NBER WORKING PAPER SERIES

SPECULATIVE ATTACK AND THE EXTERNAL CONSTRAINT IN A MAXIMIZING MODEL OF THE BALANCE OF PAYMENTS

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Working Paper No. 1437

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 1984

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ABSTRACT

This paper analyzes inevitable transitions between fixed and floating exchange-rate regimes in a balance-of-payments model where individual preferences are explicitly specified. The goal is to assess the analogy between speculative attacks in foreign exchange markets and attacks on official price-fixing schemes in natural resource markets. In discrete time the analogy with resource markets is only partially correct, for in a deterministic model the collapse of a fixed rate may be characterized by two, successive attacks. The two-attack equilibrium is peculiar to discrete-time analysis, however. In the continuous-time limit of discrete-time models there is a single attack timed so as to rule out an anticipated discrete jump in the exchange rate.

Balance-of-payments models differ from nonrenewable resource models in that foreign exchange reserves may be borrowed from abroad. The paper therefore asks whether there are limits to central-bank borrowing possibilities. In an idealized world where all private income is subject to lump-sum taxation, central-bank reserves can become infinitely negative with no violation of the public sector's intertemporal budget constraint. Nonetheless, a growth rate of domestic credit exceeding the world interest rate, if maintained indefinitely, leads to violation of the constraint in the paper's model.

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Introduction

When a central bank's foreign exchange reserves are limited, excessive domestic credit expansion must eventually undermine any attempt to maintain a pegged exchange rate through official intervention. A growing literature, beginning with the work of Krugman (1979), studies aspects of the process through which the collapse of the fixed exchange rate occurs.[1] Under rational expectations and unrestricted private capital mobility, the transition between fixed and floating exchange rate occurs on a well-defined date and is forced by a speculative attack in which private wealth owners suddenly acquire the entire central-bank reserve stock.

The literature on balance-of-payments crises grows directly out of Salant and Henderson's (1978) insight that a government attempt to fix the real price of an exhaustible natural resource will generally lead to a speculative attack on the official stockpile. Salant (1983) elaborates on this result and extends it in several directions. In Salant's (1983) setup, speculators maximize the discounted value of profits from the sale of an exhaustible resource; the discount rate is exogenous. When the government first pegs the resource price at a level greater that the competitive level, speculators sell their stocks to the government. But they repurchase the entire remaining government stockpile (thereby destroying the price-fixing scheme) on the first day that the resource's free-market price equals or exceeds its official price. Any delay would result in a resource-price path offering returns exceeding the exogenous market rate.[2]

It is tempting to proceed by analogy and argue that a balance-of-payments crisis must also occur as soon as the floating exchange rate that would equilibrate markets in the wake of a speculative attack equals or exceeds the official parity. There are two problems with this analogy. First, the world supply of any exhaustible resource is fixed at a point in time, whereas a central bank facing a perfect world capital market can always create (gross) foreign exchange reserves by borrowing. Thus, negative reserve positions are feasible. Second, the speculative attack in resource markets is derived essentially by imposing the condition that perfectly substitutable assets--that is, resource stocks and "other wealth"--must yield the same rate of return in equilibrium. Balance-of-payments crises, however, involve arbitrage between domestic money and foreign bonds, assets which presumably are not perfect substitutes. Continuous-time analyses such as Krugman's (1979) rely on the asset-price continuity principle (Calvo 1977), arguing that any discrete anticipated upward jump in the exchange rate would induce agents to flee the domestic currency "an instant before," and thus would be arbitraged away in equilibrium. As in the exhaustible resource literature, therefore, the speculative attack occurs as soon as the "shadow" market exchange rate exceeds the peg. But while this argument is plausible in continuous time, it is unclear that it extends to discrete-time models.[3]

This paper analyzes inevitable transitions between fixed- and floating-rate regimes in a balance-of-payments model where individual intertemporal preferences are explicitly specified. The equilibria studied are perfect-foresight equilibria in which (i) agents maximize utility, given expected prices and net government transfers; (ii) the government's policies staisfy its intertemporal resource constraint, given private money demand and shared expectations about prices and transfers; and (iii) markets clear on each date at the prices expected. Beside throwing light on the nature of balance-of-payments crises, the framework also yields insight into the limits to external borrowing by central banks.

With respect to balance-of-payments crises, the paper shows that in discrete time the analogy with resource models is partially correct. A speculative attack always occurs on the first date such that the "shadow" floating rate equals or exceeds the fixed rate; but equilibrium may be characterized by two attacks on successive dates. If there are two attacks, the first of these occurs the period before the exchange rate first floats as agents reduce their real balances in anticipation of the subsequent depreciation. A second speculative attack, in which the remaining official reserves are acquired by the market, collapses the fixed rate and validates the market's expectations. The two-attack equilibrium turns out to be peculiar to discrete-time analysis. In continuous time, there is a unique attack timed so as to rule out an anticipated discrete jump in the exchange rate. Thus, the solution procedure used in the descriptive continuous-time literature is supported.

With respect to central-bank borrowing possibilities, the paper shows that in an idealized world where all private income is subject to lump-sum taxes, central-bank reserves can become infinitely negative with no violation of the government's intertemporal budget contraint.[4] Even in this setting some limits to domestic-credit expansion exist, however. The government budget constraint implies that domestic credit must grow at a rate below the world real interest rate when the exchange rate is fixed and there is no economic growth. In this sense, an announced and credible policy of excessive domestic-credit expansion may leave the central bank unable to borrow reserves abroad.

The paper is organized as follows. Section I discusses the individual's optimization problem. Section II discusses the consolidated public-sector intertemporal budget constraint and the intertemporal material balance that must obtain for the economy in perfect-foresight equilibrium. Section III shows that domestic credit growth rates below the world interest rate need not lead to a breakdown of the fixed-rate regime or any violation of the official budget constraint. Section IV studies discrete-time equilibria in which crises occur, while section V extends this analysis to continuous time by allowing the market period of section IV to shrink to zero.

An important limitation of the present framework is the nonstochastic environment assumed. Section VI discusses this limitation (as well as some others), emphasizing that in a stochastic model any number of speculative attacks may precede the fixed exchange rate's collapse. An appendix containing technical details concludes the paper.

I. The Individual's Problem

The analysis is set in a small economy whose residents consume a single perishable commodity, available domestically in a fixed supply y. Agents may hold wealth in the form of domestic money, bonds denominated in domestic currency, and bonds denominated in foreign currency. A domestic-currency bond is an instrument that yields r units of domestic currency after a period and has a fixed face value equal to one unit of domestic money. A foreign-currency bond pays r* units of the foreign currency after a period and has a face value of one foreign-currency unit. The exchange rate S is the domestic-currency price of foreign money. In this single-good world, the domestic price level P and the foreign price level P* are linked by the relation P = SP*. P* and r* are constants, and P* = 1.

There is a single representative agent who maximizes a discounted sum of future instantaneous utilities. Real domestic monetary balances are assumed to yield direct utility (perhaps because they reduce transaction costs, as in Brock [1984], Feenstra [1984], or Gray [1984]); and, as ususal, instantaneous utility is derived also from consumption of the single commodity. If M denotes nominal money holdings and m real money holdings M/P, the individual's objective takes the form

$$\sum_{t=0}^{\infty} \beta^{t}[u(c_t) + v(m_t)], \qquad (1)$$

where $\beta = 1/(1 + \delta)$ and δ, the rate of time preference, is positive.[5]

A typical agent begins each period t holding M_{t-1} units of domestic money, B_{t-1} units of domestic-currency bonds, and B^*_{t-1} units of foreign-currency bonds. The agent then receives his endowment y, the nominal interest payments $r_{t-1}B_{t-1}$ and $r^*B^*_{t-1}$, and a money transfer H_t from the government; he decides on consumption c_t; and he allocates his implied end-of-period nominal assets A_t among M_t, B_t, and B^*_t. More formally, the individual's accumulation program satisfies the constraints

$$A_t - A_{t-1} = S_t y + r_{t-1}B_{t-1} + (r^*S_t + S_t - S_{t-1})B^*_{t-1} + H_t - S_t c_t, \quad (2)$$

$$A_t = M_t + B_t + S_t B^*_t. \quad (3)$$

An appendix shows how to derive necessary conditions for an optimal individual program. If corner solutions are not optimal, the conditions are

$$\frac{u'(c_t)}{S_t} = \frac{v'(m_t)}{S_t} + \frac{\beta u'(c_{t+1})}{S_{t+1}}, \quad (4)$$

$$\frac{u'(c_t)}{S_t} = \frac{\beta(1 + r_t)u'(c_{t+1})}{S_{t+1}}, \quad (5)$$

$$u'(c_t) = \beta(1 + r^*)u'(c_{t+1}). \quad (6)$$

Condition (4) states that an individual following an optimal program cannot raise his utility by adding a unit of domestic money to his portfolio at time t and spending it at time t+1. Condition (5) states that along an optimal program, utility is not raised if a domestic-currency bond is added to the portfolio at time t and sold at time t+1. Condition (6) is the analogous requirement for foreign-currency bonds.

Equations (4)-(6) naturally imply relations between the required marginal returns on the three assets. In particular, (5) and (6) yield the uncovered interest parity condition

$$(S_{t+1}/S_t)(1 + r^*) = (1 + r_t), \tag{7}$$

which equates the real returns on domestic- and foreign-currency bonds.

It is convenient to assume that the subjective time-preference rate δ satisfies

$$\delta = r^*. \tag{8}$$

Equations (6) and (8) imply that the representative agent will choose a <u>constant</u> level of consumption, $\tilde{c}$.[6] Optimality requires that $\tilde{c}$ be the highest constant consumption level consistent with the individual's available lifetime resources.

To derive the lifetime resource constraint, define real assets $a_t \equiv (M_t + B_t)/S_t + B_t^*$ and real transfers $h_t \equiv H_t/S_t$. Constraint (2) may now be written in real terms (using (7)) as

$$a_t = (1 + r^*)a_{t-1} + y + h_t - c_t - [r^* + (S_t - S_{t-1})/S_t]m_{t-1}. \tag{9}$$

Through successive substitutions, (9) implies

$$(1 + r^*)a_{-1} + \sum_{t=0}^{\infty}(1 + r^*)^{-t}(y + h_t) - \lim_{t\to\infty}(1 + r^*)^{-t}a_t$$

$$= \sum_{t=0}^{\infty}(1 + r^*)^{-t}\{c_t + [r^* + (S_t - S_{t-1})/S_t]m_{t-1}\}. \tag{10}$$

If $\lim_{t\to\infty}(1 + r^*)^{-t}a_t < 0$, the individual's debts grow (at least) at the rate $1 + r^*$, so that interest payments on existing liabilities are always financed though fresh borrowing. Such behavior would allow unbounded lifetime consumption, and is assumed to be prohibited. Thus, private expenditure plans must satisfy $\lim_{t\to\infty}(1 + r^*)^{-t}a_t \geq 0$. (For further discussion see Arrow and Kurz [1970, chapter VII] or, in an open-economy context, Obstfeld [1981].) By (10), this lifetime resource constraint may be written as

$$(1 + r^*)a_{-1} + \sum_{t=0}^{\infty}(1 + r^*)^{-t}(y + h_t)$$

$$\geq \sum_{t=0}^{\infty}(1 + r^*)^{-t}\{c_t + [r^* + (S_t - S_{t-1})/S_t]m_{t-1}\}. \tag{11}$$

Inequality (11) restricts the present value of spending on consumption and on the services of real balances to the value of real wealth at the beginning of period 0. The latter consists of initial real marketable assets $(1 + r^*)a_{-1}$ plus the present value of current and future endowments and government transfers. (Note that a_{-1} is the ex-dividend real value of marketable assets at the end of period -1. In period 0, before interest payments are made, the value of those assets is $(1 + r^*)a_{-1}$.) If an individual's plan is optimal, $\tilde{c}$ (and the path of real balances implied by (4)) will be such that (11) holds with equality.

II. The External Constraint

The previous section described the intertemporal budget constraint of the typical individual. That budget constraint restricts discounted lifetime expenditure to the present value of lifetime resources, and thus rules out unbounded consumption financed by ever-increasing borrowing. This section takes up the analogous intertemporal budget constraint for the public sector, viewed as a composite entity comprising both the fiscal and monetary authorities. As will be shown, there is no limit on the level of the government's external debt in a world with lump-sum taxation. However, the rate at which this debt grows must be less than $1 + r^*$ if the government is to satisfy its intertemporal budget constraint. Because domestic credit creation in excess of any increase in money demand leads to foreign reserve losses--that is, to external borrowing by the central bank--an upper bound on the feasible rate of domestic-credit expansion is implied.

The government's objective is to finance an exogenous path of real consumption $\{g_t\}_{t=0}^{\infty}$. Government consumption g does not enter the private utility function directly. To finance its consumption, the government may levy lump-sum taxes, spend the interest earnings on the central bank's foreign reserves, or print money (thus expanding somestic credit). Let D denote the nominal stock of central-bank domestic credit and R* central-bank reserves. The government's flow budget constraint is

$$g_t = r^*R^*_{t-1} - h_t + (D_t - D_{t-1})/S_t. \tag{12}$$

Implicit in (12) is the assumption that the fiscal authority does not itself issue debt to private agents, domestic or foreign.

Excessive domestic credit growth will lead to official foreign borrowing through central-bank reserve loss, however. Let M^d denote domestic demand for nominal balances and assume foreigners do not hold domestic money. By the central-bank balance sheet, the change in the money supply M^s is

$$M_t^s - M_{t-1}^s = D_t - D_{t-1} + S_t(R_t^* - R_{t-1}^*). \tag{13}$$

After equation of M^s and M^d, (12) can be written

$$g_t = r^* R_{t-1}^* - h_t + (M_t^d - M_{t-1}^d)/S_t - (R_t^* - R_{t-1}^*). \tag{14}$$

Equation (14) reveals that government purchases can be financed through money issue only to the extent that the demand for money is increasing. Private money demand thus limits the available revenue from seigniorage. Money creation in excess of this limit leads directly to reserve losses; and when official reserves are declining, government purchases are being financed in part through external borrowing.

After successive substitutions, (14) becomes

$$(1 + r^*)R_{-1}^* + \sum_{t=0}^{\infty}(1 + r^*)^{-t}\{\Delta m_t^d + [(S_t - S_{t-1})/S_t]m_{t-1}^d\}$$
$$- \lim_{t\to\infty} (1 + r^*)^{-t}R_t^* = \sum_{t=0}^{\infty} (1 + r^*)^{-t}(g_t + h_t), \tag{15}$$

where m_t^d is the demand for real balances and $\Delta m_t^d \equiv m_t^d - m_{t-1}^d$. If

$\lim_{t\to\infty} (1 + r^*)^{-t} R_t^* < 0$, the public sector does not repay its debts asymptotically; instead, it incurs new debt to meet the interest payments due on its previous borrowing. If such behavior is prohibited (as in the individual's case), $\lim_{t\to\infty} (1 + r^*)^{-t} R_t^* \geq 0$, and the government's intertemporal budget constraint is

$$(1 + r^*)R_{-1}^* + \sum_{t=0}^{\infty}(1 + r^*)^{-t}\{\Delta m_t^d + [(S_t - S_{t-1})/S_t]m_{t-1}^d\}$$
$$\geq \sum_{t=0}^{\infty}(1 + r^*)^{-t}(g_t + h_t). \qquad (16)$$

According to (16), the government's real outlays (consisting of government consumption and net transfers to the private sector) cannot exceed the sum of its initial foreign reserves $(1 + r^*)R_{-1}^*$ and its discounted seigniorage. The government collects seigniorage by supplying the money necessary to accommodate increases in desired real balances (at a constant price level), and by providing the money agents need to maintain a given level of real balances in the face of inflation.

Assume now that the public correctly anticipates the path of government transfers and, given a price path $\{S_t\}_{t=0}^{\infty}$, chooses consumption and real-balance paths maximizing (1) subject to (11), given a_{-1} and m_{-1}. Assume further that government outlays satisfy inequality (16) when $\{m_t^d\}_{t=0}^{\infty}$ is the real-balance path chosen by the public. In this perfect-foresight equilibrium, (11) and (16) may be added to yield

$$(1 + r^*)[a_{-1} + R_{-1}^* + (y/r^*)] + \sum_{t=0}^{\infty}(1 + r^*)^{-t}(m_t^d - m_{t-1}^d)$$
$$\geq \sum_{t=0}^{\infty}(1 + r^*)^{-t}(c_t + g_t + r^* m_{t-1}^d).$$

Because $a = m + (B/S) + B^*$, the foregoing inequality simplifies to

$$(1 + r^*)[(B_{-1}/S_{-1}) + B^*_{-1} + R^*_{-1} + (y/r^*)] \geq \sum_{t=0}^{\infty}(1 + r^*)^{-t}(c_t + g_t). \quad (17)$$

It follows that in perfect-foresight equilibrium, the present value of aggregate (private plus public) consumption cannot exceed the present value of the economy's total nonmonetary assets: net claims on foreigners held by the public and the central bank and the present value of output. If foreigners demanded domestic money, the government's resources could be augmented through the sale of domestic money abroad. As money is a nontraded asset, however, all seigniorage is extracted from domestic residents.

Given an exchange-rate path and the associated path of real money demand, a particular domestic-credit growth rule determines a path of external borrowing by the central bank. That path must be consistent with the government's intertemporal resource constraint in equilibrium. The next section examines the implications of this requirement for domestic-credit growth in the context of a fixed exchange rate regime.

III. Fixed Exchange Rates and the Limit to Domestic Credit Growth

The preceding discussion of the sectoral and aggregate constraints has been developed without reference to the exchange-rate regime. Its implications may, however, be exploited to throw light on the limit to domestic-credit creation under a pegged-rate regime. For this purpose, it is assumed that the central bank pegs the exchange rate on each date t at the constant level $\bar{S}$. (A moving peg could be analyzed as easily, but the fixed-rate case is assumed to economize on notation.) The central bank undertakes whatever foreign exchange market intervention is necessary to enforce the target rate $\bar{S}$.

Discussion of the limit to domestic-credit expansion under this regime requires a determination of the path of real money demand. If the central bank does not "overaccumulate" foreign reserves--that is, if $\lim_{t \to \infty} (1 + r^*)^{-t} R^*_t = 0$--then (16) holds with equality. Because (11) necessarily holds with equality along an optimal individual program, (17) must hold with equality in equilibrium. The equilibrium constant consumption level $\tilde{c}$ associated with the government consumption path in (17) is thus the highest constant conumption level such that (17) is an equality when $c_t = \tilde{c}$ for all $t \geq 0$. By (4), equilibrium real balances $\bar{m}$ under the pegged-rate regime are constant and satisfy

$$v'(\bar{m}) = (1 - \beta)u'(\tilde{c}) \tag{18}$$

for this equilibrium consumption level $\tilde{c}$. It is important to note for future reference that $\tilde{c}$ is invariant across exchange-rate regimes.[7]

Consider now the consequences for the bank's reserve position of alternative transfer policies $\{h_t\}_{t=0}^{\infty}$, given the paths of money demand and government consumption. A natural benchmark transfer rule is

$$h_t = r^*R^*_{t-1} - g_t , \tag{19}$$

under which the public sector rebates to citizens real foreign reserve earnings net of government consumption. Domestic inflation is zero under the fixed-rate regime, so (14) shows that reserves are constant under rule (19).[8]

The foregoing transfer rule is a member of the broader class

$$h_t = r^*R^*_{t-1} - g_t \quad + \quad (\sigma - 1)(D_{t-1}/\bar{S}), \tag{20}$$

where $\sigma-1 \geq 0$ is the domestic-credit growth rate. By (14), the reserve flow under (20) is

$$R_t^* - R_{t-1}^* = (1 - \sigma)(D_{t-1}/\bar{S}),$$

so that

$$R_t^* = R_{-1}^* + (1 - \sigma)(D_{-1}/\bar{S})(1 + \sigma + \sigma^2 + \ldots + \sigma^t)$$

$$= R_{-1}^* + (D_{-1}/\bar{S})(1 - \sigma^{t+1}). \quad (21)$$

If σ exceeds unity, the public sector in effect finances part of its spending by borrowing abroad. Because equilibrium real money demand is fixed, money transfers in excess of the value given by (19) must be purchased by the central bank to maintain the fixed exchange rate. It follows that excessive transfer payments lead to a private capital outflow which has as its counterpart the official reserve loss given by (21).

What limits does the intertemporal budget constraint (16) place on such official foreign borrowing? The program described by (20) is consistent with (16) if and only if

$$\lim_{t\to\infty} (1 + r^*)^{-t} R_t^* = -(D_0/\bar{S}) \lim_{t\to\infty} [\sigma/(1 + r^*)]^t \geq 0,$$

that is, if and only if $\sigma < (1 + r^*)$. Thus, provided the rate of domestic credit growth is smaller than the world real interest rate, the implied path of external borrowing is consistent with the government's resource constraint. In particular, there is no lower bound on the <u>level</u> of international reserves: these can become increasingly negative over time as the central bank defends the exchange rate. But there are rates of domestic-credit expansion sufficiently high that defense of the fixed exchange rate is inconsistent with the government's budget constraint. Only in these cases is there a sense in which excessive domestic-credit expansion necessarily leads to the collapse of the exchange-rate regime.

The intuition underlying this result is quite simple. As the central bank borrows reserves so that it can buy money from domestic residents, the government's external interest bill grows and the public's external interest income grows by the same amount. The government can therefore service external debt by taxing away the public's foreign interest earnings. If domestic credit is growing, however, the government is financing part of its expenditures through fresh external borrowing. Only if the rate of domestic credit growth is smaller than the world interest rate r^* is the accumulated interest on official external borrowing paid off entirely through domestic taxes over the government's planning horizon.

More formally, write the discounted present value of transfer payments to the individual as

$$\sum_{t=0}^{\infty}(1+r^*)^{-t}h_t = \sum_{t=0}^{\infty}(1+r^*)^{-t}r^*R^*_{t-1} + \sum_{t=0}^{\infty}(1+r^*)^{-t}\left[(\sigma-1)\left(\frac{D_{t-1}}{\bar{S}}\right) - g_t\right]. \quad (22)$$

With the aid of (21), the first sum on the right-hand side of (22) can be written

$$\sum_{t=0}^{\infty}(1+r^*)^{-t}r^*R^*_{t-1} = (1+r^*)R^*_{-1} + \left(\frac{D_{-1}}{\bar{S}}\right)\sum_{t=0}^{\infty}\frac{r^*(1-\sigma^t)}{(1+r^*)^t} .$$

The second sum on the right-hand side of (22) equals

$$-\sum_{t=0}^{\infty}(1+r^*)^{-t}g_t + \left(\frac{D_{-1}}{\bar{s}}\right)\sum_{t=0}^{\infty}\frac{(\sigma - 1)\sigma^t}{(1+r^*)^t} .$$

If $\sigma < (1 + r^*)$, (22) can now be written

$$\sum_{t=0}^{\infty}(1+r^*)^{-t}h_t = (1+r^*)R^*_{-1} - \sum_{t=0}^{\infty}(1+r^*)^{-t}g_t$$

$$+ \left(\frac{D_{-1}}{\bar{s}}\right)\left((1+r^*) - \frac{r^*(1+r^*)}{1+r^*-\sigma} + \frac{(\sigma-1)(1+r^*)}{1+r^*-\sigma}\right)$$

$$= (1 + r^*)R_{-1} - \sum_{t=0}^{\infty}(1 + r^*)^{-t}g_t .$$

Provided the external debt does not rise "too quickly," the path of transfers individuals expect has a present value just equal to the government's initial assets minus the present value of future government consumption. In the long run, therefore, all official foreign debt is repaid through domestic taxation.

The preceding analysis has assumed an idealized environment in which lump-sum taxes are available and all private income is subject to taxation. In practice, however, the limits to domestic-credit growth will be much more stringent than indicated above. Consider, for example, an economy in which output is produced according to the linear technology $y = \omega\ell$, where ℓ is labor supply and agents derive utility from leisure $\bar{\ell} - \ell$. If only labor income is subject to taxation, the amount of revenue that can be generated from taxes is limited. In this case, even rules of the form (20) with $1 < \sigma < (1 + r^*)$ are inconsistent with the official resource constraint, for the latter requires that tax revenue grow without bound.

The foregoing example is an extreme one, but it illustrates an important practical point. Ongoing domestic credit growth, even if moderate, causes a continual redistribution of external interest income from the government to the private sector. A necessary condition for government solvency is that private external interest earnings be observable by the government and taxable. It is precisely because this necessary condition does not hold in practice that massive capital flight so often foreshadows external financing difficulties for the government involved.

IV. Equilibrium with a Speculative Attack

This section studies the equilibrium path of an economy whose fixed exchange rate must eventually be abandoned. The previous section demonstrated that a continually declining stock of official foreign reserves need not, in itself, force the abandonment of a fixed exchange rate $\bar{S}$. For some paths of domestic credit growth, the central bank can borrow indefinitely and meet its interest charges through domestic taxation. However, I will assume here that this is not the case, and that the central bank is unable to borrow externally after its reserves, initially positive, reach zero. For example, credit might be unavailable to the bank if it announced its intention of allowing domestic credit to grow at a rate $\sigma \geq (1 + r^*)$ forever. Potential creditors would then know that the government planned to avoid paying its foreign debts in the long run. The assumption that the central bank cannot borrow reserves is quite arbitrary, though. A satisfactory discussion of the credit constraints faced by governments would require a much more complex model.[9]

The exchange rate must be allowed to float once reserves have been exhausted, and I will assume here that the floating-rate regime that follows the fixed rate's collapse is permanent.[10] To characterize the transition between regimes, it is first necessary to characterize the economy's equilibrium path under the float. This can be done with the help of the solution to the individual's problem developed above.

From equation (4), the equilibrium path of real balances must satisfy

$$u'(\tilde{c})m_t = v'(m_t)m_t + \beta u'(\tilde{c})m_{t+1}(M_t^s/M_{t+1}^s) \tag{23}$$

under the foating-rate regime. Because central-bank reserves are zero if the exchange rate floats, high-powered money is backed entirely by domestic credit:

$$M_t^s = D_t = \sigma^t D_0. \tag{24}$$

Implicit in (24) is the assumption of conveneience that the breakdown of the fixed-rate regime entails no change in the domestic-credit growth rule. Together, (23) and (24) yield

$$[u'(\tilde{c}) - v'(m_t)]m_t = (\beta/\sigma)u'(\tilde{c})m_{t+1}. \tag{25}$$

As in Brock (1974) and Obstfeld and Rogoff (1983), standard regularity conditions ensure that the unstable difference equation (25) possesses a unique positive stationary solution $\tilde{m}$. This stationary solution is the economy's saddle-path equilibrium. Given the path (24) of monetary growth, $\tilde{m}$ determines the equilibrium path $\tilde{S}_t = (\sigma^t D_0)/\tilde{m}$ for the floating exchange rate. It is assumed that while the exchange rate floats, the economy is at its saddle-path equilibrium.[11, 12]

Let T denote the first period of the floating-rate regime. If the regime-invariant growth rate of domestic credit σ were 0, we would have $T = \infty$ in the present model. I will now argue that for $\sigma > 0$, there is a unique $T < \infty$ consistent with perfect-foresight equilibrium. As will be shown below, the equilibrium path involves a sharp reduction in private money balances (and hence in official reserve holdings) at time T-1. Any reserves remaining in the central bank's portfolio after this speculative attack are acquired by the public in a second attack at time T.[13]

The argument again centers around Euler condition (4) and the fact that c_t must equal $\tilde{c}$, for all t, in equilibrium.[14] Consider the unique t such that

$$v'(\sigma^{t-1}D_0/\bar{S}) \geq u'(\tilde{c})[1 - (\beta\bar{S}\tilde{m}/\sigma^{t}D_0)] \tag{26}$$

but

$$v'(\sigma^{t}D_0/\bar{S}) < u'(\tilde{c})[1 - (\beta\bar{S}\tilde{m}/\sigma^{t+1}D_0)]. \tag{27}$$

Figure 1 illustrates its determination, demonstrating existence and uniqueness subject to familiar regularity conditions.[15] The figure implies that the date determined by conditions (26) and (27) is the unique date T on which the exchange rate first floats. This is in fact the case, and the proof must show that (i) the economy is not in equilibrium on every date if the exchange rate first floats before period T, (ii) the economy is not in equilibrium on every date if the exchange rate first floats after period T, and (iii) the economy is in equilibrium on every date if the exchange rate first floats in period T. The date of the initial speculative attack, T-1, is thus uniquely determined.

(i) Write the central bank's balance sheet as

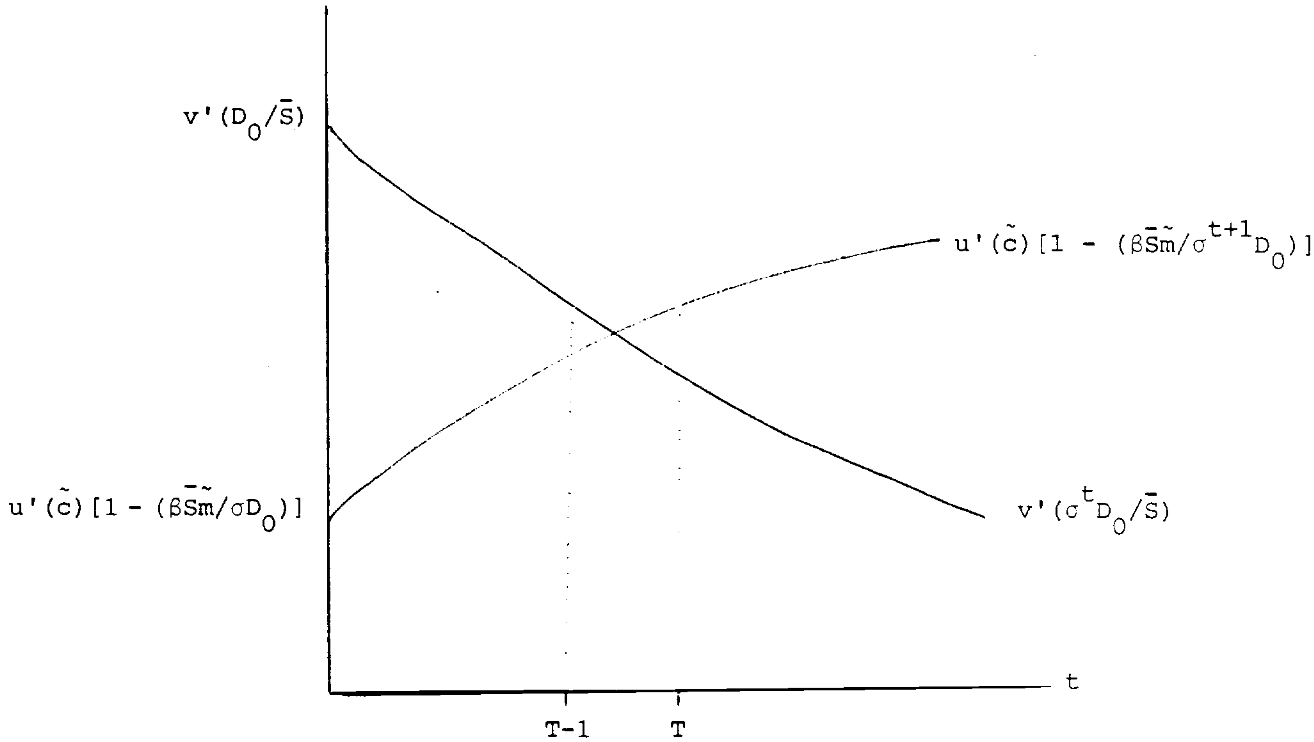
$v'(D_0/\bar{S})$
$u'(\tilde{c})[1 - (\beta\bar{S}\tilde{m}/\sigma^{t+1}D_0)]$
$u'(\tilde{c})[1 - (\beta\bar{S}\tilde{m}/\sigma D_0)]$
$v'(\sigma^t D_0/\bar{S})$
t
T-1
T

Figure 1

$$M_t^s = \bar{S}R_t^* + D_t. \tag{28}$$

Suppose the exchange rate first floats in period $T' < T$. This means that $R_{T'}^* = 0$. Because (27) does not hold for $t = T'$, (28) implies

$$\frac{u'(\tilde{c})}{\bar{S}} \leq \frac{v'(M_{T'}^s/\bar{S})}{\bar{S}} + \frac{\beta u'(\tilde{c})}{\tilde{S}_{T'+1}} . \tag{29}$$

If (29) holds as an equality, the economy is in equilibrium if $\tilde{S}_{T'} = \bar{S}$. The central bank can thus succeed in holding the exchange rate fixed at $\bar{S}$ by selling all the reserves remaining at the end of period $T'-1$. This implies that the exchange rate does not float until period $T'+1$, in contradiction to what was assumed. If (29) is a strict inequality, individual plans cannot be optimal at time T'--that is, equation (4) cannot hold--unless the market-determined exchange rate $\tilde{S}_{T'} < \bar{S}$. (This follows from the assumed strict concavity of $v(m)$.) We therefore need to ask whether the exchange rate can appreciate in the first period of floating along an equilibrium path. The answer is that it cannot. In period $T'-1$, there will be a private capital inflow--and a rise in official reserves--in anticipation of the appreciation at T'. If the exchange rate is to float in period T', agents must have an incentive to purchase the central bank's reserve stock at the period's beginning. But if they expect the exchange rate to appreciate to $\tilde{S}_T'$ instantaneously, no such incentive exists. Therefore the central bank can continue to peg through period T' if (29) is a strict inequality.

(ii) Suppose the exchange rate first floats in period $T' > T$.

As figure 1 shows, (27) must then hold for $t = T'-1$. Since the exchange rate, by hypothesis, does not float until period T', it is fixed at $\bar{S}$ during T'-1 and further, $R^*_{T'-1} \geq 0$. Therefore, by (28) and concavity,

$$\frac{u'(\tilde{c})}{\bar{S}} > \frac{v'(M^S_{T'-1}/\bar{S})}{\bar{S}} + \frac{\beta u'(\tilde{c})}{\tilde{S}_{T'}} . \tag{30}$$

An exchange rate path satisfying (30) cannot be an equilibrium, as it violates Euler equation (4). Even if reserves were zero at time T'-1, individuals would have an incentive to reduce their real balances by one unit in period T'-1, consuming the proceeds in that period and replenishing their real balances on date T'. Since consumers cannot do this in the aggregate without moving the exchange rate, the exchange rate would have to float on date T'-1, contrary to what was assumed.

(iii) Consider now the case in which the exchange rate first floats at time T. Prior to date T-1 equation (4) holds if $m_t = \bar{m}$ (see equation (18)). Up until T-1, therefore, the nominal money supply is constant and reserves steadily decline. On date T-1 there is a sharp reduction in the nominal money stock in anticipation of next period's depreciation. But by (26) there is a nonnegative reserve level R^*_{T-1} such that

$$\frac{u'(\tilde{c})}{\bar{S}} = \frac{v'[(\bar{S}R^*_{T-1} + \sigma^{T-1}D_0)/\bar{S}]}{\bar{S}} + \frac{\beta u'(\tilde{c})}{\tilde{S}_T} . \tag{31}$$

Thus (4) can hold through time T-1 with no collapse of the exchange rate, although it is possible that $R^*_{T-1} = 0$. Equation (27) now implies that on date T, there would be an excess supply of real balances at the exchange rate $\bar{S}$ even if the central bank used all its remaining reserves to purchase domestic money. The equilibrium floating exchange rate $\tilde{S}_T$ therefore exceeds $\bar{S}$, as agents expected at time T-1. In contrast to the case discussed above in the proof of (i), the public now has an incentive to acquire any remaining central-bank reserves, and does so. At the beginning of period T, agents sell domestic money in an attempt to bring their real balances to the desired level. The central bank buys as much as it can, given its remaining reserves, but this does not stem the exchange rate's rise. Agents lucky enough to have purchased reserves from the bank at price $\bar{S}$ reap a capital gain $\tilde{S}_T - \bar{S}$ per unit. If it so happens that $R^*_{T-1} = 0$, there is no second attack: because agents cannot reduce their nominal balances in this case, the money market is equilibrated entirely through the depreciation of the currency. In any event (28) and the definition of $\tilde{m}$ imply

$$\frac{u'(\tilde{c})}{\tilde{S}_T} = \frac{v'(M^S_T/\tilde{S}_T)}{\tilde{S}_T} + \frac{\beta u'(\tilde{c})}{\tilde{S}_{T+1}},$$

so that (4) holds on each date. Accordingly, the path along which the exchange rate first floats on date T is an equilibrium path for the economy.[16]

The proof that the first date of floating is uniquely determined is now complete. A potentially bothersome aspect of the equilibrium constructed is that the central bank may hold positive reserves at the end of period T-1 even though speculators who acquire them at the beginning of period T profit at the bank's expense.[17] It is worthwhile making sure that an individual cannot gain by reducing his money balances in period T-1 through the additional purchase of official reserves. Consider an individual who reduces his nominal balances by a unit in period T-1, buying reserves and then selling them for money on the market at time T. This individual forgoes monetary services worth $v'(m_{T-1}/\bar{S})/\bar{S}$ in terms of utility, but acquires interest-bearing assets worth $u'(\tilde{c})/\bar{S}$ ex dividend in utility terms (recall that $P^* = 1$). The time-T cost of repurchasing the money sold at time T-1 has a present value of $\beta u'(\tilde{c})/\tilde{S}_T$. In terms of present-value utility at time T-1, the sequence of transactions described yields a net gain of

$$\frac{u'(\tilde{c})}{\bar{S}} - \frac{v'(m_{T-1})}{\bar{S}} - \frac{\beta u'(\tilde{c})}{\tilde{S}_T} .$$

But by (31), this net gain equals zero. The individual therefore has no incentive to acquire further central-bank reserves in T-1, even if $R^*_{T-1} > 0$.

How does the domestic nominal interest rate behave along a path involving a crisis? Because real balances are constant at $\tilde{m}$ from time T onward, the exchange rate must rise at the same rate as the nominal money supply. Thus $(1 + r) = \sigma(1 + r^*)$ for $t \geq T$ is implied by the interest-parity condition (7). Similarly, $(1 + r) = (1 + r^*)$ for $t <$ T-1. So it remains to determine the nominal interest rate in period T-1.

First note that, by (25), $\tilde{m}$ is given implicitly by

$$v'(\tilde{m}) = u'(\tilde{c})[1 - (\beta/\sigma)]. \tag{32}$$

Second note that, by (31), the ratio $\tilde{S}_T/\bar{S}$ can be no greater than σ; for if it were, we would have $m_T < m_{T-1} < \tilde{m}$, contradicting the fact that $m_T = \tilde{m}$.[18] Finally note that if $R^*_{T-1} = 0$, $\tilde{S}_T/\bar{S} = \sigma$, and so $\sigma^{T-1}D_0/\bar{S} = \tilde{m}$. In this case the nominal money supply grows exactly by the factor σ between T-1 and T, and if prices rose by less than this factor we would need $\tilde{S}_{T+1}/\tilde{S}_T < \tilde{S}_T/\bar{S} < \sigma$ to induce agents to hold higher real balances in period T than in T-1. This again contradicts the fact that the economy must be in its new steady state, given by (32), in period T.

The upshot of this is that if $R^*_{T-1} = 0$, $(1 + r_{T-1}) = \sigma(1 + r^*)$. If $R^*_{T-1} > 0$, $(1 + r^*) < (1 + r_{T-1}) < \sigma(1 + r^*)$. In the first case, the economy is in its new steady state after the single speculative attack in period T-1. As will be shown in the next section, this is always true in continuous time.

V. Speculative Attacks in Continuous Time

To study attack equilibria in continuous time, I briefly rework the preceding discussion in terms of a market period of arbitrary length n. (The previous sections implicitly assumed n = 1.) Let t now denote calendar time measured in steps of length n. A continuous-time model will be obtained by taking the limit of discrete-time models as $n \to 0$.

The representative agent must consume at a constant rate c_t over period t, and he derives utility $nu(c_t)$ from doing so. Similarly, he derives utility $nv(m_t)$ from holding a constant level of real balances m_t over period t. The agent's intertemporal objective takes the form

$$\sum_{t=0}^{\infty}(1 + n\delta)^{-t/n} n[u(c_t) + v(m_t)],$$

and it is maximized subject to the constraints

$$A_t - A_{t-n} = S_t ny + nr_{t-n}B_{t-n} + (nr^*S_t + S_t - S_{t-n})B^*_{t-1} + H_t - S_t nc_t$$

and (3). The argument in the appendix can be extended to yield necessary conditions for an optimal individual plan. The ones that will concern us here are the analogues of (4) and (6), respectively:

$$\frac{u'(c_t)}{S_t} = \frac{nv'(m_t)}{S_t} + \frac{u'(c_{t+n})}{(1 + n\delta)S_{t+n}}, \tag{33}$$

$$u'(c_t) = [(1 + nr^*)/(1 + n\delta)]u'(c_{t+n}). \tag{34}$$

If once again $\delta = r^*$ (assumption (8)), (34) implies that the path of consumption is flat. The equilibrium constant consumption level $\tilde{c}$ is determined by a national intertemporal resource constraint similar to (17).

To analyze attacks in continuous time, it is again helpful to first analyze the behavior of the economy under a regime of pure floating. Let the fixed, regime-invariant growth rate of domestic-credit growth be μ; thus,

$$D_{t+n}/D_t = 1 + n\mu. \tag{35}$$

The analogue of equation (25) follows from (33) and (35):

$$[u'(\tilde{c}) - nv'(m_t)]m_t = u'(\tilde{c})m_{t+n}/(1 + n\delta)(1 + n\mu). \tag{36}$$

Equation (36) has a unique positive stationary value, denoted $\tilde{m}(n)$. This constant level of real balances defines the economy's saddle-path equilibrium. $\tilde{m}(n)$ is given implicitly by the equation

$$v'[\tilde{m}(n)] = \frac{u'(\tilde{c})[n(\delta + \mu) + n^2\delta\mu]}{n(1 + n\delta)(1 + n\mu)} . \tag{37}$$

By taking the limit of the right-hand side of (37) as $n \to 0$, one finds that the equilibrium level of real balances in the continuous-time floating-rate model, $\tilde{m}(0)$, satisfies

$$\frac{v'[\tilde{m}(0)]}{u'(\tilde{c})} = \delta + \mu. \tag{38}$$

We may now study the transition between the fixed- and floating-rate regimes. Consider again a model with market period of length n. As in the previous section (equation (26)), the exchange rate first floats on the latest date T(n) such that

$$\frac{v'[(1 + n\mu)^{[T(n)-n]/n} D_0/\bar{S}]}{u'(\tilde{c})} \geq n^{-1}\{1 - [\bar{S}/(1 + n\delta)\tilde{S}_{T(n)}]\}. \quad (39)$$

T(n) is a continuous function of the market period n.

As was demonstrated at the end of the previous section, the exchange rate on the first day of floating, $\tilde{S}_{T(n)}$, must satisfy the inequality

$$\tilde{S}_{T(n)}/\bar{S} \leq 1 + n\mu. \quad (40)$$

Only when $R^*_{T(n)-n} = 0$ is (40) an equality; and in this case (39) holds with equality as well. It is clear that $\tilde{S}_{T(n)}$ is a continuous function of n and that

$$\lim_{n\to 0} \tilde{S}_{T(n)} = \bar{S}. \quad (41)$$

Now $\tilde{S}_{T(n)} = [(1 + n\mu)^{T(n)/n} D_0]/\tilde{m}(n)$, and, by (37), $\tilde{m}(n)$ is a continuous function of n for $n \geq 0$. Equation (41) may therefore be written

$$\lim_{n\to 0} [(1 + n\mu)^{T(n)/n} D_0]/\tilde{m}(n) = e^{\mu T(0)} D_0/\tilde{m}(0) = \bar{S}. \quad (42)$$

According to (42), the regime change for a continuous-time model occurs on the unique date T(0) such that $e^{\mu T(0)} D_0/\bar{S} = \tilde{m}(0)$. This date alone has the property that a single speculative attack in which the public acquires all the central bank's reserves moves the economy to its new saddle-path equilibrium with no jump in the exchange rate.

To summarize, the present continuous-time maximizing model supports the exchange-rate continuity condition invoked by Krugman (1979) and others to study crises in descriptive models. In contrast to the previous discrete-time analysis the regime change involves a single speculative attack, timed so as to move the economy to its floating-rate equilibrium with no discrete jump in the exchange rate. With continuous trading and perfect foresight, therefore, the analogy between attacks in resource markets and in the foreign exchange market is quite close. The reason is that it is only in continuous time that an anticipated discrete asset-price jump generally entails "abnormal" profit opportunities.

VI. Limitations and Extensions

This paper has described the collapse of a fixed exchange rate in an intertemporal general-equilibrium model with perfect foresight. The goal of the exercise was to examine critically the analogy between speculative attacks in resource markets, as studied by Salant and Henderson (1978) and Salant (1983), and attacks on a moribund fixed exchange rate. While the analogy was found to be close in continuous time, a discrete-time model led to the possibility of two, successive speculative attacks, the first occurring the period before the exchange rate begins to float.

An important limitation of the analysis is the deterministic environment assumed throughout. A feature of the discrete-time equilibria studied above is that there is exactly one period in which the exchange rate is fixed but the equilibrium domestic interest exceeds the exogenous world rate. In this period before the exchange rate first floats, domestic residents reduce their real balances, possibly (but not necessarily) driving the central bank's reserves to zero. In a stochastic model such as that of Flood and Garber (1984a), the domestic interest rate exceeds the world rate whenever there is a possibility of an exchange-rate collapse next period. If speculation is defined as any private acquisition of reserves in anticipation of a possible collapse, it is clear that a stochastic setup in which money demand is a decreasing function of the home interest rate can be characterized by arbitrarily many speculative "attacks." The sharp contrast with the single attack of the resource model arises here because money and bonds are not perfect substitutes. But it is possible that a similar result could be derived in resource models such as Salant's (1983) if additional assets with stochastic returns were introduced and if the speculator's portfolio decision were addressed explicitly.

A further limitation of the analysis is that it is restricted to a policy environment of a particularly simple nature: the central bank pursues a constant rate of domestic credit expansion regardless of the exchange-rate regime, and makes no effort to avoid the fixed rate's collapse other than intervening until its reserves are exhausted. No claim is made that this policy is an any sense optimal. Rather, the aim is to create a situation, similar to that arising in nonrenewable resource markets, in which a breakdown of the price-fixing scheme is inevitable. It would be enlightening to introduce official preferences and study whether speculative attacks can result from the authorities' pursuit of optimal policies. As a preliminary step it would be useful to extend the analysis of this paper to more general, but still exogenous, policy environments.

The paper devoted considerable attention to the official inter-temporal budget constraint and its implications for feasible domestic-credit growth under a fixed exchange rate. In principle there need be no lower bound on a central bank's foreign reserves. External borrowing in support of the exchange rate can continue indefinitely provided domestic taxes rise sufficiently to service the debt incurred. There are rates of domestic-credit growth so high, however, that the indefinite support of the exchange rate is inconsistent with public-sector solvency.

Appendix

This appendix shows how to derive the necessary conditions (4)-(6) in the text. Let λ_t denote the utility shadow price of nominal saving (2) at time t and γ_t the Lagrange multiplier for the portfolio constraint (3). Necessary conditions for an optimal individual program are found by maximizing the Lagrangian expression

$$\sum_{t=0}^{\infty}\beta^t\{[u(c_t) + v(m_t)] - \lambda_t[A_t - A_{t-1} - S_t y - r_{t-1}B_{t-1}$$
$$- (r^*S_t + S_t - S_{t-1})B^*_{t-1} - H_t + S_t c_t] + \gamma_t(A_t - M_t - B_t - S_t B^*_t)\}$$

with respect to c_t, M_t, B_t, B^*_t, and A_t, for all t. This yields

$$u'(c_t) = \lambda_t S_t, \tag{A1}$$

$$v'(m_t) = \gamma_t S_t, \tag{A2}$$

$$\beta\lambda_{t+1} r_t = \gamma_t, \tag{A3}$$

$$\beta\lambda_{t+1}(r^*S_{t+1} + S_{t+1} - S_t) = \gamma_t S_t, \tag{A4}$$

$$\beta\lambda_{t+1} + \gamma_t = \lambda_t. \tag{A5}$$

Equation (4) follows immediately from (A1), (A2), and (A5). One obtains (5) by combining (A1), (A4), and (A5). Similarly, combination of (A1), (A3), and (A5) leads to (6).

Footnotes

*Financial support from the National Science Foundation (under grant SES-8318747) is acknowledged with thanks.

1. See, for example, Blanco and Garber (1982), Connolly and Taylor (1984), Cumby and van Wijnbergen (1983), Flood and Garber (1984a), Obstfeld (1984a, b), and Wyplosz (1983). Flood and Garber (1984b) study a related topic, the possible collapse of a gold-standard monetary regime.

2. In the free-market equilibrium of Salant's (1983) model, the resource price rises at a rate equal to the exogenous discount rate; consumption demand is a declining function of price; and the initial equilibrium price equates the total supply to cumulative consumption demand over the horizon of the economy. If an attack occurs on the first date that the "shadow" free-market price of the resource exceeds the official price, the remaining stock equals cumulative demand if the market price rises at the rate of interest thereafter. If speculators delay their attack (by one period, say), next period's consumption, a function of the official price, exceeds the level that would have prevailed if the attack had already occurred. Since the remaining supply of the resource is then lower as well, its price must jump by more than the rate of interest when the attack takes place. Otherwise, cumulative future demand would exceed the initial supply. In continuous time, the integer problem peculiar to discrete-time modeling disappears and the speculative

attack takes place with no jump in the resource price. A similar integer problem lies behind the "two-attack" equilibrium of the monetary model discussed below. That problem, too, disappears in continuous time. Note that Salant (1983), in an appendix, offers a maximizing rationale for his model.

3. Flood and Garber (1984a), for example, argue in a discrete-time context that the attack must occur as soon as the "shadow" market rate exceeds the peg $\bar{S}$ because "agents may purchase reserves from the central bank at price $\bar{S}$ and resell those reserves immediately on the post-collapse market" at a higher price. (A similar argument appears in Flood and Garber 1984b.) The foregoing argument shows at best that _if_ the collapse of the fixed rate is expected, it will not pay for a single individual to refrain from joining the attack. But it does not demonstrate that there is no other equilibrium. Obstfeld (1984b) gives an example of an economy with multiple equilibria in which the "shadow" exchange rate can exceed the peg without necessarily inducing a run. Because of the particular policy environment assumed by Flood and Garber (1984a), the exchange rate must indeed float on the first day that the "shadow" rate passes through $\bar{S}$. For a complete proof, see Obstfeld (1984b). Not surprisingly, the proof parallels the discussion in section IV, below.

4. McCallum (1984) derives a similar result in a closed-economy model. He shows that even if the inflation tax is not used, the stock of government debt can rise forever (at a rate less than the real interest rate) provided other taxes are raised over time to meet debt service. See also Liviatan (1984) and Sargent (1984).

5. As ususal, u(c) and v(m) are strictly concave, twice continuously differentiable, and satisfy the Inada conditions.

6. With an arbitrary nonseparable utility function U(c,m), it is no longer true that the constancy of the marginal utility of consumption implies that consumption is constant. Thus, the separability assumption embodied in (1) purchases simplicity, but it is not innocuous. This issue is pursued further, in context, below; see footnote 14.

7. This follows from the assumed sparability of the instantaneous utility function. See footnote 6.

8. If P* were rising, or if the pegged exchange rate were adjusted upward over time by the central bank, the authorities would be able to collect seigniorage equal to $\bar{m}$ times the rate of domestic price inflation.

9. Imperfect information problems of the sort studied by Stiglitz and Weiss (1981) would be at the heart of such a discussion.

10. Obstfeld (1984a) studies a transitory floating-rate regime.

11. Obstfeld and Rogoff (1983) show that the saddle-path equilibrium is the unique equilibrium of the economy if the government guarantees some minimal real redemption value for its currency.

12. Once reserves are zero and the exchange rate floats, the transfer rule (20) becomes $h_t = -g_t + (\sigma - 1)(D_{t-1}/\tilde{S}_t) = [(\tilde{S}_t - \tilde{S}_{t-1})/\tilde{S}_t](M^s_{t-1}/\tilde{S}_{t-1})$ $= [(\tilde{S}_t - \tilde{S}_{t-1})/\tilde{S}_t]\tilde{m}$. Thus (14) holds in the saddle-path equilibrium, with all government consumption being financed through lump-sum taxes and seigniorage. Constraint (15) holds trivially because reserves are constant at zero.

13. One could argue that even if the exchange rate remains at $\bar{S}$ in period T-1, T-1 is really the first period of floating if the initial attack drives reserves to zero. The proofs below could be modified to reflect this alternative definition, but no substantive differences would emerge. This inconsequential ambiguity disappears in continuous time.

14. If the instantaneous utility function in (1) were not separable in c and m, this would not be true (see footnote 6). Even though equilibrium consumption is invariant with respect to unanticipated, permanent changes in inflation, it is not invariant with respect to anticipated changes if $U_{cm} \neq 0$. While consumption would therefore be flat after the regime change, both consumption and real balances would be moving beforehand in anticipation of the heightened inflation to come. Further, the current account would not be balanced. The constant value of consumption prevailing after the regime change would depend on the country's stock of net foreign claims at the commencement of floating, and thus on the history of the current account.

15. If the two curves in figure 1 do not intersect at a positive t, an attack must occur at t = 0.

16. Along with the necessary conditions for individual optimality, the ususual sufficient conditions also hold along the path constructed (Arrow and Kurz 1970).

17. These profits disappear in the continuous-time model. Strictly speaking, the analysis of the individual's problem should have incorporated his expectation of being able to buy reserves from the bank on date T at a price below the market price. Aside from the usual distributional issues, this modification would raise no problems provided the individual anticipated buying a finite, exogenous quantity of underpriced official foreign reserves. In particular, the necessary conditions exploited above would still hold: by offering speculators a one-sided bet on date T, the central bank in effect offers them a transfer payment. It should be clear that for this reason, the equilibrium consumption level $\tilde{c}$, which is determined by (17), is unaffected.

18. This result will play a key role in the next section.

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