BRIEF COMMUNICATIONS

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Spiky Langmuir solitons in a dense ultrarelativistic electron–positron plasma

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Spiky and short-duration Langmuir solitons are found to exist in an ultrarelativistic electron–positron plasma. Nonlinear Landau damping and pulsar radiation are discussed.

Recently Surko and Murphy[1] mentioned the production of a single-component positron plasma in the laboratory. Tsytovich and Wharton[2] discussed the possibility of creating an electron–positron plasma using intense relativistic electron beams, and they studied the waves and instabilities in such a plasma. The electron–positron system has a considerable history in plasma physics. Alfvén has discussed the role that electron–positron plasmas might play in a astrophysical situation.[3] New insight into certain astrophysical phenomena (such as pulsars) may also be gained in appropriate laboratory investigations.

Theoretical models[4,5] have been developed to predict the production of electron–positron plasma in the pulsar magnetosphere. A mechanism of cascade generation of particles has been suggested. The electron pulled off in the stellar surface, accelerates along the curved lines of force of magnetic field. Since the potential for the Crab Pulsar parameter is of the order of 10^{12} V, the relativistic factor of the accelerated electron is in the range of $\gamma_0 \sim 10^7$. The particles moving along the curved trajectory, emit so called curvature radiation in the direction of motion. In the case of dipole magnetic field, the accelerated electrons emit gamma rays with energies 10^9 eV. The gamma quantum, propagating at an angle to the magnetic field, creates an electron–positron pair. It is suggested that for the Crab Pulsar about 10^7 electron–positron pairs are created for each primary particle. Thus a dense ultrarelativistic electron–positron plasma might present in the pulsar magnetosphere. The problem of short pulsar radiation and its microstructure stimulates to investigate the nonlinear modes in such a plasma.

In this Brief Communication, we consider the nonlinear propagation of the Langmuir wave in a hot ultrarelativistic electron–positron plasma. The microscopic state of the plasma is described by the kinetic equation.

$$\frac{\partial f_s}{\partial t}+\frac{c^2}{\mathscr{E}_S}\mathbf{p}_S\frac{\partial}{\partial \mathbf{r}}f_S+e_S\mathbf{E}\frac{\partial f_S}{\partial \mathbf{p}_S}=0, \quad (1)$$

where $f_S(\mathbf{r},\mathbf{p},t)$ is a relativistic invariant, $S(=e,p)$ denotes the species, electron and positron, respectively: $\mathscr{E}_S=c(P_S^2+m_S^2c^2)^{1/2}$.

Linearizing (1) with respect to small perturbation $f_{S_1}\sim\exp(-i\omega t+ikx)$, we obtain

$$f_{S_1}=-\frac{ie_S(\mathbf{E}\cdot\mathbf{V}_S)}{\omega-\mathbf{k}\cdot\mathbf{V}_S}\frac{\partial f_{S0}}{\partial \mathscr{E}_S}, \quad (2)$$

where $f_{S0}(P_S)$ is the equilibrium plasma distribution function. From the Maxwell equation

$$\mathbf{J}=\sum_S e_S\int \mathbf{V}_S f_{S_1}\,d\mathbf{p}=\frac{i\omega}{4\pi}\mathbf{E}, \quad (3)$$

we find the plasma dielectric function

$$\mathscr{E}^l(\omega,k)=1+\sum_S\frac{4\pi e_S^2}{\omega k^2}\int d\mathbf{p}\,\frac{(\mathbf{k}\cdot\mathbf{V})^2}{\omega-\mathbf{k}\cdot\mathbf{V}}\frac{\partial f_{S0}}{\partial\mathscr{E}_S}. \quad (4)$$

For an ultrarelativistic ($P_S^2 \gg m_S^2c^2$) plasma, we consider the relativistic local Maxwellian distribution[6]

$$f_{S0}=(n_Sc^3/8\pi T_S^3)e^{-\mathscr{E}_S/T_S}. \quad (5)$$

Then

$$\mathscr{E}^l(\omega,k)=1+\sum_S\frac{4\pi e_S^2 n_S}{k^2T_S}\left(1-\frac{\omega}{2kc}\ln\left|\frac{\omega+kc}{\omega-kc}\right|\right); \quad (6)$$

$\epsilon^l(\omega,k)=0$, defines the possible electrostatic modes in the ultrarelativistic electron–positron plasma, which are

$$\omega^2=\frac{c^2}{6\lambda_D^2}\left[1+\left(1+\frac{36}{5}k^2\lambda_D^2\right)^{1/2}\right],\quad \text{for } \frac{\omega}{k}>c, \quad (7a)$$

$$\omega=kc\{1+2\exp[-2(1+k^2\lambda_D^2)]\},\quad \text{for } \frac{\omega}{k}\sim c, \quad (7b)$$

$$\omega_r\cong 0,\quad \omega_i=\frac{2}{\pi}kc(1+k^2\lambda_D^2),\quad \text{for } \frac{\omega}{k}<c; \quad (7c)$$

where

$$\frac{1}{\lambda_D^2}=\sum_S \frac{1}{\lambda_{D_S}^2}, \quad \lambda_{DS}^2=\frac{T_S}{4\pi e_S^2 n_S}.$$

It shows that in an ultrarelativistic electron–positron plasma electrostatic modes (Langmuir waves) propagate with phase velocities higher than the speed of light, so linear Landau damping is not possible here. The waves with the phase velocities $\omega/k < V_t \sim c$ are heavily damped. Therefore, we shall study the nonlinear propagation of the mode (7a) in detail.

The nonlinear evolution of the wave is described by the nonlinear Schrödinger equation[7]

$$i(\partial_t+v_g\partial_x)E+\tfrac{1}{2}v_g' E-\alpha|E|^2E=0, \tag{8a}$$

where

$$v_g=\frac{d\omega}{dk}=\frac{3\sqrt{6}}{5}\cdot\frac{(k\lambda_D)c}{\beta\sqrt{1+\beta}}, \quad \beta=\left(1+\frac{36}{5}k^2\lambda_D^2\right)^{1/2}, \tag{8b}$$

$$v_g'=\frac{d^2\omega}{dk^2}=\frac{3\sqrt{6}}{5}\cdot\frac{c\lambda_D}{\beta\sqrt{1+\beta}}\left(1-\frac{36(k\lambda_D)^2}{5\beta^2}-\frac{18(k\lambda_D)^2}{5\beta(1+\beta)}\right), \tag{8c}$$

and

$$\alpha=\frac{\partial\omega}{\partial|E|^2}.$$

The nonlinear frequency shift (α) is caused by the density fluctuation $\delta n(|E|^2)$ due to the high-frequency (ω) wave propagation. The slow plasma response, in this context, can easily be described by fluid equations[8]

$$\partial_t(\gamma_S n_S)+\partial_x(\gamma_S n_S U_S)=0, \tag{9}$$

$$(\partial_t+U_S\partial_x)(\gamma_S m_S^* U_S)=-e_S\partial_x\phi+f_{p_S}-\frac{1}{\gamma_S n_S}\partial_x p_S, \tag{10}$$

where

$$\gamma_S=(1-U_S^2/c^2)^{-1/2}, \quad m_S^*=m_S\frac{T_S}{m_S c^2}\gg m_S, \quad p_S=n_S T_S$$

and

$$f_{p_S}=-\langle U_S\partial_x(\gamma_S m_S^* U_S)\rangle=-\frac{e_S^2}{2\gamma_S m_S^*\omega^2}\partial_x|E|^2 \tag{11}$$

is the ponderomotive force due to the high-frequency Langmuir wave.

For simplicity, we consider the isothermal plasma state ($T_e=T_p=T$) and also consider $\gamma_e=\gamma_p\cong\gamma_0$. In this case ponderomotive force (11) is charge independent and we can neglect the ambipolar field (ϕ) in (10).

Considering $n=n_0+\delta n$, $T=T_0+\delta T$ ($\delta n\ll n_0$, $\delta T\ll T_0$), from the adiabatic law for ultrarelativistic gas ($n_S/T_S^3=\text{const}$), we find $\delta T/T_0=\frac{1}{3}\delta n/n_0$ and $\partial_x p=\frac{4}{3}T_0\,\partial_x\delta n$. Furthermore, neglecting the mass inertia[9] (which means the time variation of low-frequency fluctuation is less than the plasma frequency) in Eq. (10), we obtain

$$\frac{\delta n}{n_0}=-\frac{3}{32}\frac{e^2c^2|E|^2}{\omega^2T_0^2}. \tag{12}$$

Thus the evolution equation of the wave takes the form

$$i(\partial_t+v_g\partial_x)E+\tfrac{1}{2}v_g'\,\partial_x^2E+Q|E|^2E=0, \tag{13}$$

with

$$Q=\frac{3}{128\sqrt{6}}\frac{e^2c^3\sqrt{1+\beta'}}{\omega^2T_0^2\lambda_D\beta}.$$

For $k\lambda_D\gg 1$, $v_g'\simeq -c/k\sqrt{k\lambda_D}$, so $v_g'Q<0$. Thus for waves with the wave length smaller than the plasma Debye length, we have periodic wave trains. But for $k\lambda_D\ll 1$, $v_g'>0$ and $v_g'Q>0$. In this case the wave is modulationally unstable and admits a solution[10]

$$E=E_0\,\text{sech}\left[\left|\frac{Q}{2v_g'}\right|^{1/2}E_0(x-v_gt)\right]\exp\left(-\frac{iQE_0^2}{2}\right), \tag{14}$$

where $E_0=|A/Q|^{1/2}$ ($A=\text{const}$) is the soliton amplitude and $\delta=|2v_g'/QE_0^2|^{1/2}$ is the soliton pulse width.

An analysis of (14) shows that

$$E_0^2\sim(n_0T_0)^{1/2} \quad\text{and}\quad \delta\sim(T_0/n_0)^{1/4}. \tag{15}$$

Thus in a dense ultrarelativistic electron–positron plasma electrostatic modes with wavelength greater than the plasma Debye length produce Langmuir solitons that are spiky in nature. One essential feature of these solitons is that they cannot form an energy flow toward smaller size, since they cannot merge with each other. To merge, the soliton would have to give energy to the sound waves, which do not exist in electron–positron plasma. In the one-dimensional case the only process occurring in soliton gas is nonlinear interaction with electrons and positrons, i.e., nonlinear Landau damping, which stops the soliton without changing its amplitude. The stopping length is of the order of the soliton width, which is very small for a dense plasma. Thus the soliton will be stopped in a short time, of the order of the time of their creation. These spiky short-duration Langmuir solitons might be related to the pulsar radiation and with its microstructures.

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