

Spin and polarization: a new direction in relativistic heavy ion physics

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Abstract. Since the first evidence of a global polarization of Λ hyperons in relativistic nuclear collisions in 2017, spin has opened a new window in the field, both at experimental and theoretical level, and an exciting perspective. The current state of the field is reviewed with regard to the theoretical understanding of the data, reporting on the most recent achievements and envisioning possible developments. The intriguing connections of spin physics in relativistic matter with fundamental questions in quantum field theory and applications in the non-relativistic domain are discussed.

1. Introduction

The observation of a global spin polarization of Λ hyperons in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV [1] confirmed, at the relativistic and subatomic level, the link between spin and rotation predicted more than a century ago and experimentally observed in the Barnett and Einstein-De Haas effects [2, 3]. Besides, it has opened a new and promising direction in the field of relativistic heavy ion physics. Since the first positive evidence, there has been a considerable progress in understanding this phenomenon both at theoretical and experimental level. Over the past few years, the experiments have been able to confirm the first observations [4] and demonstrated the capability of probing the dependence of spin polarization on momentum [5]. Besides, the measurement of the global polarization of the Ξ hyperon [6], in good agreement with hydrodynamic predictions, confirmed that this phenomenon is not driven by specific hadron-dependent couplings or properties, like in pp collisions, but by collective properties of the system, what is often referred to as vorticity-induced polarization. Spin polarization has been observed at very low energy [7, 8] and at the highest energy of the LHC [9, 10]. Hence, spin polarization is proving to be a practical new degree of freedom which is at our disposal to investigate the formation and evolution of the Quark Gluon Plasma (QGP).

The theory of spin physics in a relativistic fluid also had a significant progress over the past few years. While the first observation was a successful verification

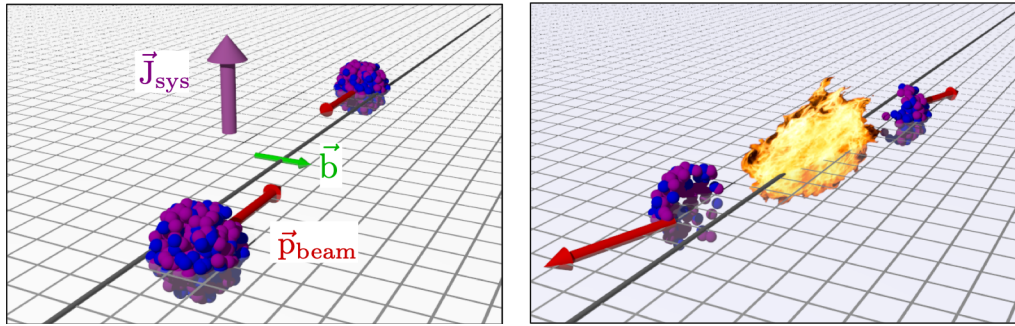


Figure 1. Two colliding nuclei at a finite impact parameter \mathbf{b} produce a Quark Gluon Plasma inheriting a fraction of their initial total relative orbital angular momentum \mathbf{J}_{sys} (from ref. [16]). The angular momentum vector is perpendicular to the plane containing the momenta of the nuclei and their impact parameter, the so-called reaction plane.

of the quantitative prediction of relativistic hydrodynamics and local equilibrium [11, 12, 13] spin polarization as a function of momentum did not meet the expectations from this model. The discrepancy motivated an intense theoretical effort in many different directions which has led to a remarkable advance in the understanding of spin thermodynamics and kinetics in a relativistic fluid. In fact, because of its quantum nature, spin has forced us to reexamine the foundations of relativistic hydrodynamics and kinetic theory within a quantum field theory framework, widening its scope and further clarifying its foundations. Furthermore, as I will discuss in this work, the study of spin in its native regime - which is relativistic - may have interesting and highly non-trivial implications for other fields where spin is at the focal point, like spintronics [14, 15].

This paper is certainly not a complete overview of the state of the field and of all the ongoing developments (interested readers may find a more detailed review in [16], which is quickly getting out of date though). I shall instead take the special point of view of theory and phenomenology and focus on the recent advances of our theoretical understanding of the observed phenomena and on the still open issues. What I find mostly exciting in this growing subfield of heavy ion physics is the strong interplay between theory, phenomenology and experimental measurements which was typical of the golden age of particle physics.

Notation

In this paper we adopt the natural units, with $\hbar = c = K = 1$. The Minkowskian metric tensor g is $\text{diag}(1, -1, -1, -1)$; for the Levi-Civita symbol we use the convention $\epsilon^{0123} = 1$. We will use the relativistic notation with repeated indices assumed to be saturated. Operators in Hilbert space will be denoted by a wide upper hat, e.g. \widehat{H} .

2. Spin and hydrodynamics

The Barnett effect shows that a rotation of a solid body induces a spin polarization in the direction of the angular velocity vector [2]. It is therefore to be expected that vorticity in a fluid induces a spin polarization in the direction of the local vorticity vector. Such an effect has been indeed observed few years ago in an experiment using liquid mercury [17], where a spin current induced by a vorticious flow was detected. In heavy ion collisions, two colliding nuclei at finite impact parameter (fig. 1) have, at high energy, a large relative angular momentum, whose a relatively large fraction can be inherited by the produced QGP in the overlapping region. It is then reasonable to expect that the angular momentum gives rise to a finite vorticity in the QGP. The measurements in heavy ion collisions extend, in a sense, the aforementioned observation to the relativistic regime, if the QGP formed in relativistic nuclear collisions is an actual relativistic fluid as it is commonly accepted.

Indeed, the hydrodynamic model has become a paradigm for the description of the QGP formed in relativistic nuclear collisions. It is by now generally accepted that the QGP is a strongly-interacting relativistic fluid which stays close to local thermodynamic equilibrium for most of its evolution (the quasi-perfect fluid picture), until it breaks up and hadronizes. The success of the statistical-hydrodynamic model in reproducing hadron abundances and spectra is seen as an evidence of the achievement of local equilibrium prior to the pseudo-phase transition and QGP hadronization. Indeed, particle momentum spectra are very well described by the formula:

$$\varepsilon \frac{dn_h}{d^3p} = (2S_h + 1) \int_{\Sigma} d\Sigma_{\mu} p^{\mu} \frac{1}{\exp [(u^{\mu}(x)p_{\mu} - \sum_i q_{ih}\mu_i(x)) / T(x)] \pm 1} \quad (1)$$

where ε is the energy and p^{μ} the four-momentum of the particle h , S_h its spin; Σ is the hadronization hypersurface, the sign $+$ applies to fermions and $-$ to bosons, and the hydrodynamic input is the temperature T , the relativistic four-velocity u^{μ} and the chemical potentials μ_i coupled to the hadronic charges q_{ih} . This equation is a consequence of local equilibrium and is an excellent leading order approximation in the classical limit; subleading terms are dissipative (out-of-equilibrium) corrections as well as quantum relativistic equilibrium corrections [18].

If the local equilibrium picture works well for the momentum, it is a quite natural idea to extend it to the other, space-time related, degree of freedom of a relativistic particle, the spin. Indeed, the counterpart of the eq. (1) for the mean spin vector, or spin polarization vector, was derived for a relativistic fluid at local equilibrium in ref. [19] and its leading order approximation in the thermal vorticity (see below) reads:

$$S_{\varpi}^{\mu}(p) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_{\sigma} \frac{\int_{\Sigma} d\Sigma_{\lambda} p^{\lambda} n_F (1 - n_F) \varpi_{\nu\rho}}{\int_{\Sigma} d\Sigma_{\lambda} p^{\lambda} n_F}, \quad (2)$$

where n_F is a shorthand for the Fermi-Dirac distribution function, i.e. the integrand function of (1) with the sign $+$:

$$n_F = \frac{1}{\exp [(u^{\mu}(x)p_{\mu} - \sum_i \mu_i(x)) / T(x) q_{ih}] + 1}$$

The thermal vorticity ϖ is defined as:

$$\varpi_{\nu\rho} = -\frac{1}{2}(\partial_\nu\beta_\rho - \partial_\rho\beta_\nu). \quad (3)$$

where $\beta^\mu = (1/T)u^\mu$ is the four-temperature vector, which plays a crucial role in relativistic hydrodynamics. The equation (2) implies that polarization states are not evenly filled and, therefore, the factor $2S_h + 1$ in the (1) is just an approximation. Indeed, it is quite an accurate one being the polarization of the order of few percent. The equation (2) also demonstrates the peculiar feature of spin polarization; unlike the momentum spectrum (1) it depends on the *gradients* of the thermo-hydrodynamic fields. Such characteristics makes spin polarization a very powerful probe of the hydrodynamic picture, because it requires the model to reproduce not just the final configuration of temperature and velocity, but its space-time dependence as well, at least near the hadronization hypersurface.

It is worth discussing the various terms of the eq. (2) by decomposing the gradients of the four-temperature vector. Since:

$$\partial_\nu\beta_\rho = u_\rho\partial_\nu\left(\frac{1}{T}\right) + \frac{1}{T}\partial_\nu u_\rho \quad (4)$$

and introducing the four-acceleration A^μ and the four-vorticity vector ω^μ with the appropriate definitions:

$$A^\mu = u \cdot \partial u^\mu \quad \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu u_\rho u_\sigma \quad (5)$$

the antisymmetric part of the tensor $\partial_\nu u_\rho$ can be expressed as a function of A and ω :

$$\frac{1}{2}(\partial_\nu u_\rho - \partial_\rho u_\nu) = \frac{1}{2}(A_\rho u_\nu - A_\nu u_\rho) - \epsilon_{\nu\rho\sigma\tau}\omega^\sigma u^\tau$$

The above decomposition, by using the eqs. (3),(4) and (5), makes the integrand of the eq. (2):

$$\epsilon^{\mu\nu\rho\sigma}p_\sigma\partial_\nu\beta_\rho = \epsilon^{\mu\nu\rho\sigma}p_\sigma\partial_\nu\left(\frac{1}{T}\right)u_\rho + 2\frac{\omega^\mu u \cdot p - u^\mu \omega \cdot p}{T} - \frac{1}{T}\epsilon^{\mu\nu\rho\sigma}p_\sigma A_\nu u_\rho \quad (6)$$

where the \cdot stands for the relativistic scalar product of vectors. Hence, polarization stems from three contributions: a term proportional to the gradient of temperature, a term proportional to the vorticity ω , and a term proportional to the acceleration. Further insight into the nature of these terms can be gained by choosing the particle rest frame, where $p = (m, \mathbf{0})$ and restoring the natural units. The eq. (6) then certifies that the spin polarization vector \mathbf{S}^* of a spin 1/2 particle, in its rest frame, at some point in the fluid, is proportional to the following combination:

$$\mathbf{S}^* \propto \frac{\hbar}{KT}2\gamma(\boldsymbol{\omega}^* - (\boldsymbol{\omega}^* \cdot \mathbf{u}^*)\mathbf{u}^*/\gamma^2 c^2) + \frac{\hbar}{KT}\mathbf{A}^* \times \mathbf{u}^*/c^2 + \frac{\hbar}{KT^2}\mathbf{u}^* \times \nabla T \quad (7)$$

where, for the occasion, we restored the natural constant \hbar and the Boltzmann constant K . In the eq. (7) $\mathbf{u}^* = \gamma\mathbf{v}^*$, \mathbf{A}^* , $\boldsymbol{\omega}^*$ are the space components of the four-vectors in eq. (5) and $\gamma = 1/\sqrt{1 - v^{*2}/c^2}$; all of the three-vectors in eq. (7) are measured in the particle

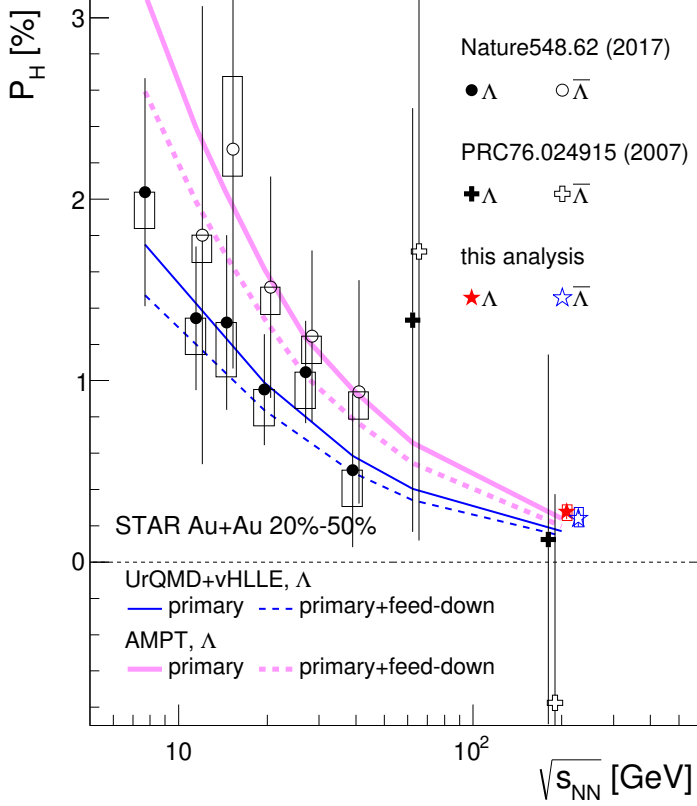


Figure 2. Comparison between the measured global polarization of Λ hyperons in relativistic heavy ion collisions at a nucleon centre-of-mass energy of 200 GeV and the predictions of equation (2). Red and blue lines refer to different models to calculate thermal vorticity field at the hadronization (from ref. [4]).

rest frame. The decomposition (7) makes it clear what are the thermodynamic "forces" responsible for polarization: the first term is the vorticity term, which corresponds to the well known Barnett effect [2]; the second term is an acceleration-driven polarization, and corresponds to the Thomas precession, which is expected as the particle is dragged by an accelerated flow; finally, the last term is a polarization induced by a combination of temperature gradient and hydrodynamic flow and should be, to the best of our knowledge, a newly found effect (see later).

Indeed, the eq. (2) successfully reproduced the global spin polarization, integrated over momentum (see fig. 2), which yields a vector directed along the initial angular momentum of the collision (see fig. 1). Yet, the same equation failed to reproduce the spin dependence on momentum. The components of the spin polarization vector of the Λ hyperons along the total angular momentum direction and along the beam line (longitudinal polarization), as a function of the hyperon azimuthal angle were measured by the STAR collaboration [5, 20] for hyperons around zero rapidity and found to strongly disagree with the predictions of the formula (2) (see e.g. the discussion in ref. [16]).

This puzzle has lasted several years. Recently, however, a new local equilibrium term which is linear in the gradients, hence comparable with the thermal vorticity contribution, was found out with two independent derivations [21, 22] (and later confirmed in the study of ref. [23]). The formula in ref. [21] reads:

$$S_{\xi}^{\mu}(p) = -\frac{1}{4m} \epsilon^{\mu\rho\sigma\tau} \frac{p_{\tau} p^{\lambda}}{\varepsilon} \frac{\int_{\Sigma} d\Sigma_{\nu} p^{\nu} n_F (1 - n_F) \hat{t}_{\rho} \xi_{\sigma\lambda}}{\int_{\Sigma} d\Sigma_{\nu} p^{\nu} n_F} \quad (8)$$

where \hat{t} is the time direction in the QGP frame and ξ is the symmetric derivative of the four-temperature, defined as *thermal shear* tensor:

$$\xi_{\mu\nu} = \frac{1}{2} (\partial_{\mu} \beta_{\nu} + \partial_{\nu} \beta_{\mu}). \quad (9)$$

Indeed, the formula (8) involves some geometrical approximations of the freeze-out hypersurface; in the derivation in ref. [22] a different approximation scheme was used and, as a result, the \hat{t} direction is replaced by the local four-velocity \ddagger . Hence, altogether, the spin polarization vector at local equilibrium is the sum of the right hand sides of the eqs. (2) and (8):

$$S_{\text{LE}}^{\mu} = S_{\omega}^{\mu}(p) + S_{\xi}^{\mu}(p) \quad (10)$$

The inclusion of (8) proved to be able to consistently reduce the discrepancies [24] and indeed fully restore the agreement with the data at top RHIC energy at $\sqrt{s_{\text{NN}}} = 200$ GeV, for both the components of polarization, once the temperature gradients are removed to take into account that the hadronization hypersurface is $T = \text{const}$ [25]. This latter assumption is a very reasonable one at very high energy, where the chemical potentials basically vanish and the only thermodynamic parameter governing the hadronization transition is temperature. In conclusion, even though this picture needs to be confirmed by more studies (see e.g. [26] and [27]) and tests at different energies, it is fair to conclude that the term (8) is a likely solution of the discrepancies between the azimuthal dependence of polarization and the local equilibrium hydrodynamic predictions at very high energy. In general, the term (8) is to be introduced in all calculations of spin polarization.

It should be stressed that the additional term (8) is not a dissipative correction like the viscous stress tensor for the stress-energy tensor. In fact, this term is a quantum correction to the local equilibrium, hence it is non-dissipative, which vanishes at global equilibrium where $\xi = 0$. The non-dissipative nature of this term is also revealed by the absence of any dynamical transport coefficient in the right hand side of eq. (8); it can be seen that it just depends on free-particle quantities, just like the (2). As such, this term is an unexpected contribution, even more so as the only non-relativistic known formula relates the spin vector to the angular velocity [28] (in this respect, the equation (2) appears to be a more typical relativistic extension of a non-relativistic formula). In

\ddagger It should also be noted that the eq. (8) is weakly dependent on the pseudo-gauge choice, even if the spin potential differs from thermal vorticity, see Section 3. Particularly, it was shown in ref. [21] that for spin 1/2 particles, the right hand side is the same for both the canonical and the Belinfante pseudo-gauges.

fact, the eq. (8) has a non-trivial non-relativistic limit which is a single term involving the gradient of the temperature field [21]:

$$\mathbf{S}_\xi = \frac{1}{8} \mathbf{v} \times \frac{\int d^3x n_F (1 - n_F) \nabla \left(\frac{1}{T} \right)}{\int d^3x n_F} \quad (11)$$

where \mathbf{v} is the velocity of the particle. Interestingly, the eq. (2) features the same term in its non-relativistic limit by going to the local rest frame of the fluid where $u = (1, \mathbf{0})$, so that the non-relativistic limit of (10) includes twice the right hand side of the eq. (11). This equation predicts that a particle travelling in a medium with a temperature gradient will get polarized, thus giving rise to a tiny magnetic field and a spin current along its motion §. Thermal spin effects have been predicted and observed in condensed matter [29] but it is not clear whether this particular one has been observed yet.

There are many more measurements that can be done to further test the hydrodynamic-local equilibrium picture. For instance, in ref. [30], a proposal of measuring the polarization dependence on the azimuthal angle at finite rapidity instead of midrapidity to probe the vorticity field. To date, the spin polarization has been measured by the experiments also as a function of transverse momentum, centrality and rapidity [4, 5, 10]. An interesting study of the hydrodynamic predictions on these dependences has been recently carried out [31], where the new term (8) and a similar local equilibrium term proportional to the gradient of μ_B/T [32] (which is, in general, more relevant at low energy, see below) were included. Therein, the authors showed that the effect of the term in eq. (8) on the global polarization (integrated over all momenta) is almost negligible, thus confirming that the agreement between global polarization and the predictions based solely on the term (2) is not accidental. Furthermore, the authors systematically studied the dependence of the hydrodynamic predictions on the initial longitudinal flow velocity, showing that the spin polarization can be used as a major probe of the initial hydrodynamic conditions, which is itself a remarkable fact.

Finally, there have been very interesting studies of polarization at very low energy. The experiment STAR has been able to push the lowest energy polarization measurement down to $\sqrt{s_{NN}} = 3$ GeV in order to test the typical prediction of the hydrodynamic model of an increasing polarization as energy decreases [13]. It is not yet clear if the same hydrodynamic model which is used at high energy can be applied at such a low energy, however there are calculations [33, 34, 35, 36] based on hydrodynamics and other models predicting the appearance of a peak of global polarization as a function of collision energy. The magnitude measured by the collaboration STAR [7] seems to favour the three-fluid model [33], however it should be stressed that none of the above calculations included the spin-shear term (8). In fact, it has not been determined if the spin-shear term can sizeably contribute to the polarization at low energy. However, a recent study showed that at low energy, where the baryon chemical potential is large,

§ In this case, we define spin current as the motion of polarized particles. The proper definition of spin current in quantum field theory requires some care.

the effect of the local equilibrium term proportional to $\partial(\mu_B/T)$ (dubbed as Spin Hall Effect [29] in ref. [32]) may play an essential role, with an almost a factor 2 increase of the global polarization magnitude [38]. Another reason for the interest of low energy polarization studies is the apparent splitting between Λ and $\bar{\Lambda}$ polarization (see fig. 2), for which the electro-magnetic field is possibly responsible [16, 37].

2.1. Beyond local equilibrium

What if deviations from local equilibrium picture eventually hold? If the discrepancies between predictions and data are significant but small (what seems to be a fair conclusion after the inclusion of the new term (8), as discussed), then it can be argued that dissipative corrections could cure them. The theoretical problem would be alike to the calculation of the dissipative corrections of the particle momentum distribution (1) with viscous terms, which is a well known problem in the hydrodynamic modelling of heavy ion collisions (see e.g. [39]). The main difficulty, in the spin case, is the identification of the involved dissipative coefficients, whether they are the known ones (shear and bulk viscosity, thermal conductivity) associated to the stress-energy tensor, or new ones which have not yet been classified. Indeed, an expression has been derived for the dissipative corrections of the spin polarization vector of a system of massless fermions [40] (see also ref. [41]) with terms proportional to the viscous part of the stress-energy tensor and to the diffusion currents. Recently, an expression of the dissipative corrections for massive fermions, involving several new coefficients, has been obtained based on relativistic kinetic theory with spin [42]. Apparently, these terms have been derived with a special choice of the spin tensor (see below), and it is not yet clear if they are independent thereof, what promises to be a really intriguing question.

If, on the other hand, large discrepancies will appear, local equilibrium could not be longer considered a good approximation. In this case, to get a better description, there are two possible options: extending the notion of local equilibrium by introducing a spin potential (which will be discussed in detail in Section 3) or a full non-equilibrium approach, such as a quantum kinetic description. Both these cases imply a separation of scales between the spin relaxation time and the typical interaction time.

The kinetic theory with spin is an extension of relativistic kinetic theory and had a remarkable progress over the past few years, with many contributors. The most suitable tool is quantum kinetic theory with collisions [43, 44, 45, 46, 47, 48], which is very useful to obtain the constitutive equations of spin hydrodynamics [49] and other theoretical insights. Yet, it is not clear if the kinetic approach, based on the picture of quasi-free colliding particles [50], requiring the mean free path to be much larger than the thermal wavelength, is phenomenologically viable for the QCD plasma near the pseudo-critical temperature. The very low value of the viscosity/entropy density ratio entails comparable magnitudes for the mean free path and thermal wavelength (the so-called strongly-interacting QGP), implying that the collisional picture is questionable. Nevertheless, if the spin relaxation time is much longer than the typical interaction time

scale, perhaps a kinetic description limited to spin-changing interactions could be still a good approximation.

3. The spin tensor, the spin potential and spin hydrodynamics

Right about the same time evidence was found of polarization in relativistic heavy ion collisions, the question arose about if and how including this new degree of freedom in the hydrodynamic model of the QGP. From this problem a new line of research has sprung, the so-called relativistic spin hydrodynamics, with many contributions.

The basic idea of relativistic spin hydrodynamics (see e.g. [51]) is to describe the dynamics of the relativistic fluid with an additional tensor of rank 3, the spin tensor $\mathcal{S}^{\lambda,\mu\nu}$, which is anti-symmetric in the last two indices, besides the familiar stress-energy tensor $T^{\mu\nu}$. The spin tensor fulfills a continuity equation dictated by the conservation of angular-momentum:

$$\partial_\lambda \mathcal{S}^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \quad (12)$$

(the stress-energy tensor not being necessarily symmetric) which is the additional fundamental equation of relativistic spin hydrodynamics.

There is a fundamental issue though, namely whether the spin tensor has a real physical meaning. This is a sort of ever-returning highly-debated question in several fields, like e.g. proton spin studies [52], and it has to do with the possibility of a unique separation of the orbital and the spin angular momentum in relativity or, tantamount, the definition of a unique stress-energy tensor. Ultimately, this question is related to a crucial feature of the conserved currents in quantum field theory, the so-called *pseudo-gauge invariance*, that is the possibility of changing the spin and the stress-energy tensors at the same time with a suitable linear transformation preserving the continuity equations and the total energy-momentum and angular momentum [53, 54], which are obtained by integrating the densities over an arbitrary 3D space-like hypersurface:

$$\begin{aligned} \hat{P}^\nu &= \int d\Sigma_\mu \hat{T}^{\mu\nu}(x) \\ \hat{J}^{\lambda\nu} &= \int d\Sigma_\mu \left(x^\lambda \hat{T}^{\mu\nu}(x) - x^\nu \hat{T}^{\mu\lambda}(x) + \hat{\mathcal{S}}^{\mu,\lambda\nu} \right) \end{aligned} \quad (13)$$

Otherwise stated, the stress-energy tensor and the spin tensor, in relativistic quantum field theory, are much alike to gauge potentials, which are physically unmeasurable. This is confirmed by the fact that the operator $\hat{S}^\mu(p)$, whose mean value is the spin polarization vector measured in the experiment, can be expressed in terms of creation and annihilation operators of the quantum field and does not depend on the spin tensor [55]:

$$\hat{S}^\mu(p) = \sum_i [p]_i^\mu D^S(J_i)_{rs} \hat{a}^\dagger(p)_r \hat{a}(p)_s$$

In the above equation, $D^S(J_i)$ are the representation matrices of the generators of rotation for the spin S , $[p]$ is the so-called standard Lorentz transformation taking the

four-vector $(m, \mathbf{0})$ to the four-momentum p and the $\hat{a}(p)_s$ are the annihilation operators of a particle with four-momentum p and spin state s .

The spin tensor, which is charge-conjugation even, contains information about the spin density and the spin current within matter at some point, but it should be emphasized that it is necessary only in a quantum relativistic theory with anti-particles, to describe a situation where both particles and antiparticles can be polarized in the same direction. Indeed, in the ordinary matter, without anti-particles, a finite spin density can be described by the magnetization tensor (e.g. in the Dirac theory $\bar{\Psi}[\gamma^\mu, \gamma^\nu]\Psi$) which is charge-conjugation odd. This means that the Barnett effect, i.e. magnetization by rotation in ordinary matter, is completely insensitive to the spin tensor and does not really break pseudo-gauge invariance.

It appears to be somewhat surprising that the stress-energy tensor is a sort of gauge field, but it should be kept in mind that, strictly speaking, the experiments cannot actually measure densities in space. In fact, they can only measure momentum spectra or spin polarization as a function of momentum. Therefore, all the information on the local state of the fluid, in terms of the hydrodynamic-thermodynamic fields, is inferred through relations like the eq. (1) or the eq. (2). These fields, particularly the four-temperature field $\beta(x)$, are defined by enforcing local thermodynamic equilibrium conditions, which turn out to be pseudo-gauge dependent [56], as it will become clear below. Therefore, the densities, the currents and the thermodynamic fields are pseudo-gauge dependent, meaning that they are physically objective only up to small, yet finite quantum terms.

The most apparent manifestation of pseudo-gauge invariance is the possibility to make the spin tensor vanishing altogether, the so-called Belinfante pseudo-gauge. This choice seems to imply that the very notion of "spin density" (beware the difference with magnetization density) is unphysical but in fact it means that in a relativistic fluid particles and anti-particles cannot be polarized with the same sign without a thermal vorticity, i.e. without a rotation [56, 57]. In other words, in the Belinfante pseudo-gauge, a relativistic fluid cannot present itself in a state where particles and anti-particles are polarized in the same direction without rotation, or better, with vanishing thermal vorticity. To describe such a situation, the prepared quantum state needs to break pseudo-gauge invariance, and indeed a density operator or a superposition of states can - at least in principle - be constructed which explicitly depends on the pseudo-gauge couple $(T^{\mu\nu}, \mathcal{S}^{\lambda,\mu,\nu})$ [56, 49, 58].

In a kinetic-based approach, where particles are the fundamental objects, a situation with particles and anti-particles polarized in the same direction without an associated thermal vorticity pertains to a system where the spin relaxation time is longer than momentum relaxation time scale [56, 57], so that spin can be considered as an independent hydrodynamic slow mode [59] which is not locked to thermal vorticity. In this case, local equilibrium of angular momentum density requires a new intensive thermodynamic quantity, an anti-symmetric tensor called the spin potential $\Omega_{\mu\nu}$ ||,

|| This quantity is called spin *chemical* potential in some papers. I find the adjective "chemical"

which is unnecessary in the Belinfante pseudo-gauge. The spin potential couples to the spin tensor in the exponent of the local equilibrium density operator [56], but the problem is which spin tensor (and, consequently, which stress-energy tensor) among all the possible pseudo-gauge choices, is to be chosen for the implementation of the local angular momentum density constraint. One can appeal to an external extended gravitational theory [57] or to other arguments [60], but in essence the choice is free in quantum field theory in flat spacetime.

The final goal of spin relativistic hydrodynamics is to determine the spin potential $\Omega_{\mu\nu}$ at the particle freeze-out with given initial conditions. Hence, spin relativistic hydrodynamics is an extension of relativistic hydrodynamics with six additional fields to be evolved besides the four-temperature β and chemical potentials. The effective number of independent dynamical variables of the spin potential might be reduced [57] if additional symmetries of the spin tensor are present, like for the canonical Dirac spin tensor which is completely anti-symmetric. The conservation equation of the mean value of the stress-energy tensor:

$$\partial_\mu T^{\mu\nu}[\beta, \zeta, \Omega] = 0$$

(the square brackets stand for a *functional* dependence, that is on the function and all the derivatives at the point x) is supplemented by the continuity equation of the mean value of the spin tensor (12) which makes it possible to find a solution for all the fields once the initial and boundary conditions are provided.

The spin potential enters the spin polarization vector expression, modifying the eq. (10) [61, 62] with an additional contribution, for the canonical spin tensor:

$$\Delta S^\mu(p) = \frac{\epsilon^{\lambda\rho\sigma\tau} \hat{t}_\lambda (p_\tau p^\mu - \delta_\tau^\mu m^2) \int_\Sigma d\Sigma_\nu p^\nu n_F(1 - n_F)(\varpi_{\rho\sigma} - \Omega_{\rho\sigma})}{m\varepsilon \int_\Sigma d\Sigma_\nu p^\nu n_F}, \quad (14)$$

which makes it apparent that a non-vanishing spin polarization is possible even with $\varpi = \xi = 0$ (see discussion above).

The topic of spin tensor and spin hydrodynamics has attracted a lot of interest among theorists, because of the enjoyable work of deriving the constitutive equations and the dissipative terms of the spin tensor [63, 64, 65, 66, 67, 57]. It is of course a very interesting subject, and a possible solution of a disagreement between data and the predictions of the eq. (10) but the price to be payed, from the point of view of phenomenology, is high. Besides the general problem of picking a particular spin tensor to be coupled to the spin potential, the introduction of six additional fields $\Omega_{\mu\nu}(x)$ requires the specification of their initial conditions, which are essentially unknown. The only observable which is sensitive enough to the spin potential is the spin polarization, through the above equation (14). Therefore, any measurement of spin polarization would serve to adjust the initial conditions of the spin potential rather than to test a theoretical prediction; put it simply, it would be a great loss of predictive power of the hydrodynamical model. This is indeed a real possibility, but a very disappointing inappropiate because it has nothing to do with the internal quantum numbers and it is in fact common to all particle species.

one. So far, there is no evidence that a spin potential different from thermal vorticity is necessary to describe the data, but further tests and more accurate measurements may lead to a different conclusion.

4. Spin alignment

The local equilibrium picture predicts that all sort of particles come out polarized from the hadronizing plasma, with a polarization depending on the hydrodynamic fields and their spin [28]. The mean spin polarization vector S^μ , in practice, can be measured in heavy ion experiments only for weakly decaying hyperons, such as Λ , Ξ and Ω . For spin 1 mesons, which do not decay weakly, a polarization-related quantity which is accessible through the decay process is the so-called alignment:

$$\Theta_{00} - 1/3$$

where Θ_{00} is the diagonal component of the spin density matrix, 0 being the eigenvalue of the third component of the spin operator $\widehat{S}_z(p)$. The possibility of detecting alignment in peripheral heavy ion collisions attracted the attention [68] almost as early as polarization [69, 70].

The alignment has been measured for the K^* and the ϕ vector mesons at $\sqrt{s_{\text{NN}}} = 200$ GeV [71] and $\sqrt{s_{\text{NN}}} = 2.76$ TeV [72] with different outcomes: a consistently negative value for the K^* at low p_T at both energies, whereas, for the ϕ , a slightly positive value at the lower energy and a negative value at low p_T at the higher energy. For all of these measurements the magnitude of the alignment (especially for the K^*) is apparently too large to be accomodated within a local equilibrium picture, which predicts a quadratic dependence on thermal vorticity [73] and, most likely, on the thermal shear too. Besides, it is difficult to explain the change of sign of the ϕ alignment from 200 to 2760 GeV. Several theoretical attempts have been made to reproduce these measurements which do not rely on local equilibrium, but on quark coalescence [74], collective effects [75] or, more recently, on calculations of quantum kinetic theory in weakly coupled QCD [76].

In my view, spin alignment is still an unsolved problem. It should be pointed out that the change of sign of ϕ alignment between the two explored energies is itself a puzzling evidence, difficult to reproduce in most models.

5. What is the spin good for?

As I have tried to emphasize, the study of spin-related phenomena in relativistic matter is a commendable effort for relativity is a more general and natural viewpoint for spin and it can lead to a deeper understanding of the non-relativistic phenomena, which are of course most likely to have practical applications. Still, spin may have other fruitful applications in the very field of relativistic heavy ion physics.

Indeed, as spin is the newest chapter for this field, thus far most efforts went and are still going into checking that spin measurements are understood within a commonly

accepted theoretical framework, what has been the subject of previous sections. As the theoretical picture consolidates, the question arises whether spin can be used as a tool to attack fundamental unanswered or partially answered questions on the QCD matter, the QGP and its properties and evolution. Indeed, there have been interesting proposals in this respect and I would like to mention three of them.

A first proposal is to use the measurement of helicity - that is the projection of the mean spin vector in the particle rest frame onto the momentum direction - to detect local parity violation in the QCD matter, the long-sought phenomenon with the Chiral Magnetic Effect (CME) [77]. The idea of revealing local parity violation with the helicity correlation of Λ hyperons was put forward in ref. [78] and has been recently proposed again in ref. [79] where a formula was found connecting the helicity to the axial chemical potential μ_A developed as a consequence of the local parity violation induced at high temperature [80]:

$$S_X^\mu(p) \simeq \frac{g_h \int_\Sigma d\Sigma_\lambda p^\lambda (\mu_A/T) n_F (1 - n_F) \varepsilon p^\mu - m^2 \hat{t}^\mu}{2 \int_\Sigma d\Sigma_\lambda p^\lambda n_F} \frac{1}{m\varepsilon} \quad (15)$$

where \hat{t} is the unit vector in the QGP center-of-mass frame, as before, and g_h is the axial charge of the Λ . The detection of local parity violation through the measurement of the helicity azimuthal correlation function of pairs of Λ hyperons does not require a coupling with the electro-magnetic field and it is thus complementary to the CME. The feasibility of such a measurement, in terms of statistics, experimental backgrounds and errors, is still under investigation.

A second possibility offered by spin measurements is to investigate the energy loss of highly energetic partons, i.e. jets, in the QGP. It has been pointed out by Serenone et al. [81] that a fast parton losing energy within the plasma would induce a vorticity field (more generally, a gradient of the hydrodynamic fields). Such a non-trivial field pattern can be probed by a measurement of the polarization of the emitted Λ hyperons as a function of their angular distance from the jet axis. The authors also showed that the amplitude of such induced polarization is very sensitive to the shear viscosity/entropy density ratio η/s .

A third, very recent idea, is to use spin polarization to search for the QCD critical point. It has been observed by the authors of ref. [82] that, in the proximity of the critical point, the change in the equation of state and the scaling behaviour of the transport coefficients, affects the thermal vorticity field to a sizeable extent, hence the final polarization of hyperons. Particularly, it turns out that the dependence of $S_j^* = \mathbf{S}_0 \cdot \hat{\mathbf{J}}$ spin component along the total angular momentum on the rapidity of the Λ is markedly different if the critical point is there, with a relatively strong suppression at midrapidity. As the magnitude of polarization at lower energy is larger than at high energy, a measurement is likely to be feasible with a sufficiently high statistics of Λ hyperons at the forthcoming heavy ion facilities NICA and FAIR.

6. Conclusions

Spin has opened a new window in the physics of the QGP and relativistic heavy ion collisions. As I have tried to emphasize, its sensitivity to the gradients of the thermodynamic fields make it a powerful probe of the hydrodynamic model of the plasma and of its initial conditions. Besides, the potentialities of the study of spin in this field with respect to a variety of phenomena and other fields such as spintronics, are still to be fully explored and understood.

While I am writing, theory and experiments are still in a quickly developing stage, with a fast and intense mutual and beneficial interaction. Over the past year, the progress in phenomenology has been remarkable and, in spite of the high level of accuracy which is demanded in the spin-related calculations, several groups in the world proved to be able to make consistent numerical simulations. At the same time, the experiments proved to be able to carry out more and more differential measurements. In the future, a further increase of collected statistics will make it possible to test even more detailed observables, such as spin-spin correlations [83] which will probe the hydrodynamic, as well as other models, in more depth.

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