Spectrum Sharing with Distributed Interference Compensation

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Abstract— **We consider a spectrum sharing problem in which each wireless transmitter can select a single channel from a set of available channels, along with the transmission power. An Asynchronous Distributed Pricing (ADP) scheme is proposed, in which users exchange "price" signals, that indicate the negative effect of interference at the receivers. Given this set of prices, each transmitter chooses a channel and power level to maximize its net benefit (utility minus cost). We show that a sequential version of this Single-Channel (SC)-ADP algorithm converges with two users and an arbitrary number of channels, and observe via simulation that it exhibits rapid convergence with more users in the network. The pricing algorithm always outperforms the heuristic algorithm in which each user picks the best channel without exchanging interference prices. In a dense network with heavy interference, the SC-ADP algorithm can also perform better than the iterative water-filling algorithm where each user transmits over multiple channels but the users do not exchange any information. The performance of the SC-ADP algorithm is also compared with a Multi-Channel (MC)-ADP algorithm in which users can transmit over multiple channels and exchange interference prices over each channel.**

I. Introduction

A major challenge to realizing the potential benefits of dynamic spectrum sharing is interference management. Namely, transmit powers must be carefully controlled to limit the interference to neighboring receivers sharing the same spectrum. This can be difficult in distributed networks where there is little or no central control over the allocation of wireless resources (i.e., power and bandwidth) across nodes.

We consider a spectrum sharing scenario in which each wireless transmitter can choose from among several available channels. These channels could represent different commercial bands, which are offered on secondary markets, or spectrum owned by government agencies (such as public safety or broadcast television), which are made available to other service providers, provided that constraints on interference to incumbent users are satisfied [1]. The channels could also represent smaller sub-bands contained within those larger bands. Each transmitter must therefore decide on which channel to use, and with how much power to transmit. This decision should depend not only on the Signal-to-Interference Plus Noise Ratio (SINR) in each band, but also on the degradation in service experienced by other users due to the interference associated with the transmission.

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To mitigate the effects of interference externalities in a distributed network, it is clearly beneficial for the wireless nodes to exchange information that reflects interference levels on each available channel at different locations. Here we focus on a particular scheme in which the wireless nodes exchange interference "prices", namely, the marginal loss in utility (assumed to be a known concave, increasing function of the received SINR) due to a marginal increase in interference. Each user then determines the transmitted power by maximizing the received utility minus the total cost of the associated interference. Because the users can update prices and powers asynchronously, this algorithm has been called an Asynchronous Distributed Pricing (ADP) algorithm for power control [2], [3]. In prior work, we have characterized the convergence of the ADP algorithm in an ad hoc, peer-to-peer network [3], and have studied its performance with limited information exchange [4].

Here we study the performance of ADP in the spectrum sharing scenario where each transmitter is constrained to choose a *single* channel from among a set of available channels. This is in contrast to the multi-channel model considered in [3], in which each user optimizes a power *distribution* over the entire set of available channels. As in [3], we assume a distributed peer-to-peer wireless network in which multiple transmitter-receiver pairs are distributed over a geographic area. It has been shown in [3] that when users spread power over all available channels, the ADP updates converge to a set of globally optimal power distributions across channels and users for a class of utility functions (i.e., they must be sufficiently concave).

The constraint that each transmitter choose only one channel leads to an optimization problem with integer constraints, which complicates the analysis. (This, of course, also applies to previous studies of dynamic channel allocation, e.g., see [5].) Consequently, we are unable to prove an analogous convergence result for an arbitrary number of users. For two users, we show that the Single-Channel (SC)-ADP algorithm converges with sequential updates across users given certain constraints on the utility functions. We also show simulation results that compare the performance of the SC-ADP algorithm with other distributed power control schemes. Those other schemes include selecting the channel with the best channel gain, iteratively selecting the channel with the best SINR, and iterative water filling [6], which do not require any information exchange. We also compare the performance of the SC-ADP algorithm with the Multi-Channel (MC)-ADP scheme proposed in [3].

Related work on dynamic channel allocation for cellular systems is reviewed in [7]. A summary of various integer/combinatorial optimization approaches and heuristic algorithms are described in [5]. Recently, channel selection has received attention in discussions related to the IEEE 802.11 protocol (see [8] and the references therein), where there are several non-overlapping available channels. Our work differs in that we consider peer-to-peer users which can exchange limited information, and we adopt a utility objective, which can account for different Qualify of Service (QoS) requirements.

Related work on power control in CDMA cellular and ad hoc networks includes [9]–[13]. In most prior work on ad hoc networks, a transmission is assumed to be successful if a fixed minimum SINR requirement is met. This is true for fixed-rate communications. However, this is not the case for "elastic" data applications, which can adapt transmission rates. In this paper, we focus on rate-adaptive users, where the goal of power control is to maximize total network performance instead of guarantee interference margins for each user.

II. System Model

We consider a set of $\mathcal{K} = \{1, \ldots, K\}$ spectrum agile users that seek to share a set of $\mathcal{M} = \{1, \cdots, M\}$ available channels (open spectrum bands). Each user corresponds to a distinct pair of nodes: one dedicated transmitter and one dedicated receiver. Our main focus is on the case where each user is constrained to transmit over at most *one* spectrum band; this could be due to policy and/or technical limitations. For simplicity, every spectrum band is modeled as having the same bandwidth and the same background noise power of n_0. Over the time-period of interest, we assume that the channel gains are fixed and that the users want to transmit continually. For channel m, the gain between user k's transmitter and user j's receiver is denoted by h_{kj}^m.[1] An example of a network with four users (pairs of nodes) is shown in Fig. 1. For simplicity, we only show the channel gains for one channel and suppress the channel superscripts.

Let $\varphi(k) \in \mathcal{M}$ denote the spectrum band selected by user k. In addition to selecting a band, each user can determine its transmission power $p_k^{\varphi(k)}$ within the band. This transmission power must lie be in a feasible set $\mathcal{P}_k^{\varphi(k)} = [\check{P}_k^{\varphi(k)}, \hat{P}_k^{\varphi(k)}]$, with $0 \leq \check{P}_k^{\varphi(k)} \leq \hat{P}_k^{\varphi}(k)$. The power constraints may vary with the selected band, for example to model different regulatory constraints. Note that a special case is when $\check{P}_k^{\varphi(k)} = \hat{P}_k^{\varphi(k)}$, in which case a user always transmits with maximum power on its selected band. We consider a spread spectrum system, where this power is spread over the entire band and interference from other users in the same band is treated as noise. Each user k's QoS is characterized by a utility function $u_k\left(\gamma_k^{\varphi(k)}\right)$, which is an increasing and strictly concave function of the received SINR on the chosen channel. The SINR of user k on channel $m \in \mathcal{M}$ is

$$\gamma_k^m\left(\boldsymbol{p}^m\right) = \frac{p_k^m h_{kk}^m}{n_0 + \sum_{j \neq k} p_j^m h_{jk}^m}, \tag{1}$$

where $\boldsymbol{p}^m = (p_k^m, k \in \mathcal{K})$ is a vector of the users' transmission powers on channel m.[2] One example utility function we will use is $u_k\left(\gamma_k^{\varphi(k)}\right) = \theta_k \log\left(1 + \gamma_k^{\varphi(k)}\right)$, which is proportional to the Shannon capacity of user k's channel weighted by a user dependent priority parameter, θ_k.

From a network perspective, our objective is to determine each user's channel selection and power allocation to maximize the total utility summed over all users, i.e.,

$$\max_{\left\{\varphi(k), p_k^{\varphi(k)}\right\}} u_{tot}\left(\boldsymbol{p}\right) = \sum_{k=1}^{K} u_k\left(\gamma_k^{\varphi(k)}\left(\boldsymbol{p}^{\varphi(k)}\right)\right). \tag{P1}$$

This is an integer and possibly non-convex optimization problem, which is typically difficult to solve. Moreover, in a spectrum sharing environment it may not be feasible for a single entity to acquire the global information needed to solve this problem. Next we present the SC-ADP algorithm, a simple, distributed heuristic algorithm that attempts to solve Problem P1.

III. Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm

In the SC-ADP algorithm each user $k \in \mathcal{K}$ communicates the negative externality due to interference by announcing an "interference price", $\pi_k^{\varphi(k)}$ for the channel $\varphi(k)$ on which it is currently transmitting. This price is given by

$$\pi_k^{\varphi(k)} = \left|\frac{\partial u_k\left(\gamma_k^{\varphi(k)}\left(\boldsymbol{p}^{\varphi(k)}\right)\right)}{\partial\left(\sum_{j \neq k} p_j^{\varphi(k)} h_{jk}^{\varphi(k)}\right)}\right|, \tag{2}$$

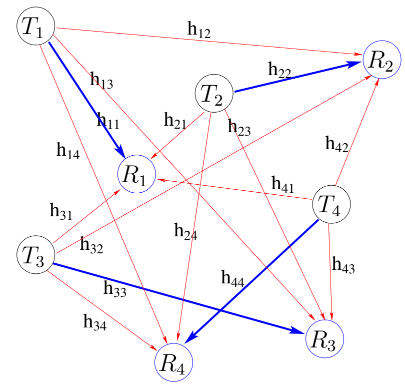

Fig. 1. Ad hoc network with four users (pairs of nodes). T_k and R_k denote the transmitter and receiver for user k, respectively. (Channel index m is suppressed for simplicity.)

[1] Note that in general $h_{kj}^m \neq h_{jk}^m$, since the latter represents the gain on channel m between user j's transmitter and user k's receiver.

[2] We assume that any reduction in interference due to bandwidth spreading is incorporated in the cross channel gains h_{jk}^m.

which reflects the marginal increase of user k's utility if its received interference is decreased by one unit. Based on the current interference prices and the current level of interference, each user $k \in \mathcal{K}$ selects a channel $\varphi(k)$ and a feasible power allocation $p_k^{\varphi(k)} \in \mathcal{P}_k^{\varphi(k)}$ that maximizes its surplus

$$\begin{aligned} & s_i\left(\varphi(k), p_k^{\varphi(k)}, p_{-k}^{\varphi(k)}, \pi_{-k}^{\varphi(k)}\right) \\ = \; & u_k\left(\gamma_k^{\varphi(k)}\left(\boldsymbol{p}^{\varphi(k)}\right)\right) - p_k^{\varphi(k)} \sum_{j \neq k} \pi_j^{\varphi(k)} h_{kj}^{\varphi(k)}. \end{aligned} \quad (3)$$

Here $p_{-k}^{\varphi(k)} = \left(p_j^{\varphi(k)}, j \in \mathcal{K} \text{ and } j \neq k\right)$ denotes the vector of powers of every user except user k in channel $\varphi(k)$; $\pi_{-k}^{\varphi(k)}$ is similarly defined. The algorithm progresses by having each user update its price announcement and channel/power allocation according to these rules. In general these updates can be asynchronous across users. For each $k \in \mathcal{K}$, let $\mathcal{T}_k$ be an unbounded set of positive time instances at which user k updates its price and channel/power allocation. The updates at these time instances are specified in Algorithm 1.

Algorithm 1 The SC-ADP Algorithm

1) *Initialization*: For each user $k \in \mathcal{K}$, select an initial channel $\varphi(k) \in \mathcal{M}$ and an initial power allocation $p_k^{\varphi(k)} \in \mathcal{P}_k^{\varphi(k)}$.
2) At each $t \in \mathcal{T}_k$, user k
   - 2.a) Selects $\varphi(k) \in \mathcal{M}$ and $p_k^{\varphi}(k) \in \mathcal{P}_k^{\varphi(k)}$ to maximize its surplus in (3),
   - 2.b) Announces price $\pi_k^{\varphi(k)}$ according to (2).

In [2], we considered a special case of this model where there is only one channel available (i.e., $M = 1$, and $\varphi(k) = 1$ for all $k \in \mathcal{K}$) and for all k, $\check{P}_k^1 < \hat{P}_k^1$. In that case, when each user's utility functions satisfies certain technical conditions, the SC-ADP algorithm is shown to globally converge to the optimal (i.e. utility maximizing) solution under arbitrary asynchronous updates. With multiple channels the convergence result from [2] no longer holds and in general, the SC-ADP may not converge to the optimal solution to Problem P1. In the next section, we establish the convergence of the SC-ADP when there are only two users present and the updates are performed sequentially.

IV. Convergence of SC-ADP Algorithm

In this section, we consider the convergence of the SC-ADP algorithm for a two-user, M-channel system, with $M > 1$. We also restrict ourselves to the case where the users update their channel selection/power allocation and prices *sequentially*, i.e. if $t \in \mathcal{T}_1$ then $t \notin \mathcal{T}_2$. Also, for $K = 2$ users, without loss of generality we further assume that these updates are performed in a round-robin order. We also assume that the users initialize sequentially by choosing the best non-empty channel and allocating the maximum power to this channel. Clearly, if both users prefer a different channel when no other users are present, then the algorithm will be at a fixed point after this initialization phase. Furthermore, this fixed point will be optimal. If both users prefer the same channel, then we can show convergence when the utility functions satisfy certain restrictions.

Let $\gamma_k^{\min} = \min\{\gamma_k^m(\boldsymbol{p}^m) : p_j^m \in \mathcal{P}_j^m, \forall j \in \mathcal{K}, \forall m \in \mathcal{M}\}$ and $\gamma_k^{\max} = \max\{\gamma_k^m(\boldsymbol{p}^m) : p_j^m \in \mathcal{P}_j^m, \forall j \in \mathcal{K}, \forall m \in \mathcal{M}\}$ for all $k \in \mathcal{K}$. Also, define $G_k(\gamma_k) = -\gamma_k u_k''(\gamma_k)/u_k'(\gamma_k)$. Then, an increasing and strictly concave utility function $u_k(\gamma_k)$ is defined to be

- *Type I* if $G_k(\gamma_k) \in [1, 2]$, $\forall \gamma_k \in \left[\gamma_k^{\min}, \gamma_k^{\max}\right]$;
- *Type II* if $G_k(\gamma_k) \in (0, 1]$, $\forall \gamma_k \in \left[\gamma_k^{\min}, \gamma_k^{\max}\right]$.

The term $G_k(\gamma_k)$ is called the *coefficient of relative risk aversion* in economics [14] and measures the relative concaveness of $u_k(\gamma_k)$ (a larger value indicates a "more concave" function). Many common utility functions are either Type I or Type II. Examples of Type I utility functions include $\theta_k \log(\gamma_k)$ and $\theta_k \gamma_k^\alpha/\alpha$ (with $\alpha \in [-1, 0)$). Examples of Type II utility functions include $\theta_k \log(\gamma_k)$, $\theta_k \log(1 + \gamma_k)$, $1 - e^{-\theta_k \gamma_k}$ (with $\theta_k < n_0/(P_k^{\max} h_{kk})$) and $\theta_k \gamma_k^\alpha/\alpha$ (with $\alpha \in (0, 1]$).

Proposition 1: For a two-user M-channel system with $M > 1$, the SC-ADP algorithm with sequential updates converges in the following two cases:

a.) both users have Type II utility functions and $0 \leq \check{P}_k^m < \hat{P}_k^m$ for all m and k;

b.) both users have either a Type I or Type II utility function, and $0 < \check{P}_k^m = \hat{P}_k^m$ for all m and k.

The proof of this is omitted due to space constraints. The basic idea is to show in both cases that if one user switches to the channel occupied by the other user, then it will never switch out of that channel. The user already occupying the channel may switch channel in the next time-step; after that it can be shown that the algorithm must have reached a fixed point.

V. Numerical Results

In this section, we study the convergence and performance of the SC-ADP algorithm through some numerical examples. Throughout the section, we assume that each user k has a utility function $u_k = \log\left(1 + \gamma_k^{\varphi(k)}\right)$, i.e., maximizing utility is equivalent to choosing a channel to maximize the user's achievable rate (in bits per channel use). Also, the noise level $n_0 = 10^{-2}$ and the feasible power set $\mathcal{P}_k^m = [0, 1]$, for each user k and channel m. As in Section IV, we consider sequential updates and assume that the users initialize sequentially by choosing the best un-occupied channel. When $K > M$, after all the channels are occupied, we initially assign each remaining user k to the channel with the largest value of h_{kk}^m.

Figure 2 shows results for a network with five users and two channels. The transmitters and receivers are uniformly placed in a 3m $\times$ 3m area. Figures 2(a) and 2(b) show the magnitudes of the channel gains across the two users, selected as $h_{kj}^m = d_{kj}^{-4} \alpha_{kj}^m$, where d_{kj} is the distance between transmitter k and receiver j, and the α_{kj}^m's are independent,

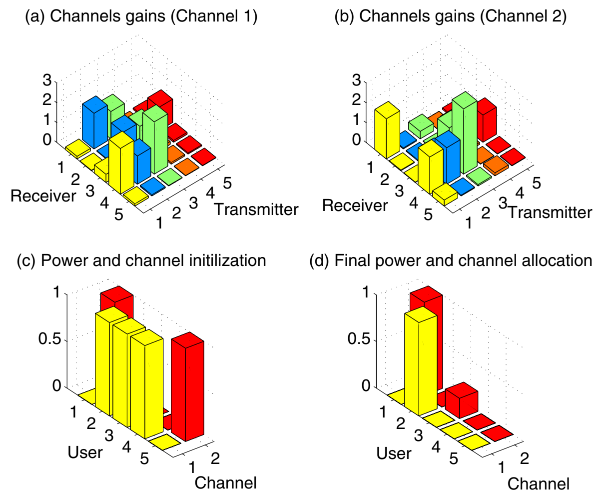

Fig. 2. Channel selections and power allocations achieved by SC-ADP in a network with five users, two channel.

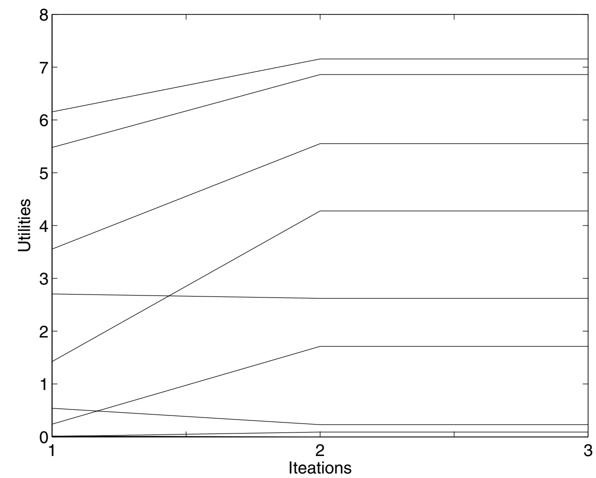

Fig. 3. Convergence of the SC-ADP algorithm for ten randomly placed users, and four channels.

unit-mean exponential random variables that model frequency-selective fading across channels. Figures 2(c) and 2(d) show the users' channel selections and the magnitude of the transmit powers. Figure 2(c) shows the channel selections after the initialization phase of the SC-ADP algorithm. Namely, user 1 selects channel 2 since $h_{11}^1 < h_{11}^2$, user 2 selects channel 1, since that is the only vacant channel, and users 3, 4 and 5 each choose the channel $\varphi(k) = \arg\max_{m\in\{1,2\}} h_{kk}^m$. All users transmit at maximum power $p_k^{\varphi(k)} = 1$ after initialization. Figure 2(d) shows the power allocation given by the SC-ADP algorithm after it converges in 3 iterations. (Each iteration is equivalent to one round of channel and power updates across all users.) Here users 1 and 3 share channel 2 (user 3 transmits with low power to mitigate the interference to user 1), user 2 transmits in channel 1, and neither user 4 nor 5 transmit due to the large interference prices announced by the active users.

Figure 3 shows plots of utilities versus iterations for different users with the SC-ADP algorithm, assuming specific (randomly chosen) channel realizations. For this example, there are 10 transmitters and 10 receivers randomly and uniformly placed in the 3m × 3m area. The total number of channels is 4, and the rest of the parameters are the same as in Figure 2. Most (but not all) of the users' utilities increase with the number of iterations. The SC-ADP algorithm again converges in 3 iterations in this case. Although convergence is not guaranteed in general, the fast convergence seen in Figure 3 is typical even when the number of users is large (i.e., > 50), and the number of channels is relatively small.[3]

Next we compare the performance of the SC-ADP algorithm with the following algorithms:

1) *Multi-channel Asynchronous Distributed Pricing (MC-ADP)* [3]: Each user k distributes power across all of the available M channels to maximize the surplus
$$\sum_{m=1}^{M} \log\left(1+\gamma_k^m\right) - \sum_{m=1}^{M} p_k^m \sum_{j\neq k} \pi_j^m h_{kj}^m,$$
subject to a total power constraint $\sum_{m=1}^{M} p_k^m \le P_i^{\max}$.

2) *Iterative Water-filling (IWF)* [6]: Each user k allocates power across channels to maximize
$$u_k = \sum_{m=1}^{M} \log\left(1+\gamma_k^m\right),$$
subject to a total power constraint $\sum_{m=1}^{M} p_k^m \le P_i^{\max}$. No information is exchanged among users, and the power allocation across channels for each user is determined by water-filling regarding the interference as noice.

3) *Best SINR*: Each user k transmits with full power $p_k^{\varphi(k)} = P_k^{\max}$ in a single channel m, which yields the largest SINR, i.e.,
$$\varphi(k) = \arg\max_{m\in\mathcal{M}} \frac{h_{kk}^m}{n_0 + \sum_{j\neq k} p_j^m h_{jk}^m}.$$

4) *Best Channel*: Each user k transmits with full power $p_k^{\varphi(k)} = P_k^{\max}$ in a single channel m, which has the largest channel gain, i.e.,
$$\varphi(k) = \arg\max_{m\in\mathcal{M}} h_{kk}^m.$$

In addition, we consider two versions of the SC-ADP algorithm, in which each user k can either choose any power in the interval $\mathcal{P}_k = [0, P_k^{\max}]$, or maximum power $\mathcal{P}_k = \{P_k^{\max}\}$. These power constraints are the same for each channel m. All algorithms except the Best Channel algorithm are iterative. That is, users sequentially update their channel selections, and power levels and prices (when part of the algorithm) until either the algorithm converges, or a total of 50 sequential iterations have been executed.

[3]The convergence is easy to achieve in a network with a large number of channels, where users could simply choose to transmit on different channels to avoid interferences.

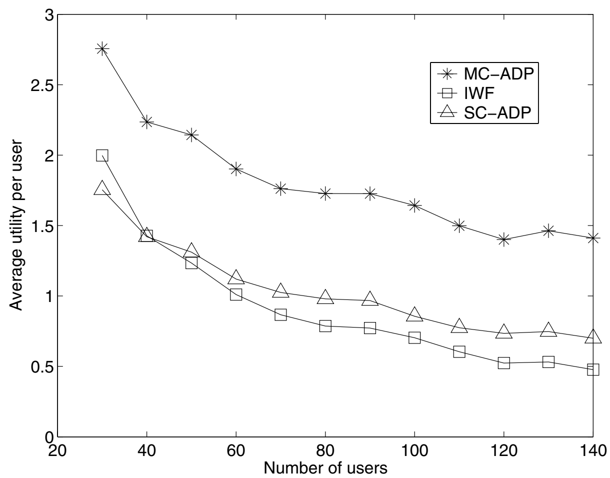

Fig. 4. Average utility versus number of users for the MC-ADP, IWF and SC-ADP algorithms.

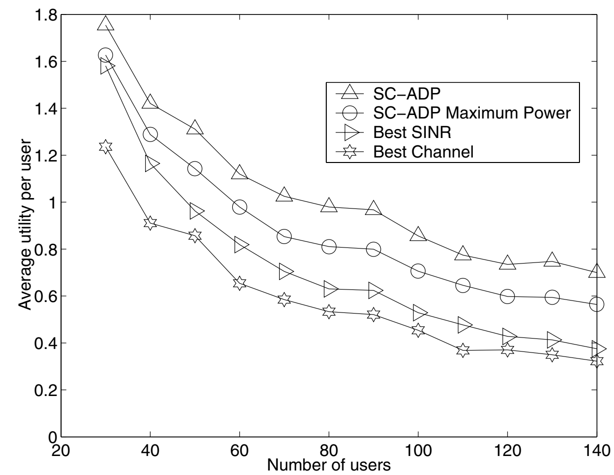

Fig. 5. Average utility versus number of users for the SC-ADP (continuous power control and maximum power), Best SINR and Best Channel algorithms.

For the results that follow, the transmitters are uniformly and randomly distributed in a 10m×10m square area, and each receiver is randomly placed within a 6m×6m square centered around the corresponding transmitter. Each simulation point is an average over 20 random network topology and channel realizations.

Figures 4 and 5 show average utility per user versus the number of users in the network with 4 channels. As the number of users increases, the interference increases, and the average utility per user decreases. Figure 4 shows that the MC-ADP algorithm achieves a significantly higher utility than the other algorithms, since it takes into account the interference prices, and has the flexibility of allocating power across multiple channels. The SC-ADP algorithm outperforms IWF in a dense network (i.e., more than 40 users), where the interference prices help to mitigate the effects of interference. Figure 5 shows that the SC-ADP with continuous power control achieves significantly more utility than with only maximum power, which achieves significantly more utility than the Best SINR algorithm. Of course, the Best Channel algorithm performs the worst since interference is not taken into account.

Figures 6 and 7 show average utility versus the number of channels in the network with 140 users in the network. Figure 6 shows that the SC-ADP outperforms IWF with a small number of channels, where the interference is relatively large, and that the gain due to the exchange of interference information (as in SC-ADP) outweighs the flexibility of transmitting over multiple channels (as in IWF). As the number of channels increase, MC-ADP achieves much higher utility than SC-ADP, due to the former's ability to exploit the presence of multiple good channels. Figure 7 shows that SC-ADP achieves a utility level that is more than twice that achieved by Best SINR when there are only two channels, and the performance gain is about 40% when there are 10 channels available.

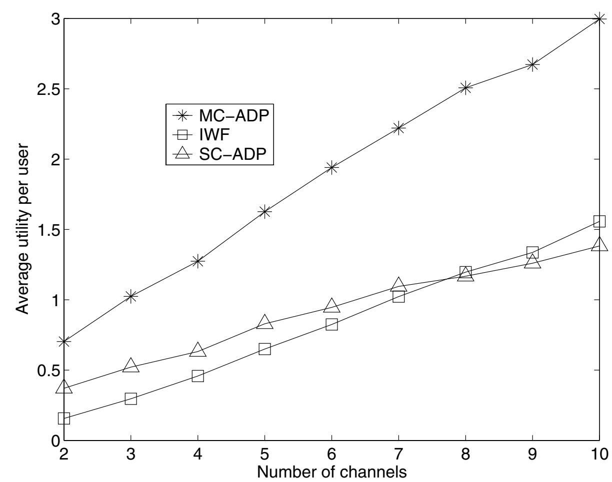

Fig. 6. Average utility versus number of channels for the MC-ADP, IWF and SC-ADP algorithms.

VI. CONCLUSIONS

We have studied the performance of a distributed channel selection and power control scheme for spectrum sharing. The scheme relies on the exchange of interference prices, which reflect the negative externality due to interference. We have proved the convergence of the SC-ADP algorithm with two users, and have observed from simulations that it converges rapidly with many more users, corresponding to a dense network. Our numerical results show that the SC-ADP algorithm can offer a significant increase in total rate relative to the algorithm in which each user picks the best channel without exchanging interference prices. Further numerical results show that the relative difference decreases with the total channel number (although the utility itself increases). In a dense network with heavy interference, the SC-ADP algorithm performs better than IWF, which allows users to spread power across all channels, but does not directly

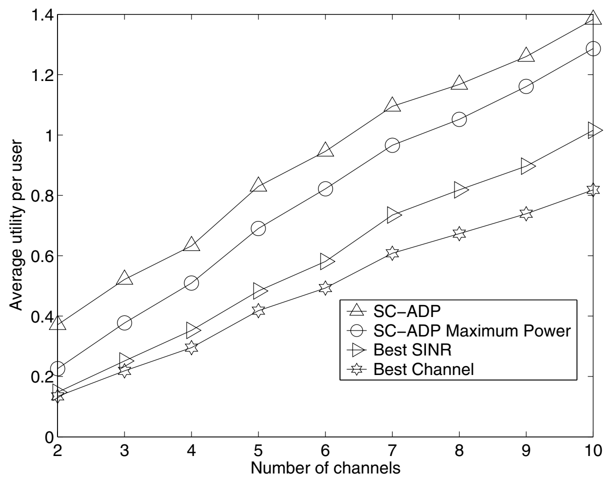

Fig. 7. Average utility versus number of channels for the SC-ADP (continuous power control and maximum power), Best SINR and Best Channel algorithms.

account for interference externalities. The efficiency loss of SC-ADP compared with MC-ADP can be substantial with a large number of channels, but diminishes as the number of channels decreases.

Although our results assume a static network with stationary channel gains, the SC-ADP algorithm can be applied to a dynamic spectrum sharing scenario, provided that the exchange of prices occurs on a slower time scale than the variations in interference. In that case, computation of the interference prices may be based on time-averaged interference. Assessing the benefits of exchanging interference prices in a dynamic spectrum sharing scenario with time-varying traffic remains a topic for further study.

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