Spice Thermal Subcircuit of Multifinger HBT derived from Ritz Vector Reduction Technique of 3D Thermal Simulation for electrothermal modeling

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Abstract—This paper deals with the integration of a reduced thermal model based on tree dimensional Finite Element (FE) thermal simulation into circuit simulator for accurate prediction of electrothermal behavior of power devices. The reduced thermal model based on the Ritz vectors approach is easily usable in any kind of circuit simulator because it is described by a spice format subcircuit. The model has been successfully experimented with the ADS simulator. Electrical based thermal measurements of transient temperature response have successfully validated our approach.

I – Introduction

GaAs based Heterojunction Bipolar Transistors have been widely accepted by the microwave industry as an excellent candidate for many high frequency applications. Due to its high current density, the HBT has been used to fabricate high performance microwave amplifier modules. On the other hand, the HBT's performance is also limited by its thermal characteristics [1]. The combination of his high current density during operation and the relatively low thermal conductivity of the GaAs substrate elevates the junction temperature severely, which may lead to the failure of the power HBT's, or to instabilities known as « collapse phenomenon » (due to the non homogeneous temperature distribution along the emitter fingers). Another drawback of the thermal behavior is the contribution of dynamic thermal effects to RF performance in pulsed radar for example.

An accurate prediction of these behaviors is possible by simulating the coupled electrical and thermal systems simultaneously.



Fig.1: Coupled electrothermal system

The simplest network usually implemented consists of a thermal resistance and capacitance cell for the whole transistor, which appears to be unsufficient to take account for the temperature distribution along the fingers and the different time constants in the temperature response. It's why we introduce a thermal admittance matrix **Yth** issued from 3D thermal simulations, which verifies:

$$\mathbf{F} = \mathbf{Y} \mathbf{t} \mathbf{h} \cdot \mathbf{d} \tag{1}$$

Where **F** represents the dissipated power vector and **d** the temperature vector at each finger for example. The originality of our model relies on the fact that on one hand it is fully physical based on a 3D analysis of thermal equation (1) and on the other hand it is fast and easily usable in any kind of circuit simulator because it is described by a spice format subcircuit.

$$div(\kappa(T) \cdot gradT) + g = \rho C_p \frac{\partial T}{\partial t}$$
⁽²⁾

In equation (1) κ is the thermal conductivity, T the temperature, ρ the density, C_p the specific heat, and g is the volumetric heat generation. Assuming κ constant, this equation discretized by FE leads to the following system: $M\dot{d} + Kd = F$ (3) where K and M are respectively the mass and the stiffness matrix, d the temperature vector at the n mesh nodes, and F the load vector which takes account of the power generation and boundary conditions. We can express equation (2) in the *Fourier* domain as

$$(j\omega \mathbf{M} + \mathbf{K}) \cdot \mathbf{d} = \mathbf{Y} \mathbf{t} \mathbf{h} \cdot \mathbf{d} = \mathbf{F}$$
⁽⁴⁾

where Yth represents the *n*-by-*n* thermal admittance matrix.

Unfortunately, **Yth** dimensions are very large (n several thousands) which makes prohibitive its direct integration into a circuit simulator. It is why a reduction technique has been used. Moreover we do not need to know the temperature at all mesh nodes for simulating the transient or steady state response.

II. Thermal Model Reduction

Most of reduction techniques (Pade, Schur technique, ...) are based on extracting a small set of dominant poles (eigenvalues and eigenvectors) to represent the original system which may contain thousands of poles. In general these methods are computation intensive or unstable. It has been shown [2] [3] that the Ritz vectors approach will assure that important response modes are not neglected and yields improved accuracy with fewer vectors as compared to the use of eigenvectors. The procedure for generating orthogonal Ritz vectors leads to an m-by-n projection matrix constituted by m Ritz vectors.

$$\boldsymbol{\Phi}_{m} = \left[\boldsymbol{\varphi}_{1} \cdots \boldsymbol{\varphi}_{m}\right] \tag{5}$$

The first Ritz vector represents the static response to the load vector F, and the other ones concern the dynamic response of the structure, m is directly linked to the precision of the transient response and is several order lower than n.

This matrix satisfies the relationships:

$$\mathbf{d} = \boldsymbol{\Phi}_m \mathbf{p} \text{ and } \boldsymbol{\Phi}_m^T \mathbf{M} \boldsymbol{\Phi}_m = \mathbf{I}_m \tag{6}$$

where p is an *m*-by-1 temperature vector, \mathbf{I}_m the *m*-by-*m* identity matrix. With (5) and (2) we easily obtain:

$$\Phi_m^T \mathbf{M} \Phi_m \dot{\mathbf{p}} + \Phi_m^T \mathbf{K} \Phi_m \mathbf{p} = \Phi_m^T \mathbf{F}$$

$$\mathbf{I}_m \dot{\mathbf{p}} + \mathbf{K}^* \mathbf{p} = \Phi_m^T \mathbf{F}.$$
(7)

 $\Gamma_m \mathbf{p} + \mathbf{k} \mathbf{p} = \Psi_m \mathbf{r}$. The next step consists of finding the eigenvalues λ_i in order to obtain a set of *m* independent differential equations. If Ψ is the *m*-by-*m* projection matrix, which satisfies $\mathbf{p} = \Psi \mathbf{t}$, where t is an *m*-by-1 temperature vector in the diagonalized system. The diagonalized reduced system is then

$$\dot{\mathbf{t}} + \boldsymbol{\Lambda}_m \mathbf{t} = \boldsymbol{\Psi}^T \boldsymbol{\Phi}_m^T \mathbf{F} \text{ where } \boldsymbol{\Lambda}_m = diag[\boldsymbol{\lambda}_1 \cdots \boldsymbol{\lambda}_m].$$
 (8)

In the electrothermal simulation, only *r* nodes corresponding to the boundary conditions and to the power injection will be retained, so we introduce an *r*-by-*n* selection matrix S which satisfies $\mathbf{d}_r = \mathbf{S} \cdot \mathbf{d}$ and $\mathbf{E} = \mathbf{S}^T \mathbf{E}$

$$\mathbf{F} = \mathbf{S}^T \mathbf{F}_r$$

The power dissipation profile in a multi-finger transistor may change during operation, so Ritz vectors evaluated for a specified load vector direction will not always lead to an accurate temperature response. It is why, in order to account for the load vector direction's evolution we express the load vector \mathbf{F}_r as the superposition of unitary

spatial load vectors $\mathbf{F}\mathbf{u}_i = \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \end{bmatrix}^T$ where Pi the power dissipated at node *i* able to change during

 $\mathbf{F}_{r} = \sum_{i=1}^{r} Pi(\boldsymbol{\omega}) \cdot \mathbf{F} \mathbf{u}_{i}$ (9)

The global response corresponds to the linear combination of each individual response with the Pi coefficients.

III. Implementation

The reduction software has been written in C-ANSI and use the BLAS and LAPACK libraries. The inputs are F, M, K and the nodes corresponding to the fixed baseplate temperature. These data proceed from the FE tool MODULEF. The outputs are the eigenvalues of K* and the r matrices product $A = S\Phi\Psi$ obtained for each unitary power injection. These outputs are used in order to automatically generate a spice format file for the thermal subcircuit. This subcircuit is composed by resistors (1/ λ ij) in parallel with unitary capacitors, CCCSs to inject power and VCVSs to collect temperatures as presented in fig.1 for r equal 2 but can be easily extended to a greater value. The variables and the symbols are respectively :

- Pi is the input power at node $i(1 \le i \le r)$

- Ai = S $\Phi\Psi$ where *i* is the index of the node of injected power ($1 \le i \le r$), Ai_{kl} is the Ai[k, l] term of the matrix Ai

- t_{ij} is the contribution of input power *i* for the reduced variable t. $(1 \le i \le r, 1 \le j \le m)$

 $-\mathbf{d}_{ij}$ is the contribution of input power *i* for the temperature node *j*. $(1 \le i \le r, 1 \le j \le r)$ $-\mathbf{d}_i$ is the whole contribution of all input power (superposition) for node *i*. $(1 \le i \le r)$.



Fig. 2: Scheme of the spice subcircuit for 2 power nodes

III. Results for reduced thermal models

The reduction technique has been tested on various HBTs. A $2x2x30\mu$ m AlGaAs/GaAs HBT (HBT1) and a $8x2x40 \mu$ m GaInP/GaAs HBT (HBT2) have been simulated [4]. The transistors are constituted by a 100 μ m width GaAs substrate brazed on a Molybdenum 2mm width baseplate at 300°K. The emitters are grounded to an Au backside through via-holes. The dissipated power has been considered uniform and localized in a volume of $2x0.6x26\mu$ m³ for HBT1 (400mW) and $2x0.6x36\mu$ m³ for HBT2 (600mW) in the collector. Figure 2 shows the comparison between transient junction temperature response to a power pulse of 400mW evaluated by measurement[5], and the FE based simulation for HBT1. The difference between measurement and simulated response at increasing temperature is due to non-linear thermal behavior of the GaAs, which is not taken account in the simulation. The junction temperature measurement has been achieved thanks to an electrical method [6]. The FE response has been obtained by simulating the HBT structure using a 23000 nodes mesh with MODULEF and leads to a thermal resistance of 296°K/W for HBT1 and 80°K/W for HBT2.



Fig. 3: HBT1 transient response comparison

Fig. 4: HBT2 transient comparison response

Figures 2 and 3 show the junction temperature plotted using the reduced thermal model with respectively 20, 30, 50 Ritz vectors and the FEM based response. The good agreement between curves exhibits the power of the reduction method, which divide the node number by more 400 and give always the same final response. Note

that a transient simulation in a circuit simulator like ADS for the HBT1model using 50Ritz vectors takes about 1s to be achieved.

IV. Results from electrothermal simulations

For example, we have performed an electrothermal simulation of the transistor HBT2 in the ADS circuit simulator. According to figure 1, we have connected the reduced thermal model to the distributed non linear electrical model. Each elementary model corresponding to a finger is derived from the global model of the transistor which is extracted from pulse measurement. It accounts for the temperature dependence of the physical parameters such as saturation currents, and current gain. The scheme depicted figure 5 represents the simulation performed using an transient analysis. Figure 6,7,8 represent the results from this analysis and shows the temperature elevation, the collector current, and the Vbe voltage decreasing during a pulse of 50µs.



IV. Conclusion

We have presented a method based on a 3D physical study to obtain a reduced thermal equivalent model for accurate prediction of electrothermal behavior of power HBTs. The reduction process based on Ritz vector approach leads to the generation of a spice format subcircuit file that can be used in many circuit simulators. Excellent results have been validated by transient measurements of temperature. An application of this model will be the slow dynamic effects in pulsed high power amplifiers for radar applications.

References

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