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PION FORM FACTOR FROM PARTIALLY UNITARIZED

HARD PION CURRENT ALGEBRA

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A B S T R A C T

Using a consistency condition for the transverse part of the primitive AAV-vertex function, the electromagnetic form factor of the pion is derived from partially unitarized hard pion current algebra and the Schnitzer-Weinberg parametrization ( $\delta$  model). The result is in very good agreement with the Orsay measurements and in good enough agreement with the scarce data for spacelike arguments. The difference between the position of the peak of the form factor and the resonance energy is discussed.

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## INTRODUCTION

The partial unitarization of the hard pion method <sup>1)</sup> has given new predictions <sup>2),3)</sup> from current algebra. In particular, this approach allows one to calculate the electromagnetic form factor of the pion in the region of the  $\rho$  resonance where increasing experimental information is available from colliding beam experiments <sup>4),5)</sup>. The basic ingredient of the method is something like elastic unitarity for the spectral function of the vector current propagator, in other words the simple  $\rho$  pole is replaced by the two-pion contribution. Two consistency conditions may then be imposed. First, one requires that the resonance occurs at the right position. Second <sup>6)</sup>, the width of the pion form factor is set equal to the decay width of  $\rho \rightarrow 2\pi$  as given by the hard pion method, taking into account that  $\rho$  dominance of the electromagnetic current is expected to hold at least near the resonance energy and that the branching ratio for the  $2\pi$  decay of the  $\rho$  meson is practically 100%. Both conditions are used to limit the number of undetermined parameters stemming on the one hand from the parametrization of the primitive vertex function for one vector and two axial-vector currents, on the other hand from the possible subtraction constants for the inverse pion form factor. In both cases one has to make assumptions. For the vertex function the famous "smoothness" assumption is usually invoked which, when employed in its simplest form, leads to the original hard pion model of Schnitzer and Weinberg <sup>1)</sup> ( $\delta$  model). As for the asymptotic behaviour of the pion form factor the situation is even less clear, since the present experimental data <sup>7),8)</sup> lie in a very limited region of the variable  $t$  and are not even there very conclusive.

In Section 1, a brief summary of the basic representation for the pion form factor  $F(t)$  <sup>2)</sup> and of the consistency condition for the primitive vertex function is given. For the actual calculation the  $\delta$  model of Schnitzer and Weinberg is used which requires at least three subtractions for the inverse pion form factor  $F^{-1}(t)$ . The validity of this assumption and its relevance for the form factor in the regions where data are available is discussed in some detail and compared with previous calculations <sup>2),6)</sup> where a cut-off was employed for the integral over the absorptive part of  $F^{-1}(t)$ .

Comparison of the resulting form factor with experiment is made in Section 2. The result turns out to depend crucially on the exact value of  $g_\rho$ , the coupling constant of the vector current to the  $\rho$  meson. Using the consistency conditions mentioned before, as well as the limited information on the  $A_1\rho\pi$  system and the assumption that the charge radius of the pion is not less than the  $\rho$  dominance value, only the Orsay value for  $g_\rho$  is shown to be consistent within our approach. The calculated form factor is then shown to be in very good agreement with the Orsay measurements in the timelike region and to differ only slightly from the  $\rho$  dominance prediction in the spacelike region.

Finally, in Section 3, we comment briefly on the shift of the peak of the form factor with respect to the resonance position. This effect is, of course, due to the energy-dependent width which comes out automatically in this approach. With a simple approximation, this shift is calculated as a function of the parameter  $\delta$  and for all acceptable values of  $\delta$  it is shown to be in qualitative agreement with the predictions of various phenomenological expressions for the form factor in the resonance region. This result is independent of the number of subtractions for  $F^{-1}(t)$  and therefore provides, at least in principle, another test for the Schnitzer-Weinberg model.

## 1. - DERIVATION OF THE FORM FACTOR

The Ward identities for the three-point functions of one vector and two axial-vector currents (Fig. 1), which are a consequence of  $SU(2) \times SU(2)$  current algebra, CVC, PCAC, and the assumption of no  $I = 1$  operator Schwinger terms in the equal-time commutators, imply the following equation <sup>2) \*)</sup>

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\*) The notation is as in Ref. 6) and differs from that of Ref. 2) only by the use of the metric  $(1, -1, -1, -1)$ .

$$F_{\pi}^2 (F_{\lambda}(q,p) - Q_{\lambda}) = q^{\mu} p^{\nu} F_{\mu\nu\lambda}(q,p) - \frac{1}{2} \int \frac{dm^2 \rho_V(m^2)}{m^2(k^2 - m^2)} \{ k^2 Q_{\lambda} - (p^2 - q^2) k_{\lambda} \} \quad (1)$$

where  $Q_{\lambda} = p_{\lambda} + q_{\lambda}$ ,  $F_{\pi}$  is the pion decay constant, and  $F_{\lambda}(q,p)$ ,  $F_{\mu\nu\lambda}(q,p)$  are related to the proper vertex functions  $\Gamma_{\lambda}(q,p)$ ,  $\Gamma_{\mu\nu\lambda}(q,p)$  of Schnitzer and Weinberg;  $\rho_V(m^2)$  is the spectral function of the vector current propagator.

Writing the most general solution of the Ward identities for the primitive vertex function  $\Gamma_{\mu\nu\lambda}(q,p)$  in the form

$$\Gamma_{\mu\nu\lambda}(q,p) = \frac{k_{\lambda}}{k^2} g_{\rho} g_{\pi}^2 C_{\nu}^{-1} \{ \Delta_{\pi\mu\nu}^{-1}(q) - \Delta_{\pi\mu\nu}^{-1}(p) \} + (q_{\lambda\sigma} - \frac{k_{\lambda} k_{\sigma}}{k^2}) B_{\mu\nu}^{\sigma}(q,p) \quad (2)$$

where  $B_{\mu\nu}^{\sigma}(q,p)$  is arbitrary except for a boundary condition<sup>9)</sup> at  $k^2 = 0$ , and defining the pion form factor  $F(t)$  by

$$F_{\lambda}(q,p) \equiv F(t) Q_{\lambda} \Big|_{p^2=q^2=m_{\pi}^2, t=k^2} \quad (3)$$

Equation (1) can be written

$$F(t) = 1 - \frac{1}{2F_{\pi}^2} \int \frac{dm^2 \rho_V(m^2)}{m^2(t - m^2)} \left\{ t + g_{\pi}^{-2} g_{\rho}^{-1} C_{\pi}^2 m^2 F(t) \right\} \quad (4)$$

with

$$q^{\mu} p^{\nu} B_{\mu\nu\lambda}(q,p) \equiv \frac{1}{2} F(t) Q_{\lambda} \quad (5)$$

for  $p^2 = q^2 = m_{\pi}^2$ ,  $t = k^2$ .

With  $\rho_V(m^2)$  given by the two-pion contribution one obtains an integral equation for  $F(t)$  the most general solution of which is

$$F^{-1}(t) = 1 + c_1 t + \dots + c_{n-1} t^{n-1} - \frac{t^n}{8\pi a_{11}} \int_{4m_\pi^2}^{\infty} \frac{dx}{(x-t)x^{n-1}} \left( \frac{x-4m_\pi^2}{x} \right)^{3/2} \left\{ 1 + g_\pi^{-2} g_\rho^{-1} C_\pi^2 H(x) \right\} \quad (6)$$

where the  $c_i$  are subtraction constants,  $F^{-1}(0) = 1$  is fixed by vector current conservation, and  $a_{11} = 12\pi F_\pi^2$ .

Two consistency conditions are now imposed on the form factor. First the  $I = 1$  p wave  $\pi\pi$  phase shift goes through  $\pi/2$  at  $t = m_\rho^2$ , defining the position of the resonance. Secondly, the width of the form factor as determined from Eq. (6) is required to equal the decay width  $\Gamma(\rho \rightarrow 2\pi)$  resulting from the usual hard pion method. The resulting equations are <sup>(6)</sup>

$$\text{Re } F^{-1}(m_\rho^2) = 0 \quad (7)$$

$$m_\rho^2 y(m_\rho^2) B(m_\rho^2) = -y \quad (8)$$

where

$$y(m_\rho^2) = \frac{d}{dt} \text{Re } F^{-1}(t) \Big|_{t=m_\rho^2} ,$$

$$B(x) = 1 + g_\pi^{-2} g_\rho^{-1} C_\pi^2 H(x) , \quad y = 2F_\pi^2 m_\rho^2 / g_\rho^2$$

In deriving (8) it has been assumed that the real part of the pole of  $F(t)$  on the second sheet is equal to the resonance value  $m_\rho^2$ . This is not necessarily true, as will be discussed in more detail in Section 3, but the error is approximately given by the shift of the

peak of the form factor which is certainly much smaller than any of the masses squared appearing in the theory. As one would expect the correction to the consistency equation (8) is indeed negligible.

In order to evaluate the integral in (6) one has to make some assumptions about the transverse part of the primitive vertex function, which determines  $A(x)$ . Although the extrapolations involved may be questioned <sup>10)</sup> the most popular assumption is still the original  $\delta$  model of Schnitzer and Weinberg. Without any further input, this model can only be expected to hold in the resonance region. However, Schnitzer and Wise <sup>9)</sup> have conjectured on the basis of the Bjorken limit that the only model involving the polynomial expansion of the scalar functions appearing in  $B_{\mu\nu\lambda}(q,p)$ , which is consistent with the minimal algebra of fields <sup>11)</sup> equal-time commutators, is the  $\delta$  model. In their paper this conjecture is proved for polynomials of at most the fourth degree in the momenta. This really seems to be the only hint so far whether and under which assumptions the original hard pion model might be valid for high energies.

Thus the situation does not seem to be very encouraging. However, if we look at the integral in (6), it is evident that the main contribution for fixed  $t$  above threshold will come from the region in the vicinity of  $t$  because of the factor  $x-t$  in the denominator, as long as the integral converges at all. Therefore, one may expect that with enough subtractions for the integral to converge, the resulting form factor should be a rather good approximation for  $t \lesssim 1 \text{ GeV}^2$  and not be too sensitive to the high energy behaviour of  $A(t)$ .

Another possibility, which at first sight seems to be rather reasonable, would be the use of a cut-off in the integral emphasizing that the  $\delta$  model is actually designed to describe the behaviour of the vertex functions in the vicinity of the respective resonances and is not necessarily a parametrization for high energies. This point of view was taken by Brehm, Golowich and Prasad <sup>2)</sup>, together with the assumption of only one subtraction for  $\tilde{F}^{-1}(t)$ . However, this

cut-off cannot be chosen arbitrarily because of the consistency conditions (7) and (8). As a matter of fact, it was shown in Ref. 6) that the cut-off can be of the order of  $100 m_{\rho}^2$ , which is rather far away from the  $\rho$  resonance, for instance. As the integral diverges linearly, one picks up substantial contributions from regions where the chosen parametrization is not expected to be valid. It is not too surprising then that one obtains in that case rather strong disagreement with the accepted values of  $\delta$ ; moreover, the consistency conditions put a limit on  $g_{\rho}$  which turns out to exclude the Orsay value.

For the  $\delta$  model we find

$$B(x) = 1 - \frac{(2-y)}{4m_{\rho}^2} (1+\delta)x \quad (9)$$

where Weinberg's first sum rule <sup>12)</sup> and  $m_A^2 = 2m_{\rho}^2$  have been used. For the vector Schwinger term  $C_v$  the narrow width approximation is assumed (corrections due to the finite width of the  $\rho$  are small for  $C_v$ ), but we do not fix  $y$  to be one, which is the KSFR value <sup>13)</sup>.

In view of the arguments made above we adopt the minimal number of subtractions so that the dispersion integral converges. Therefore, three subtraction constants are necessary, one of which is given by vector current conservation. If one wants to adhere to the usual philosophy of dispersion theory, one may expect that part of the uncertainty in the high energy behaviour of  $A(t)$  is taken into account by the remaining two subtraction constants  $c_1$  and  $c_2$ .

The dispersion integral can now be evaluated and the resulting inverse pion form factor is given by

$$F^{-1}(t) = 1 + c_1 t + c_2 t^2 + \frac{t^2}{80\pi m_{\pi}^2 a_{\parallel}} - \frac{t}{4\pi a_{\parallel}} B(t) \left\{ \frac{4}{3} - \frac{4m_{\pi}^2}{t} + \frac{1}{2} \left( 1 - \frac{4m_{\pi}^2}{t} \right)^{3/2} \left[ \ln \frac{1 - \left( 1 - \frac{4m_{\pi}^2}{t} \right)^{1/2}}{1 + \left( 1 - \frac{4m_{\pi}^2}{t} \right)^{1/2}} + i\pi \right] \right\} \quad (10)$$

for  $t \geq 4m_{\pi}^2$ , with the obvious analytic continuation for  $t \leq 4m_{\pi}^2$ .

Although our main interest will be in the region  $-20m_{\pi}^2 \lesssim t \lesssim 60m_{\pi}^2$ , one may note in passing that the asymptotic behaviour of  $F^{-1}(t)$  for  $t \rightarrow -\infty$  is  $\sim t^2 \ln(-t/m_{\pi}^2)$ . However, this prediction must not be taken too seriously, as in this case the asymptotic behaviour of  $A(t)$  and the assumption of minimal subtractions play a crucial role. It is not easy to compare this asymptotic behaviour with the predictions of more reliable models since there hardly exist any such models. The best reputation at present is probably enjoyed by the numerous Veneziano models for the pion form factor, most of which predict an asymptotic behaviour for  $F(t)$  of the form  $|t|^{-n/2}$ , where  $n$  is an odd integer<sup>14)</sup>. It seems that there are no theoretical arguments in favour of a certain  $n$ , although  $n = 3$  appears to be the preferred choice<sup>15)</sup>. However,  $n = 5$  has also been considered<sup>16)</sup>. Thus, in a way, the present approach "interpolates" between two competing Veneziano expressions, which should neither be counted as a success nor as a failure.

## 2. - COMPARISON WITH EXPERIMENT

The pion form factor as given by Eq. (10) contains four parameters, namely the subtraction constants  $c_1$  and  $c_2$ , the Schnitzer-Weinberg parameter  $\delta$ , and the parameter  $y$ , which measures the strength of the vector current coupling to the  $\rho$  meson. Making use of the consistency equations (7) and (8), which are of course independent, we are left with two parameters. Those parameters are conveniently chosen as  $y$  and  $p$ , where  $c_1 = -1/m_{\rho}^2 (1+p)$ ; with the pion charge radius  $r_{\pi}$  defined, as usually, by  $r_{\pi}^2 = 6(dF(t)/dt)|_{t=0}$ , we have  $r_{\pi}^2 = 6/m_{\rho}^2 (1+p)$ , so that  $p$  measures the deviation from the simple  $\rho$  pole prediction for  $r_{\pi}$ , which is  $r_{\pi} = (6/m_{\rho}^2)^{\frac{1}{2}} = 0.64F$ .

The consistency equations may now be solved for  $\delta$  and  $c_2$  as functions of  $y$  and  $p$ . It turns out that a reality condition for the solutions implies an upper bound on  $y$  where this upper bound is a function of  $p$ .



$$y \leq \frac{(p-0.85)^2}{0.33} + 0.94 - p \quad (11)$$

If  $r_\pi$  does not exceed the  $\rho$  dominance value by more than 20%, all three values of  $y$ , which are either taken from the colliding-beam experiments <sup>4),5)</sup> or from theoretical predictions <sup>13)</sup> and which will be considered in the following, are below this bound. The results for  $\delta$  are given in the Table.

So far we have used no experimental information on the  $A_1 \rho \pi$  system and the pion charge radius. If the Schnitzer-Weinberg model makes sense at all, very generous limits on the  $\rho$  and  $A_1$  widths imply  $-1 \leq \delta \leq 0$ , with  $\delta \simeq -\frac{1}{2}$  being the preferred choice. A recent measurement <sup>17)</sup> of the ratio between the transverse and the longitudinal coupling constants in the  $A_1 \rho \pi$  decay is also consistent with  $\delta \simeq -\frac{1}{2}$ . However, an earlier measurement <sup>18)</sup> of the same quantity gave  $\delta \simeq -\frac{3}{2}$ . Thus, the question of the validity of the model has not been completely settled yet.

The existing data on the charge radius of the pion are not very conclusive, either, mainly due to theoretical uncertainties concerning the one-pion exchange mechanism in pion-electroproduction. Excluding rapid variation of  $F(t)$  near  $t = 0$ , one may, however, conclude from the data that  $r_\pi$  is not smaller than  $0.64F$ , the  $\rho$  dominance value.

Accepting both  $-1 \leq \delta \leq 0$  and  $r_\pi \geq 0.64F$ , one infers from the Table that only the Orsay value for  $y$  is acceptable. Of course,  $y$  could be bigger than 0.7, but within the above-mentioned limits, one has  $y < 1$ . It is maybe interesting to note that the solutions are very sensitive to the exact value of  $y$ .

The pion form factor in the timelike and the spacelike regions is plotted in Figs. 2 and 3 with  $y = 0.7$ ,  $p = 0.1$  ( $r_\pi = 0.67F$ ),  $\delta = -0.45$ . The curve is in very good agreement with the Orsay data and does not differ much from the simple  $\rho$  pole form factor for spacelike  $t$ .

Two final remarks concern the shape of the form factor. Choosing a slightly larger value of  $\delta$  (and therefore smaller  $p$ ), the Orsay peak may be reproduced with a corresponding smaller width. This is evident from the dependence of  $\Gamma(\rho \rightarrow 2\pi)$  on  $\delta$ , the consistency condition  $\Gamma_\rho = \Gamma(\rho \rightarrow 2\pi)$ , and the equation <sup>\*</sup>)

$$\Gamma_\rho |F(m_\rho^2)|^2 = 6\pi g_\rho^2 / P_\rho^3 \quad (12)$$

which must hold for all our solutions <sup>6)</sup>.

Finally, the asymptotic behaviour of  $F(t)$  for large spacelike  $t$  has obviously little influence on the behaviour for  $0 \geq t \geq -0.5 \text{ GeV}^2$ , since the curve in Fig. 3 is very close to  $(m_\rho^2)/(m_\rho^2 - t)$ , while on the other hand  $F^{-1}(t) \underset{t \rightarrow -\infty}{\sim} t^2 \ln(-t)$ . In view of the arguments of Section 1, this behaviour was to be expected.

### 3. - PEAK OF THE FORM FACTOR AND RESONANCE ENERGY

This last Section is devoted to a discussion of the shift of the peak of the pion form factor with respect to the position of the resonance, which is defined by the phase shift going through  $\pi/2$  at that energy. In agreement with most phenomenological expressions <sup>19)</sup> for the form factor near the  $\rho$  resonance, the present model predicts such a shift to the left of the resonance energy. In the following an approximate calculation of the shift is presented.

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<sup>\*</sup>)  $P_\rho$  is the momentum of either of the pions in the  $\rho$  rest frame.

As usual, the pion form factor  $F(z)$  is assumed to be an analytic function in the complex energy plane (with a cut from  $4m_\pi^2$  to  $\infty$ ), which has a pole on the second sheet at  $z_0 = t_0 - iu_0$ , where  $u_0 \cong m_\rho \sqrt{s}$  and  $t_0$  is not much different from  $m_\rho^2$ .

$F^{-1}(z)$  is developed into a power series around  $z = t_0$

$$F^{-1}(z) = F^{-1}(t_0) + (z - t_0) \frac{d}{dz} F^{-1}(z) \Big|_{z=t_0} + \dots \quad (13)$$

Now we assume that to evaluate  $F^{-1}(z)$  at  $z = z_0$ , Eq. (13) with only the first two terms in the expansion taken along can be used. This approximation will be reasonable as long as the form factor does not differ too drastically from a simple Breit-Wigner expression.

Because of  $F^{-1}(z_0) = 0$

$$\operatorname{Re} F^{-1}(t_0) + u_0 \frac{d}{dt} \operatorname{Im} F^{-1}(t_0) = 0 \quad (14)$$

$$\operatorname{Im} F^{-1}(t_0) - u_0 \frac{d}{dt} \operatorname{Re} F^{-1}(t_0) = 0 \quad (15)$$

To this order of approximation

$$\frac{d}{dt} |F^{-1}(t)|^2 \Big|_{t=t_0} = 2 \left\{ \operatorname{Re} F^{-1}(t_0) \frac{d}{dt} \operatorname{Re} F^{-1}(t_0) + \operatorname{Im} F^{-1}(t_0) \frac{d}{dt} \operatorname{Im} F^{-1}(t_0) \right\} = 0 \quad (16)$$

because of Eqs. (14), (15), so that  $|F(t)|$  has its maximum at  $t_0 = \operatorname{Re} z_0$ .

As the phase of  $F(t)$  is equal to the  $I = 1$  p wave  $\pi\pi$  phase shift because of elastic unitarity, the resonance energy  $t_{\text{res}}$  is defined by  $\operatorname{Re} F^{-1}(t_{\text{res}}) = 0$ , or, again using (13) \*)

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\*) If Eq. (13) can be used at  $z = z_0$ , it will be an even better approximation at  $z = t_{\text{res}}$ , because  $|t_{\text{res}} - t_0| \ll |z_0 - t_0| = m_\rho \sqrt{s}$ .

$$\operatorname{Re} F^{-1}(t_0) + (t_{\text{res}} - t_0) \frac{d}{dt} \operatorname{Re} F^{-1}(t_0) = 0 \quad (17)$$

Thus

$$t_{\text{res}} - t_0 = - \frac{\operatorname{Re} F^{-1}(t_0)}{\frac{d}{dt} \operatorname{Re} F^{-1}(t_0)} = u_0^2 \frac{\frac{d}{dt} \operatorname{Im} F^{-1}(t_0)}{\operatorname{Im} F^{-1}(t_0)} \quad (18)$$

This equation reproduces the well-known fact that the energy shift is caused by the energy dependence of the width. What is maybe not as well-known is that the sign of  $(d/dt)\operatorname{Im} F^{-1}(t_0)$ , to the order of this approximation, determines the sign of the shift, since  $\operatorname{Im} F^{-1}(t_0) = -[\operatorname{Im} F(t_0)/|F(t_0)|^2]$  is negative because of elastic unitarity.

Below the inelastic threshold

$$\operatorname{Im} F^{-1}(t) = - \frac{1}{8a_{11}} \frac{(t - 4m_{\pi}^2)^{3/2}}{t^{1/2}} B(t) \Theta(t - 4m_{\pi}^2) \quad (19)$$

and with <sup>6)</sup>

$$\Gamma(\rho \rightarrow 2\pi) = \frac{P_{\rho}^3}{y a_{11}} [B(m_{\rho}^2)]^2 \quad (20)$$

one obtains

$$t_{\text{res}} - t_0 = \frac{P_{\rho}^6}{y^2 a_{11}^2} [B(m_{\rho}^2)]^3 [B(m_{\rho}^2) + m_{\rho}^2 \frac{d}{dt} B(m_{\rho}^2)], \quad (21)$$

neglecting orders of  $m_{\pi}^2/m_{\rho}^2$  in (19).

Defining the mass shift  $\Delta m = \sqrt{t_{\text{res}}} - \sqrt{t_0}$ , we finally get

$$\Delta m = \frac{P_p^6}{2m_p y^2 a_{\parallel}^2} [B(m_p^2)]^3 \left[ B(m_p^2) + m_p^2 \frac{d}{dt} B(m_p^2) \right] \quad (22)$$

It should be emphasized that to the order of approximation made above, Eq. (22) is independent of both higher mass contributions to  $\rho_V(m^2)$  and the high energy behaviour of the primitive vertex function  $\Gamma_{\mu\nu\lambda}(q,p)$ . Thus it provides a direct check on the parametrization of  $\Gamma_{\mu\nu\lambda}$  in the resonance region, if  $\Delta m$  is known to sufficient accuracy from experiment.

To get an idea of the magnitude of the shift as predicted by (22) let us specialize to the  $\delta$  model with the Orsay value for  $g_p$ . It is easily shown that  $\Delta m$  is positive for  $\delta \lesssim 0.5$  (i.e., the peak is shifted to the left for all acceptable values of  $\delta$ ) and that  $\Delta m$  increases rapidly with decreasing  $\delta$ , with  $\Delta m(\delta=0) = 2.5$  MeV,  $\Delta m(\delta=-\frac{1}{2}) = 9.3$  MeV, and  $\Delta m(\delta=-1) = 23.4$  MeV. A fit of the Orsay data<sup>20)</sup> using the phenomenological form factor of Gounaris and Sakurai<sup>19)</sup> gives a shift of  $\sim 10$  MeV to the left.

A final check on our approximation assumption can be made by comparing the prediction (22) with the explicitly calculated form factor of Sections 1, 2. It turns out that for the acceptable values of  $\delta$  the difference is always smaller than 15%. As an example, the peak of the form factor plotted in Fig. 2 is shifted 9.5 MeV instead of 8.5 MeV predicted by Eq. (22).

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$y \backslash p$	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3
0.7 4)	0.4	0.2 <sub>5</sub>	0.0 <sub>5</sub>	-0.1 <sub>5</sub>	-0.4 <sub>5</sub>	-0.8 <sub>5</sub>	-1.4
1.0 13)	-0.2	-0.5	-0.8 <sub>5</sub>	-1.3	-1.9	-2.7	-4.0
1.2 5)	-0.8 <sub>5</sub>	-1.3	-1.9	-2.6	-3.5	-5.0	-7.3

TABLE : Solutions of Eqs. (7) and (8) for the Schnitzer-Weinberg parameter  $\delta$ .

$$y = 2F_{\pi}^2 m_{\rho}^2 / g_{\rho}^2$$

$$r_{\pi}^2 = \frac{6}{m_{\rho}^2} (1 + \rho)$$

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FIGURE CAPTIONS

Figure\_1      Vertex for one vector (V) and two axial-vector currents (A).

Figure\_2      Absolute square of the pion form factor in the timelike region for  $y = 0.7$ ,  $p = 0.1$ . Data points are from Ref. 4) (circles) and Ref. 5) (triangles).

Figure\_3       $F(t)$  for spacelike  $t$  with the same parameters as in Fig. 1. Data are from Ref. 7) (open circles) and Ref. 8) (black circles and triangles). The dashed curve is the  $\rho$  pole prediction.

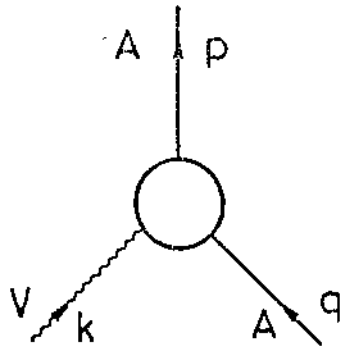


FIG.1

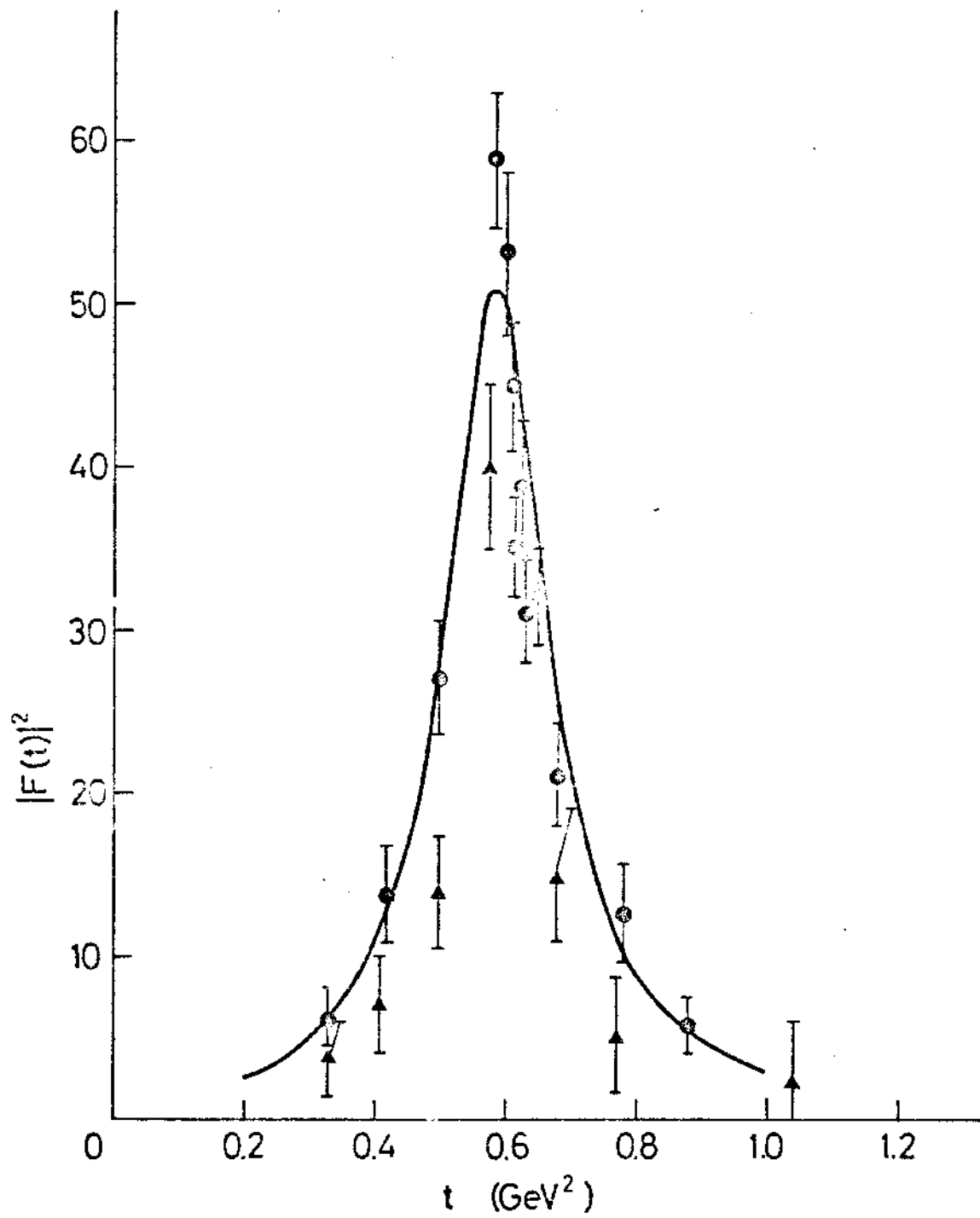


FIG. 2

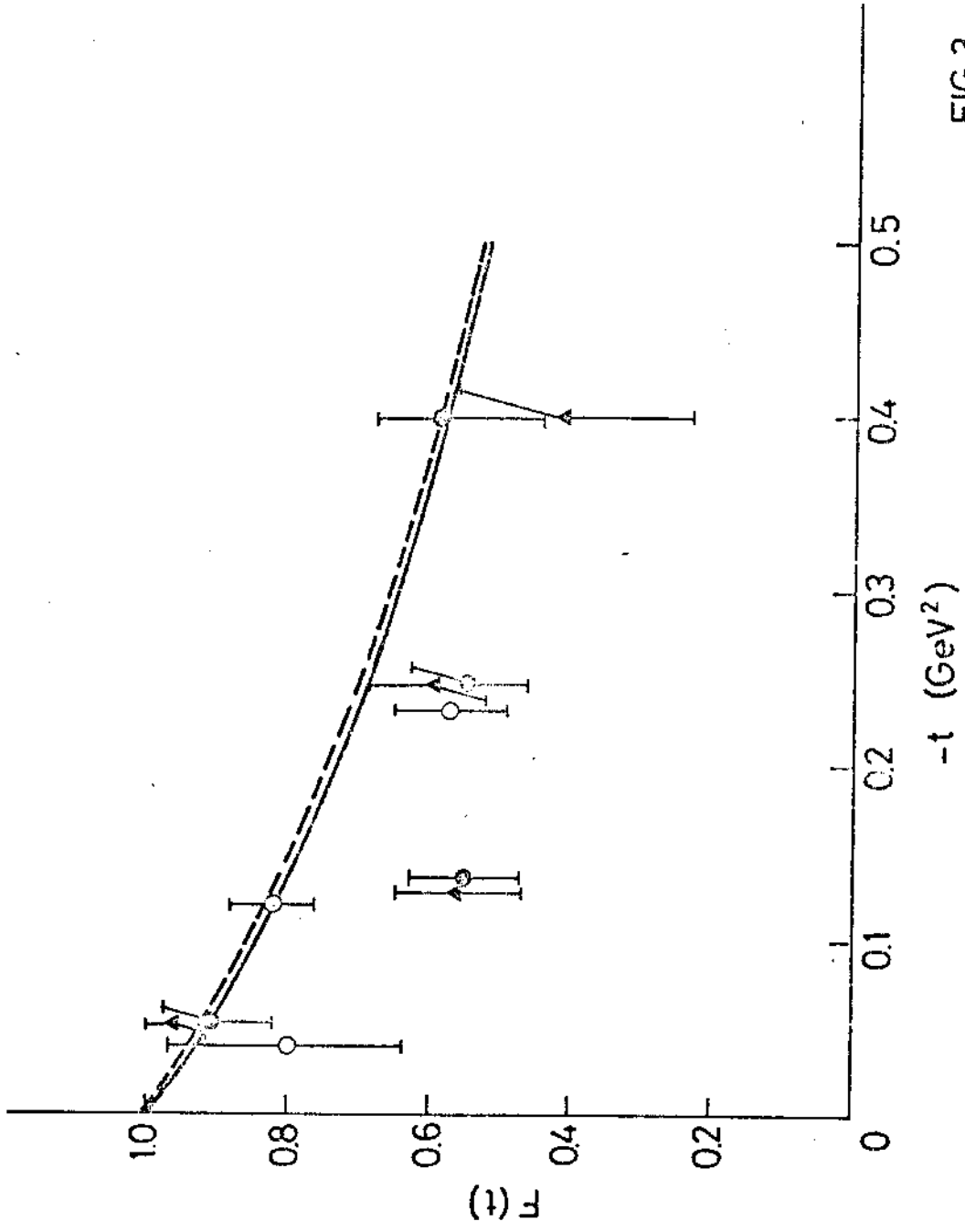


FIG.3