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Spin-down of rapidly rotating, convective stars

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SUMMARY

We construct a model for the spin-down of rapidly rotating, convective stars. We assume that rotation and convection cause a star to rotate differentially, that differential rotation and convection generate a magnetic dynamo, that a magnetic dynamo gives rise to mass loss and a magnetically controlled stellar wind, and that such a wind results in angular momentum loss and hence stellar spin-down. Using the model, we show that a protostar accreting from a circumstellar disc reaches an equilibrium rotation period significantly below break-up. We also show that the spin-down rates we derive are consistent with those required to drive the orbital evolution of cataclysmic variables.

Key words: accretion, accretion discs – convection – MHD – stars: mass-loss – novae, cataclysmic variables – stars: rotation.

1 INTRODUCTION

It has long been assumed that stars, as they form, accrete most of their material through an accretion disc (von Weizsäcker 1948; Lüst 1952; Lynden-Bell & Pringle 1974; see the review by Shu, Adams & Lizano 1987). It has also been realized that material deposited on a central object from an accretion disc arrives with high angular momentum and so should lead to a rapid spin-up of the central object (Papaloizou & Pringle 1978; Pringle 1988; Hartmann & Stauffer 1989). For these reasons it might have been expected that the youngest stars, in particular the T Tauri stars, should be rotating close to break-up. In fact, although a few T Tauri stars do rotate rapidly, it has become evident that most young stars do not. Instead, they have mean rotational velocities of around one-tenth of break-up speed (Vogel & Kuhi 1981; Smith, Beckers & Barden 1983; Hartmann *et al.* 1986; Hartmann & Stauffer 1989; see the review by Bouvier 1991). This implies that either during the accretion phase or within about 10^5 yr thereafter, substantial angular momentum loss must take place from the newly formed star (Hartmann *et al.* 1986). An obvious way in which this can be achieved is by magnetic wind braking (Pringle 1988; Hartmann & Stauffer 1989). [For an alternative possibility, see Königl (1991).]

In this paper, we investigate angular momentum loss from a rapidly rotating, fully convective protostar. Although protostars formed by spherical accretion are maintained in a radiative state by the entropy of the accreted material (Larson 1969; Stahler, Shu and Taam 1980a,b, 1981), those formed by accretion from a circumstellar disc appear to be fully convective regardless of initial conditions (Mercer-Smith, Cameron & Epstein 1984; Tout, in preparation). We construct a model which is fully self-consistent in the sense that the rotation plus convection give rise to dynamo action within the star. This in turn gives rise to a magnetic field which governs the wind structure, and energy deposition at the surface of the star which drives the wind, thus leading to angular momentum loss and spin-down. Since most of the physical processes we consider here are poorly understood, it is necessary to make what we hope are reasonable assumptions as we proceed. For this reason, the precise numerical values we derive and the details of the functional dependencies we find should not be taken at face value. Our aim in this paper is to demonstrate that such a fully consistent picture is a physical possibility, and moreover that credible and sufficient spin-down rates can be achieved in such a picture.

In Section 2 we discuss the details of the magnetic wind. As surmised by Hartmann & Stauffer (1989), we shall find that the wind is not rotationally driven. Rather, at the high mean loss rates we derive [being ~ 0.1–1.0 times the accretion rate; *cf.* Edwards *et al.* (1987), Hartmann & Raymond (1989)], and for the surface fields which we find ($\sim 10^3$ G), the Alfvén radius is only a few stellar radii. In Section 3 we consider angular momentum transfer through the star, and in particular the generation of shear within the star by a combination of rotation and convection, and also the magnitude of 'eddy viscosity' which is the result of convective motions, limited by rotation. In Section 4 we present our schematic

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dynamo model. We do not consider the details of the dynamo process, nor do we attempt to consider time variations, solar-type cycles and the like. Rather, we consider general (time- and space-averaged) balances between the relevant physical terms. To be specific, we assume that the shear converts poloidal flux to toroidal, and that the standard process of convection and rotation converts toroidal to poloidal at some rate which we parametrize. These are the growth terms. We assume that the dominant loss mechanism by which magnetic flux leaves the star is magnetic buoyancy (Durney & Robinson 1982; Robinson & Durney 1982) and that this energy loss process leads to energy deposition at the stellar surface and the driving of a stellar wind. In Section 5, we tie all the above ideas together in a specific model. We calculate the shear present in the star self-consistently, by balancing shear production from rotation and convection with shear loss to the dynamo. Our general finding is that a rate of angular momentum loss is attainable which is sufficient to ensure that even a young pre-main-sequence star emerging from its accretion phase can be spinning significantly below break-up. In Section 6, we address briefly the criticism (e.g., Bouvier 1991) that such models seem to necessitate a chance coincidence between angular momentum gain and loss in a situation where the gain and loss mechanisms are not directly physically related. Note however, that at high accretion rates, this criticism loses some of its force because the accretion drives the stellar luminosity, and hence the convection and ultimately the mass loss. In this section, however, we assume that the driving of shear by anisotropic turbulence (convection plus rotation) is less efficient than we had assumed, and consider the shear to be driven by accretion of high angular momentum material at the stellar equator and wind-driven angular momentum loss from higher stellar latitudes. Under this assumption there is less shear and hence less mass loss. Even so, we find that rotation rates below break-up are achievable. In Section 7 we undertake a critical discussion of the above model and also consider its application to other types of stars. In particular, we show that the angular momentum loss rates derived from the model are comparable to those required from the secondary stars in cataclysmic variables, to drive the mass transfer and hence the evolution of those systems.

2 MAGNETIC WIND

We consider a protostar with mass M, radius R_* and angular velocity Ω. We assume that it is losing mass in a wind at a rate $\dot{M}_w$. For the cases under consideration, we shall find that the stars are slow rotators in the sense that the radius, R_A, out to which the wind is forced by the field to corotate with the star is smaller than the corotation radius, R_Ω, at which centrifugal force on the (corotating) wind is balanced by the star's gravity. We define R_Ω by $R_\Omega\Omega^2 = GM/R_\Omega^2$. If we define the 'break-up' angular velocity for the star by

$$\Omega_K = (GM/R_*^3)^{1/2}, \tag{2.1}$$

then we have

$$R_\Omega/R_* = (\Omega_K/\Omega)^{2/3}. \tag{2.2}$$

In effect, we assume that the wind consists of two zones.

(i) The inner zone is given by $R_* < R < R_A$. In this zone the wind is accelerated to of order the escape velocity by strong energy deposition at its base. Thus we shall take the wind to have a constant velocity, v_w, which is of order the escape velocity:

$$v_w \sim (2GM/R_*)^{1/2}. \tag{2.3}$$

This requires the rate of energy input into the wind to be

$$L_w \sim \frac{GM\dot{M}_w}{R_*}. \tag{2.4}$$

(ii) The outer zone comprises the region $R > R_A$. The Alfvén radius, R_A, is defined by the radius at which

$$v_w^2 \sim B^2/4\pi\rho_w, \tag{2.5}$$

where B is the magnetic field and ρ_w the density of wind material. Outside this radius the magnetic field can no longer provide communication between the wind and the star, and the wind then escapes freely. The angular momentum loss rate in the wind is

$$\dot{J}_w \sim \dot{M}_w R_A^2 \Omega. \tag{2.6}$$

Using equation (2.4), we find

$$L_w \sim \Omega \dot{J}_w (R_*/R_A)^2 (\Omega_K/\Omega)^2. \tag{2.7}$$

Thus far the physics of the wind is more or less inevitable, but we now come to a major uncertainty: the structure of the magnetic field and of the outflow. Most discussions in the literature concern themselves with low-density outflows (as for the Sun) in which the Alfvén radius is much larger than the corotation radius. For such flows there is an intermediate zone, $R_\Omega < R < R_A$, in which the wind is centrifugally accelerated and the field structure may need to be modified to reflect this. Belcher & MacGregor (1976) adopt what they call an 'equatorial approach', treating the flow as essentially two-dimensional. Verbunt (1984) argues, from Weber & Davis (1967), that the wind flows along radial field lines and so takes $B \propto R^{-2}$, equivalent to a monopole field. He also assumes that flow streamlines are radial and that the wind is spherically symmetric. Mestel & Spruit (1987) adopt a more complicated picture with a mixture of inner dipole and outer radial field lines. We take the view that the stellar magnetic field is likely to fall off faster than a monopole, and also wish to make some allowance for divergence of the flow in the sense that the open field lines may not connect to the whole stellar surface. To model this in a simpler manner, we shall assume that the field strength depends on radius in the form:

$$B = B_*(R_*/R)^n, \tag{2.8}$$

where n is a parameter. We work mostly with a 'dipolar' field taking $n=3$, but comment below on the effect of varying n (*cf.* Taam & Spruit 1989).

Then, using equations (2.5) and (2.8), and assuming roughly spherical outflow at the Alfvén surface so that we may write at that radius

$$\rho_w \sim \dot{M}_w/(4\pi R_A^2 v_w), \tag{2.9}$$

we obtain

$$\frac{R_A}{R_*} = \left(\frac{B_*^2 R_*^2}{\dot{M}_w v_w}\right)^{1/(2n-2)} \tag{2.10}$$

3 ANGULAR MOMENTUM

One of the driving terms for the stellar dynamo is the shear, $d\Omega/dR$, which converts poloidal field to toroidal (see below). Thus we need to estimate $d\Omega/dR$. We shall work in terms of a quantity $\Delta\Omega$, which we take to be an appropriate mean within the star of the quantity $R(d\Omega/dR)$. Roughly, $\Delta\Omega$ should be the change in angular velocity between the star's centre and its outer edge.

In non-accreting single stars, the differential rotation is presumably maintained by an interaction between convection and rotation (e.g., Rüdiger 1989). This must be true for the regions in the Earth where the terrestrial dynamo is maintained (Moffatt 1978; Cook 1980). For stars there is also the possibility that the stellar wind provides a latitude-dependent torque and so maintains differential rotation. In either case, the energy for driving the dynamo comes essentially from the rotational kinetic energy of the star. We shall assume therefore that the effect of convection in a differentially rotating star is to give rise to a non-isotropic viscosity, which in turn would, in the absence of other processes, give rise to a variation of Ω through the star. In other words, we assume that a convective, rotating and non-magnetic star has differential rotation such that $\Delta\Omega \sim \Omega$, and such that the shear energy, $E_{\rm sh}$, is comparable to the rotation energy of the star, $E_{\rm rot}=\frac{1}{2}k^2MR_*^2\Omega$, where k is the dimensionless stellar radius of gyration. Thus the rate, L_+, at which energy is fed into the shear is

$$L_+ \sim \tfrac{1}{2}k^2MR_*^2\Omega^2/\tau_\nu, \tag{3.1}$$

where τ_ν is the viscous time-scale. We expect

$$\tau_\nu \sim R_*^2/\nu, \tag{3.2}$$

where ν is a measure of the kinematic (convective) viscosity.

In a non-rotating star, the convective viscosity would be of the form:

$$\nu \sim \tfrac{1}{3}v_{\rm c}l_{\rm c}, \tag{3.3}$$

where $v_{\rm c}$ is the convective velocity and $l_{\rm c}$ the mixing length. However, in a rapidly rotating star, in which the rotation period is less than the convective turnover time-scale, $\tau_{\rm c} \sim l_{\rm c}/v_{\rm c}$, the effect of angular momentum transport by the largest convective cells is likely to be curtailed (see also the discussion in Section 4). This problem was considered by Goldreich & Keeley (1977; see also Campbell & Papaloizou 1983, Goldman & Mazeh 1991) who argued that rotation does not affect the convective motions, nor indeed the convective turbulent spectrum and concluded, by assuming a Kolmogorov spectrum for the turbulence, that the viscosity is reduced by a factor of $(\Omega\tau_{\rm c})^2$ for $\Omega\tau_{\rm c}>1$. We show below that an upwardly (or downwardly) moving fluid element rotates once after it has gone a distance of $\sim(\Omega\tau_{\rm c})^{-1}H_{\rm p}$, where $H_{\rm p}(\sim l_{\rm c})$ is the local scaleheight. Thus it seems reasonable to assume that advection of angular momentum is restricted by a factor of $\Omega\tau_{\rm c}$, while advection of energy is essentially unchanged. In view of this, we adopt a viscosity of the form:

$$\nu \sim \tfrac{1}{3}v_{\rm c}l_{\rm c}(\Omega\tau_{\rm c})^{-p}, \tag{3.4}$$

where $p \ge 0$. For the above reasons, we shall adopt $p=1$ in this paper and take

$$\nu \sim \tfrac{1}{3}v_{\rm c}^2\Omega^{-1}. \tag{3.5}$$

To estimate the convective velocity, $v_{\rm c}$, we use standard mixing-length theory (Schwarzschild 1958) and obtain

$$L_* \sim \eta M v_{\rm c}^3/R_*, \tag{3.6}$$

where the constant $\eta \sim 3R_*/l_{\rm c} \sim 30$ (*cf.* Campbell & Papaloizou 1983).

4 THE DYNAMO MODEL

The dynamo equations (e.g., Moffatt 1978; Parker 1979; Cowling 1981) can be written in a schematic form as:

$$\frac{dB_\phi}{dt}=\Delta\Omega\, B_{\rm p}-B_\phi/\tau_\phi, \tag{4.1}$$

and

$$\frac{dA_\phi}{dt}=\Gamma B_\phi - A_\phi/\tau_{\rm p}. \tag{4.2}$$

Here B_ϕ is a measure of the toroidal field strength in the star, and A_ϕ is a measure of the azimuthal component of the magnetic vector potential, and so represents the poloidal field, $B_{\rm p}$. Indeed, since $B_{\rm p}\sim A_\phi/R_*$, we may rewrite (4.2) schematically as

$$\frac{dB_{\rm p}}{dt}=\left(\frac{\Gamma}{R_*}\right)B_\phi - B_{\rm p}/\tau_{\rm p}. \tag{4.3}$$

The quantities $\tau_{\rm p}$ and τ_ϕ are the time-scales on which poloidal and toroidal flux are lost or destroyed, and the quantity Γ is the standard dynamo regeneration term (also called α).

The usual form for the loss terms is some effective magnetic diffusivity, and the usual assumption about Γ is in terms of convective motions (e.g., Noyes, Weiss & Vaughan 1984). Thus the standard dynamo equations are effectively linear in the magnetic field and are unable to predict the field strength. For this reason, from a physical point of view, some kind of non-linearity must be introduced in order to model a realistic dynamo (e.g., Weiss, Cattaneo & Jones 1984). The simplest assumption to make about flux loss is that the dominant mechanism for the star as a whole is loss caused by magnetic buoyancy (i.e., the Parker instability). Although magnetic diffusivity must be acting on the smaller scales [in particular the Γ-mechanism cannot operate without it at some level (Parker 1979)], we take the view that magnetic flux is escaping from the star on a time-scale comparable to the growth time-scale for the Parker instability. Thus we follow Parker's assumption (Parker 1977) that the field escapes at a velocity some fraction of the Alfvén speed (see also Horiuchi *et al.* 1988; Matsumoto *et al.* 1988). We shall take the velocity at which flux escapes to be $\sim 0.1v_{\rm A}$, where $v_{\rm A}$ is an appropriate mean Alfvén speed in the star (Matsumoto *et al.* 1990). We note that the exact fraction assumed, here 0.1, does not play an important role in the resulting model. Thus we conclude that $\tau_{\rm p}\sim\tau_\phi\sim 10\tau_{\rm A}$, where $\tau_{\rm A}\sim R_*/v_{\rm A}$ is the Alfvén-wave crossing time in the star.

We note further that since we expect the shear term ($\Delta\Omega$) to act more rapidly than the regeneration term (Γ), we also expect that $B_\phi \gg B_{\rm p}$. We shall write $B_{\rm p}\sim\varepsilon B_\phi$ and expect that

$\varepsilon \ll 1$. This also means that we may write

$$v_A \sim B_\phi/(4\pi\rho_*)^{1/2}, \tag{4.4}$$

where

$$\rho_* \sim M/(4\pi R_*^3/3). \tag{4.5}$$

The origin of the Γ (or α) term is discussed by Cowling (1981), and the usual idea goes back to Parker's (1955) suggestion of cyclonic turbulence. If convective or turbulent motions are present, then rising eddies expand and so rotate relative to the local medium. This rotation is the basis of the twist which produces poloidal flux from toroidal, and so completes the feed-back loop and enables the self-sustaining dynamo. In these circumstances the standard formula for Γ is of the form (Cowling 1981)

$$\Gamma \sim 0.3\tau_t v_t \omega_t, \tag{4.6}$$

where τ_t is the turbulent turnover time-scale, v_t the velocity of a typical turbulent cell, ω_t the component of the vorticity parallel to v_t in the cell induced by the vertical motion, and the factor 0.3 comes from a specific model. Within an upwardly moving parcel of matter one expects

$$\omega_t \sim \Omega(d/H_p), \tag{4.7}$$

where d is the vertical distance travelled and H_p the pressure scaleheight. Again, for a turbulent cell one expects $d \sim v_t \tau_t$. For a slowly rotating star the angle through which a cell twists before it dissolves is small compared to π – this is the implicit assumption used by Parker (1979) in his derivation of Γ. However, for a rapidly rotating star there is the possibility that a cell could twist many times before dissolving. We shall assume therefore that a typical turbulent cell dissolves before it twists once. This corresponds to assuming that $\tau_t \omega_t$ is fixed and less than or of order unity. From equation (4.3), we can see that we may write

$$\Gamma \sim \gamma v_t, \tag{4.8}$$

where γ is an unknown parameter which measures the efficiency of the regeneration term (*cf.* Parker 1979). We assume that the regeneration of field is inefficient and adopt a canonical value of $\gamma \sim 10^{-2}$ (see below).

For a fully convective star we take v_t to be the convective velocity within the star, v_c, which is given roughly in terms of the stellar luminosity L_* by equation (3.6). We note that for consistency we shall require $v_c \gtrsim 0.1 v_A$.

We now assemble the various ideas and assumptions and look for a steady-state solution of the schematic dynamo equations. From equation (4.3) we find that

$$\varepsilon \sim \tau_p \Gamma/R_*, \tag{4.9}$$

and hence using equations (4.4) and (4.8), the definitions of ε and the assumption about τ_p we obtain

$$B_p \sim 10\gamma v_c(4\pi\rho_*)^{1/2}. \tag{4.10}$$

Similarly, from equation (4.1) we find

$$\varepsilon \sim 0.1(\Delta\Omega)^{-1} v_A/R_*. \tag{4.11}$$

The dynamo process not only creates magnetic field, but also leads to a continual expulsion of flux from the star. It is this mechanical output of energy that we assume drives the stellar wind, at least in the inner zone (*cf.* Hartmann & MacGregor 1980). Thus we write approximately:

$$L_w \sim \frac{d}{dt}\left(\frac{B_\phi^2}{8\pi}\right)\bigg|_{\rm loss} \frac{4}{3}\pi R_*^3 \sim \frac{B_\phi^2}{4\pi}\tau_\phi^{-1}\frac{4}{3}\pi R_*^3 \sim 0.1 M v_A^3/R_* \tag{4.12}$$

(*cf.* equation 3.6).

5 APPLICATION

We are now in a position to tie the above ideas together. From equation (3.1), convective forces put energy into the differential rotation at a rate L_+. Energy is removed from the shear by the magnetic field at a rate (equation 4.1)

$$L_- \sim \frac{d}{dt}\left(\frac{B_\phi^2}{8\pi}\right)\bigg|_{\rm gain} \frac{4}{3}\pi R_*^3 \sim \Delta\Omega(B_p B_\phi/4\pi)\frac{4}{3}\pi R_*^3 \sim \varepsilon\Delta\Omega M v_A^2. \tag{5.1}$$

Equating L_+ (equation 3.1) and L_-, and using equations (4.11) and (4.12), we then find

$$L_w \sim \tfrac{1}{6}k^2 M\Omega v_c^2 \tag{5.2}$$

and that, assuming $k^2 \approx 0.1$,

$$v_A^3 \sim \tfrac{1}{6}(R_*\Omega)v_c^2. \tag{5.3}$$

For illustration, we consider the case of a fully convective protostar descending the Hayashi track with mass $M = 1\,M_\odot$, radius $R_* = 3\,R_\odot$ and luminosity $L_* = 4\,L_\odot$. From equation (2.1) we have the Keplerian angular velocity:

$$\Omega_K = 0.61(M/M_\odot)^{1/2}(R_*/3R_\odot)^{-3/2}\ \text{rad d}^{-1}. \tag{5.4}$$

From equation (3.6), we find

$$v_c \sim 3.8\times 10^3(L_*/4L_\odot)^{1/3}(\eta/30)^{-1/3}(M/M_\odot)^{-1/3}(R_*/3R_\odot)^{1/3}\ \text{cm s}^{-1}. \tag{5.5}$$

From equation (5.3), we find

$$v_A \sim 3.9\times 10^4 f^{1/3}(L_*/4L_\odot)^{2/9}(\eta/30)^{-2/9}(M/M_\odot)^{-1/18} \times (R_*/3R_\odot)^{1/18}\ \text{cm s}^{-1}, \tag{5.6}$$

where we have defined $f \equiv \Omega/\Omega_K$. Hence, using equation (4.5), viz.

$$\rho_* = 0.052(M/M_\odot)(R_*/3R_\odot)^{-3}\ \text{g cm}^{-3}, \tag{5.7}$$

we find

$$B_\phi = 3.2\times 10^4 f^{1/3}(L_*/4L_\odot)^{2/9}(\eta/30)^{-2/9}(M/M_\odot)^{4/9} \times (R_*/3R_\odot)^{-13/9}\ \text{G}. \tag{5.8}$$

Using equation (4.10), we find

$$B_p \sim 3.1\times 10^2(L_*/4L_\odot)^{1/3}(\eta/30)^{-1/3}(M/M_\odot)^{1/6} \times (R_*/3R_\odot)^{-7/6}(\gamma/10^{-2})\ \text{G}, \tag{5.9}$$

and hence,

$$\varepsilon \sim 9.6\times 10^{-3} f^{-1/3}(L_*/4L_\odot)^{1/9}(\eta/30)^{-1/9}(M/M_\odot)^{-5/18} \times (R_*/3R_\odot)^{5/18}(\gamma/10^{-2}), \tag{5.10}$$

which is much less than unity.

From equation (5.2), and again using $k^2 \approx 0.1$, we find

$$L_{\mathrm{w}} = 5.7 \times 10^{34} f (L_*/4L_\odot)^{2/3} (\eta/30)^{-2/3} (M/M_\odot)^{5/6} \times (R_*/3R_\odot)^{-5/6}\ \mathrm{erg\ s^{-1}} \tag{5.11}$$

and hence (equation 2.4),

$$\dot{M}_{\mathrm{w}} = 1.4 \times 10^{-6} f (L_*/4L_\odot)^{2/3} (\eta/30)^{-2/3} (M/M_\odot)^{-1/6} \times (R_*/3R_\odot)^{1/6} M_\odot\ \mathrm{yr^{-1}}. \tag{5.12}$$

We note in passing that if one assumes that surface emission from a magnetically active star is proportional to L_{w}, which we assume to be the rate at which the dynamo deposits energy at the stellar surface, then we predict a surface emissivity per unit area of the form $(L_{\mathrm{w}}/R^2) \propto \Omega M^{1/3} R^{-4/3} L^{2/3}$. This dependence of surface emissivity on rotation period is similar to the one found by Vilhu & Rucinski (1983; see also Rucinski 1985).

We now identify B_* with B_{p}. That is, we assume that the magnetic field in the wind is predominantly caused by the poloidal component of the stellar field. Then using equations (5.9) and (5.12) together with the expression (equation 2.3), viz.,

$$v_{\mathrm{w}} \sim 3.6 \times 10^7 (M/M_\odot)^{1/2} (R_*/3R_\odot)^{-1/2}\ \mathrm{cm\ s^{-1}}, \tag{5.13}$$

we obtain from equation (2.10), using $n=3$ (i.e., a dipole-like field):

$$R_{\mathrm{A}}/R_* = 1.1 f^{-1/4} (\gamma/10^{-2})^{1/2}. \tag{5.14}$$

We note that $R_{\mathrm{A}} \gtrsim R_*$, provided that $f \lesssim 1.3(\gamma/10^{-2})^2$. We also note that

$$R_{\mathrm{A}}/R_\Omega \sim 1.1 f^{5/12} (\gamma/10^{-2})^{1/2}, \tag{5.15}$$

which is less than unity for $f \lesssim 0.9(\gamma/10^{-2})^{-6/5}$. We now define the spin-down time-scale, τ_{sd}, as

$$\tau_{\mathrm{sd}} \sim \frac{k^2 M R_*^2 \Omega}{\dot{J}_{\mathrm{w}}}, \tag{5.16}$$

where $\dot{J}_{\mathrm{w}}$ is given by equation (2.6). Thus we obtain

$$\tau_{\mathrm{sd}} \sim 6.2 \times 10^4 f^{-1/2} (\gamma/10^{-2})^{-1} (\eta/30)^{2/3} (L_*/4L_\odot)^{-2/3} \times (M/M_\odot)^{7/6} (R_*/3R_\odot)^{-1/6}\ \mathrm{yr}. \tag{5.17}$$

For a protostar accreting at a rate

$$\dot{M}_{\mathrm{acc}} \sim 6.3 \times 10^{18} \dot{m}\ \mathrm{g\ s^{-1}}, \tag{5.18}$$

where $\dot{m}$ is the accretion rate in units of $10^{-7} M_\odot\ \mathrm{yr^{-1}}$, we may define a spin-up time-scale, τ_{su}, as

$$\tau_{\mathrm{su}} = \frac{k^2 M R_*^2 \Omega}{\dot{M}_{\mathrm{acc}} (G M R_*)^{1/2}} = 1.0 \times 10^6 (M/M_\odot) f \dot{m}^{-1}\ \mathrm{yr}. \tag{5.19}$$

For an equilibrium rotation rate, we equate τ_{su} and τ_{sd} to find

$$f = 0.16 \dot{m}^{2/3} (\gamma/10^{-2})^{-2/3} (\eta/30)^{4/9} (L_*/4L_\odot)^{-4/9} (M/M_\odot)^{1/9} \times (R_*/3R_\odot)^{-1/9}. \tag{5.20}$$

We note that, strictly speaking, the luminosity we have used for a star descending the Hayashi track is valid only if the accretion rate is small enough. We may write the accretion energy as

$$L_{\mathrm{acc}} \sim G M \dot{M}_{\mathrm{acc}}/R_* \sim 4.0 \times 10^{33} (M/M_\odot)(R_*/3R_\odot)^{-1} \dot{m}\ \ \mathrm{erg\ s^{-1}}. \tag{5.21}$$

As a matter is accreted on to the star, enforced gravitational contraction generates an internal stellar luminosity of order L_{acc}. Thus the above equations hold only if $L_{\mathrm{acc}} \lesssim L_*$, i.e., if

$$\dot{m} \lesssim \dot{m}_{\mathrm{crit}} \sim 3.8 (L_*/4L_\odot)(M/M_\odot)^{-1} (R_*/3R_\odot). \tag{5.22}$$

At higher accretion rates we substitute $L \sim L_{\mathrm{acc}}$ in equation (5.20) to obtain

$$f \sim 0.29 \dot{m}^{2/9} (\gamma/10^{-2})^{-2/3} (\eta/30)^{4/9} (M/M_\odot)^{-1/3} (R_*/3R_\odot)^{1/3}. \tag{5.23}$$

6 ACCRETION-DRIVEN SHEAR

From the calculations of Section 5, it can be shown that the amount of shear required in the star, for $\dot{m} < \dot{m}_{\mathrm{crit}}$, is

$$\frac{\Delta\Omega}{\Omega} \sim 1.6 \times 10^{-2} f^{-1/3} \left(\frac{\gamma}{10^{-2}}\right)^{-1} (L_*/4L_\odot)^{-1/9} (\eta/30)^{-1/9} \times (M/M_\odot)^{-5/18} (R_*/3R_\odot)^{5/18}. \tag{6.1}$$

This shear comes about by a balance between an increase caused by the combination of convection and rotation and a decrease caused by magnetic torques. In such a situation one expects $d\Omega/dR < 0$. In an accreting protostar, however, there is another source of shear which is caused by the star being spun up at the equator by accretion and being spun down by the stellar wind torques operating at higher latitudes. The accretion-induced shear $\Delta\Omega_{\mathrm{acc}}$ can be estimated by equating the rate at which angular momentum is added to the star, $\dot{J}_{\mathrm{acc}} \sim \dot{M}_{\mathrm{acc}} (G M R_*)^{1/2}$, to the rate at which the convective viscosity can transfer the angular momentum through the star, viz.

$$\dot{J}_{\mathrm{acc}} \sim \Delta\Omega_{\mathrm{acc}} \rho \nu 4\pi R_*^2 R_*. \tag{6.2}$$

From this we deduce

$$\frac{\Delta\Omega_{\mathrm{acc}}}{\Omega} \sim 1.1 \times 10^{-3} \dot{m} (L_*/4L_\odot)^{-2/3} (\eta/30)^{2/3} (M/M_\odot)^{1/6} \times (R_*/3R_\odot)^{-1/6}. \tag{6.3}$$

Note that for such a shear we expect $d\Omega/dR > 0$.

We conclude that neglect of accretion-induced shear in Section 5 was justified. However, it is instructive to ask how the model outlined above might operate if this mechanism, that we have assumed gives rise to convection-induced shear, either does not operate or is less efficient than we suppose. In this case, if we use the same assumptions as above, but use the accretion-induced value for $\Delta\Omega$, we find fairly similar results. In particular, we find that

$$\dot{M}_{\mathrm{w}}(\mathrm{acc}) \sim 2.6 \times 10^{-8} \dot{m}^{3/2} \left(\frac{\gamma}{10^{-2}}\right)^{3/2} f^{3/2} (L_*/4L_\odot)^{-1/2} (\eta/30)^{1/2} \times (M/M_\odot)^{1/2} (R_*/3R_\odot)^{-1/2} M_\odot\ \mathrm{yr^{-1}}. \tag{6.4}$$

We find a spin-down time-scale of

$$\tau_{\rm sd}({\rm acc}) \sim 4.6\times 10^5 \dot{m}^{-3/4}\left(\frac{\gamma}{10^{-2}}\right)^{-7/4} f^{-3/4}(L_*/4L_\odot)^{-1/12} \times(\eta/30)^{1/12}(M/M_\odot)^{5/6}(R_*/3R_\odot)^{1/6}\ {\rm yr}, \quad (6.5)$$

and hence deduce an equilibrium rotation rate of

$$f_{\rm acc} \sim 0.64\dot{m}^{1/7}\left(\frac{\gamma}{10^{-2}}\right)^{-1}(L_*/4L_\odot)^{-1/21}(\eta/30)^{1/21}(M/M_\odot)^{-2/21} \times(R_*/3R_\odot)^{2/21}. \quad (6.6)$$

7 DISCUSSION

We have constructed a model to investigate the spin-down rate of a rapidly rotating fully (or mainly) convective star. The model is self-consistent in the sense that the convectively driven shear coupled with the rotation gives rise to dynamo action within the star. The energy loss from the dynamo gives rise to a magnetically driven, and magnetically controlled, stellar wind which results in stellar angular momentum loss and hence in spin-down. For a fully convective protostar accreting from a circumstellar disc, the balance between gain and loss of angular momentum leads to an estimate of the stellar rotation rate for a given accretion rate. We find that such rotation rates can be significantly below break-up (equations 5.20 and 5.23).

However, in order to construct such an all-embracing model it has been necessary to gloss over a large number of physical uncertainties. To do so, we have made what we hope are reasonable or plausible assumptions. Indeed, many of the ideas adopted in various parts of the models are, as we have indicated, to be found already in the literature. Nevertheless, there are three major areas of uncertainty in the model as a whole. The first two are intimately linked and are concerned with the effect of convection on the generation of poloidal magnetic fields.

The first, the interaction between convection and rotation, is an area about which much has been written [see, for example, Durney (1983, 1985), Hathaway (1984), and the reviews in Tassoul (1978) and Rüdiger (1989)], but about which few firm conclusions have been drawn. For the regime we are considering, the Rossby number $[R_0 \equiv (\Omega\tau_c)^{-1}]$ is given by

$$R_0 = 1.5\times 10^{-3}(L_*/4L_\odot)^{1/3}(\eta/30)^{2/3} f^{-1}(M/M_\odot)^{-5/6} \times(R_*/3R_\odot)^{5/6}, \quad (7.1)$$

and is much less than unity. We have taken the viscosity $\nu \propto R_0^{-p}$, with $p=1$ (equation 3.5). We note that since R_0 is so small in this case, our results are sensitive to the precise value of p. By taking $p=1$, we assume in effect that although rotation has some effect, it is not as strong at these small Rossby numbers as would be implied by taking the extreme value of $p=2$ which can be found in the literature. We feel that the assumptions we have adopted here are close to the mainstream of current thinking, but certainly do not encompass all the possibilities. We stress, however, that the calculations in this paper are valid only in the limit of rapid rotation and should not be extrapolated to slow rotators.

The second uncertainty, the rate of regeneration of poloidal flux, is of course highly uncertain and widely discussed. Here we have made an assumption consistent with that adopted for the convective viscosity, but have also included the parameter, γ, which we have carried through the calculations. We adopted a canonical value of $\gamma = 10^{-2}$ for no reasons other than the fact that we expect γ to be smaller than unity (i.e., field regeneration to be inefficient) and this value seems to give rise to reasonable model parameters.

The third major area of uncertainty is in how the magnetic energy generated by the dynamo manages to drive the stellar wind, and what the field strength and structure in the wind is. We have assumed that all the magnetic energy generated by the dynamo process (and the major contributor is toroidal flux generated by shear) is advected to the stellar surface by the Parker instability and there dissipated, being converted efficiently into driving the stellar wind. This is of course over-optimistic, and we expect therefore the stellar wind mass-loss rates derived in this paper to be corresponding overestimates. Conversely, we have taken the magnetic field in the wind to be derived from the general poloidal field within the star. This is probably an underestimate, since much of the poloidal flux at the base of the wind is likely to come from the non-linear development of the Parker instability operating on the stellar toroidal field (*cf.* Shibata *et al.* 1990). Our assumption of a dipole-like radial flux distribution within the wind may go some way towards compensating for this, since the actual surface field is likely to have a higher multipole structure. However, since the Alfvén radius is not far from the stellar surface in the models presented here, the exact power law with which the magnetic field drops with radius does not play a large role in determining the outcome. It would of course have been possible to introduce further unknown parameters to allow for some of these incalculable effects, but we feel the proliferation of free parameters would have served merely to distract from the clarity of the paper without adding anything of useful or physical significance. Suffice it to say that in view of the above remarks, we expect the stellar mass-loss rates predicted here to be overestimates and the stellar surface field strengths (taken to be $B_{\rm p}$) to be underestimates. We hope that the net outcome is to give plausible estimates of spin-down time-scales and rotation rates.

Even so, we should note that with spin-down rates as rapid as those predicted by equation (5.17), we would have expected all completely or deeply convective stars to have spun down on time-scales much shorter than their evolutionary time-scales. However, in the young clusters α Persei and the Pleiades (ages around 6×10^7 yr), although most of the stars do rotate slowly, there is a large minority (about 30 per cent) which rotate rapidly with $f \approx 0.1$–0.5 (Jones 1991, personal communication; Prosser 1991; see also Stauffer 1988). As Hartmann (1991) remarks: 'It is difficult to understand why stellar wind angular momentum loss is *efficient* on time-scales $\lesssim 10^6$ yr in the T Tauri phase, but inefficient on time-scales $\gtrsim 10^7$ yr during post-T Tauri contraction.' Soderblom *et al.* (in preparation) point out that all models of spin-down appear to have difficulty in accounting for the almost bimodal distribution of rotation rates, which is independent of stellar mass, and suggest that there must be something complicated going on, if such similar stars appear to be undergoing such different spin-down histories. Such complications, they consider, might include intermittent, late-time accretion which might spin up perhaps just the outer layer of a few stars, and/or differential rotation between a rapidly spinning (radiative) core and a convective envelope which have erratic and occasionally efficient coupling between them. It is worth stressing, for example, that magnetic dynamos are in reality a highly non-linear and chaotic phenomenon (e.g., Weiss 1985), and that the simple modelling in terms of time- and space-averaged quantities which we have used here may simply not apply straightforwardly to all stars. Thus it may not be unreasonable in practice to expect that superficially similar stars have different modes of dynamo activity and so have different spin-down histories.

Although most of this paper has been concerned with the rotation of pre-main-sequence stars, we note that angular momentum loss from rapidly rotating stars is thought to play a crucial role in determining the evolution of cataclysmic variables. These stars consist of a low-mass quasi-main-sequence star (the secondary) filling its Roche lobe and transferring material on to a white dwarf (the primary). For the longer period systems, it is thought that the evolution (i.e., the mass transfer) is driven predominantly by an unseen magnetic wind from the mainly convective secondary star. The wind removes spin angular momentum from the secondary, and since the secondary is strongly tidally coupled, this has the net effect of removing angular momentum from the orbit. This process was introduced by Verbunt & Zwaan (1981) and later used by Rappaport, Verbunt & Joss (1983) to make detailed calculations of the evolution of these systems. We also note that for these systems, it has been hypothesized that when the secondary star's mass has been reduced so much that it becomes fully convective, the magnetic braking process becomes suddenly less effective (Rappaport *et al.* 1983; Spruit & Ritter 1983; Taam & Spruit 1989). Thus the arguments in this paper, in favour of efficient magnetic braking of fully convective stars, are somewhat in contradiction to some suggested theories of cataclysmic variable evolution. Nevertheless it is instructive to compare the spin-down rates required in cataclysmic variables and those computed here. The torque formula used by Rappaport *et al.* is of the form

$$\dot{J}_{\rm cv}=3.8\times 10^{-30} M R_\odot^4 (R_*/R_\odot)^\Gamma \Omega^3 \text{ dyn cm}, \tag{7.2}$$

where the parameter Γ is taken in the range 2 to 4. This is to be compared with our derivation that

$$\dot{J}_{\rm w}=5.2\times 10^{38} f^{3/2} (\gamma/10^{-2})(L_*/4L_\odot)^{2/3}(\eta/30)^{-2/3}(M/M_\odot)^{1/3} \times (R_*/3R_\odot)^{2/3} \text{ dyn cm}. \tag{7.3}$$

The comparison between these two is not straightforward. This is because in the case of cataclysmic variables, as the evolution proceeds, the mass M, radius R_* and spin rate Ω (or equivalently binary orbital period $P=2\pi/\Omega$) are all strongly correlated. Thus the required functional dependence of $\dot{J}_{\rm cv}$ on each of these quantities separately is not well determined. Indeed, it is apparent from figs 2 and 3 of Rappaport *et al.* that as the evolution proceeds we have the approximate relation $M\propto R_*\propto \Omega^{-1}$, and thus the effective dependence of $\dot{J}_{\rm cv}$ on Ω is much weaker than the $\dot{J}_{\rm cv}\propto\Omega^3$ apparent from equation (7.1). To make a more direct comparison, we evaluate the formulae (7.1) and (7.2) for two particular models given by Rappaport *et al.* (1983). For the first, we take $M=0.3M_\odot$, $R_*=0.3R_\odot$, $L=10^{-2}L_\odot$, $\Gamma=4$ and $P=3$ hr. For such binaries where the mass ratio q is of order 0.1, we find $f\approx 0.3$. Then, evaluating the above we find $\dot{J}_{\rm cv}\simeq 8.7\times 10^{34}$ dyn cm and $\dot{J}_{\rm w}=2.2\times 10^{35}$ dyn cm. For the second model we take $M=0.3M_\odot$, $R_*=0.4R_\odot$, $L=2\times 10^{-2}L_\odot$, $\Gamma=2$ and $P=4$ hr. For this model we find $j_{\rm cv}\simeq 7.3\times 10^{35}$ dyn cm and $\dot{J}_{\rm w}\simeq 5.2\times 10^{35}$ dyn cm. We conclude that the rates of angular momentum loss predicted by our model are not greatly out of line with those required for driving the evolution of cataclysmic variable systems.

In summary, it is evident that the model presented here should be regarded as neither definitive nor inevitable. In each part of the model there are a number of other possibilities which need to be explored. We regard it as encouraging, however, that such a model might be able to account for the observed relatively slow rotation rates of the youngest low-mass stars, and to tie in with the evolution of cataclysmic variables. We hope therefore that at the least the above analysis will provide a useful framework for future discussion.

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