

Daniel D. Stancil • Anil Prabhakar

Spin Waves

Theory and Applications

e) Springer

Contents

1 Introduction to Magnetism	1
1.1 Magnetic Properties of Materials	1
1.1.1 Diamagnetism	3
1.1.2 Paramagnetism	3
1.1.3 Ferromagnetism	3
1.1.4 Ferrimagnetism and Antiferromagnetism	4
1.2 Spinning Top	5
1.3 Magnetism	8
1.3.1 Equation of Motion	8
1.3.2 Gyromagnetic Ratio	10
1.4 Angular Momentum in Quantum Mechanics	12
1.4.1 Basic Postulates of Quantum Mechanics	13
1.4.2 Eigenvalue Equations	14
1.4.3 Angular Momentum	14
1.4.4 Addition of Angular Momenta	20
1.5 Magnetic Moments of Atoms and Ions	23
1.5.1 Construction of Ground States of Atoms and Ions	23
1.6 Elements Important to Magnetism	28
Problems	28
References	31
2 Quantum Theory of Spin Waves	33
2.1 Charged Particle in an Electromagnetic Field	33
2.2 Zeeman Energy	36
2.3 Larmor Precession	38
2.4 Origins of Exchange: The Heisenberg Hamiltonian	39
2.5 Spin Wave an a Linear Ferromagnetic Chain	46
2.6 Harmonic Oscillator	50
2.6.1 Harmonic Oscillator Eigenfunctions	50
2.6.2 Raising and Lowering Operators	52

2.7 Magnons in a 3D Ferromagnet: Method of Holstein and Primakoff	55
2.7.1 Magnon Dispersion Relation	55
2.7.2 Magnon Interactions	60
Problems	64
References	65
3 Magnetic Susceptibilities	67
3.1 Diamagnetism	67
3.2 Paramagnetism	70
3.3 Weiss Theory of Ferromagnetism	73
3.4 Neel Theory of Ferrimagnetism	76
3.5 Exchange Field	81
3.5.1 Uniform Magnetization	82
3.5.2 Non-uniform Magnetization	83
3.6 Magnetocrystalline Anisotropy	84
3.6.1 Uniaxial Anisotropy	84
3.6.2 Cubic Anisotropy	86
3.6.3 Coordinate Transformations	87
3.7 Polder Susceptibility Tensor	91
3.7.1 Equation of Motion for the Magnetization	91
3.7.2 Susceptibility Without Exchange or Anisotropy	91
3.7.3 Susceptibility with Exchange and Anisotropy	93
3.8 Magnetic Damping	94
3.9 Magnetic Switching	102
3.9.1 Stoner—Wohlfarth Particle	102
3.9.2 Damped Precession	104
Problems	106
References	108
4 Electromagnetic Waves in Anisotropic Dispersive Media	111
4.1 Maxwell's Equations	111
4.2 Constitutive Relations	112
4.3 Instantaneous Poynting Theorem	114
4.4 Complex Poynting Theorem	116
4.5 Energy Densities in Lossless Dispersive Media	117
4.6 Wave Equations	119
4.7 Polarization of the Electromagnetic Fields	122
4.8 Group and Energy Velocities	124
4.9 Plane Waves in a Magnetized Ferrite	127
4.9.1 Propagation Parallel to the Applied Field	128
4.9.2 Propagation Perpendicular to the Applied Field	130
4.10 The Magnetostatic Approximation	132
Problems	134
References.....	137

5 Magnetostatic Modes	139
5.1 Walker's Equation	139
5.2 Spin Waves	141
5.3 Uniform Precession Modes	144
5.3.1 Normally Magnetized Ferrite Film	144
5.3.2 Tangentially Magnetized Ferrite Film	145
5.3.3 Ferrite Sphere	146
5.4 Normally Magnetized Film: Forward Volume Waves	151
5.5 Tangentially Magnetized Film: Backward Volume Waves	158
5.6 Tangentially Magnetized Film: Surface Waves	162
Problems	166
References	167
6 Propagation Characteristics and Excitation of Dipolar Spin Waves	169
6.1 Energy Velocities for Dipolar Spin Waves	169
6.2 Propagation Loss	171
6.2.1 Relaxation Time for Propagating Modes	171
6.2.2 Surface Waves	173
6.2.3 Volume Waves	174
6.2.4 Summary of the Phenomenological Loss Theory	176
6.3 Mode Orthogonality and Normalization	178
6.3.1 Forward Volume Waves	178
6.3.2 Backward Volume Waves	180
6.3.3 Surface Waves	182
6.4 Excitation of Dipolar Spin Waves	183
6.4.1 Common Excitation Structures	183
6.4.2 Forward Volume Waves	188
6.4.3 Backward Volume Waves	194
6.4.4 Surface Waves	195
6.4.5 Discussion of Excitation Calculations	197
Problems	199
References	201
7 Variational Formulation for Magnetostatic Modes	203
7.1 General Problem Statement	203
7.2 Calculus of Variations	204
7.2.1 Formulation for One Independent Variable	204
7.2.2 Extensions to Three Independent Variables	206
7.3 Small-Signal Functional for Ferrites	208
7.4 Interpretation of the Functional	210
7.5 Stationary Formulas	212
7.6 Stationary Formula Examples with Forward Volume Waves	214
7.6.1 Large k-limit	215

XIV Contents

7.6.2 Improved Approximation	216
7.6.3 Effect of Medium Inhomogeneity	218
7.7 Finite Element Analysis	218
Problems	219
References	221
8 Optical-Spin Wave Interactions	223
8.1 Symmetric Dielectric Waveguides	224
8.1.1 TE Modes	224
8.1.2 TM Modes	227
8.1.3 Optical Mode Orthogonality and Normalization	228
8.2 Magneto-Optical Interactions	231
8.2.1 Can You Tell the Difference Between T_i and T_o	231
8.2.2 Definition of Magnetization at High Frequencies	234
8.2.3 Symmetry Requirements on the Permittivity	235
8.3 Coupled-Mode Theory	236
8.3.1 Coupled-Mode Equations	237
8.3.2 Energy Conservation	238
8.3.3 Solutions to the Coupled-Mode Equations	239
8.4 Scattering of Optical-Guided Modes by Forward Volume Spin Waves	241
8.4.1 Coupled-Mode Equations	241
8.4.2 Coupling Coefficients	245
8.4.3 Tightly Bound Optical Mode Approximation	250
8.4.4 Cotton -Mouton Effect	252
8.5 Anisotropic Bragg Diffraction	253
Problems	256
References	260
9 Nonlinear Interactions	263
9.1 Large-Amplitude Spin Waves	263
9.1.1 Foldover and Bistability	267
9.2 Hamiltonian Equations of Motion	271
9.3 Spin Wave Interactions	273
9.3.1 Decay Instability	280
9.3.2 $H^{(2)}$ Coefficients	282
9.4 Nonlinear Schrödinger Equation	284
9.4.1 Modulational Instability and Solitons	285
9.4.2 Split-Step Fourier Method	287
9.4.3 Anomalous Dispersion	289
9.4.4 Other Aspects	291
9.5 Routes to Chaos	293
9.5.1 Center Manifold Theory	293
9.5.2 Quantizing Low-Dimensional Chaos	296
Problems	302
References	305

10 Novel Applications	309
10.1 Nano-Contact Spin-Wave Excitations	309
10.1.1 Current-Induced Spin Torque	310
10.1.2 Magnetic Precession	315
10.2 Magnetic Precession in Patterned Structures	322
10.3 Inverse Doppler Effect in Backward Volume Waves	325
Problems.....	329
References	330
Appendix A: Properties of YIG	333
References	334
Appendix B: Currents in Quantum Mechanics	335
B.1 Density of States	335
B.2 Electric and Spin Current Densities	337
B.3 Reflection and Transmission at a Boundary	338
B.4 Tunneling Through a Barrier	339
References	341
Appendix C: Characteristics of Spin Wave Modes	343
C.1 Constitutive Tensors	343
C.1.1 Polder Susceptibility Tensor	343
C.1.2 Permeability Tensor	344
C.2 Uniform Precession Mode Frequencies	344
C.3 Spin Wave Resonance Frequencies	344
C.4 General Magnetostatic Field Relations	344
C.5 Forward Volume Spin Waves	345
C.6 Backward Volume Spin Waves	346
C.7 Surface Spin Waves	347
Appendix D: Mathematical Relations	349
D.1 Trigonometrie Identities	349
D.2 Vector Identities and Definitions	349
D.3 Fourier Transform Definitions	350
Index	351