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A SPINOR FORMALISM FOR DUAL RESONANCE MODELS

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ABSTRACT

A spinor formalism for the factorization of dual resonance models is proposed. It is seen to lead to an SU(2) classification of the states and introduces a new degeneracy. The symmetry of the propagator is discussed.

This past year has seen considerable work devoted to dual resonance models.(1) In particular, the factorizability(2) of the general dual n-body scalar amplitude has opened the way for the building of a fully unitarized dual theory. Unfortunately, progress in this direction has been hampered by two disturbing facts. On the one hand, a new type of divergence was noted(3) to appear in the computation of higher order diagrams while on the other hand the factorization program revealed(2) the presence of ghost states in the theory. However, the discovery(2) of a non-trivial invariance group for the n-body scalar amplitude led to a natural, albeit partial, ghost compensation mechanism. Although we do not provide an answer to either of these troublesome questions, we believe a necessary first step to lie in the creation of a natural formalism in which to discuss the intriguing symmetries of the n-body amplitude.

It has been shown[2] that the (r+1)+(s+1)-point scalar amplitude

$$A_{r+1,s+1} = \int_0^1 d\bar{x}\phi(\bar{x},\bar{p}) \int_0^1 dy\phi(y,q) \int_0^1 dz z^{-\alpha(s)-1}(1-z)^{-c} \cdot \\ \cdot \exp \sum_{n=1}^{\infty} \left\{ \frac{z^n}{n} \bar{P}^{(n)} Q^{(n)} \right\} \quad (1)$$

corresponding to Figure 1a, is identical to the "charge-conjugated" amplitude

$$CA_{r+1,s+1}C^{-1} = \int_0^1 dx\phi(x,p) \int_0^1 d\bar{y}\phi(\bar{y},\bar{q}) \int_0^1 dz z^{-1-\alpha(s)}(1-z)^{-c} \cdot \\ \cdot \exp \left\{ \sum_{n=1}^{\infty} \frac{z^n}{n} P^{(n)} \bar{Q}^{(n)} \right\} \quad (2)$$

corresponding to Figure 1b. Here $\phi(\bar{x},\bar{p})$ and $\phi(y,q)^*$ are the usual integrals of the (r+2)- and (s+2)-point amplitudes respectively and $P^{(n)}$, $\bar{P}^{(n)}$, $Q^{(n)}$, $\bar{Q}^{(n)}$ are the four vectors introduced in Ref. (2a).

A particularly simple method has been proposed[4] to study the general factorization properties of the scalar amplitude in which one associates with the vectors $\bar{P}_\mu^{(n)}$ and $Q_\mu^{(n)}$ (or $P_\mu^{(n)}$ and $\bar{Q}_\mu^{(n)}$) harmonic oscillator operators $a_\mu^{(n)\dagger}$ and $a_\mu^{(n)}$. In this letter, we generalize this method to incorporate the "charge-conjugation" symmetry of the scalar amplitude. To wit we introduce a fundamental two-component object,

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$^*dx\phi(x,p) = d\bar{x}\phi(\bar{x},\bar{p})$; $dy\phi(y,q) = d\bar{y}\phi(\bar{y},\bar{q})$

$$T_p^{(n)} \equiv \frac{1}{2}\begin{bmatrix} P^{(n)} + \overline{P}^{(n)} \\ P^{(n)} - \overline{P}^{(n)} \end{bmatrix} \tag{3}$$

to which we associate in the manner of Ref. (4a) a spinor harmonic oscillator operator $a_{\mu\xi}^{(n)}$ where $\mu = 0,1,2,3$ is the Lorentz index and $\xi = 1,2$ in the spinor index. They obey the usual commutation relations

$$\left[a_{\mu\xi}^{(n)}, a_{\nu\eta}^{(m)\dagger}\right] = g_{\mu\nu}\delta^{n,m}\delta_{\xi,\eta} . \tag{4}$$

Noting that**

$$F \equiv \exp\left\{\sum_n \frac{z^n}{n}\overline{P}^{(n)}Q^{(n)}\right\} = \exp\left\{\sum_n \frac{z^n}{n}(T_q^{(n)\dagger}\eta T_p^{(n)})\right\} \tag{5a}$$

and

$$\overline{F} \mp \exp\left\{\sum_n \frac{z^n}{n}P^{(n)}\overline{Q}^{(n)}\right\} = \exp\left\{\sum_n \frac{z^n}{n}(T_q^{(n)\dagger}\eta^\dagger T_p^{(n)})\right\} \tag{5b}$$

where $\eta = \sigma_3 - i\sigma_2$ is a 2x2 matrix, it follows that we can rewrite equations (5a) and (5b) as follows ***

$$F = \langle 0|\ \exp\left\{\sum_n \left(\frac{T_q^{(n)\dagger}}{\sqrt{n}}\ a^{(n)}\right)\right\} S(\eta)\ z^R \exp\left\{\sum_n \left(a^{(n)\dagger}\ \frac{T_p^{(n)}}{\sqrt{n}}\right)\right\}|0\rangle \tag{6a}$$

$$\overline{F} = \langle 0|\ \exp\left\{\sum_n \left(\frac{T_q^{(n)\dagger}}{\sqrt{n}}\ a^{(n)}\right)\right\} S(\eta^\dagger)\ z^R \exp\left\{\sum_n \left(a^{(n)\dagger}\ \frac{T_p^{(n)}}{\sqrt{n}}\right)\right\}|0\rangle \tag{6b}$$

where $|0\rangle$ is the ground state of all the a's, i.e. $a_{\mu\xi}^{(i)}|0\rangle = 0$,

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**We use the notation $(x^\dagger y) \equiv \sum_\mu \left[x_1^{\ \mu} y_{\mu 1} + x_2^{\ \mu} y_{\mu 2}\right]$

***We understand $a^{(n)}$ to be a column spinor and $a^{\dagger(n)}$ to be a row spinor.

and use has been made of the following identities

$$e^{fa} z^{a^\dagger a} = z^{a^\dagger a} e^{faz} \tag{7a}$$

$$e^{fa} e^{ga^\dagger} = e^{ga^\dagger} e^{fa} e^{fg} \tag{7b}$$

$$\langle o|\ \exp\{(T^\dagger \eta a)\} = \langle o|\exp\{(T^\dagger a)\} s(\eta) \tag{7c}$$

where

$$s(\eta) = \prod_{n=1}^{\infty} \left\{ \frac{\left[(a^{\dagger(n)} \eta a^{(n)})\right]^{(a^{(n)\dagger} a^{(n)})}}{(a^{\dagger(n)} a^{(n)})!} \right\} \tag{8}$$

and

$$R = \sum_{n=1}^{\infty} n(a^{(n)\dagger} a^{(n)}) . \tag{9}$$

Inserting F and $\bar{F}$ into equations (1) and (2) and performing the integrations over z, we obtain

$$A_{r+1,s+1} = \langle 0|G(q,a) S(\eta) D(R,s) G^\dagger(p,a\ )|0\rangle \tag{10a}$$

$$CA_{r+1,s+1} C^{-1} = \langle 0|G(q,a) S(\eta^\dagger) D(R,s) G^\dagger(p,a\ )|0\rangle\ , \tag{10b}$$

where the new vertex function is

$$G(q,a) = \int dy \phi(y,q)\ \exp \sum \left\{ \frac{1}{\sqrt{n}}\ (T_q{}^{(n)\dagger} a^{(n)}) \right\}\ , \tag{11}$$

and D(R,s) is the usual propagator introduced in Reference (4a). Clearly this is a factorized form. In contrast with the previous results, the reflection invariance of the scalar amplitude is clearly and naturally associated with the symmetry properties of the propagator and not with the vertex functions G. To bring out the symmetry of the propagator, it is useful to consider the operators $S(\eta)$ and $S(\eta^\dagger)$ in a more familiar context. Following the elegant work of Schwinger[5], we identify the bilinear form

$$(a^{\dagger(n)} \frac{\sigma_i}{2} a^{(n)}) \tag{12}$$

with the ith component of the angular momentum operator, $J_i^{(n)}$. Clearly we have

$$\left[J_i^{(n)}, J_k^{(m)}\right] = i\epsilon_{ikj} J_j^{(n)} \delta_{n,m} \, , \tag{13}$$

We may, therefore, label our states with the usual SU(2) quantum numbers

$$|j^{(n)}, m^{(n)}\rangle \equiv \frac{\left[a_1^{\dagger(n)}\right]^{(j^{(n)}+m^{(n)})}}{\left[(j^{(n)}+m^{(n)})!\right]^{\frac{1}{2}}} \frac{\left[a_2^{\dagger(n)}\right]^{(j^{(n)}-m^{(n)})}}{\left[(j^{(n)}-m^{(n)})!\right]^{\frac{1}{2}}} |0\rangle \tag{14}$$

which is, of course, appropriate for this infinite dimensional direct-product representation of SU(2). It follows that

$$R \prod_{n=1}^{\infty} \otimes |j^{(n)}, m^{(n)}\rangle = \left(2 \sum_{n=1} n j^{(n)}\right) \prod_{n=1}^{\infty} \otimes |j^{(n)}, m^{(n)}\rangle \tag{15}$$

so that D(R,s) is invariant under SU(2) rotations. However, we see that $S(\eta)$ and $S(\eta^\dagger)$ are just rotation matrices. Thus, the "charge-conjugation" invariance acquires a natural interpretation as a rotation by π about the 3-axis, i.e.

$$s(\eta) = e^{-i\pi \sum_n J_3^{(n)}} S(\eta^\dagger) \, e^{i\pi \sum_n J_3^{(n)}} \tag{16}$$

The Ward-like identities of Ref. (2) can now be stated in our operator language as

$$\langle o|G(q,a)e^{-i\pi J_3} D(R,S) \; S(\eta)e^{i\pi J_3} \, G^\dagger(p,a)|o\rangle =$$

$$\langle o|G(q,a)D(R,S)S(\eta)G^\dagger(p,a)|o\rangle \tag{17}$$

where, $J_3 = \sum_n J_3^{(n)}$. (18)

A by-product of this spinor factorization scheme is the introduction of an additional degeneracy over that found in Refs. (2) and (4). Indeed the partition state $\Pi_i|\lambda_i\rangle$ of Reference (4a) is now in our formalism $\Pi_i(\lambda_i+1)$-fold degenerate since $\lambda_i = 2j^{(i)}$. This new degeneracy is due to the magnetic quantum numbers $m^{(i)}$. Their physical significance is linked with the transformation properties of the states under charge-conjugation.

At present we are examining more complicated vertex functions. We hope that the formulation of dual resonance models in the familiar SU-(2) language will prove useful in understanding the gauge-like problem. In addition the roles the new quantum numbers $j^{(i)}$, $m^{(i)}$ as well as their possible connection with quark models need to be assessed.

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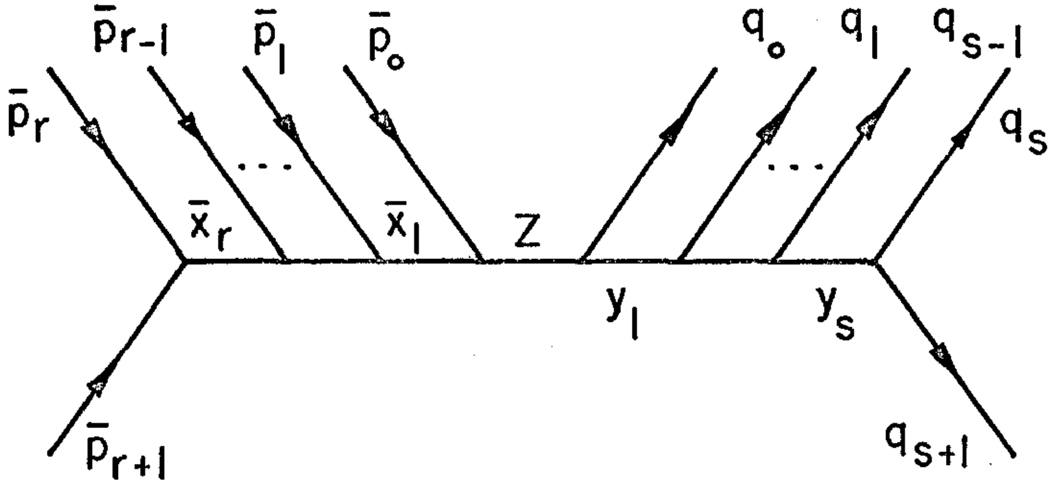
FIGURE CAPTIONS

Figure 1a (r + 1) + (s + 1) - point scalar amplitude.

Figure 1b (r + 1) + (s + 1) - point scalar "charge-conjugated" amplitude.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$\bar{p}_1$
$\bar{p}_o$
q_o
q_1
q_{s-1}
$\bar{p}_r$
q_s
$\bar{x}_r$
$\bar{x}_1$
Z
y_1
y_s
$\bar{p}_{r+1}$
q_{s+1}

Fig. 1a

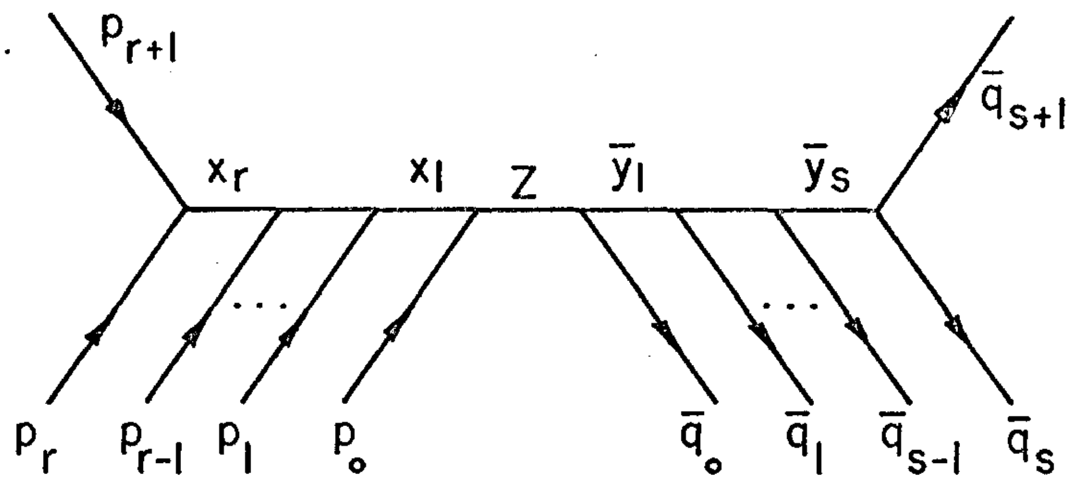
p_{r+1}
$\bar{q}_{s+1}$
x_r
x_1
Z
$\bar{y}_1$
$\bar{y}_s$
p_r
p_{r-1}
p_1
p_o
$\bar{q}_o$
$\bar{q}_1$
$\bar{q}_{s-1}$
$\bar{q}_s$

Fig. 1b

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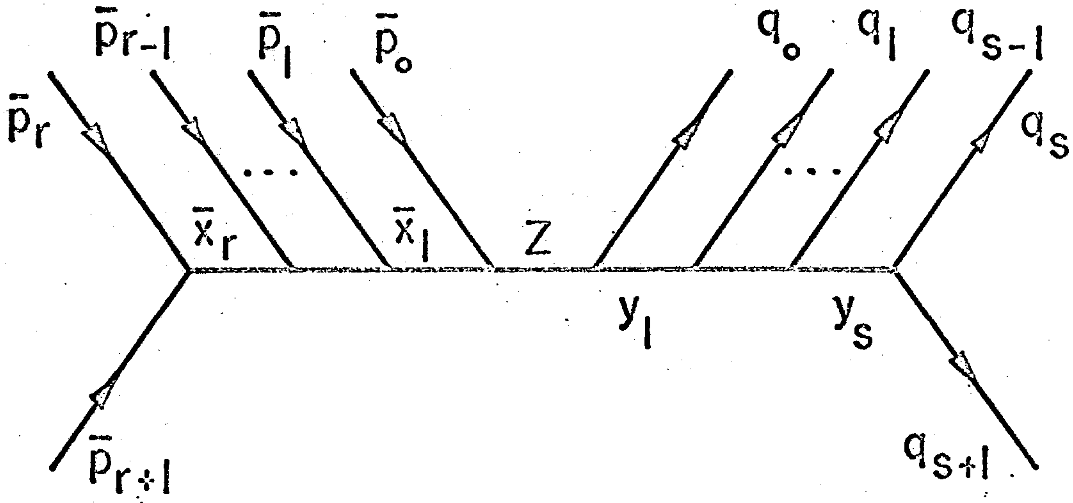
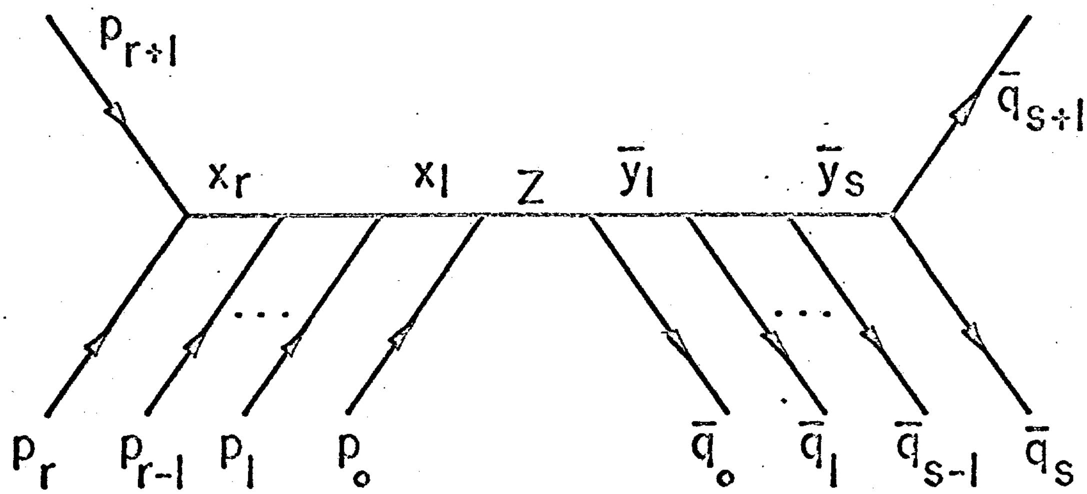


p̄r
p̄r-1
p̄1
p̄o
qo
q1
qs-1
qs
x̄r
x̄1
Z
y1
ys
p̄r+1
qs+1

Fig. 1a



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q̄s+1
xr
x1
Z
ȳ1
ȳs
pr
pr-1
p1
po
q̄o
q̄1
q̄s-1
q̄s

Fig. 1b