[z^{N_h+\tilde{N}_h-1} e^{-z(k'+1)}\right] \quad (15)$$

where $\widehat{\sum}_{k'}[\cdot] \triangleq \sum_{k'=0}^{Q-1} \binom{Q-1}{k'} (-1)^{k'} \sum_{\substack{l_1,l_2,\cdots,l_{N_h} \\ l_1+\cdots+l_{N_h}=k'}} \left(\frac{(k')!}{l_1! l_2! \cdots l_{N_h}!}\right) \prod_{t=0}^{N_h-1} \left(\frac{1}{t!}\right)^{l_{t+1}} [\cdot]$, and $\tilde{N}_h \triangleq \sum_{t=0}^{N_h-1} t l_{t+1}$. Moreover, using the binomial and multinomial identities, the CDF of the RV Z is given by

$$F_Z(z) = 1 + \tilde{F}_Z(z) = 1 + \widehat{\widehat{\sum}}_{k'}\left[z^{\tilde{N}_h} e^{-zk'}\right] \quad (16)$$

where we define $\tilde{F}_Z(z) \triangleq \widehat{\widehat{\sum}}_{k'}\left[z^{\tilde{N}_h} e^{-zk'}\right]$ with the notational definition

$$\widehat{\widehat{\sum}}_{k'}[\cdot] \triangleq \sum_{k'=1}^{Q} \binom{Q}{k'} (-1)^{k'} \sum_{\substack{l_1,l_2,\cdots,l_{N_h} \\ l_1+\cdots+l_{N_h}=k'}} \left(\frac{(k')!}{l_1! l_2! \cdots l_{N_h}!}\right) \prod_{t=0}^{N_h-1} \left(\frac{1}{t!}\right)^{l_{t+1}} [\cdot]. \quad (17)$$

Using (15) and Assumption 1 for fading channels, the CDF of γ_{SC} is derived in (18) at the top of the next page. In (18), we have defined

$$\tilde{F}_{\gamma_{\mathrm{SC}}}(\gamma) \triangleq -\widehat{\widehat{\sum}}_{k'}\left[\left(\mu\gamma/\tilde{I}_p\right)^{\tilde{N}_h} e^{-(\mu\gamma/\tilde{I}_p)k'}\right] + \frac{Q}{\Gamma(N_h)} \widetilde{\sum}\widehat{\sum}_{k'}\left[\left(\gamma/\tilde{I}_p\right)^{N_h+\tilde{N}_h} \left(\gamma(k'+1)/\tilde{I}_p + \tilde{\beta}\right)^{-(N_h+\tilde{N}_h+\tilde{l})} \Gamma\left(N_h + \tilde{N}_h + \tilde{l}, \mu\left(\gamma(k'+1)/\tilde{I}_p + \tilde{\beta}\right)\right)\right]. \quad (19)$$

A detailed derivation of (18) is provided in Appendix C.

$$F_{\gamma_{\mathrm{SC}}}(\gamma) = 1 + \tilde{F}_Z\left(\gamma\mu/\tilde{I}_p\right) - \frac{Q}{\Gamma(N_h)}\widetilde{\sum\sum}\widehat{\sum\sum}_{k'}\Big[\left(\gamma/\tilde{I}_p\right)^{N_h+\tilde{N}_h}\left(\gamma(k'+1)/\tilde{I}_p+\tilde{\beta}\right)^{-(N_h+\tilde{N}_h+\tilde{l})}$$
$$\Gamma\Big(N_h+\tilde{N}_h+\tilde{l}, \mu\big(\gamma(k'+1)/\tilde{I}_p+\tilde{\beta}\big)\Big)\Big] = 1 - \tilde{F}_{\gamma_{\mathrm{SC}}}(\gamma). \quad (18)$$

IV. Performance Analysis

In this section, we derive the outage probability, ergodic capacity, and ABER using the newly derived CDFs of the instantaneous post-processing SNRs.

A. Outage Probability

The outage probability at a pre-determined SNR threshold γ_{th} can be readily obtained as

$$\begin{aligned} P^{\mathrm{out}}_{\mathrm{MRC}}(\gamma_{th}) &= 1 - \tilde{F}_{\gamma_{\mathrm{MRC}}}(\gamma_{th}) \text{ and} \\ P^{\mathrm{out}}_{\mathrm{SC}}(\gamma_{th}) &= 1 - \tilde{F}_{\gamma_{\mathrm{SC}}}(\gamma_{th}). \end{aligned} \quad (20)$$

It follows that the closed-form expression for $P^{\mathrm{out}}_{\mathrm{MRC}}(\gamma_{th})$ is given by

$$P^{\mathrm{out}}_{\mathrm{MRC}}(\gamma_{th}) = F_Y\left(\gamma_{th}\mu/\tilde{I}_p\right) - \frac{1}{\Gamma(N_hQ)}\left(\gamma_{th}/\tilde{I}_p\right)^{N_hQ} \widetilde{\sum}\Big[\left(\gamma_{th}/\tilde{I}_p+\tilde{\beta}\right)^{-(N_hQ+\tilde{l})} \Gamma\Big(N_hQ+\tilde{l}, \mu\big(\gamma_{th}/\tilde{I}_p+\tilde{\beta}\big)\Big)\Big]. \quad (21)$$

Similarly, we can readily derive the closed-form expression for $P^{\mathrm{out}}_{\mathrm{SC}}(\gamma_{th})$. Although the exact outage probabilities for both combining protocols can be obtained, their complex forms provide no insights into diversity gain.

To characterize the impact of the number of PU-Rx's, fading severities proportional to m_ks between the SU-Tx and all the PU-Rx's, the number of receiving antennas, and the number of multipath channels on the outage probability, we proceed to derive the asymptotic outage probability in the region of high $\tilde{P}_T$ [35]. To this end, we first use the following asymptotic CDFs for each combining protocol:

$$\begin{aligned} \hat{F}_Y\left(\gamma/\tilde{P}_T\right) &\overset{\tilde{P}_T\to\infty}{\approx} \frac{1}{\Gamma(N_hQ+1)}\left(\gamma/\tilde{P}_T\right)^{N_hQ} \text{ and} \\ \hat{F}_Z\left(\gamma/\tilde{P}_T\right) &\overset{\tilde{P}_T\to\infty}{\approx} \frac{1}{\left(\Gamma(N_h+1)\right)^Q}\left(\gamma/\tilde{P}_T\right)^{N_hQ}. \end{aligned} \quad (22)$$

Note that $\hat{F}_Y\left(\gamma/\tilde{P}_T\right)$ and $\hat{F}_Z\left(\gamma/\tilde{P}_T\right)$ are in the form of $\hat{F}_Y\left(\gamma/\tilde{P}_T\right) \propto \left(\gamma/\tilde{P}_T\right)^{N_hQ}$ and $\hat{F}_Z\left(\gamma/\tilde{P}_T\right) \propto \left(\gamma/\tilde{P}_T\right)^{N_hQ}$. Using (22), the asymptotic outage probabilities for each combining protocol are given by

$$\begin{aligned} P^{\mathrm{as,out}}_{\mathrm{MRC}}(\gamma_{th}) &= (\beta_1+\beta_2-\beta_3)\left(\tilde{P}_T\right)^{-N_hQ} \text{ and} \\ P^{\mathrm{as,out}}_{\mathrm{SC}}(\gamma_{th}) &= (\beta_4+\beta_5-\beta_6)\left(\tilde{P}_T\right)^{-N_hQ} \end{aligned} \quad (23)$$

where $\beta_1 \triangleq \frac{1}{\Gamma(N_hQ+1)}\sum\left[\mu^{\tilde{l}}e^{-\tilde{\beta}\mu}(\gamma_{th})^{N_hQ}\right]$, $\beta_2 \triangleq \frac{1}{\Gamma(N_hQ+1)}\widetilde{\sum}\left[\tilde{l}(\tilde{\beta})^{-(N_hQ+\tilde{l})}\Gamma(N_hQ+\tilde{l}, \tilde{\beta}\mu)\right]$, and $\beta_3 \triangleq \frac{1}{\Gamma(N_hQ+1)}\widetilde{\sum}\left[(\tilde{\beta})^{-(N_hQ+\tilde{l})}\Gamma(N_hQ + \tilde{l} + 1, \tilde{\beta}\mu)\right]$. Also, we defined $\beta_4 \triangleq \frac{1}{(\Gamma(N_h+1))^Q}\sum\left[\mu^{\tilde{l}}e^{-\tilde{\beta}\mu}(\gamma_{th})^{N_hQ}\right]$, $\beta_5 \triangleq \frac{1}{(\Gamma(N_h+1))^Q}\widetilde{\sum}\left[\tilde{l}(\tilde{\beta})^{-(N_hQ+\tilde{l})}\Gamma(N_hQ + \tilde{l}, \tilde{\beta}\mu)\right]$, and $\beta_6 \triangleq \frac{1}{(\Gamma(N_h+1))^Q}\widetilde{\sum}\left[(\tilde{\beta})^{-(N_hQ+\tilde{l})}\Gamma(N_hQ+\tilde{l}+1, \tilde{\beta}\mu)\right]$. Note that (23) verifies that an asymptotic outage diversity gain is determined by the number of receiving antennas and the number of multipath channels between SU-Tx and receiving antennas. The number of PU-Rx's and the fading severity of a channel from the SU-Tx to the PU-Rx have no impact on the asymptotic outage diversity gain. Both combining protocols have the same asymptotic outage diversity gain. A derivation of (23) is provided in Appendix D.

B. Ergodic Capacity Analysis

The ergodic capacity of the proposed network is defined as [36]

$$\begin{aligned} C_{\mathrm{MRC}} &= \frac{1}{\log(2)}\int_0^\infty \frac{1-F_{\gamma_{\mathrm{MRC}}(\gamma)}}{1+\gamma}d\gamma \\ &= \frac{1}{\log(2)}\int_0^\infty \frac{\tilde{F}_{\gamma_{\mathrm{MRC}}(\gamma)}}{1+\gamma}d\gamma \text{ and} \\ C_{\mathrm{SC}} &= \frac{1}{\log(2)}\int_0^\infty \frac{1-F_{\gamma_{\mathrm{SC}}(\gamma)}}{1+\gamma}d\gamma \\ &= \frac{1}{\log(2)}\int_0^\infty \frac{\tilde{F}_{\gamma_{\mathrm{SC}}(\gamma)}}{1+\gamma}d\gamma \end{aligned} \quad (24)$$

which follows (25) derived at the top of the next page after some manipulations. In (25), we have defined $c_2 \triangleq \frac{1}{(\hat{I}_p-1)^{N_hQ+\tilde{l}-m}}$, $c_{3,l} \triangleq \frac{(-1)}{(\hat{I}_p-1)^{N_hQ+\tilde{l}-m-l+1}}$, and $d_m \triangleq \frac{1}{\Gamma(m+1)}\left(\frac{\mu}{\tilde{I}_p}\right)^m$. In addition, we have defined $c_4 \triangleq \frac{1}{\left(\hat{I}_p/(k'+1)-1\right)^{N_h+\tilde{N}_h+\tilde{l}-m}}$, $c_{5,l} \triangleq \frac{(-1)}{\left(\hat{I}_p/(k'+1)-1\right)^{N_h+\tilde{N}_h+\tilde{l}-m-l+1}}$, and $e_m \triangleq \frac{\left(\frac{\mu(k'+1)}{\tilde{I}_p}\right)^m}{\Gamma(m+1)}$ with $\hat{I}_p \triangleq \tilde{\beta}\tilde{I}_p$. Also, $\mathbb{U}(\cdot,\cdot;\cdot)$ denotes the confluent hypergeometric function [37, Eq. 9.211.4]. A detailed derivation of (25) is provided in Appendix E.

C. Average Bit Error Rate

Here, we derive the ABER for BPSK modulation based on the newly derived CDFs. The ABER is given as [36]

$$\begin{aligned} P_{b,\mathrm{MRC}} &= \int_0^\infty P_b(e|\gamma_{\mathrm{MRC}})f_{\gamma_{\mathrm{MRC}}}(\gamma)d\gamma \\ &= \frac{1}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{\mathrm{MRC}}}\left(t^2/2\right)e^{-t^2/2}dt \text{ and} \\ P_{b,\mathrm{SC}} &= \int_0^\infty P_b(e|\gamma_{\mathrm{SC}})f_{\gamma_{\mathrm{SC}}}(\gamma)d\gamma \\ &= \frac{1}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{\mathrm{SC}}}\left(t^2/2\right)e^{-t^2/2}dt \end{aligned} \quad (26)$$

$$C_{\mathrm{MRC}} = \frac{1}{\log(2)} \sum_{k=0}^{N_hQ-1} \left(\mu/\tilde{I}_p\right)^k \mathbb{U}\left(k+1, k+1; \mu/\tilde{I}_p\right) + \frac{1}{\log(2)\Gamma(N_hQ)} \widetilde{\sum} \Big[\left(\tilde{I}_p\right)^{\tilde{l}} \Gamma(N_hQ+\tilde{l})$$
$$e^{-\mu\tilde{\beta}} \sum_{m=0}^{N_hQ+\tilde{l}-1} d_m \Big[c_2\Gamma(N_hQ+1)\mathbb{U}\left(N_hQ+1, N_hQ+1; \mu/\tilde{I}_p\right) +$$
$$\sum_{l=1}^{N_hQ+\tilde{l}-m} c_{3,l}\left(\hat{I}_p\right)^{N_hQ+1-l}\Gamma(N_hQ+1)\mathbb{U}\left(N_hQ+1, N_hQ+2-l; \tilde{\beta}\mu\right)\Big]\Big] \text{ and}$$
$$C_{\mathrm{SC}} = -\frac{1}{\log(2)} \widehat{\widehat{\sum}}_{k'} \Big[(\mu/\tilde{I}_p)^{\tilde{N}_h}\Gamma(\tilde{N}_h+1)\mathbb{U}(N_h+\tilde{N}_h+1, \tilde{N}_h+1; \mu k'/\tilde{I}_p)\Big] +$$
$$\frac{Q}{\Gamma(N_h)} \widetilde{\sum}\widehat{\sum}_{k'} \Big[\left(\tilde{I}_p\right)^{\tilde{l}} (k'+1)^{-(N_h+\tilde{N}_h+\tilde{l})}\Gamma(N_h+\tilde{N}_h+\tilde{l})e^{-\mu\tilde{\beta}}$$
$$\sum_{m=0}^{N_h+\tilde{N}_h+\tilde{l}-1} e_m\Big(c_4\Gamma(N_h+\tilde{N}_h+1)\mathbb{U}\left(N_h+\tilde{N}_h+1, N_h+\tilde{N}_h+1; \mu(k'+1)/\tilde{I}_p\right) +$$
$$\sum_{l=1}^{N_h+\tilde{N}_h+\tilde{l}-m} c_{5,l}\Big(\frac{\hat{I}_p}{(k'+1)}\Big)^{N_h+\tilde{N}_h+1-l}\Gamma(N_h+\tilde{N}_h+1)\mathbb{U}(N_h+\tilde{N}_h+1, N_h+\tilde{N}_h+2-l; \mu\tilde{\beta})\Big)\Big]. \quad (25)$$

$$P_{b,\mathrm{MRC}} = 0.5 - \frac{1}{2\sqrt{\pi}} \sum_{k=0}^{N_hQ} \frac{1}{\Gamma(k+1)} \left(1/\tilde{P}_T\right)^k \Gamma(k+1/2)\left(1+1/\tilde{P}_T\right)^{-(k+1/2)} -$$
$$\frac{1}{2\sqrt{\pi}\Gamma(N_hQ)} \widetilde{\sum}\Big[(\tilde{I}_p)^{\tilde{l}}\Gamma(N_hQ+\tilde{l})e^{-\mu\tilde{\beta}} \sum_{m=0}^{N_hQ+\tilde{l}-1} d_m(\hat{I}_p)^{m-\tilde{l}+1/2}\Gamma(N_hQ+1/2)$$
$$\mathbb{U}\left(N_hQ+1/2, m-\tilde{l}+3/2; \tilde{\beta}\tilde{I}_p+\mu\tilde{\beta}\right)\Big] \text{ and}$$
$$P_{b,\mathrm{SC}} = 0.5 + \frac{1}{2\sqrt{\pi}} \widehat{\widehat{\sum}}_{k'} \Big[(1/\tilde{P}_T)^{\tilde{N}_h}\Gamma\left(\tilde{N}_h+1/2\right)\left(1+k'/\tilde{P}_T\right)^{-(\tilde{N}_h+1/2)}\Big] -$$
$$\frac{Q}{2\sqrt{\pi}\Gamma(N_h)} \widetilde{\sum}\widehat{\sum}_{k'} \Big[(\tilde{I}_p)^{\tilde{l}}(k'+1)^{-(N_h+\tilde{N}_h+\tilde{l})}\Gamma(N_h+\tilde{N}_h+\tilde{l})e^{-\mu\tilde{\beta}}$$
$$\sum_{m=0}^{N_h+\tilde{N}_h+\tilde{l}-1} e_m\Big(\frac{\hat{I}_p}{k'+1}\Big)^{m-\tilde{l}+1/2}\Gamma\left(\breve{N}_h\right)\mathbb{U}\Big(\breve{N}_h, m-\tilde{l}+3/2; \frac{\tilde{\beta}(\tilde{I}_p+\mu(k'+1))}{k'+1}\Big)\Big]. \quad (27)$$

---

where $P_b(e|\gamma_{\mathrm{MRC}})$ and $P_b(e|\gamma_{\mathrm{SC}})$ are conditional BERs conditioned on γ_{MRC} and γ_{SC} for MRC and SC, respectively. In addition, $f_{\gamma_{\mathrm{MRC}}}(\gamma)$ and $f_{\gamma_{\mathrm{SC}}}(\gamma)$ denote the PDFs of γ_{MRC} and γ_{SC}. Note that the final forms in (26) make it possible to derive the ABER without exact knowledge of PDFs. Substituting (12) and (18) into (26), yields (27) at the top of this page. In (27), we defined $\breve{N}_h \stackrel{\triangle}{=} N_h + \tilde{N}_h + 1/2$. A detailed derivation of (27) is provided in Appendix F. Using again asymptotic CDFs, $\hat{F}_{\gamma_{\mathrm{MRC}}}(\gamma)$ and $\hat{F}_{\gamma_{\mathrm{SC}}}(\gamma)$, asymptotic ABERs are given, respectively, as

$$\hat{P}_{b,\mathrm{MRC}} \overset{\tilde{P}_T\to\infty}{\approx} \frac{1}{\sqrt{2\pi}}\int_0^\infty \hat{F}_{\gamma_{\mathrm{MRC}}}\left(t^2/2\right)e^{-t^2/2}dt \text{ and}$$
$$\hat{P}_{b,\mathrm{SC}} \overset{\tilde{P}_T\to\infty}{\approx} \frac{1}{\sqrt{2\pi}}\int_0^\infty \hat{F}_{\gamma_{\mathrm{SC}}}\left(t^2/2\right)e^{-t^2/2}dt \quad (28)$$

which are computed as

$$\hat{P}_{b,\mathrm{MRC}} = c_6\left(\tilde{P}_T\right)^{-N_hQ} \text{ and } \hat{P}_{b,\mathrm{SC}} = c_7\left(\tilde{P}_T\right)^{-N_hQ} \quad (29)$$

where $c_6 \stackrel{\triangle}{=} \frac{1}{2\Gamma(N_hQ+1)}\Big(\widetilde{\sum}\Big[\frac{\Gamma(N_hQ+1/2)}{\sqrt{\pi}}\mu^{\tilde{l}}e^{-\tilde{\beta}\mu} + \tilde{\beta}^{-(N_hQ+\tilde{l})}\Gamma(N_hQ+\tilde{l}, \tilde{\beta}\mu) - \tilde{\beta}^{-(N_hQ+\tilde{l}+1)}\Gamma(N_hQ+\tilde{l}, \tilde{\beta}\mu)\Big]\Big)$ and $c_7 \stackrel{\triangle}{=} \frac{1}{2(\Gamma(N_h+1))^Q}\Big(\widetilde{\sum}\Big[\frac{\Gamma(N_hQ+1/2)}{\sqrt{\pi}}\mu^{\tilde{l}}e^{-\tilde{\beta}\mu} + \tilde{\beta}^{-(N_hQ+\tilde{l})}\Gamma(N_hQ+\tilde{l}, \tilde{\beta}\mu) - \tilde{\beta}^{-(N_hQ+\tilde{l}+1)}\Gamma(N_hQ+\tilde{l}, \tilde{\beta}\mu)\Big]\Big)$. Note that (29) shows an asymptotic diversity gain of N_hQ independent of the combining protocols (MRC and SC). Also, the number of PU-Rx's and fading severities between the SU-Tx and PU-Rx's have no impact on the diversity gain. Along with the asymptotic outage diversity gain analysis, these results are novel compared with the previous works [2], [3], [25], [26].

V. Simulation Results

We assume $N = 256$ and $N_g = 16$ for the data symbol block size and the CP length, respectively. We use BPSK modulation and a fixed $\gamma_{th} = 3$ dB in the computation of the outage probability. To investigate the frequency selective

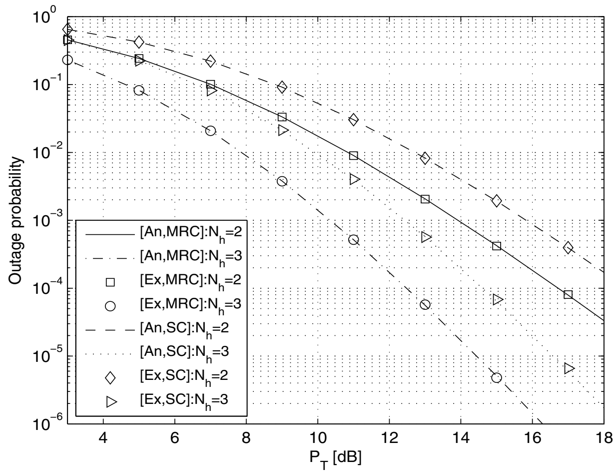

Fig. 2. Outage probability of MRC and SC for various values of N_h with $K = 2$, $Q = 2$, and $(\mathrm{M}_1, \mathrm{A}_1)$.

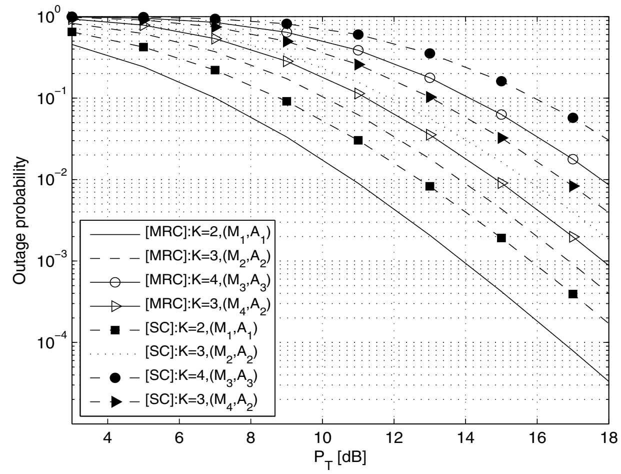

Fig. 3. Outage probability of MRC and SC for various values of K and fading severities $\{\mathrm{M}_k, \mathrm{A}_k\}$.

fading severity effects on the performance, we use various frequency selective fading sets : $(\mathrm{M}_1 \stackrel{\triangle}{=} \{m_1 = 2, m_2 = 3\}, \mathrm{A}_1 \stackrel{\triangle}{=} \{\alpha_1 = 1/0.5, \alpha_2 = 1/0.3\})$, $(\mathrm{M}_2 \stackrel{\triangle}{=} \{m_1 = 2, m_2 = 3, m_3 = 4\}, \mathrm{A}_2 \stackrel{\triangle}{=} \{\alpha_1 = 1/0.5, \alpha_2 = 1/0.3, \alpha_3 = 1/0.2\})$, $(\mathrm{M}_3 \stackrel{\triangle}{=} \{m_1 = 2, m_2 = 3, m_3 = 4, m_4 = 5\}, \mathrm{A}_3 \stackrel{\triangle}{=} \{\alpha_1 = 1/0.5, \alpha_2 = 1/0.3, \alpha_3 = 1/0.2, \alpha_4 = 1/0.1\})$, and $\mathrm{M}_4 \stackrel{\triangle}{=} \{m_1 = 3, m_2 = 4, m_3 = 5\}$. In the figures, the curves obtained from actual link simulations are denoted by **Ex**, whereas analytically derived curves are denoted by **An**. In addition, asymptotically derived curves are denoted by **Asymp**. Fading channels are generated according to Assumption 1, that is, channels between the SU-Tx and all PU-Rx's are generated to follow independent modified chi-squared distributions with different fading severities, whereas all frequency selective fading channels between the SU-Tx and Q receive antennas at the SU-Rx are generated by N_h i.i.d. complex Gaussian random variables with zero means and unit variances.

A. Outage Probability

Fig. 2 shows the outage probability for various values of N_h with $Q = 2$, $K = 2$, and $(\mathrm{M}_1, \mathrm{A}_1)$. From this figure, we observe a good match between the derived outage probabilities and the exact outage probabilities for the two combining protocols. In addition, for the same values of Q, N_h and K, MRC achieves a lower outage probability than SC, which follows the same behavior as non-spectrum-sharing networks. Also, as N_h increases, a lower outage probability is obtained in both combining protocols due to a higher outage diversity gain.

Fig. 3 shows the effects of fading severity of the fading channel between the SU-Tx and all PU-Rx's and the number of PU-Rx's (denoted by K) on the outage probability. For different M_k and A_k, we observe that the outage probability increases with increasing K. Also, for $K = 3$, a system with $(\mathrm{M}_4, \mathrm{A}_2)$ will have a worse outage probability than that of $(\mathrm{M}_2, \mathrm{A}_2)$ due to more severe fading between the SU-Tx and all PU-Rx's. However, we can readily observe that the slopes of the two curves for these two previous scenarios are same.

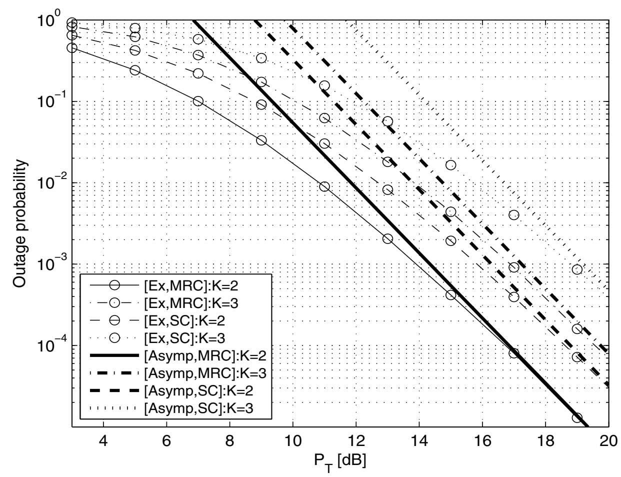

Fig. 4. Outage probability of MRC and SC for various values of K with its corresponding $(\mathrm{M}_1, \mathrm{A}_1)$ and $(\mathrm{M}_2, \mathrm{A}_2)$. We use $Q = 2$ and $N_h = 2$.

Fig. 4 shows the outage probability for various fading severity and number of PU-Rx's with fixed $Q = 2$ and $N_h = 2$. Since the slopes of all curves do not change, we find that the fading severity of the fading channel between the SU-Tx and all the PU-Rx's, as well as the number of PU-Rx's do not influence the outage diversity gain. It can be seen that only the multipath diversity gain and the receive diversity gain of the SU network simultaneously influence the outage diversity gain.

B. Ergodic Capacity

Fig. 5 shows the ergodic capacity for various values of K and fading severities. For a fixed number of PU-Rx's, a higher ergodic capacity is achieved with more antennas or more multiple channels. Furthermore, we see that MRC achieves a higher ergodic capacity compared with SC for the same network configuration.

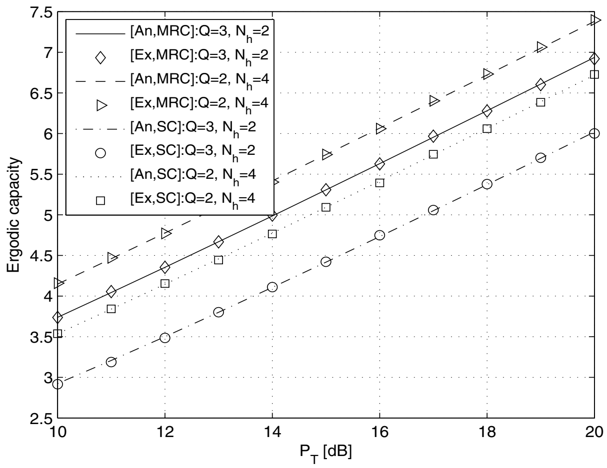

Fig. 5. Ergodic capacity of MRC and SC for various values of (Q, N_h) with $K = 2$ and $(\mathrm{M}_1, \mathrm{A}_1)$.

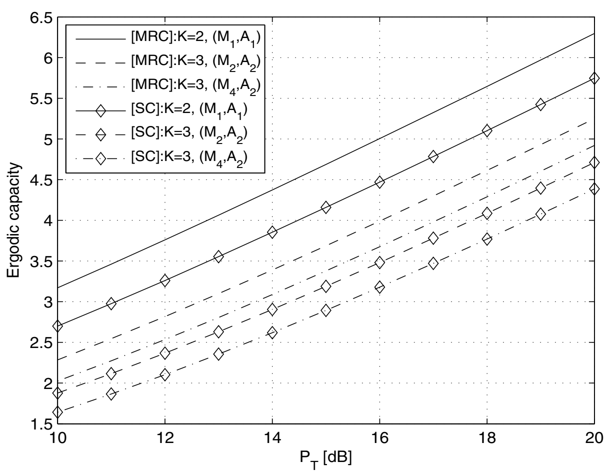

Fig. 6. Ergodic capacity of MRC and SC for various values of K with $Q = 2$ and $N_h = 2$.

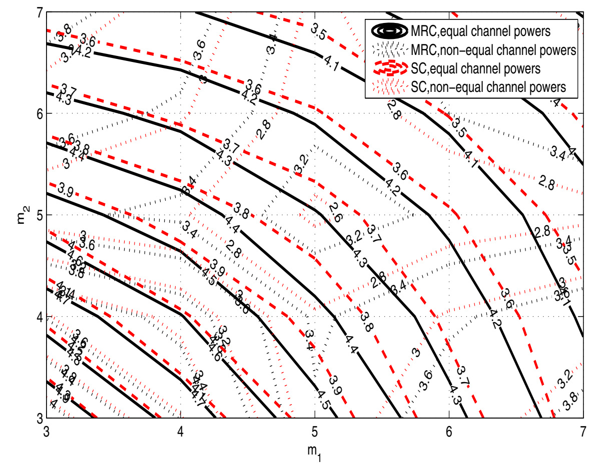

Fig. 7. Ergodic capacity of MRC and SC for various values of $(m_1, 1/\alpha_1)$ and $(m_2, 1/\alpha_2)$ with $Q = 2$ and $N_h = 3$.

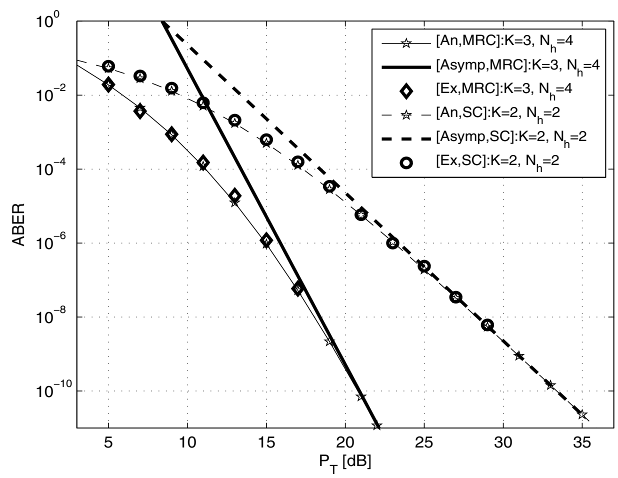

Fig. 8. ABER of MRC and SC for various values of (K, N_h) with $Q = 2$.

Fig. 6 shows that a lower ergodic capacity obtained as either K increases or fading between the SU-Tx and PU-Rx's worsens. However, we notice that the slopes of all curves are the same irrespective of K and the combining protocol. That is, the asymptotic outage diversity gain is seen to be independent of K and the combining protocol. If we measure the slope from the asymptotic curves, it is given by $G_d^{\text{outage}} = N_h Q$ with $N_h = 2$ and $Q = 2$. As $\tilde{P}_T$ increases, the exact outage probability approaches the asymptotic outage probability.

In generating Fig. 7, we calculate the exact ergodic capacity for various values of (m_1, m_2) and (α_1, α_2) with fixed $Q = 2$ and $N_h = 3$. We consider $m_1 = \{3, 4, 5, 6, 7\}$ and $m_2 = \{3, 4, 5, 6, 7\}$ with $\alpha_1 = \{1/0.8, 1/0.6, 1/0.4, 1/0.6, 1/0.8\}$ and $\alpha_2 = \{1/0.8, 1/0.6, 1/0.4, 1/0.6, 1/0.8\}$ for the non-equal channel power case and $\alpha_1 = \alpha_2 = 1.0$ for the equal channel power case. This figure shows the impacts of both the fading severity and the channel powers on the ergodic capacity. As in Figs. 5 and 6, MRC achieves a higher ergodic capacity than SC.

C. Average Bit Error Rate

To obtain the exact ABER, we employ QRD-M [34], [38]. In Fig. 8, we can see that as P_T increases, the exact ABER approaches the asymptotic ABER. If we measure the slopes from the asymptotically obtained ABER, it is given by $G_d^{\text{ABER}} = N_h Q$. That is, a larger multipath channel length is seen to have a better ABER due to the multipath diversity gain.

Fig. 9 shows the effects of the number of PU-Rx's on the ABER. This figure shows that a lower ABER for MRC is obtained for $K = 2$ and $N_h = 4$ than $K = 3$ and $N_h = 3$ due to a higher multipath diversity gain. Similarly, a lower ABER for SC is obtained for $K = 3$ and $(Q = 3, N_h = 2)$ than $K = 3$ and $(Q = 2, N_h = 2)$ due to a higher receive diversity gain. Thus, as in the outage probability, the diversity gain is determined by the product of the multipath diversity gain N_h and the receive diversity gain Q.

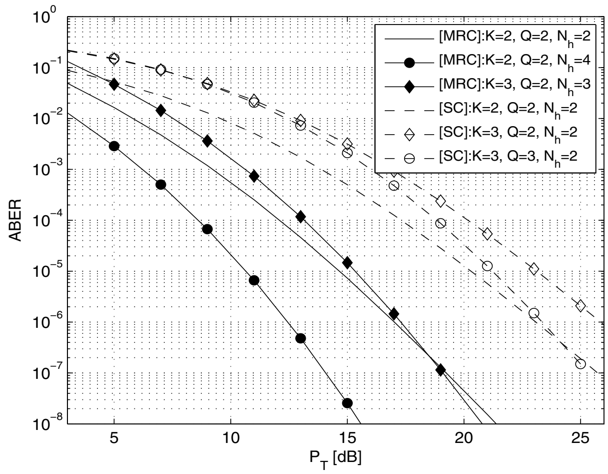

Fig. 9. ABER of MRC and SC for various values of (Q, N_h) with $K = 2$.

VI. Conclusions

We have introduced spatial diversity and proposed MRC and SC in spectrum sharing single-carrier networks. The purpose of this paper has been to showcase the joint prevalence of 1) multiple receive antennas in the secondary network, and 2) multiple users in the primary network, in the more general and complete scenario of 1) non-identical frequency selective fading between the SU-Tx and all the PU-Rx's, and 2) frequency selective fading between the SU-Tx and the SU-Rx. To facilitate this, we have derived new exact closed-form expressions for the outage, ergodic capacity, and ABER. We also derived new asymptotic closed-form expressions for the outage probability and the ABER. Our results are concise and easy-to-evaluate, and more importantly, they take into account the joint effects of the number of PUs, fading severities between the SU-Tx and all the PU-Rx's, frequency selectivity of the channels in the SU networks, the number of receiving antennas in the SU-Rx, as well as the combining protocols such as MRC and SC. From simulation and analysis, we have verified that the number of PU-Rx's and fading severities between the SU-Tx and all the PU-Rx's have no influence on the asymptotic diversity gain. We have also confirmed that the receive diversity and multipath diversity are the network parameters that determine the overall asymptotic diversity gain of MRC and SC.

Appendix A: A Detailed Derivation of Lemma 1

Let us define $X_k \stackrel{\triangle}{=} \alpha_k \|\boldsymbol{g}^k\|^2$. The CDF of X_k is $F_{X_k}(x) = \left(1 - e^{-\beta_k x} \sum_{l=0}^{m_k-1} \frac{1}{l!}(\beta_k x)^l\right) \mathrm{U}(x) = (1 - x_k)\mathrm{U}(x)$, where $x_k \stackrel{\triangle}{=} e^{-\beta_k x} \sum_{l=0}^{m_k-1} \frac{1}{l!}(\beta_k x)^l$. Due to independent fading for all links from the SU-Tx to all PU-Rx's, the CDF of $X = \max_{k=1,\cdots,K} X_k$ is given by $F_X(x) = \prod_{k=1}^{K} F_{X_k}(x)\mathrm{U}(x) = \prod_{k=1}^{K}(1-x_k)\mathrm{U}(x)$. With some manipulations, we can see that

$$\prod_{k=1}^{K}(1 - x_k) = 1 + \sum_{k=1}^{K} \frac{(-1)^k}{k!} \underbrace{\sum_{n_1=1}^{K} \cdots \sum_{n_k=1}^{K}}_{|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k| = k} \prod_{t=1}^{k} x_{n_t}. \quad \text{(A.1)}$$

Replacing x_k with its definition, we arrive at the following expression:

$$\prod_{k=1}^{K}(1 - x_k) = 1 + \sum_{k=1}^{K} \frac{(-1)^k}{k!} \underbrace{\sum_{n_1=1}^{K} \cdots \sum_{n_k=1}^{K}}_{|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k| = k} \prod_{t=1}^{k} e^{-\beta_{n_t} x} \sum_{l=0}^{m_{n_t}-1} \frac{1}{l!}(\beta_{n_t} x)^l. \quad \text{(A.2)}$$

After some simplifications, we obtain

$$\prod_{k=1}^{K}(1 - x_k) = 1 + \sum_{k=1}^{K} \frac{(-1)^k}{k!} \underbrace{\sum_{n_1=1}^{K} \cdots \sum_{n_k=1}^{K}}_{|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k| = k} \sum_{l_1=0}^{m_{n_1}-1} \cdots \sum_{l_k=0}^{m_{n_k}-1} \prod_{t=1}^{k} \left(\frac{(\beta_{n_t})^{l_t}}{l_t!}\right) x^{\sum_{t=1}^{k} l_t} e^{-(\sum_{t=1}^{k} \beta_{n_t})x} \quad \text{(A.3)}$$

which proves (11).

Appendix B: A Detailed Derivation of (12)

The CDF of γ_{MRC} is given by

$$\begin{aligned} F_{\gamma_{\mathrm{MRC}}}(\gamma) &= Pr\left(\min\left(\tilde{P}_T, \tilde{I}_p/X\right) Y < \gamma\right) \\ &= I_{\mathrm{MRC},1}(\gamma) + I_{\mathrm{MRC},2}(\gamma) \end{aligned} \quad \text{(B.1)}$$

where $I_{\mathrm{MRC},1}(\gamma) \stackrel{\triangle}{=} Pr\left(Y < \gamma/\tilde{P}_T, \tilde{I}_p > X\tilde{P}_T\right)$ and $I_{\mathrm{MRC},2}(\gamma) \stackrel{\triangle}{=} Pr\left(Y < X\gamma/\tilde{I}_p, \tilde{I}_p < X\tilde{P}_T\right)$. Since the fading between the SU-Tx and PUs is independent of the multipath fading between the SU-Tx and SU-Rx, it is easy to see that

$$I_{\mathrm{MRC},1}(\gamma) = F_Y\left(\gamma/\tilde{P}_T\right) F_X(\mu). \quad \text{(B.2)}$$

The derived expression for the defined $I_{\mathrm{MRC},2}$ is given by

$$\begin{aligned} I_{\mathrm{MRC},2}(\gamma) &= Pr\left(Y < X\gamma/\tilde{I}_p, \tilde{I}_p < X\tilde{P}_T\right) \\ &= \int_{\mu}^{\infty} F_Y\left(\gamma x/\tilde{I}_p\right) f_X(x) dx. \end{aligned} \quad \text{(B.3)}$$

Using integration by parts, (B.3) can be evaluated to the following expression:

$$I_{\mathrm{MRC},2}(\gamma) = -F_Y\left(\gamma\mu/\tilde{I}_p\right) F_X(\mu) + F_Y\left(\gamma\mu/\tilde{I}_p\right) - \frac{\left(\frac{\gamma}{\tilde{I}_p}\right)^{N_h Q}}{\Gamma(N_h Q)} \widetilde{\sum}\left[\int_{\mu}^{\infty} x^{N_h Q + \tilde{l} - 1} e^{-x(\frac{\gamma}{\tilde{I}_p} + \tilde{\alpha})}\right] dx. \quad \text{(B.4)}$$

Now collecting (B.2) and (B.4), yields (12).

Appendix C: A Detailed Derivation of (18)

Similar to derivations of the CDF of $\gamma_{\rm MRC}$, the CDF of $\gamma_{\rm SC}$ is given by

$$\begin{aligned} F_{\gamma_{\rm SC}}(\gamma) &= Pr\left(\min\left(\tilde{P}_T, \tilde{I}_p/X\right)Z < \gamma\right) \\ &= I_{\rm SC,1}(\gamma) + I_{\rm SC,2}(\gamma) \end{aligned} \tag{C.1}$$

where $I_{\rm SC,1}(\gamma) \triangleq Pr\left(Z < \gamma/\tilde{P}_T, \tilde{I}_p > X\tilde{P}_T\right)$ is given by

$$I_{\rm SC,1}(\gamma) = F_Z\left(\gamma/\tilde{P}_T\right)F_X(\mu). \tag{C.2}$$

Also, we define $I_{\rm SC,2}(\gamma) \triangleq Pr\left(Z < X\gamma/\tilde{I}_p, \tilde{I}_p < X\tilde{P}_T\right)$. Similar to the previous derivation used in (B.4), we have

$$\begin{aligned} I_{\rm SC,2}(\gamma) = &-F_Z\left(\gamma/\tilde{P}_T\right)F_X(\mu) + F_Z\left(\gamma/\tilde{P}_T\right) - \frac{Q}{\Gamma(N_h)} \\ &\widetilde{\sum}\widehat{\sum}_{k'}\left[\left(\gamma/\tilde{I}_p\right)^{N_h+\tilde{N}_h}\right. \\ &\left.\int_\mu^\infty x^{\tilde{l}+N_h+\tilde{N}_h-1}e^{-x\left(\frac{\gamma(k'+1)}{\tilde{I}_p}+\tilde{\alpha}\right)}dx\right] \end{aligned} \tag{C.3}$$

which is evaluated as

$$\begin{aligned} I_{\rm SC,2}(\gamma) = &-F_Z\left(\gamma/\tilde{P}_T\right)F_X(\mu) + F_Z\left(\gamma/\tilde{P}_T\right) - \frac{Q}{\Gamma(N_h)} \\ &\widetilde{\sum}\widehat{\sum}_{k'}\left[\left(\gamma(k'+1)/\tilde{I}_p + \tilde{\alpha}\right)^{-(N_h+\tilde{N}_h+\tilde{l})}\right. \\ &\left(\gamma/\tilde{I}_p\right)^{N_h+\tilde{N}_h} \\ &\left.\Gamma\left(N_h+\tilde{N}_h+\tilde{l}, \mu\left(\gamma(k'+1)/\tilde{I}_p+\tilde{\alpha}\right)\right)\right]. \end{aligned} \tag{C.4}$$

Collecting (C.2) and (C.4), we arrive to the following expression:

$$\begin{aligned} F_{\gamma_{\rm SC}}(\gamma) = &F_Z\left(\gamma/\tilde{P}_T\right) - \frac{Q}{\Gamma(N_h)}\widetilde{\sum}\widehat{\sum}_{k'}\left[\left(\gamma/\tilde{I}_p\right)^{N_h+\tilde{N}_h}\right. \\ &\left(\gamma(k'+1)/\tilde{I}_p+\tilde{\alpha}\right)^{-(N_h+\tilde{N}_h+\tilde{l})} \\ &\left.\Gamma\left(N_h+\tilde{N}_h+\tilde{l}, \mu\left(\gamma(k'+1)/\tilde{I}_p+\tilde{\alpha}\right)\right)\right]. \end{aligned} \tag{C.5}$$

Appendix D: A Detailed Derivation of (23)

We first compute the PDF of the RV X. Differentiating (11) with respect to x, we can find the corresponding PDF as follows:

$$f_X(\gamma) = \widetilde{\sum}\left[\tilde{l}\gamma^{\tilde{l}-1}e^{-\tilde{\beta}\gamma}{\rm U}(\gamma)\right] - \widetilde{\sum}\left[\tilde{\beta}\gamma^{\tilde{l}}e^{-\tilde{\beta}\gamma}{\rm U}(\gamma)\right]. \tag{D.1}$$

Now we can have

$$\begin{aligned} F_{\gamma_{\rm MRC}}(\gamma) &= Pr\left(\min\left(\tilde{P}_T, \tilde{I}_p/X\right)Y < \gamma\right) \\ &\overset{\tilde{P}_T\to\infty}{\approx} \hat{I}_{\rm MRC,1}(\gamma) + \hat{I}_{\rm MRC,2}(\gamma) \triangleq \hat{F}_{\gamma_{\rm MRC}}(\gamma) \end{aligned} \tag{D.2}$$

where

$$\begin{aligned} &\hat{I}_{\rm MRC,1}(\gamma) \overset{\tilde{P}_T\to\infty}{\approx} F_Y\left(\gamma/\tilde{P}_T\right)F_X(\mu) \\ &= \frac{1}{\Gamma(N_hQ+1)}\widetilde{\sum}\left[\mu^{\tilde{l}}e^{-\tilde{\beta}\mu}\left(\gamma/\tilde{P}_T\right)^{N_hQ}\right] \text{ and} \\ &\hat{I}_{\rm MRC,2}(\gamma) \overset{\tilde{P}_T\to\infty}{\approx} \int_\mu^\infty \frac{1}{\Gamma(N_hQ+1)}\left(\gamma x/\tilde{I}_p\right)^{N_hQ} \\ &\left(\widetilde{\sum}\left[\tilde{l}x^{\tilde{l}-1}e^{-\tilde{\beta}x}\right] - \widetilde{\sum}\left[\tilde{\beta}x^{\tilde{l}}e^{-\tilde{\beta}x}\right]\right)dx. \end{aligned} \tag{D.3}$$

After some manipulations, we can derive $P_{\rm MRC}^{\rm as,out} \triangleq \hat{F}_{\gamma_{\rm MRC}}(\gamma_{th})$. Similarly, we can readily derive $P_{\rm SC}^{\rm as,out}(\gamma_{th})$ from the derivation of $\hat{F}_{\gamma_{\rm SC}}(\gamma_{th})$.

Appendix E: A Detailed Derivation of (25)

To obtain the dependence on the power of γ in $\tilde{F}_{\gamma_{\rm MRC}}(\gamma)$, we use [37, Eq. (8.352.4)] the power series expansion for incomplete gamma function. That is, $\tilde{F}_{\gamma_{\rm MRC}}(\gamma)$ is given by

$$\begin{aligned} \tilde{F}_{\gamma_{\rm MRC}}(\gamma) = &\sum_{k=0}^{N_hQ-1}\frac{1}{\Gamma(k+1)}\left(\mu/\tilde{I}_p\right)^k\gamma^k e^{-(\gamma\mu/\tilde{I}_p)} + \\ &\frac{1}{\Gamma(N_hQ)}\widetilde{\sum}\left[\left(\tilde{I}_p\right)^{\tilde{l}}\Gamma(N_hQ+\tilde{l})e^{-\mu\tilde{\beta}}\right. \\ &\sum_{m=0}^{N_hQ+\tilde{l}-1}d_m\gamma^{N_hQ}\left(\gamma+\hat{I}_p\right)^{-(N_hQ+\tilde{l}-m)} \\ &\left.e^{-\mu\gamma/\tilde{I}_p}\right]. \end{aligned} \tag{E.1}$$

Using (E.1), the ergodic capacity $C_{\rm MRC}$ is given by

$$\begin{aligned} C_{\rm MRC} &= \frac{1}{\log(2)}\int_0^\infty \frac{\tilde{F}_{\gamma_{\rm MRC}(x)}}{1+x}dx \\ &= \frac{1}{\log(2)}\sum_{k=0}^{N_hQ-1}\frac{1}{\Gamma(k+1)}\left(\mu/\tilde{I}_p\right)^k \\ &\int_0^\infty \frac{\gamma^k e^{-(\gamma\mu/\tilde{I}_p)}}{1+\gamma}d\gamma + \frac{1}{\log(2)\Gamma(N_hQ)} \\ &\widetilde{\sum}\left[\left(\tilde{I}_p\right)^{\tilde{l}}\Gamma(N_hQ+\tilde{l})e^{-\mu\tilde{\beta}}\sum_{m=0}^{N_hQ+\tilde{l}-1}d_m\mathcal{I}_1\right] \end{aligned} \tag{E.2}$$

where $\mathcal{I}_1 \triangleq \int_0^\infty \frac{\gamma^{N_hQ}e^{-\gamma\mu/\tilde{I}_p}}{(1+\gamma)\left(\gamma+\hat{I}_p\right)^{N_hQ+\tilde{l}-m}}d\gamma$. To compute $\mathcal{I}_1$, we apply the partial fraction (PF) to $\frac{1}{(1+\gamma)\left(\gamma+\hat{I}_p\right)^{N_hQ+\tilde{l}-m}}$, so that we have

$$\frac{(1+\gamma)^{-1}}{\left(\gamma+\hat{I}_p\right)^{N_hQ+\tilde{l}-m}} = \frac{c_2}{(1+\gamma)} + \sum_{l=1}^{N_hQ+\tilde{l}-m}\frac{c_{3,l}}{(\gamma+\hat{I}_p)^l}. \tag{E.3}$$

Recall that $c_2 \triangleq \frac{1}{(\hat{I}_p-1)^{N_hQ+\tilde{l}-m}}$, and $c_{3,l} \triangleq \frac{(-1)}{(\hat{I}_p-1)^{N_hQ+\tilde{l}-m-l+1}}$. Having applied (E.3) and [37, Eq. 9.211.4] to $\mathcal{I}_1$, it is evaluated to the following form:

$$\begin{aligned} \mathcal{I}_1 = &c_2\int_0^\infty \frac{\gamma^{N_hQ}e^{-\gamma\mu/\tilde{I}_p}}{(1+\gamma)}d\gamma + \\ &\sum_{l=1}^{N_hQ+\tilde{l}-m}c_{3,l}\int_0^\infty \frac{\gamma^{N_hQ}e^{-x\mu/\tilde{I}_p}}{\left(\gamma+\hat{I}_p\right)^l}d\gamma \\ &= c_2\Gamma(N_hQ+1)\mathbb{U}\left(N_hQ+1, N_hQ+1; \mu/\tilde{I}_p\right) + \\ &\sum_{l=1}^{N_hQ+\tilde{l}-m}c_{3,l}\left(\hat{I}_p\right)^{N_hQ+1-l} \\ &\Gamma(N_hQ+1)\mathbb{U}\left(N_hQ+1, N_hQ+2-l; \tilde{\beta}\mu\right). \end{aligned} \tag{E.4}$$

After some final manipulations, we have

$$C_{\rm MRC} = \frac{1}{\log(2)} \sum_{k=0}^{N_hQ-1} (\mu/\tilde{I}_p)^k \mathbb{U}\left(k+1, k+1; \mu/\tilde{I}_p\right) + \frac{1}{\log(2)\Gamma(N_hQ)} \widetilde{\sum}\left[\left(\tilde{I}_p\right)^{\tilde{l}} \Gamma(N_hQ+\tilde{l}) e^{-\mu\tilde{\beta}} \sum_{m=0}^{N_hQ+\tilde{l}-1} d_m\big[c_2\Gamma(N_hQ+1)\mathbb{U}\big(N_hQ+1, N_hQ+1; \mu/\tilde{I}_p\big) + \sum_{l=1}^{N_hQ+\tilde{l}-m} c_{3,l}\big(\hat{I}_p\big)^{N_hQ+1-l} \Gamma(N_hQ+1)\mathbb{U}\big(N_hQ+1, N_hQ+2-l; \tilde{\beta}\mu\big)\big]\right] \quad \text{(E.5)}$$

which proves (25). For SC, we have an alternative form $\tilde{F}_{\gamma_{\rm SC}}(\gamma)$, given by

$$\tilde{F}_{\gamma_{\rm SC}}(\gamma) \stackrel{\triangle}{=} -\widehat{\widehat{\sum}}_{k'}\left[\left(\mu\gamma/\tilde{I}_p\right)^{\tilde{N}_h} e^{-\left(\mu\gamma/\tilde{I}_p\right)k'}\right] + \frac{Q}{\Gamma(N_h)} \widetilde{\sum}\widehat{\sum}_{k'}\left[\left(\tilde{I}_p\right)^{\tilde{l}} (k'+1)^{-(N_h+\tilde{N}_h+\tilde{l})} \Gamma(N_h+\tilde{N}_h+\tilde{l}) e^{-\mu\tilde{\beta}} \sum_{m=0}^{N_h+\tilde{N}_h+\tilde{l}-1} e_m \gamma^{N_h+\tilde{N}_h} \left(\gamma+\frac{\hat{\beta}}{k'+1}\right)^{-(N_h+\tilde{N}_h+\tilde{l}-m)} e^{-\gamma\left(\frac{\mu(k'+1)}{\tilde{I}_p}\right)}\right]. \quad \text{(E.6)}$$

Thus, the ergodic capacity of SC is evaluated as follows:

$$C_{\rm SC} = -\frac{1}{\log(2)} \widehat{\widehat{\sum}}_{k'}\left[(\mu/\tilde{I}_p)^{\tilde{N}_h} \mathcal{I}_2\right] + \frac{Q}{\Gamma(N_h)} \widetilde{\sum}\widehat{\sum}_{k'}\left[\left(\tilde{I}_p\right)^{\tilde{l}} (k'+1)^{-(N_h+\tilde{N}_h+\tilde{l})} \Gamma(N_h+\tilde{N}_h+\tilde{l}) e^{-\mu\tilde{\beta}} \sum_{m=0}^{N_h+\tilde{N}_h+\tilde{l}-1} e_m \mathcal{I}_3\right] \quad \text{(E.7)}$$

where $\mathcal{I}_2 \stackrel{\triangle}{=} \int_0^\infty e^{-\mu k'\gamma/\tilde{I}_p} \gamma^{\tilde{N}_h} (1+\gamma)^{-1} d\gamma$ and $\mathcal{I}_3 \stackrel{\triangle}{=} \int_0^\infty \gamma^{N_h+\tilde{N}_h} \left(\gamma+\frac{\hat{I}_p}{k'+1}\right)^{-(N_h+\tilde{N}_h+\tilde{l}-m)} (1+\gamma)^{-1} e^{-\gamma\left(\frac{\mu(k'+1)}{\tilde{I}_p}\right)} d\gamma$. Again using the PF and [37, Eq. 9.211.4], $\mathcal{I}_2$ becomes

$$\mathcal{I}_2 = \Gamma(\tilde{N}_h+1)\mathbb{U}(N_h+\tilde{N}_h+1, \tilde{N}_h+1; \mu k'/\tilde{I}_p) \quad \text{(E.8)}$$

and

$$\mathcal{I}_3 = c_4\Gamma(N_h+\tilde{N}_h+1) \mathbb{U}\left(N_h+\tilde{N}_h+1, N_h+\tilde{N}_h+1; \mu(k'+1)/\tilde{I}_p\right) + \sum_{l=1}^{N_h+\tilde{N}_h+\tilde{l}-m} c_{5,l}\left(\frac{\hat{I}_p}{k'+1}\right)^{N_h+\tilde{N}_h+1-l} \Gamma(N_h+\tilde{N}_h+1) \mathbb{U}(N_h+\tilde{N}_h+1, N_h+\tilde{N}_h+2-l; \mu\tilde{\beta}). \quad \text{(E.9)}$$

Using (E.8) and (E.9), we obtain (25).

Appendix F: A Detailed Derivation of (27)

We again use (E.1) and (E.6) to obtain the dependence on the power of γ in $F_{\gamma_{\rm MRC}}(\gamma)$ and $F_{\gamma_{\rm SC}}(\gamma)$. Thus, we have

$$F_{\gamma_{\rm MRC}}(\gamma) = 1 - \sum_{k=0}^{N_hQ-1} \frac{1}{\Gamma(k+1)} \left(\mu/\tilde{I}_p\right)^k \gamma^k e^{-(\gamma\mu/\tilde{I}_p)} - \frac{1}{\Gamma(N_hQ)} \widetilde{\sum}\left[\left(\tilde{I}_p\right)^{\tilde{l}} \Gamma(N_hQ+\tilde{l}) e^{-\mu\tilde{\alpha}} \sum_{m=0}^{N_hQ+\tilde{l}-1} d_m \gamma^{N_hQ} \left(\gamma+\hat{I}_p\right)^{-(N_hQ+\tilde{l}-m)} e^{-\mu x/\tilde{I}_p}\right] \text{ and}$$

$$F_{\gamma_{\rm SC}}(\gamma) \stackrel{\triangle}{=} 1 + \widehat{\widehat{\sum}}_{k'}\left[\left(\mu\gamma/\tilde{I}_p\right)^{\tilde{N}_h} e^{-\left(\mu\gamma/\tilde{I}_p\right)k'}\right] - \frac{Q}{\Gamma(N_h)} \widetilde{\sum}\widehat{\sum}_{k'}\left[\left(\tilde{I}_p\right)^{\tilde{l}} (k'+1)^{-(N_h+\tilde{N}_h+\tilde{l})} \Gamma(N_h+\tilde{N}_h+\tilde{l}) e^{-\mu\tilde{\beta}} \sum_{m=0}^{N_h+\tilde{N}_h+\tilde{l}-1} e_m \gamma^{N_h+\tilde{N}_h} \left(\gamma+\frac{\hat{I}_p}{k'+1}\right)^{-(N_h+\tilde{N}_h+\tilde{l}-m)} e^{-\gamma\left(\frac{\mu(k'+1)}{\tilde{I}_p}\right)}\right]. \quad \text{(F.1)}$$

Having applied [37, Eqs. (3.351.3) and (9.211.4)] into (F.1), we readily obtain (27).

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