Geophys. J. R. astr. Soc. (1975), **42**, 131–174.

Speculations on the Thermal and Tectonic History of the Earth

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(Received 1974 November 25)*

Summary

The connection between the Earth's thermal history and convection in the mantle is exploited to elucidate the early evolution of the Earth. It appears probable that convection extending over almost all of the mantle has dominated vertical heat transport throughout the whole of the Earth's history. Only in boundary layers at the surface and at a depth of 650–700 km is conduction likely to be important. The resulting evolution appears to be consistent with geological observations on early Precambrian rocks.

Various arguments are put forward in favour of two horizontal scales of convective flow in the mantle at depths less than 650 km. The large scale flow is related to the motion of major plates, and must be ordered over distances of more than 5000 km. Its evolution and energetics are discussed and there are no obvious problems in maintaining the proposed convective motions. Small scale flow with an extent of the order of 500 km appears necessary both to explain the heat flow through older parts of the Earth's surface and to reconcile the geophysical observations with the results of numerical experiments. Though the existence of the small scale flow is at present speculative, various tests of its presence are proposed.

1. Introduction

Though the early model of sea-floor spreading (Hess 1962) proposed a close relationship between surface motions and the flow in the mantle which produced them, this view has now largely been abandoned. The absence of direct observations of the form of the three-dimensional flow in the mantle has produced an increasing number of speculative attempts to maintain the plate motions, none of which has been generally accepted. The principal purpose of this paper is to make yet another attempt, but to start from general physical principles and from an understanding of convection at high Rayleigh number gained from a large number of numerical experiments. These have been described by McKenzie, Roberts & Weiss (1974), referred to hereafter as Paper I, and will not be examined here. The discussion therefore differs from those of Elsasser (1969), Lliboutry (1969) and Morgan (1971), who were principally concerned in explaining observations rather than building consistent models.

In laboratory experiments the plan form of convection in a layer of liquid is immediately obvious. However it is notoriously difficult to obtain this plan form

* Received in original form 1974 May 10.

from the equations governing the motion. In addition, small variations in the properties of the convecting fluid may produce radical changes in the plan form. On the other hand, averaged quantities (such as the rate of heat transfer, the convective velocities, the temperature and viscosity and the thickness of the boundary layers) are more easily estimated from the governing equations and much less sensitive to changes in the fluid's properties. Both Runcorn (1965) and Morgan (1971, 1972) attempted to construct convective models based principally on their notions of the plan form of flow within the mantle. Since this is hard to determine theoretically, while nothing is yet known with confidence about the actual plan form of convection in the mantle, we shall not follow them. Instead we have attempted to relate predicted values of the averaged quantities obtained from numerical experiments to the available geological and geophysical observations.

A separate but related question is that of the Earth's thermal history. Various attempts have been made to determine the variation of the temperature with time at different depths, but the detailed calculations have all assumed that conduction of heat dominated convection. However, both Slichter (1941) and MacDonald (1959) noted that the vertical convection of heat would dominate conduction if slow mass motions occur in the mantle, and MacDonald estimated that his calculations would be invalid if the vertical velocities exceeded 3×10^{-2} mm yr^{-1}. Since it is now generally believed that vertical velocities of 50 mm yr^{-1} occur at least locally beneath the island arcs, studies of the Earth's thermal history which ignore convection are not likely to be useful.

The close connection between the Earth's thermal history and plate motions occurs because both are governed by the same convective motions driven by horizontal temperature differences. Therefore the vertical extent of convection and the variation

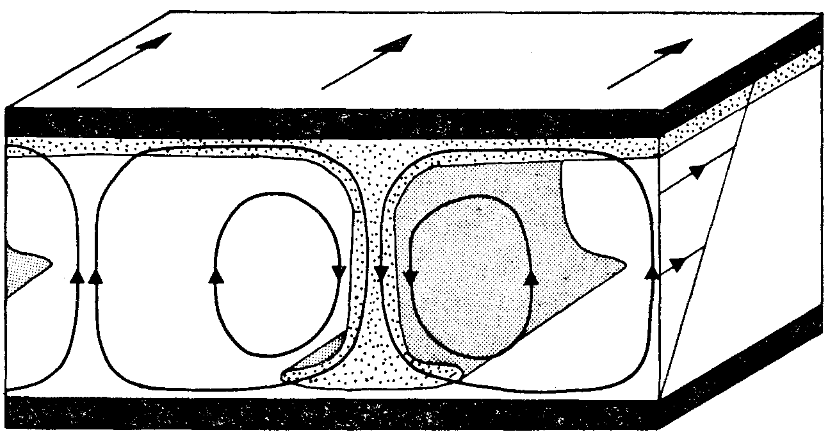

FIG. 1 Illustration of two scales of convective flow. The black horizontal slabs represent boundary layers with mechanical strength; the upper one forms part of a plate moving relative to the lower boundary which is at a depth of about 650–700 km below the surface. The arrows on the upper surface of the plate represent the relative motion between the two mechanical layers, which produces a shear flow illustrated by the arrows on the side of the box. The inclined line shows the position of the edge of the box at some later time. The plate motion and shear flow form part of the large scale circulation. The dotted regions show the thermal boundary layer associated with the small scale flow. The form of its circulation will depend on the magnitude of the shearing. The presence of strong shearing will produce rolls aligned along the direction of shear, and the projection of the particle paths onto the front of the section is shown by the curved lines with arrows. Three-dimensional small scale flow probably occurs in weakly sheared regions.

with time of the surface heat flux cannot be discussed separately. This connection permits the observations of the deformation of Precambrian rocks to be used as constraints on the Earth's thermal history.

An important difference between the models discussed below and previous convective theories lies in the assumption of two scales of convection, the larger scale flow being related to the observed motions of large plates, the smaller transporting the greater part of the heat (Fig. 1). The existence of the two scales of flow implies the existence of two types of boundary layers. The large scale flow involves the creation and desctruction of plates, and these form a boundary layer of the flow, which is both cold and strong. It will be referred to as the mechanical boundary layer, or plate. The boundary layer of the small scale flow must exist beneath the plates, and though its properties are probably affected by the variation of viscosity with temperature it cannot be rigidly attached to the plate. It will be referred to as the thermal boundary layer. The addition and removal of material to and from this boundary layer convects heat to the base of the plate and maintains the heat flow through the floor of the ocean basins. Such a model appears at present to be the only way of reconciling the results of the numerical experiments described in Paper I with the existence of large rapidly moving plates. Though the experiments produced surface velocities, deformations and gravity fields which agree in magnitude with those observed, the horizontal scale of the convective motions was comparable with the depth of the convective layer. Attempts to force the flow to have a larger horizontal scale resulted in instabilities developing in the upper boundary layer and in the formation of smaller scale flow. Though the computations were restricted to two dimensions, it appears unlikely that three-dimensional flows with large horizontal scales will be stable to small scale disturbances. Preliminary calculations with a fluid whose viscosity is a function of temperature do not suggest that large scale motions are stabilized with respect to small scale disturbances, and rather general considerations (Section 3) suggest that small scale motions must be present in the Earth. The numerical experiments described in Paper I are therefore relevant only to the small scale flow.

A naive extension of the simple boundary layer model described in Paper I may also be used to demonstrate that small scale motions are necessary. If the calculations are repeated for a cell of width L and depth l the variation of the Nusselt number Nu (=convective heat flux/conducted heat flux) with the Rayleigh number R for the Rayleigh–Bénard case is given by

$$Nu \approx R^{1/3}\left(\frac{l}{L}\right)^{2/3}. \tag{1}$$

Hence cells with large widths do not transport heat efficiently. The local Rayleigh number for the boundary layer is then given by

$$R_b \approx \left(\frac{L}{l}\right)^{2} \tag{2}$$

and therefore the boundary layer becomes less stable as L is increased. This tendency is even more pronounced if the heat producing the convection is generated within the fluid, when the corresponding expression is

$$R_b \approx R^{1/5}\left(\frac{L}{l}\right)^{16/5}. \tag{3}$$

The pronounced instability of this model's boundary layer is also clear from the numerical experiments in Paper I. These expressions suggest that flow with $L \gg l$ will be unstable to smaller scale disturbances, and therefore also favour a two-scale model for mantle convection.

A similar two-scale model has recently been put forward by Richter (1973). He considered only the form of the small scale flow and showed that rolls not aligned in the direction of the shear decay when the Rayleigh number is $\approx 10^4$. Though it seems unlikely that his analysis will be valid for mantle convection, where the Rayleigh number is probably between 10^5 and 10^6, his physical arguments concerning the plan form should still apply.

So far little attention has been paid to two-scale models of convection in the mantle. Yet multiple scales of convection are by no means uncommon. For example, both cloud patterns observed by satellites in the Earth's atmosphere and cellular convection in the Sun cover a larger range of scales than we propose for the mantle (though the governing equations are of course significantly different). In laboratory studies of Rayleigh–Bénard convection Busse & Whitehead (1974) and Busse (private communication) have demonstrated that two scales of flow occur when the Rayleigh number exceeds $2{\cdot}4 \times 10^5$. Although the Prandtl number was relatively large (63), the Reynolds number was high and the formation of two scales, with time-dependence of the resulting flow, occurred because momentum was advected. In the mantle the flow is dominated by viscosity and there is no significant advection of momentum; it is not clear from the experiments whether two scales will exist in these conditions.

Laboratory experiments on convection driven by internal heating show cells whose width increases as the Rayleigh number is increased (Tritton & Zarraga 1967). Subsequent experiments by Hooper (Tritton, private communication), using water with a Prandtl number around 6, show somewhat less elongated cells, with a width about three times the layer depth at a Rayleigh number of 4×10^5. Within these elongated cells the flow becomes time-dependent at Rayleigh numbers greater than 5×10^4: small blobs appear, falling off the thermal boundary layer at the top, and the number of blobs rises as the Rayleigh number is increased. This behaviour is consistent with the numerical experiments in Paper I, though the Reynolds number is again quite different.

Another important feature of the convective models discussed below is the existence of convection in the lower mantle below a depth of 700 km. The arguments in this paper favour flow at these depths to transport heat generated by radioactive decay and perhaps by recrystallization within the core (Section 2, Bullard 1950). The model therefore differs from that proposed by MacDonald (1963) and McKenzie (1967a), who believed that the viscosity of the lower mantle was too great to permit important convective heat transport. The earlier model required radiation to transport the heat generated, but it now seems unlikely that this mechanism is important.

Most discussions of the effect of the phase changes near 650 km on convection have been concerned with thermodynamics and have ignored its effect on mechanical properties. If the mechanical properties of the two phases are very different at the same temperature and if the heat transport is from the more to the less competent material, a cold mechanical boundary layer of competent material forms on one side of the boundary. Such a situation occurs on a small scale in the Hawaiian lava pools, and on a larger scale in plate tectonics. Whether the phase change at about 650 km depth produces a similar mechanical boundary layer depends on the change in the activation energy of creep; that in Table 1 is sufficient to produce such a shell within the lower mantle, and its existence may well be more important than the other effects of the phase change on convection in the mantle. Whether this layer breaks up into plates, as it does on the Earth, or forms a rigid shell across which heat is conducted, as it does on the Moon, must depend on whether the convective forces on it from below are sufficient to overcome its strength.

This rather complicated model appears to be the simplest consistent with the observations. It is not possible to extend the convection cells throughout the entire mantle and in this way both to increase the horizontal scale of the flow and to convect heat through the lower mantle, for two reasons. The first depends on the numerical experiments described in Paper I. These showed that the horizontal scale of the flow at large Rayleigh numbers was governed by the thermal structure of the upper boundary layer. Since its structure is controlled principally by the heat flux conducted through it, increasing the depth of the convecting layer has little effect if the heat flux remains constant, and the change from heating from below to internal heating required by such a model may even reduce the horizontal length scale as the depth is increased (see (1)).

The other argument against convection cells extending throughout the entire mantle depends on the focal mechanisms of earthquakes below 70 km (Isacks & Molnar 1969, 1971). Such shocks occur only within slabs of material which once formed plates at the Earth's surface but which have now sunk through the mantle. Their observations show that slabs which extend to depths of less than 300 km are in tension. If they extend below this depth their lower parts are in compression but their upper parts remain in tension unless the tip of the slab reaches below 600 km. Three such slabs are known, and all three are in compression throughout their length. These observations show that the cold sinking boundary layer meets with mechanical or thermodynamic resistance to its motion at depths below 600 km. This argument, together with the absence of earthquakes deeper than 700 km, appears to exclude the simple model.

Throughout this paper we shall assume that the material of the mantle deforms as does a Newtonian fluid. This assumption is valid only if the deformation is rate controlled by diffusion, either along grain boundaries or through the grains themselves. Under these conditions the variation of viscosity with temperature can be obtained, and we use the expression valid for diffusion through the grains, often called Nabarro–Herring creep. Creep of this type only occurs if the shear stress is below a critical, but at present undetermined, value. It is therefore unknown whether the creep rate is determined by diffusion, though it is clear that there are regions of the mantle beneath island arcs where it is not. Although it is of interest to know how realistic are the calculations made using a Newtonian viscosity, it is anyway of importance to understand their implications, since the behaviour of the simpler system should be used as a guide to that of the more complicated one. The success of the simple thermal models of plate creation and destruction shows that the main features of these boundaries can be understood without taking into account complex processes, such as magma generation and movement, which are known to be taking place.

2. Thermal history

Deformation of the crust and upper mantle is caused by convection and the form and vigour of the convection depends on the heat it must transport. Any discussion of the Earth's tectonic history must therefore begin with its thermal history. In this section we assume that the heat generation occurs uniformly throughout the Earth, and study the effect of different original temperature distributions. Since the Earth is large, while its thermal conductivity is small, it can only lose heat from most of its interior if that heat is transported upwards by convection. If convection is to occur, the viscosity must be sufficiently small and the temperature sufficiently high. Otherwise the heat is not transported and merely raises the temperature of the material. Once convection is occurring everywhere the heat transported adjusts quickly to any changes in the heat generation rate, and therefore the surface heat flux at any epoch equals the rate at which heat is generated. Though the details presented in Figs 2–15

Table 1

$$l = 700\,\text{km}$$

$$\kappa = \frac{k}{\rho C_P} = 8\times10^{-7}\,\text{m}^2\,\text{s}^{-1}$$

$$R = 0{\cdot}5\,\text{mm}$$

$$D_0 = 5\times10^{-4}\,\text{m}^2\,\text{s}^{-1}$$

$$V^* = 8\times10^{-30}\,\text{m}^3$$

$$\nu_0 = 2\times10^{17}\,\text{m}^2\,\text{s}^{-1}$$

$$\alpha = 2\times10^{-5}\,{}^\circ\text{C}^{-1}$$

$$\rho = 4\,\text{Mg}\,\text{m}^{-3}$$

$$C_P = 1{\cdot}2\times10^3\,\text{J}\,\text{kg}^{-1}\,{}^\circ\text{C}^{-1}$$

$$T_c = 1000\,{}^\circ\text{C}$$

Parameters used in Section 2

depend on values of the parameters in Table 1, the general behaviour of the model depends only on rapid variation of viscosity with temperature and on the thermal time constant for the whole Earth being many times its age. For various reasons a hot origin of the Earth appears most likely, and therefore the conclusion of this section is that convection has always been sufficiently vigorous to transport all the heat generated within the Earth to its outer surface. This constraint is then used in Section 3 to investigate the time dependent behaviour of convection.

Various authors, in particular Slichter (1941), Lubimova (1958) and MacDonald (1959), have attempted to construct models of the Earth's thermal history without permitting convective heat transfer. The principal difficulty facing such attempts is the long time constant for thermal diffusion through the Earth. If the conductivity is everywhere the same as that in Table 1 the thermal time constant is about 10^5 My, or about twenty times the Earth's age. In order to supply the heat at present being lost by conduction through the Earth's surface without any convective transfer either the thermal conductivity has to increase with depth by several orders of magnitude or the radioactive elements generating the heat have to be concentrated near the surface. MacDonald's models incorporated both features, but neither is compatible with our present knowledge.

The increase in conductivity with depth was believed to be due to the importance of radiative heat transfer at high temperatures. However, measurements of the thermal conductivity of rocks likely to form the mantle by Fujisawa *et al.* (1968) and more recently by Schatz & Simmons (1972) showed only small variations in conductivity with temperature. Measurements of the opacity of the constituent minerals at high temperatures by Fukao, Mizutani & Uyeda (1968) and Fukao (1969) showed that the earlier measurements were not valid at temperatures of 1300° K. Therefore in the upper mantle at least radiative conduction does not produce order of magnitude changes in the conductivity.

Since even the crystal structure of the lower mantle rocks is unknown, the importance of lattice and radiative conductivity is less clear. However, as Clark (1957) pointed out, the electrical conductivity of the lower mantle is approximately $10^2\,\Omega^{-1}\,\text{m}^{-1}$, and this is sufficient to cause the radiative contribution to the conductivity to be less than that due to phonons unless the temperature exceeds 3000°C. It is therefore probable that the thermal conductivity throughout the mantle does not differ substantially from the value in Table 1, which will therefore be used in all calculations.

The rate of radioactive heat generation within the mantle is not as well known. Probably the best estimates come from the radioactivity of oceanic rocks, and are discussed in Paper I. However, though these measurements are compatible with a uniform distribution of radioactive elements throughout the mantle, they are not yet either extensive or accurate. For simplicity we assume a constant rate of radioactive heat generation throughout the Earth.

Even without our present extensive knowledge of plate motions the new measurements of thermal conductivity require a complete re-examination of the thermal history problem. The importance of convective heat transfer is that it provides an effective thermal conductivity which is sufficient to transport heat from any part of the interior to the surface in a time which is short compared to the age of the Earth.

The method adopted below for examining the thermal history of the Earth is in some ways similar to that suggested by Tozer (1972). It depends on the rapid variation of viscosity η with temperature when creep is rate limited by body diffusion:

$$\rho v = \eta = \frac{kTR_G^2}{10m_a D} \tag{4}$$

where T is the absolute temperature, k Boltzmann's constant, R_G the mean grain radius, m_a the mass of an oxygen ion and D the diffusion coefficient:

$$D = D_0 \exp\left[-\frac{E^*+PV^*}{kT}\right] \tag{5}$$

where D_0 is a constant, E^* the activation energy, P the pressure and V^* the activation volume. The values used for the various parameters are given in Table 1 and are taken from Gordon (1965) and McKenzie (1967a). Though several of them are poorly known the uncertainties in the initial temperature distribution and in the distribution of radioactive elements have a greater influence on the Earth's thermal history than do those in the parameters in equations (4) and (5), with the one exception of the activation energy E^*. The value of E^* controls the temperature at which convection becomes the dominant form of heat transport, and the increase in E^* from 4 electron volts in the upper mantle (above a depth of about 700 km) to 6 eV in the lower mantle strongly modifies the form of the convection and the thermal history calculations. An increase in activation energy at a depth near 700 km is probable because the phase change from spinel to post-spinel must involve a major change in lattice structure. Unlike the olivine to spinel phase change at shallower depths all suggestions for the phase change near 700 km involve a major change in the environment of the oxygen ions, and their diffusion is most likely to control the creep rate at all depths. The change in activation energy is included partly for these reasons and partly also to illustrate the effects to which it gives rise.

The rapid variation of viscosity with temperature is demonstrated in Fig. 2. Because of this variation convective heat transfer is ineffective in the mantle until the temperature reaches a critical value; at lower temperatures heat is transferred by conduction alone. Furthermore the flow velocities vary rapidly with temperature and therefore once the heat transport is dominated by convection the thermal time constant becomes of the order of 100 My.

Convection in a plane layer with variable viscosity

These remarks are best illustrated by considering the convective heat transport in a layer 700 km thick and using the relation between the mean temperature difference

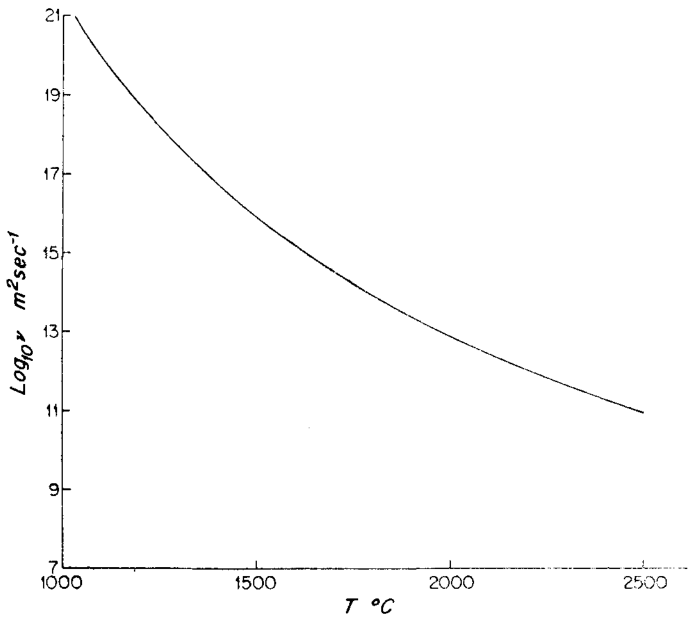

FIG. 2. The variation of viscosity with temperature (equation (4)).

across the thermal boundary layer ΔT and the convective heat flux E obtained in Paper I when half the heat is conducted in through the lower boundary and half is generated by radioactivity within the layer:

$$\log_{10} \Delta T = 0{\cdot}75 \log_{10} E + 3{\cdot}53 + 0{\cdot}25 \log_{10}(\nu/\nu_0) \tag{6}$$

where ν_0 is the reference viscosity of 2×10^{17} m^2 s^{-1}. Though equation (6) describes the results of numerical experiments using a fluid of constant viscosity, we assume that it can be used to give an approximate value of the heat flux when $\nu = \nu(T)$ provided T is the mean temperature of the convective region. Equation (6) may also be written

$$E = \left(\frac{\Delta T}{c}\right)^{4/3} \left(\frac{\nu_0}{\nu}\right)^{1/3} \tag{7}$$

where

$$\log_{10} c \approx 3{\cdot}53.$$

The boundary layer analysis in Paper I shows that

$$c = c_0 \left(\frac{\nu_0}{g\alpha\kappa^2}\right)^{1/4}$$

where g is the acceleration due to gravity, α the thermal expansion coefficient, κ the thermal diffusivity and c_0 is a numerical constant. Thus all terms in (7) are independent of the thickness of the convecting layer. If the rate of heat supply F to the layer of depth l is not equal to the rate E at which heat is being lost, then

$$\rho C_P l \frac{d\Delta T}{dt} = F - E \tag{8}$$

where C_P is the specific heat at constant pressure. Substitution of equation (7) gives

$$\rho C_P l c^{4/3} \frac{d\Delta T}{dt} = c^{4/3}\ F - \Delta T^{4/3}\left(\frac{\nu_0}{\nu}\right)^{1/3}. \tag{9}$$

In writing equation (8) in this form we have assumed that the time taken for the fluid to overturn is short compared with the time taken for its temperature to adjust to changes in F. Though this assumption may not be true for the example in Fig. 3, it is certainly true for the thermal history calculations discussed below, since the overturn time is now ≈ 200 My, and can only have been ≈ 20 My in the early Precambrian. It is convenient to let $t = \tau t'$ where

$$\tau = \rho C_P l c^{4/3}; \tag{10}$$

substitution of $l = 700$ km and the values of ρ and C_P in Table 1 gives

$$\tau = 2{\cdot}2 \times 10^3 \text{ My}.$$

Then we obtain

$$\frac{d\Delta T}{dt'} = c^{4/3}\ F - \Delta T^{4/3}\left(\frac{\nu_0}{\nu}\right)^{1/3}. \tag{11}$$

Once again (11) is independent of the layer thickness. Therefore for a convecting layer the time taken to adjust to temperature changes depends directly on its thickness l and not on l^2 as in the conductive case. This difference greatly simplifies the thermal history calculations because the time constant of a convecting layer is sufficiently short to be neglected. This result may be demonstrated by solving equation (11) numerically, using the expression for given by equation (4) evaluated at a depth of 350 km and setting $F = 0{\cdot}0585$ Wm^{-2} initially, then increasing F to $0{\cdot}585$ Wm^{-2} for 70 My and finally letting F revert to its original value (as shown in Fig. 3(a)). The behaviour of the surface heat flux, the mean temperature and the viscosity are shown in Fig. 3.

The most important feature of the parameters displayed in Fig. 3 is their rapid adjustment to changes in the rate of heat supply. This adjustment is achieved principally by changing the viscosity and in this way modifying the convective velocities. The corresponding changes of temperature are small. This stabilizing effect of a strongly temperature dependent viscosity has been discussed in some detail by Tozer (1972) and greatly simplifies thermal history calculations. Fig. 3(a) shows that the adjustment is more rapid when the rate of heat supply is increased than when it is decreased, but in both cases the time constant is of the order of 30 My. The rapid adjustment and the small variation in temperature (Fig. 3 (b)) required to produce order of magnitude variations in the rate of heat transfer suggest that the effect of convection on the thermal history can be approximated by obtaining the temperature distribution $\theta(r)$ as a function of radius that results in a constant viscosity, and then using $\theta(r)$ as the upper bound on the temperature $T(r)$. If at any depth $T < \theta$ then heat is transferred by conduction only: if $T = \theta$ over some range of radius $r_1 \leqslant r \leqslant r_2$ all heat conducted in at r_1 and generated between r_1 and r_2 must be conducted out at r_2. Such a model includes the important features of the convective model illustrated in Fig. 3, but the computations required are simpler than those which would result using equation (11). The calculations were carried out using a conservative difference scheme described in Appendix A. The procedure for including convection is valid only if the viscosity is constant along adiabats, and we assume this to be true. If it is not the viscosity of a parcel of fluid rising adiabatically changes, and under these conditions the convection probably becomes strongly time dependent.

Earth models

The model described above requires an initial temperature distribution, and knowledge of the distribution of heat sources as a function of radius and time, before it can be used to calculate Earth's thermal history. Very little is yet known about the temperature of the Earth after it had been formed, but it seems unlikely to have been less than 1000 °C. Adiabatic compression and the energy generated by shock waves from the capture of planetesimals would both increase the temperature in the interior during accretion. There is now considerable evidence (see Solomon & Toksöz 1973) that the Moon's temperature was in the region of 1000° after formation, and this

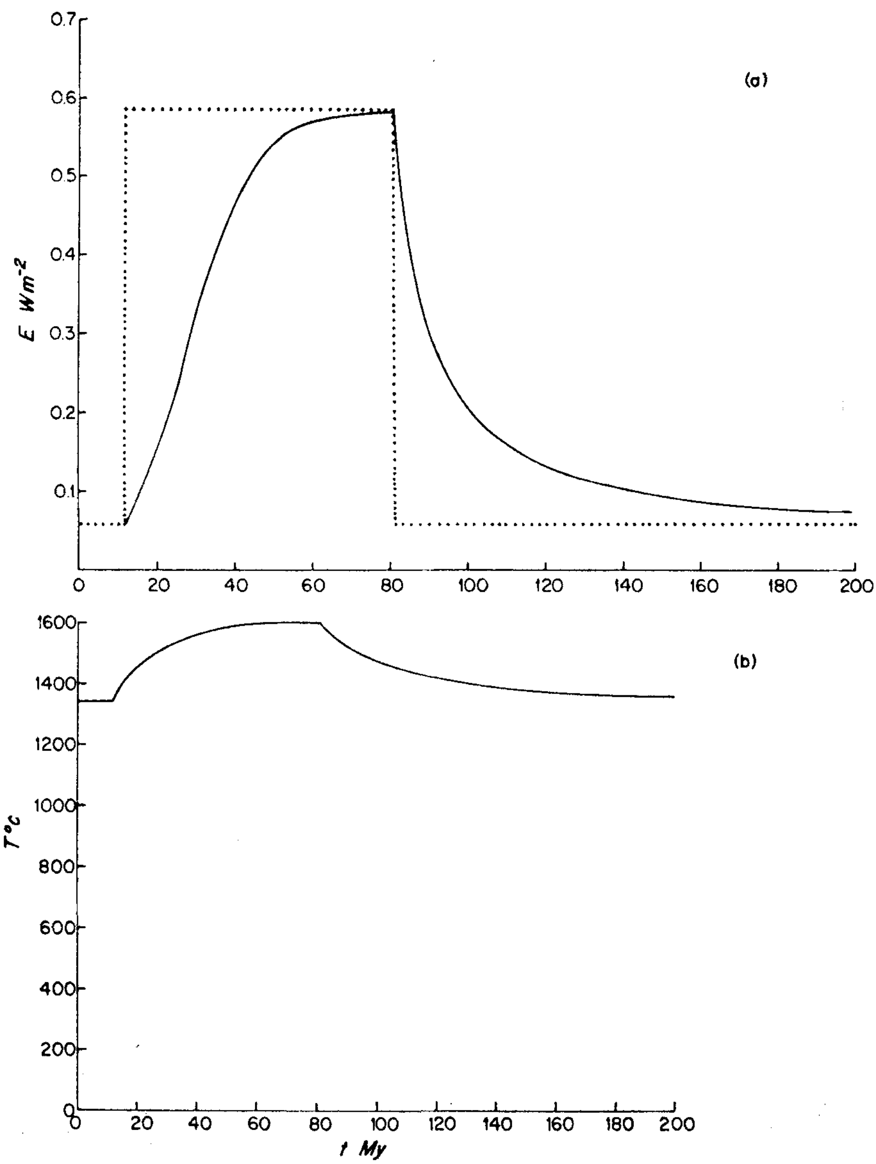

FIG. 3(a) and (b)

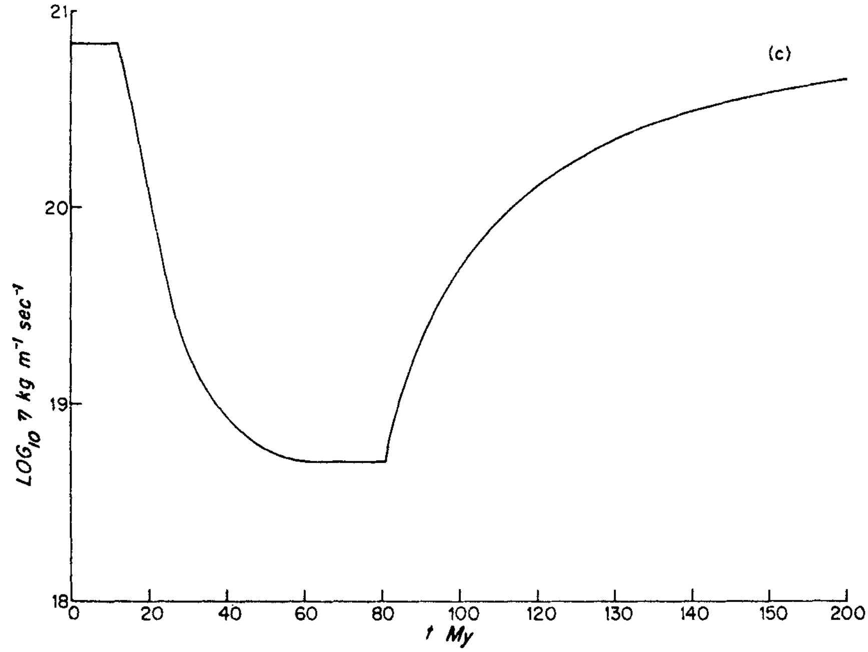

FIG. 3. The behaviour of the temperature (b) and viscosity (c) of a convecting layer 700 km thick when the heat flux is increased by a factor of ten for 70 My. The heat flux through the base of the layer is shown by the dotted line in (a), the heat flux through the top by the solid line in (a). The time required to reach steady state is short compared to the corresponding time of 10^3 My for heat to be conducted across the layer.

suggests that the more massive earth would be at least as hot. Despite this evidence, calculations were carried out for four models with very different starting temperatures.

The distribution of radioactive elements with radius as a function of time is far from clear, and, because island arcs are at present the only places where appreciable quantities of radioactive elements are transferred from the mantle to the crust, depends entirely on the chemical models of these structures. Unfortunately even the question of whether continental material was formed by differentiation early in the Earth's history or has been produced by island arc volcanism at a constant rate throughout the Earth's history is still unsettled. Dickinson & Luth (1971) believe that radioactive elements in the upper part of the oceanic plates are not returned to the mantle beneath island arcs, and that the depleted plate sinks to the base of the convecting layer to form a growing shell of exhausted material. One difficulty with this model is the manner in which the depleted upper part of the oceanic plate is separated from the lower part whose composition is the same as the mantle elsewhere. Convective motions on a scale of a few tens of kilometres are required, and these must be driven by the difference in radioactive content. Since the thermal diffusion time for such distances is short compared to the time required for radioactive heating, such a separation is not easily achieved. Moreover it is not clear whether the processes in island arcs do result in a net loss of radioactive elements from the mantle.

The constancy of composition of tholeiitic lavas throughout geological time (Hallberg & Williams 1972) does not suggest that major changes in the composition of the upper mantle have occurred during the last 3 Aeons (1 Aeon $\equiv 10^9$ yr) and the increasing evidence for large scale reworking of continental rocks by later orogenies which is now being obtained by sophisticated dating techniques suggests that the extent of continental growth in the last 500 My has been overestimated. Muelberger, Denison & Lidiak (1967) believe that at least 50 per cent of the basement of the United States is older than 2500 My, and the figure for Asia, Africa and Australia must be considerably greater (Watson 1973). We therefore assume that the composition of the mantle has not changed to any important extent during geological time, and that the radioactive elements producing the mean oceanic heat flow of $5{\cdot}8\times10^{-2}\,\mathrm{W\,m^{-2}}$ are uniformly distributed throughout the mantle and core. Though there has recently been considerable discussion about the potassium content of the core (Oversby & Ringwood 1972; Goettel 1972) this question has little influence on the thermal history.

For reasons discussed later in this section, it is probable that the rate of heat generation H within the Earth differs little from the rate of heat loss. We therefore calculate the heat generation rate from the oceanic heat flow. Because of radioactive decay, this rate has been greater in the past, but how much greater depends upon whether the ratio of potassium to uranium and thorium within the Earth is taken to be that in chondrites or that derived from measurements made on surface rocks on Earth (Birch 1965a). The calculations have been carried through for both ratios, the first using a heating rate

$$H(t) = H_0 \exp(-t/\tau_R) \tag{12}$$

where t is measured in My and $H_0 = 2{\cdot}07\times10^{-7}\,\mathrm{W\,m^{-3}}$, $\tau_R = 2219$ for the chondritic composition, while $H_0 = 1{\cdot}09\times10^{-7}\,\mathrm{W\,m^{-3}}$, $\tau_R = 3248$ for that of Wasserburg *et al.* (1964). The constants H_0 and τ_R were chosen to give correct values for $H(t)$ when $t = 0$ and $t = 4500$. Equation (12) is an approximation to the true variation, which involves the decay of several elements with different half lives. In all the figures results from the chrondritic model are shown by a solid line, those from Wasserburg *et al.*'s (1964) model, which contains less potassium, by a dashed line.

Figs 4–7 show the surface heat flow for the two different models obtained using various initial conditions. The difference between them is caused by time taken to

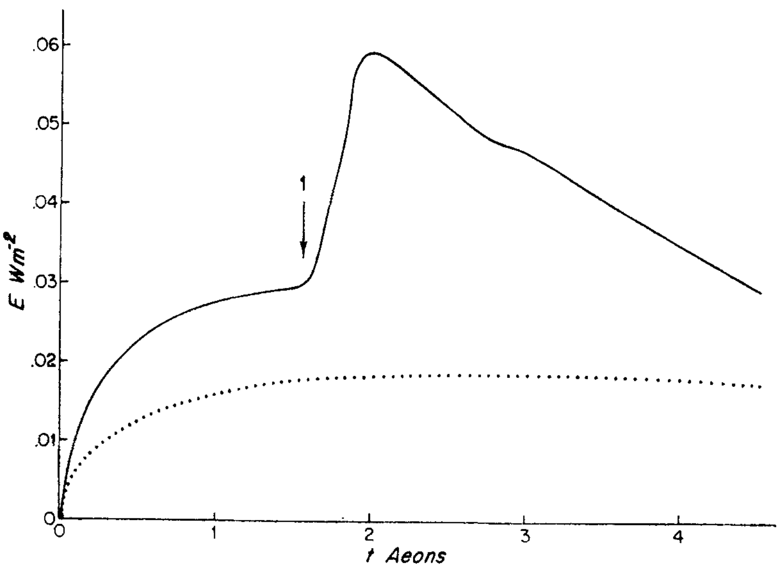

FIG. 4. Surface heat flow as a function of time for two models of the Earth. The solid line is that for a chondritic model, the dotted line that for a model with the composition suggested by Wasserburg *et al.* (1964). Both models started with a temperature of 0 °C everywhere. The arrow labelled 1 marks the time of onset of convection in the upper mantle.

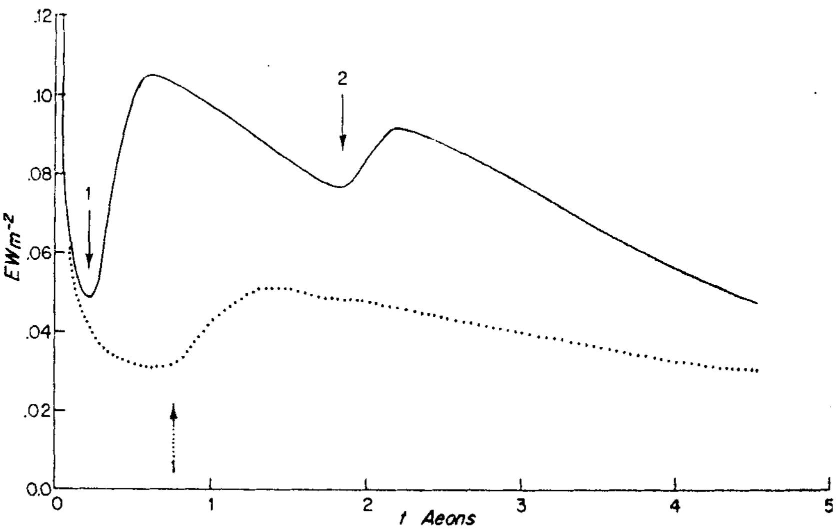

FIG. 5. Surface heat flow for the two models described in Fig. 4, but with initial temperatures of 10^3 °C. The arrow labelled 2 marks the time of onset of convection in the lower mantle.

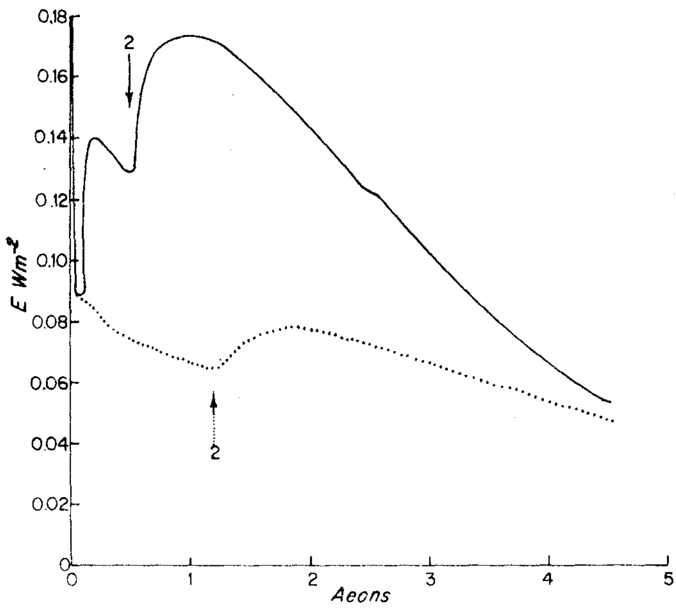

FIG. 6. Surface heat flow for the two models described in Fig. 4, but with initial temperatures of 2×10^3 °C. The arrows labelled 2 mark the onset of lower mantle convection.

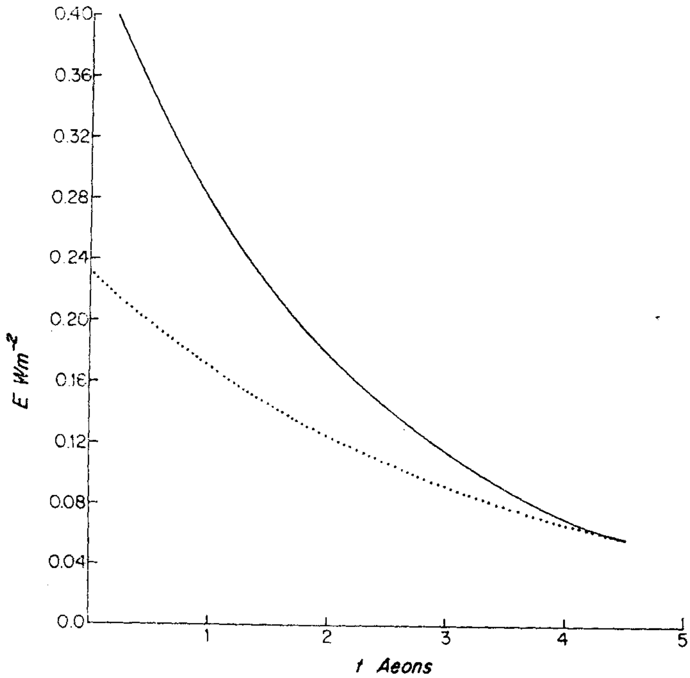

FIG. 7. Surface heat flow for the two models described in Fig. 4, but with the initial temperature sufficiently great to permit convection everywhere (see text).

warm the mantle to a temperature sufficient for convection. Until such warming has occurred almost all the heat generated in the mantle produces a rise in temperature and, except in the surface layers, none is conducted to the surface. Thus, in Figs 4 and 5 little of the radioactive heat generated in the first aeon of the Earth's history reaches the surfaces because little convection is occurring. The arrows marked '1' show the time of onset of upper mantle convection; those marked '2', the onset of lower mantle convection. In Fig. 6 the initial temperature is sufficient to cause the upper mantle to convect, but not the lower. In Fig. 7 convection occurs through the mantle at all times. An example of the variation of temperature at different times is shown in Fig. 8 for the chondritic model of Fig. 5. The dashed line shows the temperature required for convection, which is the same for all models, and the solid lines show the actual temperature at different times. Fig. 8 shows that the delay in the onset of lower mantle convection is the result of the change in activation energy at 700 km, and that the base of the lower mantle is the last part to start convecting. It is also clear from these models that lower mantle convection need not be occurring, but that such models require a solid non-convecting core and hence no magnetic field early in the Earth's history. None of these conditions appear likely to have been satisfied. The Earth's magnetic field existed at least 2600 My ago (McElhinny 1973b), and if the main field was maintained in the same way as it now is, the core must have then been molten, as it still is except at the centre. Recent observations of lateral inhomogeneities in the lower mantle (Davies & Sheppard 1972; Julian & Sengupta 1973; Needham & Davies 1973) are also easily explained by convective temperature differences. Although there is strong evidence that the sinking slabs do not penetrate the lower mantle, as discussed in Paper I, Goldreich & Toomre (1969) have shown that a highly viscous lower mantle is not required to explain the Earth's non-hydrostatic bulge. Furthermore two major sources of heat have been omitted from the calculations, principally because of our uncertainty as to when they operated and the magnitude of the heat generated.

The first of these is core formation. Tozer (1965) and Birch (1965b) (corrected in Flasar & Birch 1973) estimated the gravitational energy released by core formation from a chemically uniform earth would raise its temperature by about 2000 °C. This heat would be convected to the surface if the mantle was already hot enough to convect and the process occurred over a time of 100 My or greater. Since the temperature required to melt iron is close to that required for convection, lower mantle convection must have been occurring since the Earth had a liquid core.

The second process is tidal dissipation within the solid Earth (Munk 1968). Whether this source of energy has been important in the past depends on the history of the Moon's orbit, which is still uncertain. However any heat generated will help raise the temperature towards that required for convection; once convection starts this heat can easily be transported to the surface.

In contrast to core formation, differentiation of the Earth's crust generates a negligible amount of energy (Tozer 1965), and is important only because it transports the radioactive elements close to the Earth's surface and in this way affects the thermal history.

It is clear from this discussion that there are a number of processes which can produce large increases in the temperature of the Earth early in its history, even if it did not start hot. Because the conductivity of rocks is so small all of this heat must remain in the interior unless convection occurs. If convection was not occurring as the Earth formed, it is likely to have started very soon afterwards; and such convection is necessary to explain observations on Precambrian rocks. For these reasons we shall assume that convection in the upper and lower mantle have been taking place throughout the Earth's history and that only small variations in the temperature have taken place except in the two conductive boundary layers. These arguments in favour of a hot origin of the Earth are in agreement with those used by Hanks & Anderson (1969). Under these conditions the surface heat flow is that shown in Fig. 7 and the temperature distribution close to that shown in Fig. 8. Furthermore any heat required to maintain the Earth's magnetic field can easily be transported from the core by convection. The model therefore avoids the problems discussed by Bullard (1950). The rest of this paper is concerned with the evolution and structure of the convective and conductive regions within the Earth.

3. Small scale flow

Certainly the most speculative part of the model proposed in this paper is the small scale flow. Various arguments in favour of its existence were proposed in the introduction, all of which essentially depend on the instability of the layer of material beneath the plates when it loses heat by conduction upwards, or, equivalently, the inability of the mantle to lose heat without some form of small scale vertical convective transport. There are, however, at present no observations which unambiguously require the existence of such a flow pattern, and it is therefore important to examine the consequences of its existence in some detail and to suggest observations which may demonstrate its presence both now and in the past. Such an examination is carried out in this section using the preferred model of Section 2: that which convects upwards all the heat generated within the Earth. It shows that the temperature below the thermal boundary layer at the Earth's surface was about 200 °C hotter in the early Precrambrian than it is now, with corresponding convective velocities of $1\ \mathrm{m\ yr^{-1}}$. The stresses generated by the small scale flow were then ten times as great as they are now, and may have been sufficient to prevent large plates from forming. The model can therefore account both for the small horizontal scale of early Pre-

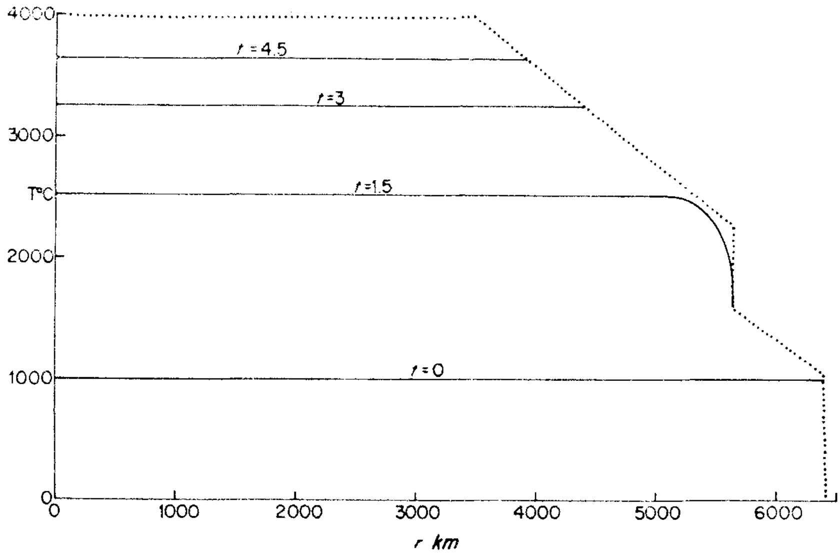

FIG. 8. The temperature as a function of radius for the chondritic model in Fig. 5. The dotted line shows the temperature necessary for convection to dominate heat transfer (see text). The values of t on each solid curve show the corresponding time elapsed in aeons. The conductive boundary layers at the surface and a depth of about 650 km are not shown.

cambrian structures and for the occurrence of hotter magmas at that time than those being erupted now. The plan form of the small scale flow can only be guessed, but existing experiments suggest it is likely to consist of rolls beneath rapidly moving plates, and cellular flow beneath those moving more slowly.

The simple boundary layer model used in Paper I and in the last section to examine the behaviour of a convecting layer can also be used to obtain the maximum horizontal, $\hat{u}$, and vertical, $\hat{w}$, velocities, the mean temperature, viscosity, and the thickness δ of the surface boundary layer. The relevant expressions are given in Paper I:

$$\left.\begin{aligned} \log_{10}\hat{u} &= 0{\cdot}49\log_{10}\left(\frac{E\nu_0}{\nu}\right)+1{\cdot}80 \\ \log_{10}\hat{w} &= 0{\cdot}59\log_{10}\left(\frac{E\nu_0}{\nu}\right)+2{\cdot}18 \\ \log_{10}\delta &= -0{\cdot}16\log_{10}\left(\frac{E\nu_0}{\nu}\right)-1{\cdot}33 \end{aligned}\right\} \tag{13}$$

where E is the rate of heat loss in W m^{-2} and $\hat{u}$, $\hat{w}$ are measured in mm yr^{-1}. A convenient method of obtaining solutions to equation (13) as a function of time is to use equations (11) and (12) first to find the variation of T, then to substitute into (13). Since the time constant is short compared to τ_R the surface heat flux from such a time-dependent calculation is always within 5 per cent of that in Fig. 7. The resulting variation in T, ν, δ, $\hat{u}$, and $\hat{w}$ is shown in Figs 9–13. For reasons explained in the last section, small variations in T produce large changes in the velocities and in this way increase the convective heat transport. The thickness of the boundary layer is that of the thermal boundary layer associated with the small scale flow, and not the

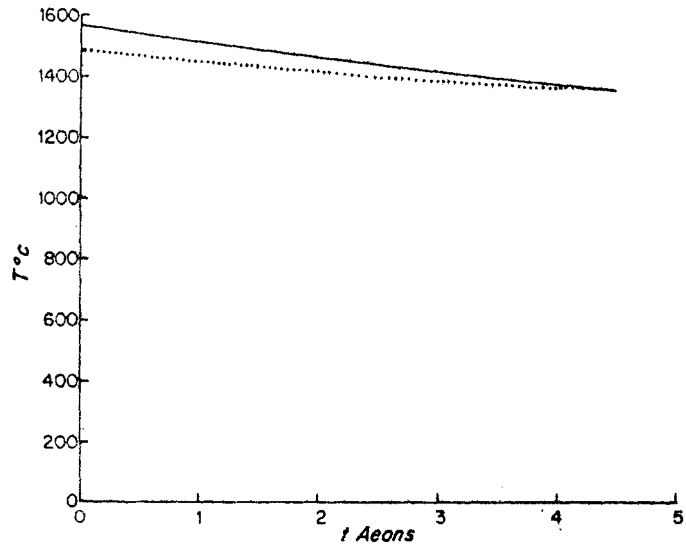

FIG. 9. The temperature at 350 km as a function of time in a mantle which is convecting at all times. The solid curve is for a chondritic model, the dotted one for that of Wasserburg *et al.* (1964).

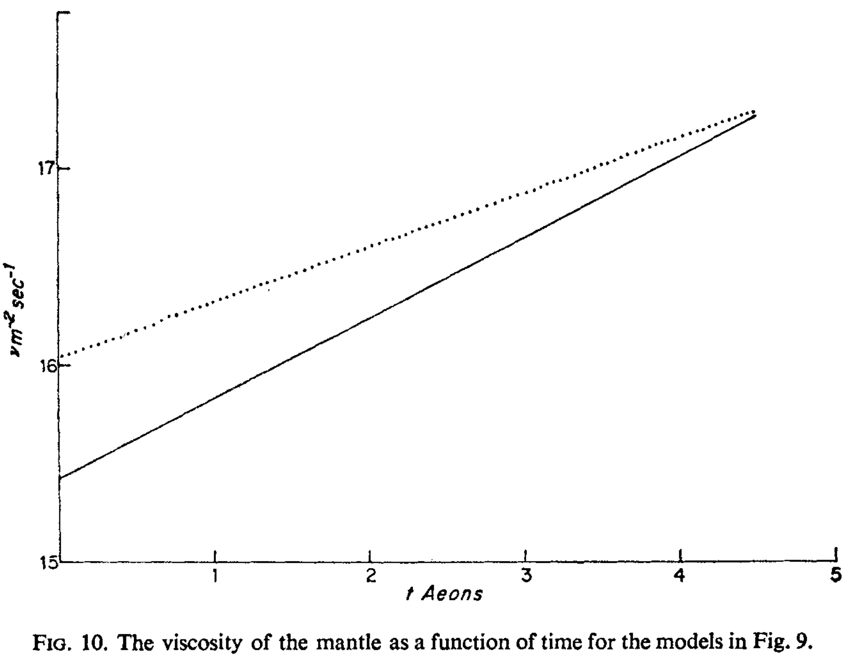

FIG. 10. The viscosity of the mantle as a function of time for the models in Fig. 9.

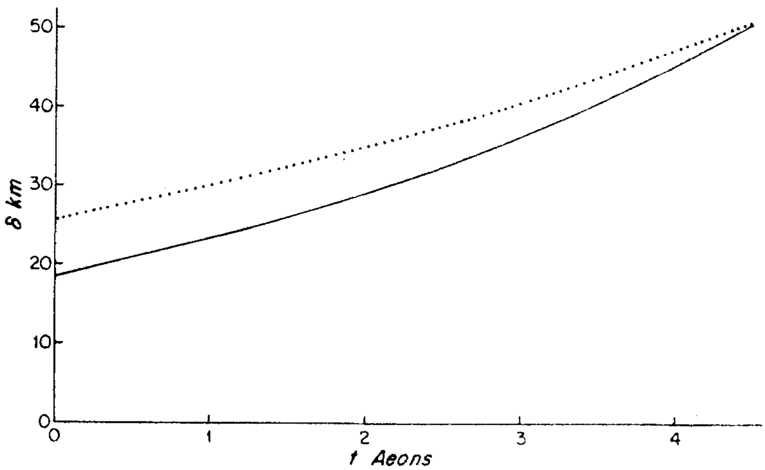

FIG. 11. The thickness of the thermal boundary layer associated with the small scale flow (see Fig. 1) as a function of time for the models in Fig. 9.

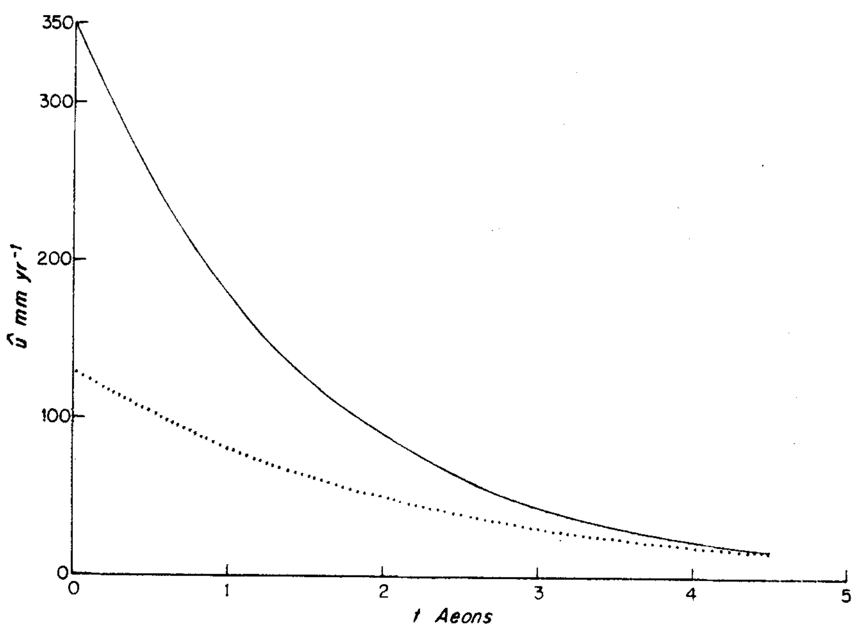

FIG. 12. The maximum horizontal velocity $\hat{u}$ as a function of time for the models in Fig. 9.

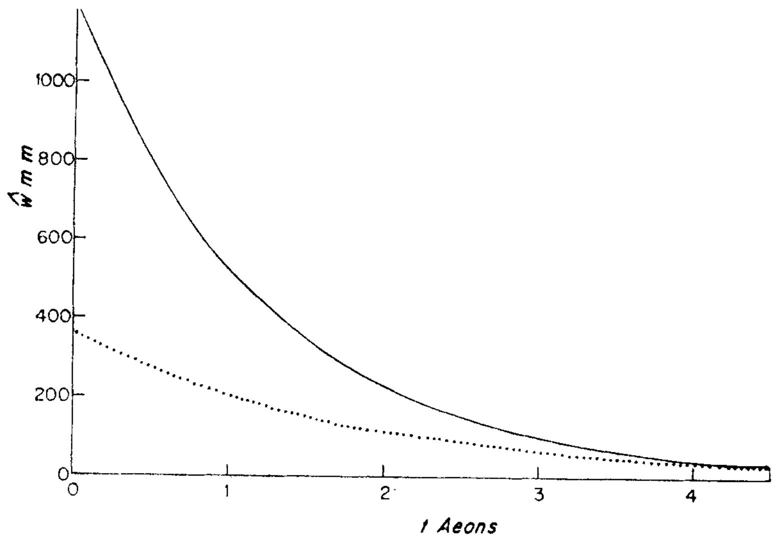

FIG. 13. The maximum vertical velocity $\hat{w}$ as a function of time for the models in Fig. 9.

thickness of the plate on top (see Fig. 1). The variation in the plate thickness Δ with time is shown in Fig. 14 and is obtained from

$$E = \frac{kT_c}{\Delta} \tag{14}$$

where T_c is taken to be 1000 °C and k to be 4 $\mathrm{Wm^{-1}\,°C^{-1}}$. The thickness of the boundary layer near 650 km may also be obtained from (14) with Tt = 780 °C and is shown in Fig. 15. Various authors have suggested that the plate thickness in the early Precambrian was considerably less than it is today. Hart *et al.* (1970) argued that the widespread tholeiitic volcanism requires shallow melting and a thin plate, and Saggerson & Owen (1969) regarded the absence of intermediate pressure and temperature metamorphic facies as evidence for temperatures of 800–900 °C at 11 km. Shackleton (1973) has reviewed several of these arguments and noted some exceptions. Though these estimates agree rather well with the plate thickness in Fig. 14 they are unlikely to be reliable. The difficulty is that most volcanism and metamorphism now occurs near plate boundaries, and in these regions the plate thickness is often much less than elsewhere. An estimate of present average plate thickness cannot be obtained from the composition of rocks erupted on ridges or in island arcs. Few, if any, metamorphic reactions occur in plates owing to the mean temperature gradient in the older parts of plates, and though some volcanism now occurs at places remote from plate boundaries it produces only a small fraction of the erupted rocks and would be difficult to recognize in a Precambrian terrain. We shall therefore put little emphasis on these estimates of plate thickness.

The occurrence of early Precambrian ultramafic magmas is fortunately open to far fewer interpretations. It used to be widely believed that ultrabasic lavas did not occur and that intrusive and extrusive rocks with this composition carried the olivine as crystals in suspension. However, recently a number of careful studies of the fabric and mode of occurrence of such lavas has clearly demonstrated that they were produced in the early Precambrian (older than 2800 My). Drever & Johnston (1957) demonstrated that the crystal habit of olivine crystals formed by rapid cooling is

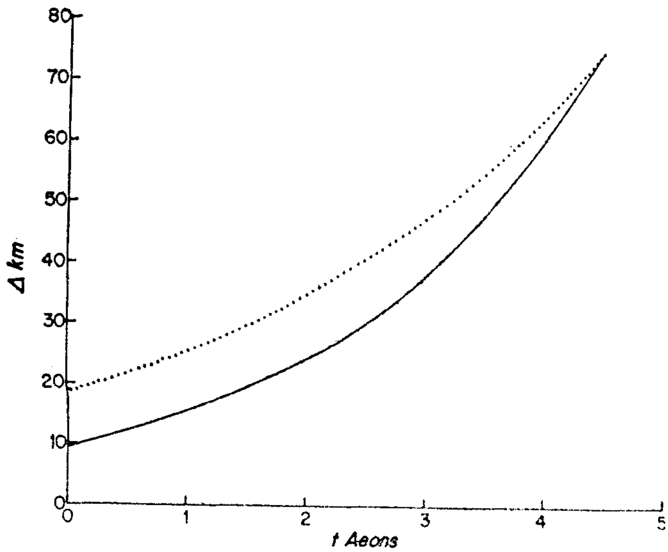

FIG. 14. The surface plate thickness as a function of time for the models in Fig. 9.

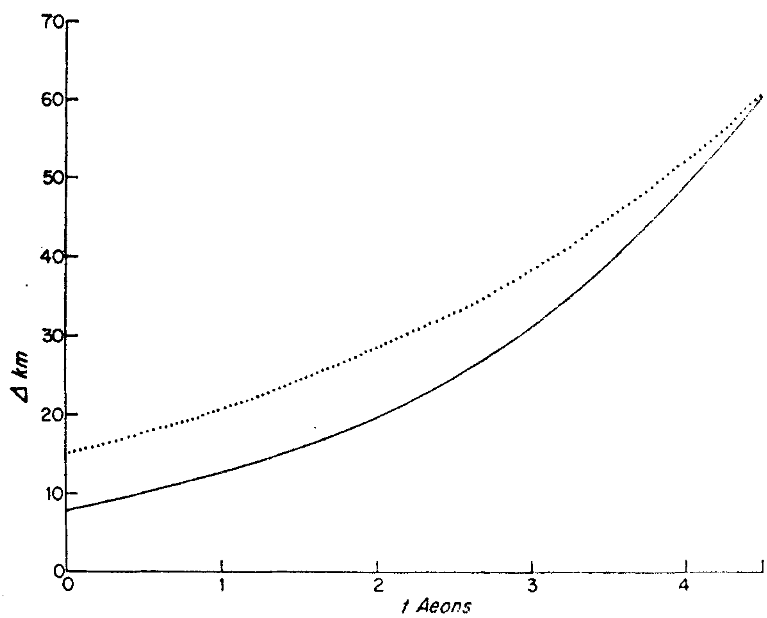

FIG. 15. The thickness of the conducting layer near a depth of 650 km as a function of time for the models in Fig. 9. The thickness of this layer depends on the change in activation energy of the creep from 4 eV–6 eV. The latter is the activation energy for MgO, which the experiments of Kumazawa *et al.* (1974) suggest is present below 700 km, together with stishovite.

distinctive, and similar crystals have been recognized in Precambrian rocks in Canada (Naldrett & Mason 1968), in Australia (Hallberg & Williams 1972) and in South Africa (Viljoen & Viljoen 1969) where they occur in pillow lavas. There now seems no doubt that melts containing up to 80 per cent of olivine occurred in the early history of the Earth, but are no longer formed. As Green (1972) has pointed out these observations require temperatures of about 1600 °C where the melting takes place, or at least 200 °C higher than that of the source regions of present day lavas. At present lavas are produced within and below the thermal boundary layer, and their temperatures do not exceed 1400 °C. This upper limit is a consequence of the adiabatic temperature distribution below the thermal boundary layer. Therefore whatever the depth from which the magmas come they will cool by adiabatic expansion to 1400 °C or less as they rise. Thus the existence of ultrabasic magmas requires a higher temperature for the main flow, and the observed difference of 200 °C is within the range calculated (Fig. 9). This explanation differs from that suggested by Green (1972) who argued that large scale excavation by meteorites could increase the thermal

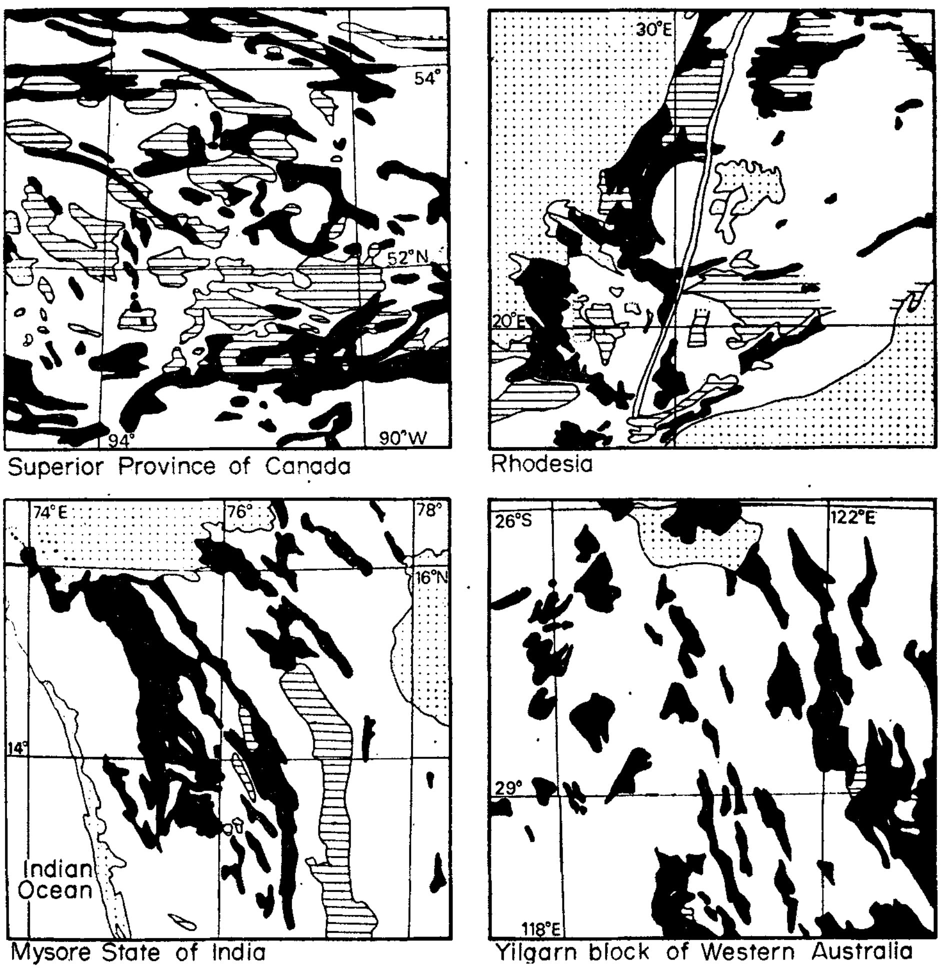

FIG. 16. Four different areas of Archean rocks, all 483 km square. Greenstone belts, shown in black, are surrounded by a ' sea of granite ' (left clear) in which younger granite rocks (ruled) have been differentiated in places. Rocks younger than Archean are dotted (Talbot 1973).

gradient sufficiently to melt olivine. Though catastrophic origins for geological structures have rarely been favoured, there is now no doubt that the Moon was being bombarded by massive meteorites 3700 My ago. Green's proposal can therefore not be rejected for this reason. However a more fundamental shortcoming is the existence of an adiabatic temperature gradient below the thermal boundary layer. Under such circumstances no amount of excavation can now produce lavas whose temperature exceeds 1400 °C. Furthermore, the large change in the thickness of the plates and thermal boundary layer shown in Figs 11 and 14, together with the increase in temperature in Fig. 9 make catastrophic origin for these rocks unnecessary. The other advantage of this proposal is that ultrabasic magmas can be produced throughout the early Precambrian. It is therefore more compatible with McGlynn & Henderson's (1970) remark that what we see preserved is the last of a long series of cycles that operated under conditions which have never again existed on Earth.

Another striking difference between the structures produced during the early Precambrian and present mountain belts is their scale. Fig. 16 from Talbot (1973) shows four examples of such greenstone belts drawn to the same scale. The Rocky Mountains or the Himalayas are very much more extensive and would cover more than the whole page when drawn to the same scale. As Anhaeusser *et al.* (1969) have pointed out, there are very close relationships between greenstone belts wherever they occur, and the structures within them are very different from those in present mountain belts such as the Alps or Himalayas. These differences could be accounted for if the surface motions were governed by the small scale convection and if large plates did not exist. This type of behaviour is possible only if the stresses produced by the small scale flow are sufficient to break up the plates. Using the model in Paper I once again the shear stress/unit area σ exerted by the flow on the plate is

$$\left.\begin{aligned}\sigma &\approx \eta u/\delta\\ &\approx \frac{\eta\kappa}{l^2}R^{3/4}.\end{aligned}\right\}\tag{15}$$

This shear stress acts on the base of the plate of thickness Δ over a distance $\approx l$, and hence the stress S within the plate is:

$$\left.\begin{aligned}S &\approx \frac{El}{kT_c}\sigma\\ &\approx \frac{(g\alpha)^{3/4}}{C_p\,T_c\,\kappa^{3/2}}\,l^2\,\nu^{1/4}\,E^{7/4};\end{aligned}\right\}\tag{16}$$

substitution for ν and E shows that for the chondritic model S has decreased by a factor of 12 during the last 4·5 Aeons, while for that of Wasserburg *et al.* (1964) the factor is 5·6. Unfortunately the stresses at present produced within the plates by the small scale flow are not known, but if they are now between 5×10^6 and 10^7 Newtons m^{-2} they would then have been around 5×10^7 Newtons m^{-2}. This stress is comparable to the largest stresses observed in present-day earthquakes, and considerably greater than that in most plate boundary shocks (Wyss 1970). It is therefore quite probable that large plates did not exist in the early Precambrian and that the small scale of the greenstone belts reflects the scale of the small scale convection. It is, however, important to remark that the absence of large plates does not imply the absence of a hard surface layer of rock; such a layer must always exist if the Earth's surface is cold. In the Precambrian, however, its strength may well have been inadequate to resist being broken up by local small scale convection. In all other respects its behaviour would be unchanged, and the relationship between tholeiitic basalts and boundary layer creation, between andesites and boundary layer destruction should still be valid in the Precambrian.

Throughout at least the second half of the Earth's history plates have probably dominated the surface deformation, and, except perhaps locally, have been sufficiently strong to withstand small scale convective stresses. Under such conditions the small scale flow is strongly affected by the plate motions, and this interaction may provide evidence of its existence. The onset of convection in a layer of fluid subject to uniform shear (plane Couette flow) has been studied extensively both in the laboratory and theoretically (see for instance Ingersoll 1966; Asai 1970). This work has shown that the stability of longitudinal rolls, whose axes lie in the direction of the shearing flow, is unaffected by the shear, whereas the critical Rayleigh number for the onset of convection as transverse rolls is increased by the shear flow. Asai (1970) has discussed the transfer of energy between the perturbation and the main flow in some detail, and has demonstrated that transverse rolls transfer energy to the main flow and for this reason are moıe difficult to excite. It is clear from both the theoretical and experimental work that longitudinal rolls are the preferred mode at moderate Rayleigh numbers. Though no analysis comparable to that of Busse (1967a) has been carried out for this problem, it appears likely that longitudinal rolls will be stabilized by the shear flow, and will not change to three-dimensional convection until the Rayleigh number considerably exceeds the value of 22 600 found for the Rayleigh–Bénard problem. Convection in a non-uniform shear flow has also been extensively studied (Ogura & Yagihashi 1969; Nakayama, Hwang & Cheng 1970; Liang & Acrivos 1970; Akiyama, Hwang & Cheng 1971) and here also longitudinal rolls are preferred. Hwang & Cheng (1972) have studied the structure of such rolls when an axial temperature gradient is present. This problem is equivalent to that of constant temperature walls and an internally heated fluid, and they find the same asymmetry in the rolls as was demonstrated in Paper I. Most of the work on convection in a sheared fluid has used fluids with Prandtl numbers less than 10. The Prandtl number of the mantle is about 10^{23}, and therefore only the limiting case for large Prandtl number is relevant to mantle convection. It is then easily demonstrated that the velocity field for longitudinal rolls is unaffected by shear. Though the Rayleigh number greatly exceeds that for which shear flow experiments have been carried out it is at least probable that longitudinal rolls are stable beneath at least those plates that are moving the most rapidly. For the reasons discussed in Paper I the rolls will probably display time dependent behaviour. In the mantle, shear is produced by the large scale flow (Fig. 1). The variation of viscosity with depth combined with the requirement that the flow should be such as to minimize viscous dissipation must result in a variation of shear with depth.

The small scale convection must take the form of longitudinal rolls only if the shear is sufficiently strong. What the critical value is for the shear to govern the flow is not known, nor has the stable pattern of convection at high Rayleigh number for internal heating been determined. However the laboratory experiments of Whitehead & Chen (1970), Hooper and Tritton, as well as the numerical experiments of Thirlby (1970) and those described in Paper I, all suggest that the pattern will consist of sinking jets of cold fluid with upwelling occurring elsewhere, and that it will be time dependent. In the upper mantle heat is conducted in from below as well as generated internally, and hence jets rising from the lower boundary are also likely to be present (see Section 5 below) in the absence of shear.

These detailed predictions of the form of the small scale flow and its dependence on shear should permit experimental tests of its existence by examining the correlation between the free air gravity anomaly and topography in the manner described by Anderson, McKenzie & Sclater (1973). Such tests should not be carried out near ridge axes because the vertical convection of heat associated with plate creation is likely to disturb the small scale flow. The small scale convection is probably most

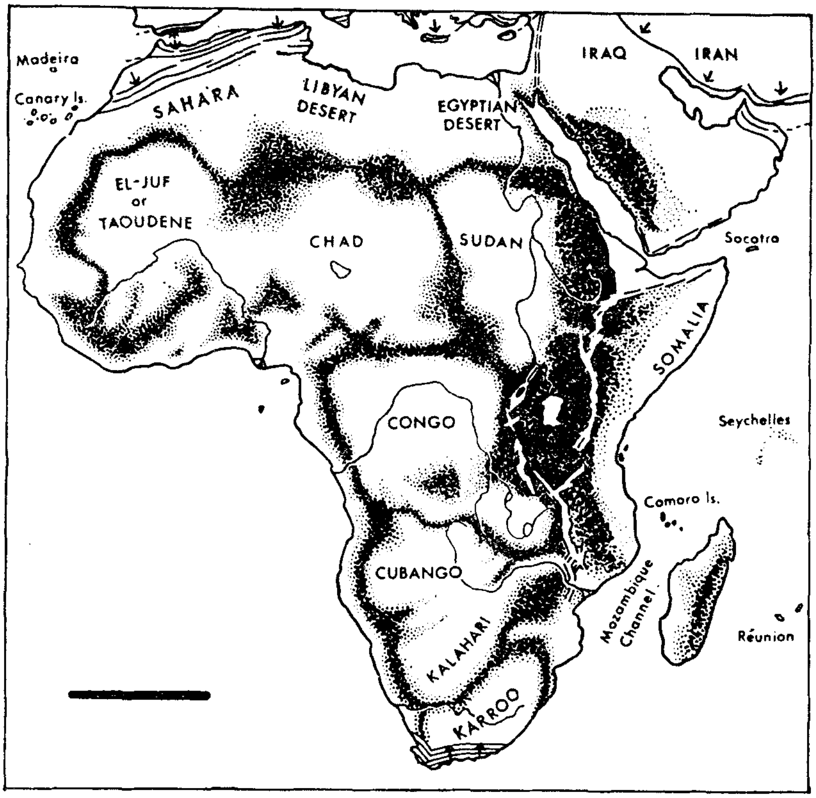

FIG. 17. Map showing the tectonic basins of Africa and intervening swells, plateaus and rift valleys (shaded). Generalized from the topography by Holmes (1965). The bar at the bottom left corresponds to 1400 km, or twice the thickness of the convecting layer of the upper mantle. The size of the basins is comparable to twice the layer thickness, and they may be the surface expression of small scale flow.

easily studied in oceanic regions where the plate age is greater than 40 My since it is in such regions that the surface heat flow must be maintained by such a flow. In such areas the correlation between the gravity and surface deformation is affected by the elastic properties of the plate on top of the convecting mantle (Appendix B) and changes sign at a wavelength of about 500 km. This feature should be particularly distinctive if it can be demonstrated to occur. Probably the best region to carry out such a test is in the Pacific, where the direction of shear is most likely to be WNW. If a long line could be obtained at right angles to this direction it should provide a test for these suggestions. In the absence of shear the surface deformation should show depressions above the sinking jets, and the basin structure of Africa (Fig. 17) may be the surface expression of such flow. It closely resembles that to be expected from Thirlby's numerical experiments if the convective layer is about 700 km thick and the flow is driven by internal heating. Fig. 17 shows little evidence of any preferred direction induced by shearing, which is in agreement with the geophysical observations on the limited movement of Africa during the last 200 My.

The discussion above has considered the form of the small scale flow beneath a large plate in regions remote from the plate boundaries. Even in these regions there are strong reasons to believe that the flow will be time dependent. Near plate boundaries the large and small scale flows will interact to produce a large variety of transient effects in three dimensions. It is therefore prudent to avoid plate boundaries when carrying out experimental tests of these suggestions.

4. Large scale flow and plate motions

In contrast to the problems discussed in the last two sections, there is now a wealth of information about the details of present plate motions and their evolution with time. It is, however, not at once obvious what parts of this knowledge are relevant to the problem of the driving mechanism of the motions. The difficulty is caused by the strength and elastic properties of the plates: except perhaps in the Precambrian, surface plate motions do not reflect in a simple manner those of the mantle below. In this section we consider the problem of how sufficient heat is converted into work to maintain the flow, and how any large scale flow can occur which produces sufficient stress to break existing plates. We show that there is no enetgetic difficulty in maintaining the large scale flow by convection. However, since continuous plate evolution always reduces the number of plates, some process must form new ones. The history of surface plate motions suggests that new plate boundaries form as ridges behind island arcs, and extend laterally by inducing large scale flow beneath unbroken plates. This process is probably restricted to the back of rapidly consuming trenches.

The conversion of heat into mechanical work by convective flow has not been discussed in detail in the literature of fluid dynamics, probably because the Boussinesq approximation corresponds to the limiting case when the conversion efficiency becomes vanishingly small (Malkus 1964; Busse 1967b). Malkus (1973) has used the Boussinesq equations to estimate the efficiency of convection in the core, but his expressions are only valid if the temperature scale height is large compared with the depth of the convecting layer. It is, however, relatively simple to obtain the corresponding equations for the general case. We shall only consider the problem of steady-state convection, and shall require the equations governing the conservation of mass and energy for a compressible fluid (see, for instance, McKenzie 1968, Appendix C)

$$\nabla . \rho \mathbf{u} = 0 \tag{17}$$

$$\rho C_P \mathbf{u} . [\nabla T - (\nabla T)_S] = \nabla . k \nabla T + H + \tau_{ij}' \frac{\partial u_i}{\partial x_j} \tag{18}$$

where τ_{ij}' is the deviatoric part of the stress tensor and $(\nabla T)_S$ is the adiabatic temperature gradient

$$(\nabla T)_S = \frac{\alpha T}{\rho C_P} \nabla P. \tag{19}$$

If the convection is confined to a volume V within a surface S such that the normal velocity vanishes on S we may integrate equation (18) over V using equation (17) and the divergence theorem to obtain an expression relating the heat generated by the shearing stress to the convected heat flux F_c when the Rayleigh number is large (Hewitt, McKenzie & Weiss 1975)

$$\frac{1}{F_c} \int_V \tau_{ij}' \frac{\partial u_i}{\partial x_j} dV = \frac{l}{H_T} (1 - \tfrac{1}{2}\mu) = \varepsilon \tag{20}$$

where H_T is the temperature scale height

$$H_T = \frac{C_P}{\alpha g} \tag{21}$$

μ the fraction of heat generated within the layer of thickness l

$$\mu = \frac{Hl}{Hl+f} \tag{22}$$

f the heat flux/unit area through the base of the layer, and ε the efficiency of conversion of heat into work. The value of ε does not depend on the relationship between τ_{ij}' and u_i, or on the details of the motion.

In the Earth $H_T \approx 2500$ km if phase changes are neglected. The large scale flow extends at least to a depth of 700 km and $\mu \approx 0{\cdot}5$, therefore $\varepsilon \approx 0{\cdot}2$. The most accurate estimate of F_c, the heat convected by the large scale flow, is from the heat lost from the Earth in plate creation on ridges. Sclater & Francheteau (1970) obtained a value of about 4×10^{12} W and this agrees with a more accurate estimate using Chase's (1972) value of $2{\cdot}9\ \mathrm{km}^2\ \mathrm{yr}^{-1}$ for the rate of plate creation. This rate could be estimated from the thermal structure of the sinking slabs, but with considerably less confidence. The rate of dissipation must therefore be less than about 10^{12} W if the convection of heat by the main flow is to maintain the motions. One major source of dissipation discussed in Paper I is the energy released by earthquakes, and is about 2×10^{11} W. The other major source is the work required to maintain the shear flow beneath the plates. It is easy to show that the viscous dissipation from the large and small scale flows are independent if the small scale flow takes the form of longitudinal rolls. Therefore if the mean velocity between the surface plates and the boundary at 650 km is $20\ \mathrm{mm\ yr}^{-1}$ and the rate of shear is constant everywhere then the dissipation is about $1{\cdot}5 \times 10^{11}$ W, if the velocity is $100\ \mathrm{mm\ yr}^{-1}$ it is about 3×10^{12} W, if the values of the parameters in Table 1 are used and the kinematic viscosity is $2 \times 10^{17}\ \mathrm{m}^2\ \mathrm{s}^{-1}$. These numbers do not suggest that there is any energetic problem in maintaining the large scale flow, even though it transports less than half the convected heat. The viscous dissipation would be greater if the shear flow was required to have no net mass transport, but would be reduced by a low viscosity layer in the upper mantle. Also the efficiency would be greater if H_T was reduced by phase changes. In the absence of such information the estimates above appear adequate.

Though the argument above shows that sufficient convective energy is probably available to maintain the flow it provides no understanding of the details of the processes by which the motion is maintained. Though the forces acting on the plates cannot be measured directly, some features of the observed motions are most easily understood if the buoyancy forces generated by the sinking slabs can be transmitted to the plate of which they previously formed part. The transmission may be achieved through elastic forces, as Elsasser (1969) suggested, or through viscous forces in the mantle acting on the base of the plates. This buoyancy force is the only obvious cause of the difference in behaviour between plates with sinking slabs and those without. If a three-dimensional model of the relative angular velocities between the major plates is constructed from the values given by Chase (1972) or by Minster *et al.* (1974), plates without slabs attached lie close to each other in angular velocity space and are joined by short vectors, whereas the Indian, Pacific and Nasca vertices are all distant from both each other and the other major plates (McKenzie & Parker 1974). This difference was also noticed by Minster *et al.* (1974) when they obtained the angular velocities of the plates relative to a frame defined by the melting spots, which they called the absolute angular velocities. The simplest explanation of this difference is that some stress is transmitted elastically from the sinking slab through the complicated shallow structure of the island arc to the plate beyond, though such a model is not easily reconciled with the increasing number of observations of earthquakes produced by normal faulting within the bending plate (Abe 1972a,b; Kanamori 1971). The other possible explanation depends on the large horizontal temperature differences beneath island arcs, which must generate vorticity on both sides of the sinking slab.

The viscous dissipation in these regions was discussed by McKenzie (1969) who showed it was greater above the sinking slab than below it. Therefore the velocity of the plate attached to the sinking slab is likely to be greater than that of the plate above the sinking slab simply because of the geometry of the flow. This explanation depends only on the viscous and not elastic forces. The importance of the sinking slabs as generators of vorticity was emphasized by McKenzie (1969), and it still appears likely that their position controls the form of the large scale flow. It is not possible to maintain the large scale flow simply by elastic forces transmitted through the lithosphere because Antarctica, Africa and South America are in relative motion but none are attached to sinking slabs of any importance.

It is also important to understand how any large scale flow originates, since it is not obviously the consequence of the small scale convection. In the Precambrian small scale flow may have been sufficiently vigorous to break the mechanical boundary layer and the Earth's surface would then have consisted of a large number of small plates somewhere between 500 and 1000 km across. There is no reason at present to suppose that large scale motions existed, superimposed on this rapid small scale motion, and it seems probable that the pattern of motion moved considerably more slowly than the individual plates. Many laboratory experiments show such behaviour even when the flow is time dependent. As the motions became less vigorous they were unable to break the mechanical boundary layer, though this change would not have occurred suddenly. Small plates must then have joined to produce larger ones but unless there was interaction between the surface motions and the convective flow to amplify large scale motions at the expense of the small, the velocities had to decrease. Since the large scale velocities are now at least as large as the small scale ones in Fig. 12 some such process must operate. Probably the detached mechanical boundary layers, or sinking slabs, generate more vorticity than the detached thermal boundary layers alone because of the greater difference in temperature, and in this way increase the velocity. The main problem is that, because ridges produce plate on both sides and trenches only consume plate on one side, any geometric pattern of ridges and trenches must evolve with time. Since a ridge and trench combine to form a single plate boundary when they meet, the number of plate boundaries, and hence plates, is steadily reduced until only one exists, covering the whole Earth. The evolution of the north eastern Pacific since anomaly 32 shows this process in action (Atwater 1970). Magnetic surveys have demonstrated that there were four plates in this area 75 My ago: the Pacific and American plates which still exist, the Farallon plate of which only fragments remain, and the Kula plate which has entirely vanished. The ridges separating the Pacific, Kula and Farallon plates migrated towards the trench systems on the edge of the American plate and the fusion of these plate boundaries has reduced the number of plates in the area and simplified the motions. Pieces of the Farallon plate remain in the north-east Pacific, and a large fragment as the Nasca plate west of South America, but if present motions continue all these fragments will be consumed or fused to the American plate. Though there may well have been long periods in the past during which there were many fewer plates than there are now, there must be some process by which new plate boundaries can form. Otherwise, since the small scale flow cannot break the mechanical boundary layer, plate tectonics ceases. Clearly since this has not happened on Earth, though it may have on other planets, there must be another process which forms new plate boundaries.

Somewhat surprisingly this process appears to be interarc spreading, which was suggested by Karig (1970) to explain the marine geology of the marginal seas behind the western Pacific island arcs. In Karig's (1971) model a new plate boundary forms close to the volcanoes because of extension produced by diapiric intrusions from below. He believed that the process differs from normal sea-floor spreading and that it probably does not produce magnetic anomalies. It now appears necessary to modify these ideas since several marginal seas have now been shown to contain presently active (Barker 1972; Sclater *et al.* 1972) or extinct (Hayes & Ringlis 1973; Ben-Avraham, Segawa & Bowin 1972; Bayer, Le Mouel & Le Pichon 1973) ridges flanked by symmetric and identifiable magnetic anomalies. These ridges are indistinguishable from those in oceans such as the Atlantic and Pacific suggesting that the observed features of ridges are the result of extension at rates of tens of millimetres a year, and will occur wherever separation is taking place. Therefore, the important feature of Karig's (1970) suggestion is that new ridges can be formed behind island arcs, and hence produce new plate boundaries to replace those being destroyed. Though no ridge formed behind a trench has yet been demonstrated to evolve into one which has formed a major ocean basin, both the Mid-Atlantic Ridge and the Carlsberg–South-east Indian Ridge and the Pacific Antarctic ridge may all have started behind the trench systems around the Pacific. The oldest part of the Mid-Atlantic ridge probably lies to the east of Middle America (Peter *et al.* 1973) and the South-east Indian ridge appears to have started east of Patagonia and the Antarctic peninsula (Fig. 18). Unfortunately the early development of both regions

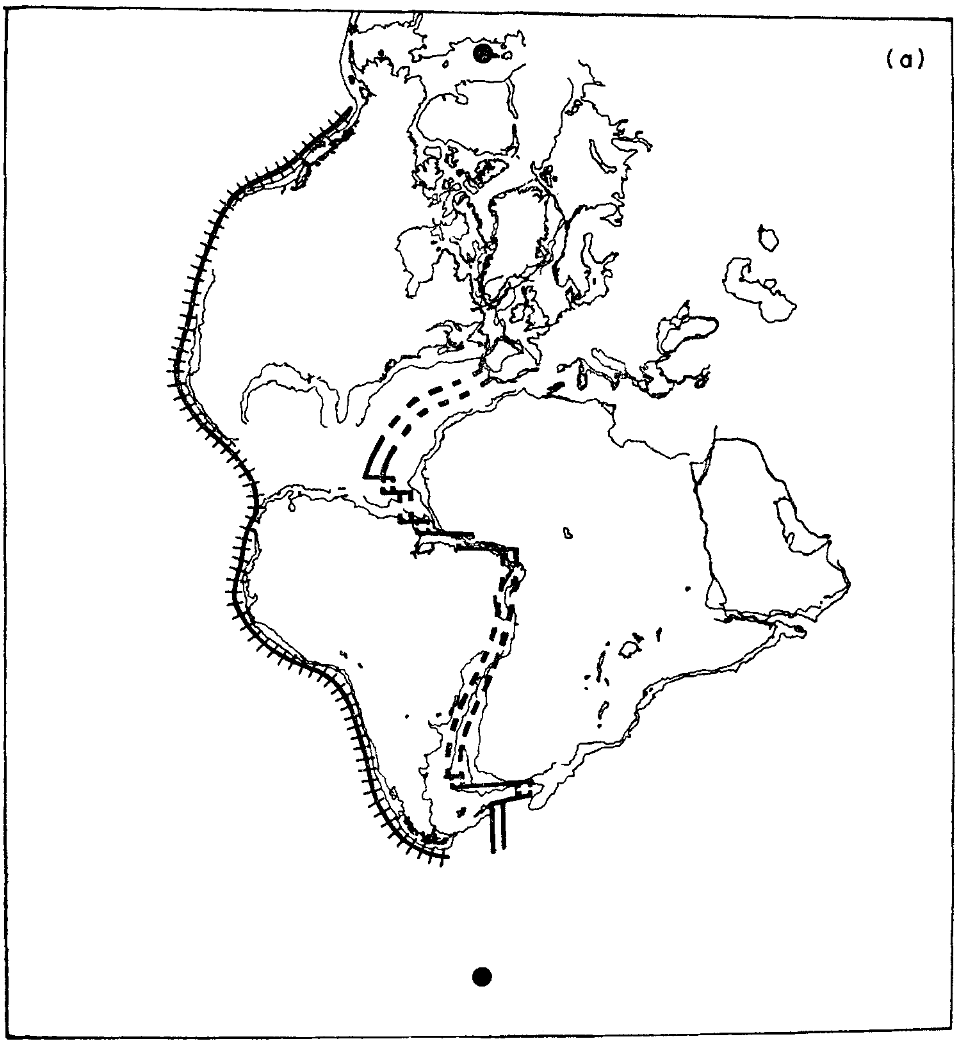

FIG. 18(a)

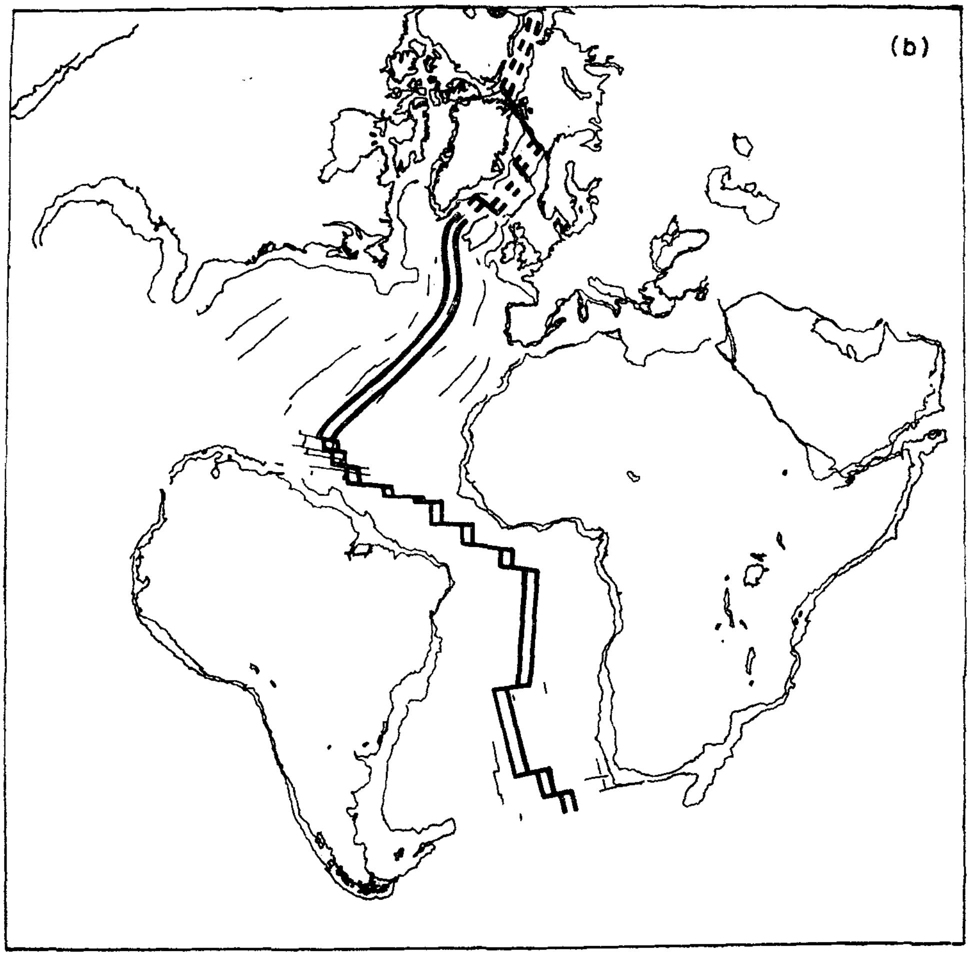

Fig. 18. The development of the Atlantic ridge system, showing established ridges (double solid lines) and their extensions (double dotted lines) at different periods (a) Mesozoic. The existence of older ridges east of the southern end of South America and of Central America is speculative (b) Anomaly 13 (Middle Tertiary). South Poles from Phillips & Forsyth (1972).

is unknown, and is not likely to be easily established because much of the sea floor may have been replaced. Once formed as short ridge segments both ridges extended beyond their bounding transform faults in a series of discrete steps. Fig. 18 shows the sequence in the Atlantic Ocean and 19 that in the Indian Ocean. The time scale for this extension is of the order of 50–100 My, and presumably is the result of the large scale flow beneath the plates on either side of the spreading ridge segment organising the flow beneath the unbroken plates beyond its limiting transform faults (Fig. 20). Once the large scale flow has been set up it can fracture the plate and start a new ridge segment. Since the break is the consequence of a large scale stress pattern it is likely to occur wherever there is an existing weakness with suitable orientation. It is therefore not surprising that the ridges round and within Africa follow Precambrian trends (Fyfe & Leonardos 1973; Dubertret 1970; McConnell 1972).

The early development of the Pacific Antarctic ridge has recently been worked out from the magnetic lineations by Molnar *et al.* (1975). This ridge also appears to have started behind island arcs close to Australia and Antarctica and therefore to be an example of the same process. No similar discussion of the East Pacific rise is possible because it is the remnant of a Jurassic ridge whose origin is at present unknown and cannot be studied using magnetic anomalies.

Though most of the Earth's surface is covered by large plates whose strength conceals the motions below, there are a few regions where the tectonics is controlled by the motion of a considerable number of small plates. The East Indies, the Caribbean, Scotia and the Mediterranean Seas are the most important, and of these the Mediterranean is the best known. In these areas the plate motions must be related to the small scale flow. The form of the convection and its velocities will be strongly affected by its ability to detach both the thermal and mechanical boundary layers from the surface, and it is therefore unlikely to resemble closely the small scale flow elsewhere. The flow in such regions probably more closely resembles the world wide pattern during the early Precambrian. The evolution of the Mediterranean is best known and displays clearly how such small ocean basins evolve. At present Africa is approaching Eurasia at a rate which increases from west to east. West of Gibraltar large thrusts are active (Fukao 1973) and the deformation is of the type that must precede the formation of an island arc. In the Eastern Mediterranean the small

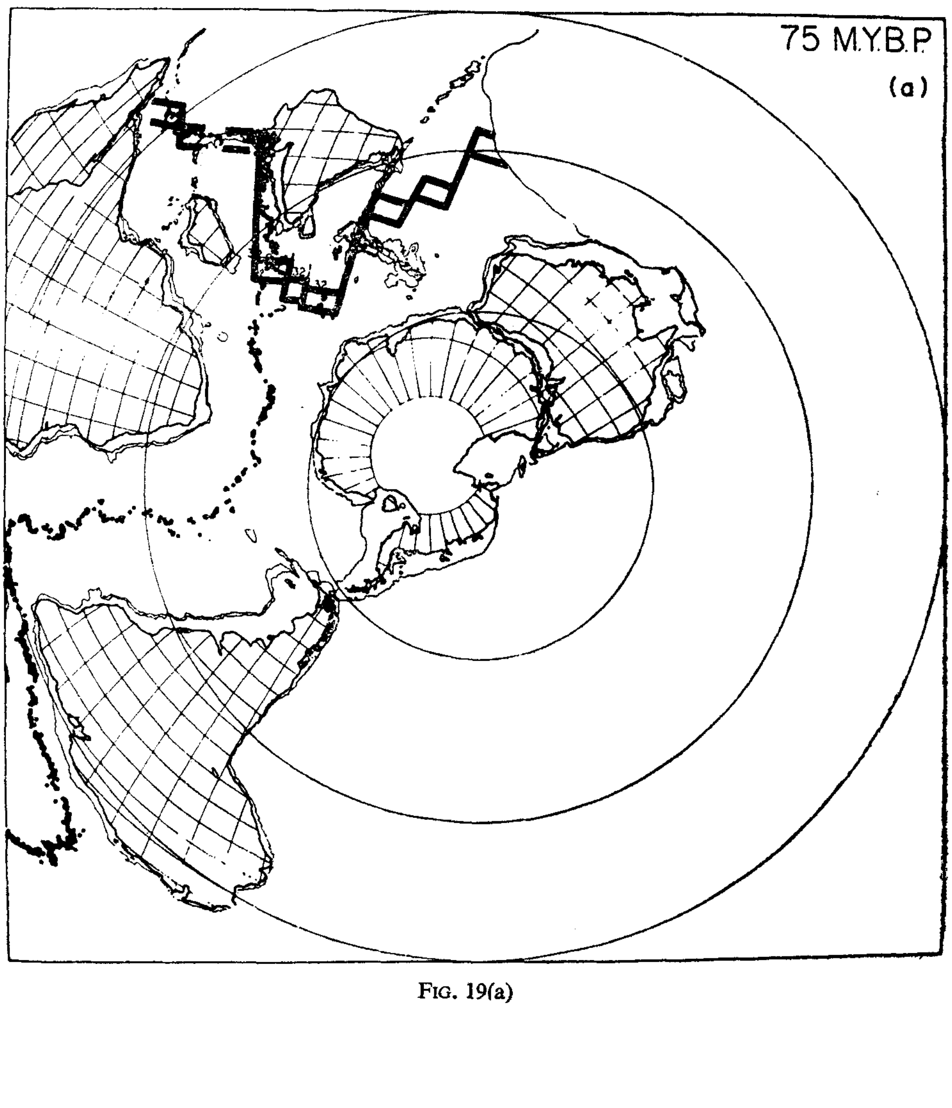

FIG. 19(a)

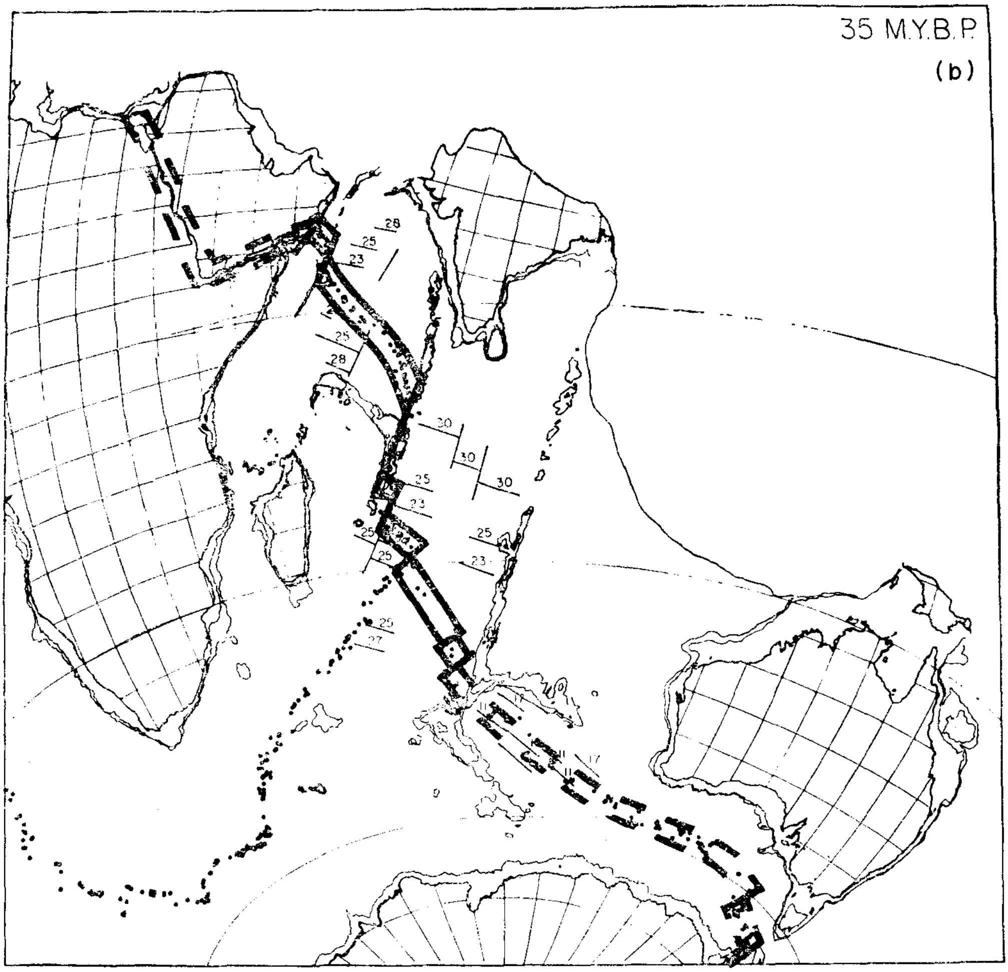

FIG. 19. The development of the Indian Ocean ridge system at different periods (a) Anomaly 32. (b) Anomaly 13. See Fig. 18.

Aegean plate has an island arc along its southern margin and a ridge along its northern (McKenzie 1972). This pattern of motion closely resembles that described by Karig (1970) except that the ridge was formed north of the volcanic arc. If continued the motion will produce a small ocean basin south of western Turkey and eastern Greece, and will carry the Aegean plate and Cretean arc south until it collides with Africa when consumption will stop. It is now known that the western Mediterranean was produced in this way, by the eastward motion of Italy, Corsica and Sardinia. A sketch illustrating this process is shown in Fig. 21. This sequence of events is started by the sinking slab, and can be repeated any number of times provided a new slab can be generated. In the Mediterranean the slow approach of Africa to Europe can continually produce slabs, and each cycle will sweep the sediment from the floor of the existing basin and generate new sediment free sea floor with magnetic anomalies (Bayer *et al.* 1973). When the motions which produced them have stopped mountain belts formed by this process should be easily recognizable because all the features of the island arc will face towards the continent rather than towards the ocean as they do in currently active arcs. An example of such a backward facing arc is known in

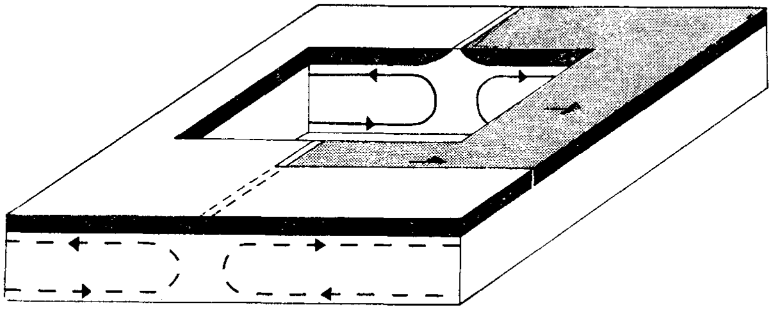

FIG. 20. The motions of two plates separated by a ridge and a transform fault must produce a large scale flow resembling that shown by the continuous flow lines. This in turn must induce flow beneath the unbroken plate beyond the transform fault shown by dashed flow lines. If the induced flow is sufficiently vigorous the ridge segment will be extended beyond the transform fault. This process could account for the observations in Figs 18 and 19.

Japan (Sugimura & Uyeda 1973), but it is not clear whether it was produced by such a cycle.

The continual creation and destruction of ocean floor by the passage of small continental fragments may also produce ophiolites sandwiched within continents. Many of the ophiolites discussed by Smith (1971) and by Dewey *et al.* (1973) were probably produced in this way, and the model described here is compatible with the ideas of these authors and of Hsu & Schlanger (1971). Furthermore, the same cycle on a larger scale could occur within a large ocean; it would merely take longer for the arcs to cross the basin. The Pacific in particular may have been crossed more than once by trenches during its long history.

It seems appropriate to end this section by examining the stress balance in the region of island arcs to determine the conditions which must be satisfied before inter-arc spreading can occur. This question was briefly discussed by Karig (1971). Since we are concerned only with the horizontal stresses within the plate behind the trench, the complicated failure of the other plate can be ignored (Fig. 22). The stress involved in thrusting between the plates is not accurately known, but is in the region of 10^7 Newtons m^{-2}. If the dip of the plane is 15° the horizontal compression resulting from this force is 4×10^7 Newtons m^{-2} which must be balanced by other forces acting close to the island arc if inter-arc spreading is to occur. The only candidates appear to be traction on the base of the plate (McKenzie 1969) and the buoyancy force resulting from the variation in plate thickness produced by vulcanism (Lliboutry 1969). The first of these arises because of the flow induced by the sinking slab, and produces a force acting towards the trench. The second exists because the density of the lava erupting from the volcanoes is less than that of the plate through which the vents must pass. This mechanism is unlikely to be able to produce a stress of greater than 2×10^7 Newtons m^{-2}, and therefore if the net force on the part of the plate between the trench and the volcanoes is to act towards the trench shear stresses on its base induced by the flow must be important. The magnitude of the effect cannot be estimated with any accuracy using the simple model (McKenzie 1969) partly because it contains a singularity and partly because the creep cannot be described by a viscosity close to the island arc. If, however, the model is valid in the region between 100 and 700 km behind the trench the resulting stress is 5×10^7 Newtons m^{-2} if the plate is 50 km thick and the consumption velocity is 100 mm yr^{-1}. A more accurate estimate is likely to be greater, but must still increase with the consumption rate. It therefore appears that the compressive forces acting on the plate margin in the region of the

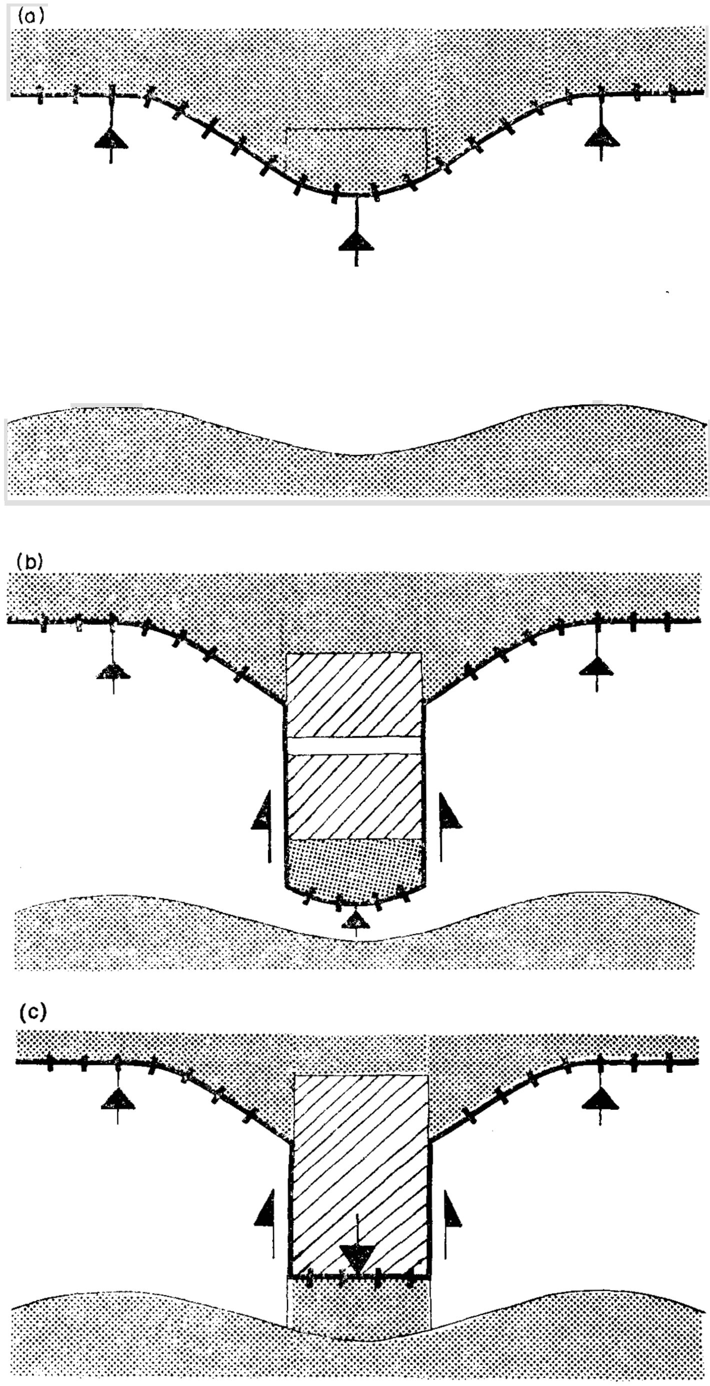

FIG. 21. The development of an oceanic basin between two approaching continents, shown dotted. (a) A piece of the continent behind the trench breaks off. (b) The resulting ridge segment forms new oceanic crust behind the trench. (c) The continental fragment collides with the other continent, consumption between them ceases and a new trench is formed elsewhere. The oceanic crust marked by diagonal lines is produced by these events, and sediment present in (a) will be removed from the ocean floor and deformed between the continental fragments.

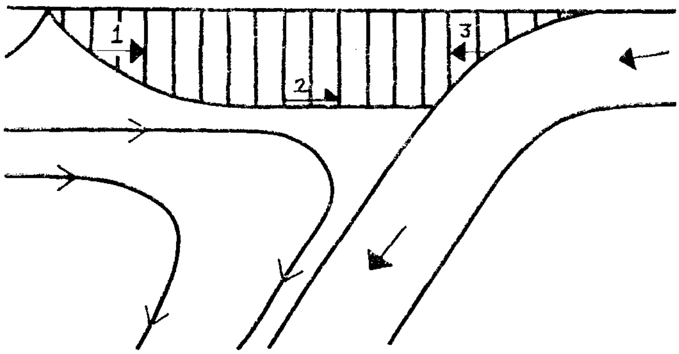

FIG. 22. The force balance on the lithosphere behind the island are (shown with vertical lines). 1. Hydrostatic force due to the presence of a spreading ridge. 2. Traction on the base of the lithosphere due to the flow induced by the sinking slab. 3. Force due to friction between the two plates which gives rise to the large shallow earthquakes with thrust mechanisms.

trench can be balanced by buoyancy and traction acting in the opposite direction, and that inter-arc spreading does not require the existence of new processes beneath island arcs. The force balance is only possible if a sinking slab with its associated flow is present, and if the consumption velocity exceeds some critical value. These restrictions could help explain why inter-arc spreading does not occur behind all arcs and why it is not always a continuous process behind those arcs where it does occur.

5. Melting spots, plumes and the boundary layer near 650 km

The existence of lines of volcanic seamounts and islands in all of the major ocean basins is a striking feature of any modern bathymetric map. Wilson (1963, 1965) proposed that they were produced as the plates passed over some fixed region in the mantle which generated large volumes of magma, here called a melting spot. Once it became clear that plate tectonics offered no obvious explanation for their existence they received little attention while the kinematic theory was developed. More recently, however, Morgan (1971, 1972) has proposed that the vulcanism forms the surface expression of plumes of hot rock rising from the core–mantle boundary. He also suggested that the plumes were stationary with respect to each other and the rotational axis, and provided the driving force required to maintain plate motions. Deffeyes (1972) has attempted to construct a model for the dynamics of the mantle which is consistent with these ideas, and a large number of papers have been published claiming plumes exist under most volcanic regions, especially those not at plate boundaries. None of this work has taken account of existing knowledge of convection in internally heated fluids, with which it is not consistent. We show that this inconsistency can be avoided if the hot rising regions originate near the boundary at 650 km, and that the existence of such regions is compatible with the model proposed above.

If Morgan's suggestions are valid, then it should be possible to determine a frame in which the melting spots remain fixed. Minster *et al.* (1974) have attempted to determine such a frame using only the azimuths of traces of the melting spots, and found some difficulty in accounting for the trace of the Icelandic melting spot. The difficulty is even greater if the Indian Ocean melting spots corresponding to the Rodrigues Ridge, and Amsterdam and St Paul's islands are used instead of the older features included by Minster *et al.* (1974). The difficulty becomes greater still when finite motions are considered, and Atwater & Molnar (1973) have demonstrated that relative motions between melting spots of a few tens of millimetres a year have occurred during the last 38 My. They therefore do not form a fixed frame. Also McElhinny (1973a) has used palaeomagnetism to show that the melting spots are not fixed with respect to the rotational axis.

At present the major volcanic features on the Earth's surface are the oceanic spreading centres. The vulcanism at such features occurs because they form lines of weakness along which mantle material upwells adiabatically and melts as it does so. The importance of lines of weakness within plates in governing the distribution of volcanoes has long been recognized by geologists, and provides a simple explanation for the linearity of some melting spots. In the Indian Ocean in particular the coincidence of the chains of volcanic seamounts forming Mauritius, Reunion and the Chagos–Laccadive Ridge on one side of India and those forming the Ninety east ridge on the other with the traces of the two largest fracture zones in the Indian Ocean (McKenzie & Sclater 1971) suggests control by lines of weakness. This explanation cannot account for the Hawaiian Island chain, but does suggest that the strength of the plates strongly affects the distribution of volcanoes.

Morgan's proposal that mantle convection is dominated by plumes extending from the core–mantle boundary was not easily reconciled with the lack of relative motion between melting spots, since if the whole mantle is convecting, it can scarcely form a reference frame, and there is no obvious substitute once the whole mantle is in motion. Perhaps a more serious problem is the convective behaviour of fluids when the heat is supplied uniformly throughout the fluid rather than at the lower boundary. A series of two-dimensional numerical experiments described in Paper I show that under these conditions the downward flow is confined to thin rapidly moving sheets, and elsewhere the hot fluid rises. Whitehead & Chen's (1970) laboratory experiments show that the three-dimensional flow takes the form of sinking jets of cold fluid with upwelling taking place elsewhere. Furthermore all experiments whether numerical or laboratory showed time-dependent motions. More recent numerical experiments on fluids with temperature dependent viscosity show the same behaviour, though the sinking regions become smaller and the velocities more rapid. This behaviour was expected for general thermodynamic reasons, since uniform heating can only produce local hot regions if work is available to concentrate the heat, and no mechanism or source of energy exists within a convecting fluid which can do so. The behaviour of a fluid heated from below is quite different: the numerical experiments described in Paper I and by Torrance & Turcotte (1971) show that hot rising plumes occur especially when the viscosity is temperature dependent. These experiments suggest that, if melting spots are the surface expression of rising jets of hot material within the mantle, they are produced at a boundary where heat is supplied from below. Though the core mantle boundary cannot yet be excluded, the boundary at a depth of 650–700 km seems at present to be more likely. Furthermore, the inability of the cold sinking slabs to penetrate below this depth does not suggest that rising plumes would easily do so. Under these conditions the small scale flow would be driven by heat conducted from below as well as supplied from within, and it is quite likely that part of the small scale convection consists of hot plumes rising through the mantle, though the flow is likely to be time dependent. The shear resulting from the large scale flow will affect the form and location of the plumes, and might well produce surface features such as the Hawaiian Islands and Emperor Seamount chain. Furthermore, if the viscosity varies with depth, flow patterns originating at 650 km in a higher layer of viscosity may well persist for 50–100 My and therefore appear to be stationary when compared to the rapid evolution of the pattern of surface motions.

It is clear that the simplifications to the problem of mantle convection proposed by Morgan (1971) must be modified in order to be consistent with geophysical observations and the observed behaviour of convecting fluids. Unfortunately the modifications remove the appealing simplicity of the original proposal. The fate of earlier proposed simplifications (Elsasser 1969; Lliboutry 1969) was similar. All are included as possible solutions to the general problem of mantle convection, but each requires modifications before it is consistent with the geophysical problem, and these modifications invalidate the approximations required to produce simplifications. The essential difficulty arises from the number of degrees of freedom necessary to describe convection at large Rayleigh number in a body as large as the Earth.

The existence of a conducting thermal boundary layer at about 650 km was shown in Section 2 to be a consequence of the change in the activation energy of the creep process. Its existence would explain both the earthquake mechanisms within the sinking slabs and also the existence of rising plumes. Though its thickness was estimated in Fig. 15, the value obtained depends entirely on the change in activation energy across the phase change, which is simply guessed by McKenzie (1967a). Whether the layer at about 650 km forms an unbroken shell or consists of caps in relative motion like the surface boundary layer is not known, though it seems likely that careful studies of the locations and mechanisms of earthquakes such as that of Barazangi *et al.* (1973) can be used to investigate this question. However, one curious feature of the deep earthquake distribution may be the result of the proposed boundary. In several places isolated shocks have occurred at a depth of between 600 and 700 km. The most famous of these is perhaps the Spanish deep earthquake (see McKenzie 1972 or Isacks & Molnar 1971), but a number are also known from South America and the south-west Pacific. They are puzzling because they do not occur in clearly defined sinking slabs which can be related to surface features. If, however, sinking slabs can penetrate part of the boundary at 650–700 km, distortion of the isotherms cannot be removed by convection and may last considerably longer than similar distortions in the convecting regions at shallower depths. The stresses resulting from such distortions could then produce shocks long after the rest of the slab had warmed up.

6. Conclusions

Perhaps the most important consequence of convection throughout the mantle is that the effective thermal time constant becomes short compared with the age of the Earth, and therefore surface regions are not thermally isolated from the deep interior. Since the rate of heat generation probably was close to the rate of heat loss throughout most of the Earth's history, the convective velocities in the early Precambrian must have been more than an order of magnitude greater on account of the greater rate of radioactive heat generation. We suggest that this difference accounts for the changes in tectonic style and in the composition of erupted lavas during the Earth's history.

There appears to be no way of reconciling the numerical and laboratory observations on convecting fluids with the behaviour of plates on the Earth's surface without requiring two scales of convective motion: a small scale motion whose horizontal extent is a few hundred kilometres, and a large scale one corresponding to the motions of the major plates. The principal surface evidence of the small scale motion is the heat conducted through the older parts of the plates whose temperature is constant. There appear to be no major problems with the energetics or stability of either scale of convection: the only difficulty which does arise is that of forming new plate boundaries. Two suggestions are made as to how this may be achieved. Little can yet be said of the plan form of the small scale flow except that it is likely to be strongly affected by the surface plate motions and to consist of some form of rolls, or cells elongated parallel to the direction of shear. Whether jets of rising fluid also occur is unknown, but their existence may indeed be the explanation of the melting spots as Morgan (1971, 1972) has supposed. It is, however, not easy to understand how such plumes could be as important as was originally supposed, nor how they could possess all the many properties ascribed to them. Fortunately recent studies of their evolution through time has required major modifications of the original ideas, and the conflict between the modified proposals and the model proposed here appears slight.

Clearly, the form of mantle convection proposed here is based on insufficient experiments, involving fluids whose properties differ radically from those of the mantle, and on a few selected observations which could prove to be wrongly interpreted. However, it does at least attempt to connect fluid dynamical ideas with geological and geophysical observations.

Acknowledgments

We are grateful to Woods Hole Institute of Oceanography and in particular the Ocean Industry Program for supporting D. P. McKenzie at Woods Hole during the period in which most of this research was carried out. Work on mantle convection at Cambridge is supported by a grant from the Natural Environment Research Council. We are grateful to Dr D. J. Tritton for allowing us to quote some unpublished experimental results.

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References

Abe, K., 1972a. Lithospheric faulting beneath the Aleutian trench, *Phys. Earth Planet. Int.*, **5**, 190.

Abe, K., 1972b. Mechanisms and tectonic implications of the 1966 and 1970 Peru earthquakes, *Phys. Earth Planet. Int.*, **5**, 367.

Akiyama, M., Hwang, G. T. & Cheng, K. C., 1971. Experiments on the onset of longitudinal vortices in laminar forced convection between horizontal plates, *J. Heat Transfer*, **93**, 335.

Anderson, R. N., McKenzie, D. P. & Sclater, J. G., 1973. Gravity, bathymetry and convection in the Earth, *Earth Planet. Sci. Letters*, **18**, 391.

Anhaeusser, C. R., Mason, R., Viljoen, M. J. & Viljoen, R. P., 1969. A reappraisal of some aspects of Precambrian shield geology, *Bull. geol. Soc. Amer.*, **80**, 2175.

Asai, T., 1970. Three-dimensional features of thermal convection in a plane couette flow, *J. Met. Soc. Japan*, **48**, 18.

Atwater, T., 1970. Implications of plate tectonics for the Cenozoic tectonic evolution of western North America, *Bull. geol. Soc. Am.*, **81**, 3513.

Atwater, T. & Molnar, P., 1973. Relative motion of hot spots in the mantle, *Nature*, **246**, 288.

Barazangi, M., Isacks, B. L., Oliver, J., Dubois, J. & Pascal, G., 1973. Descent of lithosphere beneath New Hebrides, Tonga, Fiji and New Zealand: evidence for detached slabs, *Nature*, **242**, 98.

Barker, P. F., 1972. A spreading centre in the East Scotia Sea, *Earth Planet. Sci. Letters*, **15**, 123.

Bayer, R., Le Mouel, J. L. & Le Pichon, X., 1973. Magnetic anomaly pattern in the Western Mediterranean, *Earth Planet. Sci. Letters*, **19**, 168.

Ben-Avraham, Z., Segawa, J. & Bowin, C., 1972. An extinct spreading centre in the Philippine Sea, *Nature*, **240**, 453.

Birch, F., 1965a. Speculations on the Earth's thermal history, *Bull. geol. Soc. Am.*, **76**, 133.

Birch, F., 1965b. Energetics of core formation, *J. geophys. Res.*, **70**, 6217.

Bullard, E. C., 1950. The transfer of heat from the core of the Earth, *Mon. Not. R. astr. Soc., geophys. Suppl.*, **6**, 36.

Busse, F. H., 1967a. On stability of two-dimensional convection in a layer heated from below, *J. Math. Phys.*, **46**, 140.

Busse, F. H., 1967b. The stability of finite amplitude cellular convection and its relation to an extremum principle, *J. Fluid. Mech.*, **30**, 625.

Busse, F. H. & Whitehead, J. A., 1974. Oscillatory and collective instabilities in large Prandtl number convection, *J. fluid Mech.*, **66**, 67.

Chase, C. G., 1972. The N Plate problem of plate tectonics, *Geophys. J. R. astr. Soc.*, **29**, 117.

Clarke, S. P., 1957. Radiative transfer in the Earth's mantle, *Trans. Am. geophys. Un.* **38**, 931.

Davies, D. & Sheppard, R. M., 1972. Lateral heterogeneity in the Earth's mantle, *Nature*, **239**, 318.

Deffeyes, K. S., 1972. Plume convection with an upper-mantle temperature inversion *Nature*, **240**, 539.

Dickinson, W. R. & Luth, W. C., 1971. A model for plate tectonic evolution of mantle layers, *Science*, **174**, 400.

Drever, H. I. & Johnson, R., 1957. Crystal growth of forsteritic olivine in magmas and melts, *Trans. R. Soc. Edinb.*, **63**, 289.

Dubertret, L., 1970. Review of structural geology of the Red Sea and surrounding areas, *Phil. Trans. R. Soc.* Lond., **A267**, 9.

Dewey, J. F., Pitman, W. C., Ryan, B. F. & Bonin, J., 1973. Plate tectonics and the evolution of the Alpine System, *Bull. geol. Soc. Am.*, **84**, 3137.

Elsasser, W. M., 1969. Convection and stress propagation in the upper mantle in *The applications of modern physics to the earth and planetary interiors*, ed. S. K. Runcorn, Interscience, New York.

Flasar, F. M. & Birch, F., 1973. Energetics of core formation, *J. geophys. Res.*, **78**, 6101.

Fujisawa, H., Fujii, N., Mizutani, H., Kanamori, H. & Akimoto, S., 1968. Thermal diffusivity of Mg_2SiO_4, Fe_2SiO_4, and NaCl at high pressures and temperatures, *J. geophys. Res.*, **73**, 4727.

Fukao, Y., 1969. On the radiative heat transfer and the thermal conductivity in the upper mantle, *Bull. earthq. Res. Inst.*, **47**, 549.

Fukao, Y., 1973. Thrust faulting at a lithospheric plate boundary the Portugal earthquake of 1969, *Earth Planet. Sci. Letters*, **18**, 205.

Fukao, Y., Mizutani, H. & Uyeda, S., 1968. Optical absorption spectra at high temperatures and radiative thermal conductivity of olivines, *Phys. Earth Planet. Int.*, **1**, 57.

Fyfe, W. S. & Leonardos, O. H., 1973. Ancient metamorphic-migmatite belts of the Brazilian–African coasts, *Nature*, **244**, 501.

Goettel, K. A., 1972. Partitioning of potassium between silicates and sulfide melts: experiments relevant to the earth's core, *Phys. Earth Planet. Int.*, **6**, 161.

Goldreich, P. & Toomre, A., 1969. Some remarks on polar wandering, *J. geophys. Res.*, **74**, 2555.

Gordon, R. B., 1965. Diffusion creep in the earth's mantle, *J. geophys. Res.*, **70**, 2413

Green, D. H., 1972. Archaean greenstone may include terrestrial equivalents of lunar maria? *Earth Planet. Sci. Letters*, **15**, 263.

Hallberg, J. A. & Williams, D. A. C., 1972. Archean mafic and ultramafic rock associations in the eastern goldfields region, western Australia, *Earth Planet. Sci. Letters*, **15**, 191.

Hanks, T. C. & Anderson, D. L., 1969. The early thermal history of the earth, *Phys. Earth Planet. Int.*, **2**, 19.

Hart, S. R., Brooks, C., Krogh, T. E. & Davis, G L , 1970. Ancient and modern volcanic rocks: a trace element model, *Earth Planet. Sci. Letters*, **10**, 17.

Hayes, D. E. & Ringis, J., 1973. Seafloor spreading in the Tasman Sea, *Nature*, **243**, 454.

Hess, H. H., 1962. History of the ocean basins *in Petrologic studies*: Buddington Memorial Volume, Geol. Soc. Amer., 599.

Hewitt, J., McKenzie, D. P. & Weiss, N. O., 1974. Dissipative heating in convective flows, *J. fluid Mech.*, in press.

Holmes, A., 1965. *Principles of physical geology*. Nelson.

Hsu, K. J. & Schlanger, S. O., 1971. Ultrahelvetic flysch sedimentation and deformation related to plate tectonics, *Bull. geol. Soc. Am.*, **82**, 1207.

Hwang, G. J. & Cheng, K. C., 1972. Finite amplitude convection with longitudinal vortices in plane Poiseuille flow—the effect of uniform axial temperature gradient, *Int. J. Heat Mass Transfer*, **15**, 789.

Ingersoll, A. P., 1966. Convective instabilities in plane couette flow, *Phys. Fluids*, **9**, 682.

Isacks, B. L. & Molnar, P., 1969. Mantle earthquake mechanisms and the sinking lithosphere, *Nature*, **223**, 1121.

Isacks, B. L. & Molnar, P., 1971. Distribution of stresses in the descending lithosphere from a global survey of focal-mechanism, Solutions of mantle earthquakes, *Rev. Geophys.*, **9**, 103.

Julian, B. R. & Sengupta, M. K., 1973. Seismic travel time evidence for lateral inhomogeneity in the deep mantle, *Nature*, **242**, 443.

Kanamori, H., 1971. Seismological evidence for lithospheric normal faulting—the Sanriku earthquake of 1933, *Phys. Earth Planet. Int.*, **4**, 289.

Karig, D. E., 1970. Ridges and basins of the Tonga–Kermadec Island arcs system, *J. geophys. Res.*, **75**, 239.

Karig, D. E., 1971. Origin and development of the marginal basins in the western Pacific, *J. geophys. Res*, **76**, 2542.

Kumazawa, M., Sawamoto, H., Ohtani, E. & Masaki. K., 1974. Postspinel phase of Forsterite and evolution of the Earth's mantle, *Nature*, **247**, 356.

Liang, S. F. & Acrivos, A., 1970. Stability of buoyancy-driven convection in a tilted slot, *Int. J. Heat Mass Transfer*, **13**, 449.

Lliboutry, L., 1969. Sea floor spreading, continental drift and lithosphere sinking with an asthenosphere at melting point, *J. geophys. Res.*, **74**, 6525.

Lubimova, E. A., 1958. Thermal history of the Earth with consideration of the variable thermal conductivity of the mantle, *Geophys. J. R. astr. Soc.*, **1**, 115.

McConnell, R. B., 1972. Geological development of the rift system of eastern Africa, *Bull. geol. Soc. Am.*, **83**, 2549.

MacDonald, G. J. F., 1959. Calculations on the thermal history of the Earth, *J. geophys. Res.*, **64**, 1967.

McElhinny, M. W., 1973a. Mantle plumes, palaeomagnetism and polar wandering, *Nature*, **241**, 523.

McElhinny, M. W., 1973b. *Palaeomagnetism and plate tectonics*, Cambridge University Press.

McGlynn, J. C. & Henderson, J. B., 1970. Archaen volcanism and sedimentation in the slave structural province, *Geol. Surv. Can.*, **Paper 70–40**, 31.

McKenzie, D. P., 1967a. The viscosity of the mantle, *Geophys. J. R. astr. Soc.*, **14**, 297.

McKenzie, D. P., 1967b. Some remarks on heat flow and gravity anomalies, *J. geophys. Res.*, **72**, 61.

McKenzie, D. P., 1968. The influence of the boundary conditions and rotation on convection in the Earth's mantle, *Geophys. J. R. astr. Soc.*, **15**, 457.

McKenzie, D. P., 1969. Speculations on the causes and consequences of plate motions, *Geophys. J. R. astr. Soc.*, **18**, 1.

McKenzie, D. P., 1970. Temperature and potential temperature beneath island arcs, *Tectonophys.*, **10**, 357.

McKenzie, D. P., 1972. Active tectonics of the Mediterranean region, *Geophys. J. R. astr. Soc.*, **30**, 109.

McKenzie, D. P. & Parker, R. L., 1974. Plate tectonic in ω space, *Earth Planet. Sci. Letters*, **22**, 285.

McKenzie, D. P., Roberts, J. M. & Weiss, N. O., 1974. Convection in the Earth's mantle: towards a numerical solution, *J. fluid Mech.*, **62**, 465.

McKenzie, D. P. & Sclater, J. G., 1971. The evolution of the Indian Ocean since the late Cretaceous, *Geophys. J. R. astr. Soc.*, **25**, 437.

Malkus, W. V. R., 1964. Boussinesq equations and convection energetics *WHOI geophysical Fluid Dynamics Notes.*

Malkus, W. V. R., 1973. Convection of the melting point: a thermal history of the Earth's core, *Geophys. Fluid. Dyn.*, **4**, 267.

Minster, J. B., Jordan, T. H., Molnar, P. & Haines, E., 1974. Numerical modeling of instantaneous plate tectonics, *Geophys. J. R. astr. Soc.*, **36**, 541.

Molnar, P., Atwater, T., Mammerickx, J. & Smith, S. M., 1975. Magnetic anomalies, bathymetry and the tectonic evolution of the South Pacific since the Late Cretaceous, *Geophys. J. R. astr. Soc.*, **40**, 383–420.

Morgan, W. J., 1971. Convection plumes in the lower mantle, *Nature*, **230**, 42.

Morgan, W. J., 1972. Deep mantle convection plumes and plate motions, *Bull. Am. Assoc. Pet. Geol.*, **56**, 203.

Muelberger, W. R., Denison, R. E. & Lidiak, E. G., 1967. Basement rocks in continental interior of United States, *Bull. Am. Assoc. Pet. Geol.*, **51**, 2351.

Munk, W., 1969. Once again—tidal friction, *Q. Jl R. astr. Soc.*, **9**, 352.

Nakayama, W., Hwang, G. J. & Cheng, K. C., 1970. Thermal instability in plane poiseuille flow, *J. Heat Transfer*, **92**, 61.

Naldrett, A. J. & Mason, G. C., 1968. Contrasting archaean ultramafic igneous bodies in Dundonald and Clergue Townships, Ontario, Canada, *Can. J. Earth Sci.*, **5**, 111.

Needham, R. E. & Davies, D., 1973. Lateral heterogeneity in the deep mantle from seismic body wave amplitudes, *Nature*, **244**, 152.

Ogura, Y. & Yagihashi, A., 1969. A numerical study of convection rolls in a flow between horizontal parallel plates, *J. Met. Soc. Japan*, **47**, 205.

Oversby, V. M. & Ringwood, A. E., 1972. Potassium distribution between metal and silicate and its bearing on the occurrence of potassium in the Earth's core, *Earth Planet Sci. Letters*, **14**, 345.

Peter, G., Lattimore, R. K., DeWald, O. E. & Merrill, G., 1973. Development of the Mid-Atlantic Ridge east of the Lesser Antilles Island Arc, *Nature Phys. Sci.*, **245**, 129.

Phillips, J. D. & Forsyth, D., 1972. Plate tectonics, paleomagnetism, and the opening of the Atlantic, *Bull. geol. Soc. Am.*, **83**, 1579.

Richter, F. M., 1973. Convection and the Large Scale Circulation of the Mantle, *J. geophys. Res.*, **78**, 8735.

Runcorn, S. K., 1965. Changes in the convection pattern in the Earth's mantle and continental drift: Evidence for a cold origin of the Earth, *Phil. Trans. R. Soc. London Ser. A*, **258**, 228.

Saggerson, E. P. & Owen, L. M., 1969. Metamorphism as a guide to depth of the top of the mantle in Southern Africa, *Geol. Soc. S. Africa Spec. Publ. No.*, **2**, U.M.P., 335.

Schatz, J. P. & Simmons, G., 1972. Thermal conductivity of Earth materials at high temperatures, *J. geophys. Res.*, **77**, 6966.

Sclater, J. G. & Francheteau, J., 1970. The implications of terrestrial heat flow observations on current tectonic and geochemical models of the crust and upper mantle of the Earth, *Geophys. J. R. astr. Soc.*, **20**, 509.

Sclater, J. G., Hawkins, J. W., Mammerickx, J. & Chase, C. G., 1972. Crustal extension between the Tonga and Lau Ridges: Petrologic and geophysical evidence, *Bull. geol. Soc. Am.*, **83**, 505.

Shackleton, R. M., 1973. Problems of the evolution of the continental crust, *Phil. Trans. R. Soc.*, **A273**, 317.

Slichter, L. B., 1941. Cooling of the earth, *Bull. geol. Soc. Am.*, **52**, 561.

Smith, A. G., 1971. Alpine deformation and oceanic areas of the Tethys, Mediterranean and Atlantic, *Bull. geol. Soc. Am.*, **82**, 2039.

Solomon, S. C. & Toksöz, M. N., 1973. Internal constitution and evolution of the moon, *Phys. Earth Planet. Int.*, **7**, 15.

Sugimura, A. & Uyeda, S., 1973. *Island arcs, Japan and its environs*, Elsevier.

Talbot, C. J., 1973. A plate tectonic model for the Archaean crust, *Phil. Trans. R. Soc. Lond.*, **A273**, 413.

Thirlby, R., 1970. Convection in an internally heated layer, *J. fluid. Mech.*, **44**, 673.

Torrance, K. E. & Turcotte, D. L., 1971. Thermal convection with large viscosity variations, *J. fluid Mech.*, **47**, 113.

Tozer, D. C., 1965. Thermal history of the Earth, *Geophys. J. R. astr. Soc.*, **9**, 95.

Tozer, D. C., 1972. The present thermal state of the terrestrial planets, *Phys. Earth Planet. Int.*, **6**, 182.

Tritton, D. J. & Zarraga, M. N., 1967. Convection in horizontal layers with internal heat generation experiments, *J. fluid Mech.*, **30**, 21.

Viljoen, M. J. & Viljoen, R. P., 1969. The geology and geochemistry of the lower ultramafic unit of the Onverwacht group and a proposed new class of igneous rocks, *Spec. Publ. Geol. Soc. S. Africa*, **2**, 309.

Wasserburg, G. J., MacDonald, G. J. F., Hoyle, F. & Fowler, W. A., 1964. Relative contributions of uranium, thorium and potassium to heat production in the Earth, *Science*, **143**, 465.

Watson, J. V., 1973. Effects of reworking on high-grade gneiss complexes, *Phil. Trans. R. Soc. Lond.*, **A273**, 443.

Whitehead, J. A. & Chen, M. M., 1970. Thermal instability and convection of a thin fluid layer bounded by a stably stratified region, *J. fluid Mech.*, **40**, 549.

Wilson, J. T., 1963. A possible origin of the Hawaiian Islands, *Can. J. Phys.*, **41**, 863.

Wilson, J. T., 1965. Evidence from ocean islands suggesting movements in the Earth, *Phil. Trans. R. Soc. Lond.*, **A258**, 145.

Wyss, M., 1970. Apparent stresses of earthquakes on ridges compared to apparent stresses of earthquakes in trenches, *Geophys. J. R. astr. Soc.*, **19**, 479.

Appendix A

The numerical scheme for the thermal history

The numerical scheme used to calculate the thermal history is based on the equation for heat conduction in a spherically symmetric body

$$\rho C_P \frac{\partial T}{\partial t} = \nabla . k \nabla T + H \tag{A1}$$

in conservative form this equation becomes

$$\frac{d}{dt}\left[\rho C_P \int_{r_1}^{r_2} Tr^2\, dr\right] = k\left[r_2^2\left(\frac{\partial T}{\partial r}\right)_{r_2} - r_1^2\left(\frac{\partial T}{\partial r}\right)_{r_1}\right] + \frac{H}{3}(r_2^3 - r_1^3) \tag{A2}$$

for a layer extending from r_1 to r_2. If the body is divided into N shells of equal thickness Δr, and T_n^i is the temperature mid-way between $(n-1)\Delta r$ and $n\Delta r$ at time $i\Delta t$, then a suitable finite difference form of A2 is

$$T_n^{i+1} - T_n^i = \frac{\kappa \Delta t}{\Delta r^2}\left[\frac{3}{n^3-(n-1)^3}\right](F_{n-1} - F_n) + \frac{H\Delta t}{\rho C_P} \tag{A3}$$

where F_{n-1} is the heat flux into the layer from below, F_n that out of the layer upwards:

$$F_n = -n^2(T_{n+1}^i - T_n^i). \tag{A4}$$

The value of Δt was taken to be

$$\Delta t = \frac{\Delta r^2}{6\kappa} \tag{A5}$$

since the scheme is then stable, and, in the limiting case of plane layers or $n \gg 1$, accurate to fourth, rather than second, order. The temperature is allowed to increase to θ for which the viscosity is 10^{21}, thereafter T_n^{i+1} is put equal to T_n^i to obtain F_n. If the right-hand side of (A3) is negative when $T_n^i = \theta$ then T_n^{i+1} is obtained from (A3). For all the calculations described above the core is taken to be isothermal because its effective conductivity is very much greater than that of the mantle at all temperatures. The calculations were carried out using two arrays, for T_n^i and T_n^{i+1}, updating each from the other. A buffer point above the Earth's surface was used to fix the temperature at 0 °C at $r = N\Delta r$.

Appendix B

Surface gravity and deformation from convection beneath a plate

McKenzie (1968) obtained the surface gravity field of a convecting fluid driven by an imposed horizontal temperature variation, and showed that if the viscosity was constant and the surface free, the gravity anomaly was positive over the rising fluid. If, however, the surface was not allowed to deform, the gravity anomaly was negative. A more realistic model must take into account the presence of a plate on the surface with a non-zero shear modulus. It is then obvious that convection cells whose width is large compared to the plate thickness will behave as if the plate were absent and hence the relation between gravity and surface deformation will be that given by McKenzie (1968), but if the scale is small compared to the thickness, the surface will not deform and the gravity anomaly will be negative over a rising region. The wavelength at which this change of sign occurs is important because the correlation between gravity and bathymetry is one of the few possible ways of examining mantle convection (Anderson *et al.* 1973). The wavelength can be estimated by combining a simple convective model (McKenzie 1968) with an equally simple elastic plate model (McKenzie 1967b). On the upper surface of the plate, $z = h(1+\xi \cos k'x')$, the normal and tangential stress must vanish:

$$(A+B)e^{k'} - (C+D)e^{-k'} = 1 \tag{B1}$$

$$\left[A + B\left(\frac{1}{k'}+1\right)\right]e^{k'} + \left[C + D\left(1-\frac{1}{k'}\right)\right]e^{-k'} = 0 \tag{B2}$$

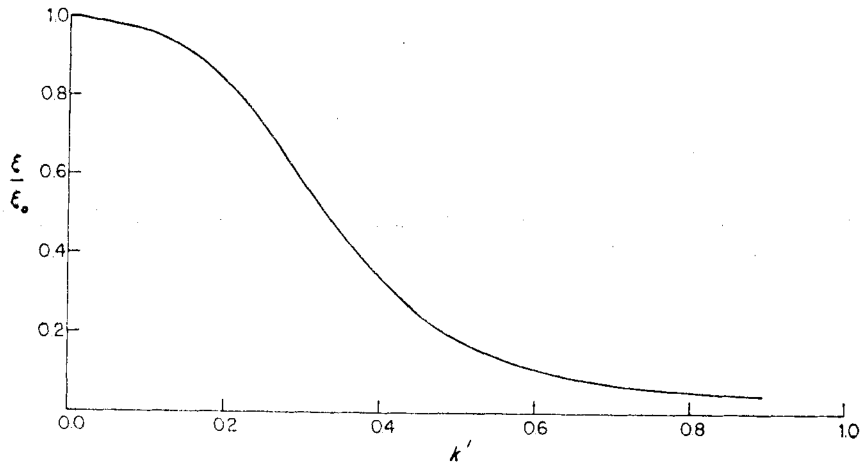

FIG. 23. The ratio of the surface deformation in the presence of a plate ξ to that in the absence of the plate ξ_0. When the deformation is produced by convection beneath the plate, thickness $2h$, by flow with a wavenumber $k = k'/h$.

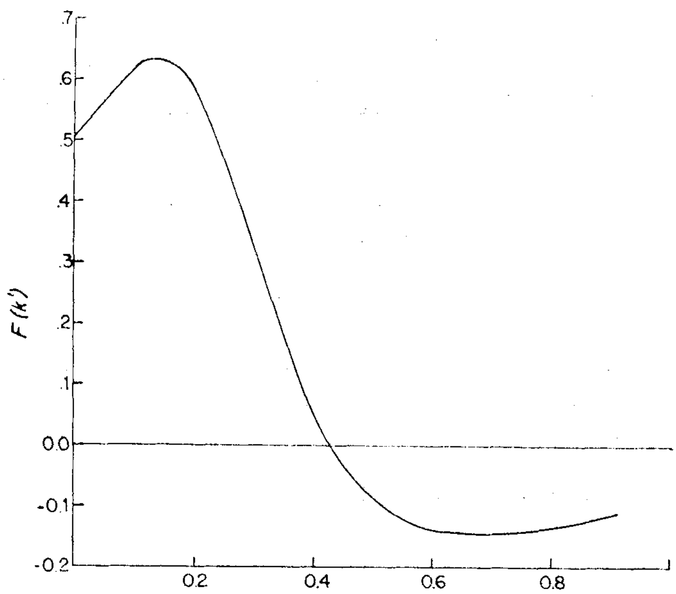

FIG. 24. $F(k')$ as a function of k' (see equation (B9)). The sign of the gravity anomaly over the rising flow is the same as that of $F(k')$, and therefore changes sign when $k' \approx 0{\cdot}4$.

where all parameters are defined in the same way as in McKenzie (1967b). On the lower surface continuity of normal and transverse stress requires

$$(A-B)e^{-k'}-(C-D)e^{k'} = \frac{s_0}{\xi} \tag{B3}$$

$$\left[A+B\left(\frac{1}{k'}-1\right)\right]e^{-k'}+\left[C-D\left(\frac{1}{k'}+1\right)\right]e^{k'} = \frac{\Sigma}{\xi} \tag{B4}$$

where S_0 and Σ must be obtained from the flow field given by McKenzie (1968) with the velocity zero at the base of the plate:

$$\Sigma = -\frac{\alpha T_0}{4k'}, \quad s_0 = \frac{3\alpha T_0}{4k'}. \tag{B5}$$

Equations B1–B4 form four equations in five unknowns, since we desire to determine ξ. The final equation is obtained by requiring the plate to be stress free when $T_0 = \xi = 0$

$$(A+B)e^{k'}+(C+D)e^{-k'} = -2Mk' \tag{B6}$$

where

$$M = \frac{\mu}{gh\rho} = \frac{V_s^2}{gh} \simeq 54 \tag{B7}$$

solution of these equations is straightforward but tedious and gives

$$\frac{\xi}{\xi_0} = \frac{3\cosh 2k'+\left(\frac{3}{2k'}-1\right)\sinh 2k'}{6k'\left[(1-2Mk')\left(1-\left(\frac{\sinh 2k'}{2k'}\right)^2\right)-\left(1-\frac{1}{2k'}\right)+\frac{e^{2k'}}{(2k')^2}\sinh 2k'\right]} \tag{B8}$$

and

$$\left.\begin{aligned} \Delta g &= \frac{3g\alpha T_0 h}{4k'a}F(k')\cos\left(\frac{k'x}{h}\right) \\ F(k') &= \frac{3}{2}\frac{\xi}{\xi_0}-e^{-2k'} \end{aligned}\right\} \tag{B9}$$

where ξ_0 is the value of ξ as $k' \to 0$ given in McKenzie (1968), g the acceleration due to gravity at the Earth's surface, a the Earth's radius, $2h$ the plate thickness and $T_0 \cos(k'x/h)$ the temperature distribution at the base of the plate driving the convection. Figs 23 and 24 show ξ/ξ_0 and $F(k')$ in (B9). Gravity becomes negative over the rising current when $k' = 0{\cdot}43$. If $2h = 75$ km the corresponding wavelength is 550 km.