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Based on the construction of generalized Halperin wave functions, we predict the possible existence of a large class of broken spin symmetry states in bilayer quantum Hall structures, generalizing the recently suggested canted antiferromgnetic phase to many fractional fillings. We develop the appropriate Chern-Simons theory, and establish explicitly that the low-lying neutral excitation is a Goldstone mode and that the charged excitations are bimerons with continuously tunable (through the canted antiferromagnetic order parameter) electric charge on the individual merons.

Recently, a canted antiferromagnetic (CAF) state has been predicted to exist in bilayer quantum Hall (QH) systems at the special filling factor $\nu=2$,[1] or more generally at $\nu=2/m$ where m is an odd integer.[2] The original theoretical prediction[1] based on a microscopic Hartree-Fock calculation has been followed up by a number of subsequent theoretical works using a quantum nonlinear σ model,[2] a bosonic spin approach,[3,4] and more detailed Hartree-Fock calculations.[5–7] Fairly persuasive experimental support for the CAF phase in $\nu=2$ bilayer QH systems also exists.[8] The basic idea underlying the CAF phase is that the competition between interlayer tunneling, Zeeman splitting, intralayer Coulomb interactions, and interlayer Coulomb interactions can cause spontaneous symmetry breaking in bilayer systems, leading to the CAF phase. This CAF phase lies in between the usual spin polarized ferromagnetic phase and the symmetric paramagnetic (or singlet) phase.

In spite of the extensive theoretical work on the problem using Hartree-Fock or related spin operator approaches,[1–7] a fundamental understanding of the precise nature of the CAF phase, either from the perspective of actual QH wave functions or from a long wavelength field theoretic viewpoint, is still lacking. In this paper we construct a microscopic wave function for the ground state of the CAF state, and develop a Chern-Simons theory to study the excitations above the CAF ground state. We find a neutral Goldstone mode associated with the breaking of the spin symmetry in the CAF phase and bimeronic charged excitations, which we discuss below. Furthermore, we establish that the type of symmetry breaking characterizing the CAF state is quite generally allowed in bilayer QH systems and may in principle exist for a large class of QH states far beyond the originally predicted $\nu=2/m$ filling factors.

As pointed out first by Wen and Zee,[9] the Halperin (m,m,m) wave functions[10] (neglecting electron spin) in bilayer QH systems have the property of fixing the total filling factor ν in the system, but not the individual filling factors of each layer. This allows one to construct wave functions that are a superposition of states with different numbers of particles in each layer (but fixed total number of particles), which leads to spontaneous interlayer coherence in the absence of interlayer tunneling.[9,11] We show that the analogous Halperin-type construction for spinful electrons in bilayer QH systems leads to a CAF phase that breaks the spin symmetry spontaneously.

The electron Hamiltonian for a bilayer QH system can be written as

$$\mathcal{H}=\int d^2x\{(1/2m)\,|(-i\vec{\partial}-e\mathbf{A}^{ex})\Psi_{a\sigma}|^2 + (u_I^c+u_O^c)(\bar{\Psi}_{a\sigma}\Psi_{a\sigma})^2+\Delta_Z\bar{\Psi}_{a\alpha}\sigma^z_{\alpha\beta}\Psi_{a\beta} + (u_I^c-u_O^c)(\bar{\Psi}_{a\sigma}\tau^z_{ab}\Psi_{b\sigma})^2+\Delta_{SAS}\bar{\Psi}_{a\sigma}\tau^x_{ab}\Psi_{b\sigma}\}. \quad (1)$$

Here a, b and α, β are layer (''isospin'') and spin indices, respectively; u_I^c is the intralayer and u_O^c is the interlayer Coulomb interaction; Δ_{SAS} is the splitting between symmetric and antisymmetric states due to interlayer tunneling; Δ_z is the Zeeman splitting. In the discussion below we choose to work with the symmetric/antisymmetric electron wave functions of definite S^z. Using such single electron wave functions, however, does not rely on assuming large interlayer tunneling and Zeeman splitting. As discussed in Ref. 1, Coulomb interaction itself generates effective tunneling and Zeeman field that are much larger than bare Δ_{SAS} and Δ_z. The appearance of such strong effective field is related to the tendency of electrons in the lowest Landau level to obey Hund's rule,[12] so that the fully polarized ferromagnetic or spin singlet phases discussed in Refs. 1–8 correspond to the system obeying Hund's rule in spin or isospin, and the CAF phase is a nontrivial phase that achieves a compromise between the two.

Of the four possible single particle states, the symmetric spin up ($S\uparrow$) state always has the lowest energy and antisymmetric spin down ($A\downarrow$) has the highest energy. Since the symmetric spin down ($S\downarrow$) and the antisymmetric spin up ($A\uparrow$) states may be close in energy, it is important to consider mixing between them.[13] We now construct a Halperin-

like wave function[10] for our spinful bilayer system. Our wave function does not fix the number of electrons in the $S\downarrow$ and $A\uparrow$ states individually, but fixes their sum. If we label $S\uparrow$ states by z, $S\downarrow$ states by u, and $A\uparrow$ states by w, we can easily write the Halperin wave function that fixes the number of electrons in the $S\downarrow$ and $A\uparrow$ states together, but not in each of them separately:

$$\Psi(\{z\}\{u\}\{w\}) = \prod (z_i - z_j)^n \prod (z_i - w_j)^l \prod (z_i - u_j)^l \times \prod (w_i - w_j)^m \prod (w_i - u_j)^m \prod (u_i - u_j)^m \times \exp\left[-\frac{1}{4}\left(\sum |z_i|^2 + \sum |u_j|^2 + \sum |w_k|^2 \right) \right]. \quad (2)$$

Here n and m are odd integers, and l can be any integer. A simple calculation then gives the total filling of this wave function: $\nu = (n+m-2l)/(nm-l^2)$. What is remarkable about such a wave function is that, since the individual filling factors in the $S\downarrow$ and $A\uparrow$ states are not fixed, we can consider wave functions that are a superposition of states with various $N_u - N_w$. They mix states with different values of S^z (the z component of spin lies along the direction of the magnetic field) and therefore describe states with spontaneously broken spin symmetry—the CAF state of Refs. 1–8. It is easy to see that taking $l=0$ and $n=m$ in Eq. (2) gives $\nu = 2/m$, i.e., the CAF state discussed in Refs. 1 and 2. In the CAF phase, the electrons in the two layers have the same z component of spin but opposite $x-y$ components. The direction of the Néel order parameter (defined as the difference in the spin expectation values in the two layers) comes from the spontaneous breaking of the S^z spin symmetry. It should be mentioned that Halperin wave functions for spontaneously broken spin symmetry states may also be constructed for single layer QH systems, leading to the possibility (at least in principle) of exotic spin states in a single layer QH system.[14]

Properties of the state (2) are conveniently discussed using a bosonic Chern-Simons theory.[15,16] For simplicity, we again assume that the $A\downarrow$ states are empty and consider only three kinds of electrons: Ψ_1 for $S\uparrow$, Ψ_2 for $S\downarrow$, and Ψ_3 for $A\uparrow$. Equation (2) tells us that the electron Ψ_1 is seen as a vortex of strength n by other Ψ_1 electrons and a vortex of strength l by electrons $\Psi_{2,3}$; electrons $\Psi_{2,3}$ are seen as vortices of strength m and l by electrons $\Psi_{2,3}$ and Ψ_1, respectively. We are therefore led to consider the following (bosonic) Chern-Simons Lagrangian:

$$\mathcal{L} = \bar{\Psi}_1(\partial_0 - i a_0)\Psi_1 + \sum_{a=2,3} \bar{\Psi}_a(\partial_0 - i\tilde{a}_0)\Psi_a + \frac{1}{2m}|[-i\vec{\partial} - n\mathbf{a} - l\tilde{\mathbf{a}} - \mathbf{A}^{ex}]\Psi_1|^2 + \frac{1}{2m}\sum_{a=2,3}|[-i\vec{\partial} - l\mathbf{a} - m\tilde{\mathbf{a}} - \mathbf{A}^{ex}]\Psi_a|^2 - (\Delta_Z + \Delta_{SAS})\bar{\Psi}_1\Psi_1 - (\Delta_{SAS} - \Delta_Z)\bar{\Psi}_2\Psi_2 + (\Delta_{SAS} - \Delta_Z)\bar{\Psi}_3\Psi_3 + u^c_{LL'}(x-y)(\rho_L(x) - \bar{\rho})(\rho_{L'}(y) - \bar{\rho}) + \mathcal{L}_{CS}(a) + \mathcal{L}_{CS}(\tilde{a}), \quad (3)$$

where $\mathcal{L}_{CS}(a) = (i/4\pi)\,\epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$, and L is a layer index (''top'' or ''bottom'') in the Coulomb interaction term. We decompose the Ψ_i's into an amplitude, a trivial phase, and a vortex part:[16,17] $\Psi_1 = \sqrt{\rho_1} e^{i\theta_1}\phi_{v1}$ and $\Psi_a = \sqrt{\rho_2} e^{i\theta_2}\phi_{v2} z_{a-1}$ for $a=2,3$, with the constraints $\bar{\phi}_{v1}\phi_{v1} = \bar{\phi}_{v2}\phi_{v2} = \bar{z}_a z_a = 1$. Then Eq. (3) can be written as

$$\mathcal{L} = i\rho_1\left(\frac{\partial_0\theta_1}{i} + \bar{\phi}_{v1}\frac{\partial_0}{i}\phi_{v1} - a_0 \right) + i\rho_2\left(\frac{\partial_0\theta_2}{i} + \bar{\phi}_{v2}\frac{\partial_0}{i}\phi_{v2} + \bar{z}_a\frac{\partial_0}{i}z_a - \tilde{a}_0 \right) + i\mathbf{J}\left(\frac{\vec{\partial}\theta_1}{i} + \bar{\phi}_{v1}\frac{\vec{\partial}}{i}\phi_{v1} - n\mathbf{a} - l\tilde{\mathbf{a}} - \mathbf{A}^{ex} \right) + i\tilde{\mathbf{J}}\left(\frac{\vec{\partial}\theta_2}{i} + \bar{\phi}_{v2}\frac{\vec{\partial}}{i}\phi_{v2} + \bar{z}_a\frac{\vec{\partial}}{i}z_a - l\mathbf{a} - m\tilde{\mathbf{a}} - \mathbf{A}^{ex} \right) + \frac{K_1}{2}|\mathbf{J}|^2 + \frac{K_2}{2}|\tilde{\mathbf{J}}|^2 + \frac{1}{2K_2}(|\vec{\partial}z|^2 + (\bar{z}\vec{\partial}z)^2) - (\Delta_Z + \Delta_{SAS})\rho_1 - (\Delta_{SAS} - \Delta_Z)\rho_2|z_1|^2 + (\Delta_{SAS} - \Delta_Z)\rho_2|z_2|^2 - \sum_{ab}\gamma_{ab}|z_a|^2|z_b|^2 + (\rho_1 + \rho_2 - \bar{\rho})(x)u(x-y)(\rho_1 + \rho_2 - \bar{\rho})(y) + \mathcal{L}_{CS}(a) + \mathcal{L}_{CS}(\tilde{a}). \quad (4)$$

Here $K_i = m/\rho_i$, terms with γ_{ab} come from the exchange part of the Coulomb interaction, and in the direct part of the Coulomb interaction we keep only the layer symmetric part of $u^c_{LL'}$, which does not vanish in the limit of $d=0$. By integrating out θ_1 and θ_2 we find that $J_\mu = (\rho_1, \mathbf{J})$ and $\tilde{J}_\mu = (\rho_2, \tilde{\mathbf{J}})$ are conserved. Therefore, we introduce dual gauge fields, b_λ and $\tilde{b}_\lambda$, such that $J_\mu = (1/2\pi)\epsilon^{\mu\nu\lambda}\partial_\nu b_\lambda$ and $\tilde{J}_\mu = (1/2\pi)\epsilon^{\mu\nu\lambda}\partial_\nu \tilde{b}_\lambda$. Then, we integrate out the statistical gauge fields, a_μ and $\tilde{a}_\mu$, and the time component of the dual gauge fields, b_0 and $\tilde{b}_0$. This gives (up to irrelevant constants)

$$\mathcal{L}=ib_\alpha J^v_\alpha+i\tilde{b}_\alpha(\tilde{J}^v_\alpha+\tilde{J}^S_\alpha)+\frac{1}{8\pi^2K_1}(\partial_0 b_\alpha)^2+\frac{1}{8\pi^2K_2}(\partial_0\tilde{b}_\alpha)^2$$
$$+\frac{1}{2K_2}(|\vec{\partial}z|^2+(\bar{z}\vec{\partial}z)^2)+i\epsilon^{\alpha\beta}b_\alpha\partial_0 b_\beta+i\epsilon^{\alpha\beta}\tilde{b}_\alpha\partial_0\tilde{b}_\beta+\frac{K_1}{2}r(x)\ln|x-y|r(y)+\frac{K_2}{2}\tilde{r}(x)\ln|x-y|\tilde{r}(y)+(r_1|z_1|^2+r_2|z_2|^2$$
$$-\gamma_{11}|z_1|^4-\gamma_{22}|z_2|^4-2\gamma_{12}|z_1|^2|z_2|^2)+\frac{1}{4\pi^2}(\epsilon^{\alpha\beta}\partial_\alpha b_\beta+\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta-\bar{\rho})(x)u(x-y)(\epsilon^{\alpha\beta}\partial_\alpha b_\beta+\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta-\bar{\rho})(y), \quad (5)$$

where we have defined vortex and skyrmion currents as in:[17] $J^v_\mu=(J^v_0,J^v_\alpha)=(1/2\pi)\epsilon^{\mu\nu\lambda}\partial_\nu(\bar{\phi}_{v1}(\partial_\lambda/i)\phi_{v1})$, $\tilde{J}^v_\mu=(\tilde{J}^v_0,\tilde{J}^v_\alpha)=(1/2\pi)\epsilon^{\mu\nu\lambda}\partial_\nu(\bar{\phi}_{v2}(\partial_\lambda/i)\phi_{v2})$, and $\tilde{J}^S_\mu=(\tilde{J}^S_0,\tilde{J}^S_\alpha)=(1/2\pi)\epsilon^{\mu\nu\lambda}\partial_\nu(\bar{z}_z(\partial_\lambda/i)z_a)$. The parameters r_1 and r_2 describe effective tunneling and Zeeman energy renormalized by the Coulomb interaction, and

$$r(x)=2\pi J^V_0-n\epsilon^{\alpha\beta}\partial_\alpha b_\beta-l\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta-\epsilon^{\alpha\beta}\partial_\alpha A^{ex}_\beta,$$

$$\tilde{r}(x)=2\pi\tilde{J}^V_0+2\pi\tilde{J}^S_0-l\epsilon^{\alpha\beta}\partial_\alpha b_\beta-m\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta-\epsilon^{\alpha\beta}\partial_\alpha A^{ex}_\beta.$$

In the ground state there are no vortices or skyrmions, so the cancellation of the long range logarithmic interaction gives two conditions,

$$\frac{1}{2\pi}\int d^2x[\epsilon^{\alpha\beta}\partial_\alpha A^{ex}_\beta+n\epsilon^{\alpha\beta}\partial_\alpha b_\beta+l\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta]=0,$$

$$\frac{1}{2\pi}\int d^2x[\epsilon^{\alpha\beta}\partial_\alpha A^{ex}_\beta+l\epsilon^{\alpha\beta}\partial_\alpha b_\beta+m\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta]=0. \quad (6)$$

Recalling that $1/2\pi\epsilon^{\alpha\beta}\partial_\alpha b_\beta$ gives the density of Ψ_1 electrons and $1/2\pi\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta$ gives the density of Ψ_2 and Ψ_3 electrons, we realize that Eq. (6) gives us the same filling fractions as the Halperin wave function (2).

From the last line of Eq. (5) it is obvious that as we change the strength of the Zeeman interaction and/or interlayer tunneling, we will stabilize various values of $|z_1|$ and $|z_2|$. Parameters γ_{ab} have been effectively computed in Hartree-Fock approximations for $n=m=1$ and $l=0$, i.e., $\nu=2$, in Refs. 1, 2, 7, and shown to obey $\gamma_{12}>\gamma_{11}+\gamma_{22}$, which implies that in this case there is no direct transition from $|z_1|=1$ to $|z_2|=1$, but there is an intermediate phase where both $|z_1|=\cos\theta_0$ and $|z_2|=\sin\theta_0$ are finite.[13] We will assume that the same holds for fractional fillings as well, although at this time we can offer no proof of this fact. The phase where both z's are finite will correspond to the CAF phase. In this phase interactions fix the absolute values of z's but not their relative phase. Therefore, when $\bar{z}_1z_2$ develops an expectation value, we have a spontaneous breaking of the $U(1)$ symmetry and the appearance of a Goldstone mode.

In the CAF phase, dynamics of the spin is determined by

$$\mathcal{L}_z=i\bar{\rho}_2\bar{z}\partial_t z-(1/2K_2)(|\vec{\partial}z|^2+(\bar{z}\vec{\partial}z)^2)+r_1|z_1|^2+r_2|z_2|^2$$
$$-\gamma_{11}|z_1|^4-\gamma_{22}|z_2|^4-2\gamma_{12}|z_1|^2|z_2|^2. \quad (7)$$

In the CAF phase only $\langle\bar{z}_1z_2\rangle$ develops a nonzero expectation value, but not $\langle z_1\rangle$ or $\langle z_2\rangle$. Therefore, we can write $z_1=|z_1|e^{i(\phi+\chi)}$ and $z_2=|z_2|e^{i(-\phi+\chi)}$. ϕ, the relative phase between z_1 and z_2, acquires an expectation value and gives rise to the Goldstone mode associated with the symmetry breaking. We also introduce $q=|z_1|^2-|z_2|^2$. Using that fluctuations of q are massive and their gradients may be neglected, we find from Eq. (7)

$$\mathcal{L}_z=i\bar{\rho}_2\delta q\ \partial_t\phi-(1/2K_2)(\vec{\partial}\phi)^2[1-q^2_{\min}]$$
$$+\alpha(\delta q)^2+i\bar{\rho}_2\ q_{\min}\partial_t\phi, \quad (8)$$

with $\alpha=(\gamma_{11}+\gamma_{22}-2\gamma_{12})/4$. Integrating out δq we get

$$\mathcal{L}_\phi=(\bar{\rho}^2_2/4\alpha^2)(\partial_t\phi)^2-(1/2K_2)(\vec{\partial}\phi)^2[1-q^2_{\min}]$$
$$+i\bar{\rho}_2\ q_{\min}\partial_t\phi. \quad (9)$$

We see that the spin wave velocity is $v_s=(\gamma_{11}+\gamma_{22}-2\gamma_{12})^2\sin^2(2\theta_0)/(8m\bar{\rho}_2)$. By introducing an infinitesimal external Zeeman field and integrating out fluctuations in ϕ, one can also calculate the S^z correlation function which explicitly shows a Goldstone resonance

$$\chi^{zz}(q,\omega)=\frac{(\bar{\rho}^2_2/2\alpha)\omega^2}{\omega^2-v^2_sk^2}-\frac{\bar{\rho}^2_2}{2\alpha}. \quad (10)$$

When the ground state of a system breaks a $U(1)$ symmetry spontaneously, vortices of the $U(1)$ phase will be the elementary excitations in the system and will carry a fractional electric charge, analogously to the merons discussed in Refs. 9 and 11. It is clear from the discussion above that such a meron corresponds to a vortex of the z field. Far away from the vortex core, $|z_1|=\cos\theta_0$ and $|z_2|=\sin\theta_0$, and the relative phase between the two $z's$ has nontrivial winding characterized by integer vorticity, n_v. In order to avoid a singularity of this phase, in the vortex core we must have either $|z_2|=0$ (S vortex) or $|z_1|=0$ (T vortex). According to the definition of $\tilde{J}^S_0$, this implies a nontrivial skyrmion winding number,

$$Q=\frac{1}{2\pi}\int d^2x\tilde{J}^S_o=\frac{1}{2\pi}\int d^2x\epsilon^{\alpha\beta}\partial_\alpha\left(\frac{\bar{z}\frac{\partial_\beta}{i}z}\right)$$
$$=\begin{cases} n_v\times\sin^2\theta_0 & S\ \text{vortex} \\ -n_v\times\cos^2\theta_0 & T\ \text{vortex}, \end{cases} \quad (11)$$

where n_v is an integer characterizing the winding of the relative phase between z_1 and z_2. From Eq. (5) the extra skyrmion charge has to be compensated by electric charge. To cancel the long-range forces, $\int[-n\epsilon^{\alpha\beta}\partial_\alpha b_\beta-l\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta]=0$ and $\int[-l\epsilon^{\alpha\beta}\partial_\alpha b_\beta-m\epsilon^{\alpha\beta}\partial_\alpha\tilde{b}_\beta]=-2\pi Q^s$, which immediately gives us the total charge of the meron,

$$Q_{\text{meron}}=\frac{1}{2\pi}\int [\epsilon^{\alpha\beta}\partial_\alpha b_\beta+\epsilon^{\alpha\beta}\partial_\alpha \tilde{b}_\beta],$$

$$=[(n-l)/nm-l^2]\times Q^s. \quad (12)$$

For $l=0$, which includes the $\nu=2/m$ states discussed in Ref. 2, we find $Q_{\text{meron}}=1/m\times Q^s$. Note that if we were to create simple quasiparticles by squezing a vortex into the ground state, $J_0^v=\delta^2(x-x_0)$ or $\tilde{J}_0^v=\delta^2(x-x_0)$, we could use the same arguments to find their charges: $q=(m-l)/(nm-l^2)$ and $\tilde{q}=(n-l)(nm-l^2)$. So, as in the simple case of a meron in the (m,m,m) state, two merons add up to a charge of 0 or the charge of a single quasiparticle, $\tilde{q}$.[18]

In the simplest case of $\nu=2$, one can give a simple picture of the meron excitation in the CAF phase using a generalization of the Berry's phase argument in Ref. 11. As suggested in Ref. 3, the CAF phase can be described by combining pairs of electrons into hard core bosons and writing the wave function as $|\Psi\rangle=\cos\theta|S\rangle+e^{i\phi}\sin\theta|T\rangle$. Here $|S\rangle$ and $|T\rangle$ denote singlet and triplet bosons, respectively; the relative phase between the two bosons, ϕ, determines the direction of the Néel vector in the $x-y$ plane. When a vortex is present, this phase winds nontrivially around the vortex core and is characterized by an integer vorticity, n_v. At the center of the core, one has to demand that there is only one kind of boson present (so as to avoid a singularity of the relative phase); therefore, one expects the appearance of two kinds of vortices: vortices with a singlet core (S) or a triplet core (T). We can now imagine taking a pair of electrons and adiabatically moving them around the vortex. In the course of such adiabatic transport, the wave function for a pair of electrons will acquire a phase $i\Gamma=\oint\langle\psi|d\psi\rangle=i\ n_v\sin^2\theta$ for an S vortex or $i\Gamma=-i\ n_v\cos^2\theta$ for a T vortex. The Berry's phase in adiabatic transport is indistinguishable from extra flux going through the system $\Delta\Phi=\Phi_0/2\times\Gamma/(2\pi)$, where the factor of 1/2 comes from the fact that we transported a pair of electrons. This extra flux can be related to the charge carried by the meron as $\Delta q=\sigma_{xy}\Delta\Phi=n_v\times\sin^2\theta$ and $\Delta q=-n_v\times\cos^2\theta$ for S and T vortices, respectively. So, the two kinds of merons in this case carry fractional charge; the charge depends on where the system is in the phase diagram, i.e., on the CAF phase order parameter θ (θ goes to 0 at the boundary of the CAF phase with the spin singlet phase, and $\pi/2$ at the boundary with the fully polarized ferromagnetic state; see Ref. 3 for details). However, two merons with opposite vorticities again add up to a charge of 0 or 1, as in the $\nu=1$ bilayer (1,1,1) state.[11]

It is also instructive to consider an explicit wave function for a meron in the CAF phase at $\nu=2$. As discussed above, the wave function of the CAF phase may be conveniently written as (in the limit when d is small) $|\Psi_0\rangle=\Pi_m(\cos\theta\quad S_m^\dagger+e^{i\phi}\sin\theta T_m^\dagger)|0\rangle$, where $S_m^\dagger=1/\sqrt{2}(c_{Sm\uparrow}^\dagger c_{Sm\downarrow}^\dagger-c_{Sm\downarrow}^\dagger c_{Sm\uparrow}^\dagger)$ and $T_m^\dagger=c_{Sm\uparrow}^\dagger c_{Am\uparrow}^\dagger$ create singlet and triplet combinations of electrons with orbital momentum m, and $|0\rangle$ is the Fock vacuum. Using the definition of the Néel order parameter, $N^a(z)=\langle\Psi|S_T^a-S_B^a|\Psi\rangle=\Sigma_{mn}\Psi_m^*(z)\Psi_n(z)\times\langle\Psi|c_{Sm\alpha}^\dagger\sigma_{\alpha\beta}^a c_{An\beta}-c_{Am\alpha}^\dagger\sigma_{\alpha\beta}^a c_{Sn\beta}|\Psi\rangle$, where $\Psi_m(z)$ is the wave function of an electron in the first Landau level with angular momentum m, one can easily prove that state $|\Psi_0\rangle$ has a uniform $\vec{N}$ in the XY plane $N^+(z)=1/2\cos\theta\sin\theta e^{i\phi}$. To have a meron we need a wave function where the direction of the Neel vector winds around as one goes around the center of the meron. This is achieved by considering the following wave function: $|\Psi_M\rangle=\Pi_m(\cos\theta S_m^\dagger+e^{i\phi}\sin\theta\tilde{T}_m^\dagger)|0\rangle$, where $\tilde{T}_m^\dagger=c_{Sm+1\uparrow}^\dagger c_{Am\uparrow}^\dagger$. For $|\Psi_M\rangle$ one finds that $N^+(z)=1/2\Sigma_m\Psi_m^*(z)\Psi_{m+1}(z)\cos\theta\sin\theta e^{i\phi_0}$. Since $\Psi_m\propto z^m\exp(-|z|^2)$ we find that $\arctan(N_y/N_x)=\arg(z)+\phi$; the direction of $\vec{N}$ winds in the XY plane following the argument of the complex coordinate z. It is also obvious from $|\Psi_M\rangle$ that it describes a state with a missing electron in the $S\uparrow$ state of $m=0$, so we have an S vortex with charge $-\sin^2\theta$.

In summary, we have developed an analytic theory for the bilayer QH CAF phase. Our theory is consistent with the original Hartree-Fock theory for $\nu=2$, but is general enough to predict a different class of *fractional* QH CAF phases as well as the correct excitation spectra.

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