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SPL: A Language and Compiler for DSP Algorithms

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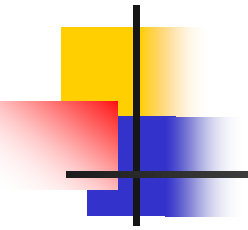
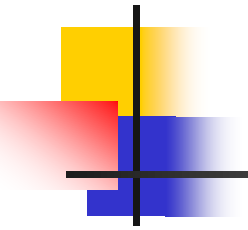


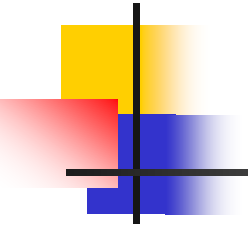
Overview

- SPL: A domain specific language
  - DSP core algorithms
  - Matrix factorization
- SPL Compiler:
  - SPL $\Rightarrow$ Fortran/C programs
  - Efficient implementation
- Part of SPIRAL(www.ece.cmu.edu/~spiral):
  - Adaptive framework for optimizing DSP libraries
  - Search over different SPL formulas using SPL compiler.



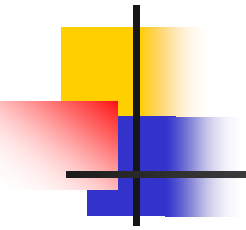
Outline

- Motivation
- Mathematical formulation of DSP algorithms
- SPL Language
- SPL Compiler
- Performance Evaluation
- Conclusion



Motivation

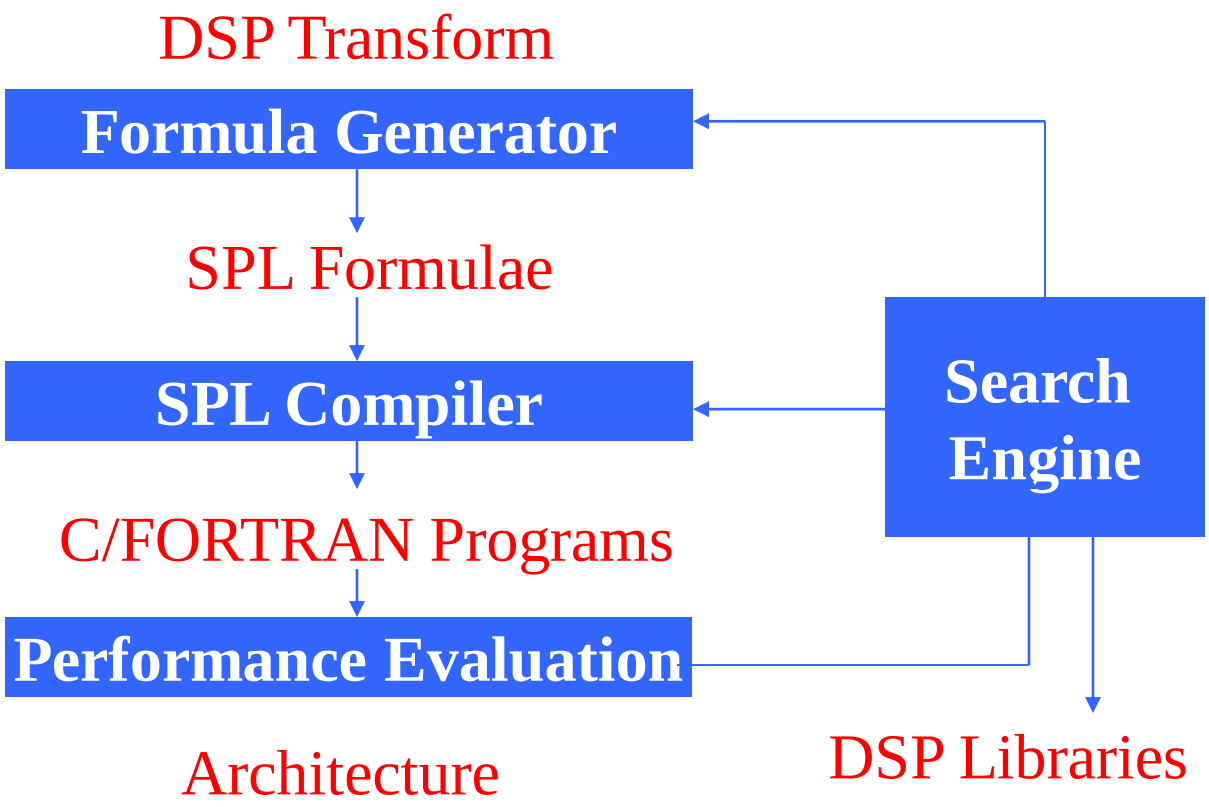
- What affects the performance?
  - Architecture features:
    - pipeline, FU, cache, …
  - Compiler:
    - Ability to take advantage of architecture features
    - Ability to handle large / complicated programs
- Ideal compiler
  - Perform perfect optimization based on the architecture
  - Practical compilers have limiations



Motivation (continue)

- Manual Performance Tuning
  - Modify the source based on profiling information
  - Requires knowledge about the architecture features
  - Requires considerable work
  - The performance is not portable
- Automatic performance tuning?
  - Very difficult for general programs
  - DSP core algorithms: SPIRAL.

SPIRAL Framework



Fast DSP Algorithms as Matrix Factorizations

- A DSP Transform:
  - $y = Mx \;\Rightarrow\; y = M_1 M_2 \ldots M_k\, x$
- Example: n-point DFT $y = F_n x$

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

$$F_4 = (F_2 \otimes I_2)\, T_2^4\, (I_2 \otimes F_2)\, L_2^4$$

$$= \begin{bmatrix} 1 & & 1 & \\ & 1 & & 1 \\ 1 & & -1 & \\ & 1 & & -1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & i \end{bmatrix} \begin{bmatrix} 1 & 1 & & \\ 1 & -1 & & \\ & & 1 & 1 \\ & & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{bmatrix}$$

Tensor Product

- A linear algebra operation for representing repetitive matrix structures

$$A_{m\times n} \otimes B_{m'\times n'} = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}_{mm'\times nn'}$$

- Loop

$$I \otimes B = \begin{bmatrix} B & & \\ & \ddots & \\ & & B \end{bmatrix}$$

Tensor Product (continue)

- Vector operations

$$A \otimes I = \begin{bmatrix} \begin{bmatrix} a_{11} & & \\ & \ddots & \\ & & a_{11} \end{bmatrix} & \cdots & \begin{bmatrix} a_{1n} & & \\ & \ddots & \\ & & a_{1n} \end{bmatrix} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} a_{m1} & & \\ & \ddots & \\ & & a_{m1} \end{bmatrix} & \cdots & \begin{bmatrix} a_{mn} & & \\ & \ddots & \\ & & a_{mn} \end{bmatrix} \end{bmatrix}$$

Rules for Recursive Factorization

- Cooley-Tukey factorization for DFT

$$F_{rs} = (F_r \otimes I_s) T_s^{rs} (I_r \otimes F_s) L_r^{rs}$$

- General K-way factorization for DFT

$$F_n = \prod_{i=1}^{k} \left[(I_{n_{i-}} \otimes F_{n_i} \otimes I_{n_{i+}}) (I_{n_{i-}} \otimes T_{n_{i+}}^{n_i n_{i+}}) \right] \cdot \prod_{i=k}^{1} (I_{n_{i-}} \otimes L_{n_i}^{n_i n_{i+}})$$

where $n=n_1 \ldots n_k$, $n_{i-}=n_1 \ldots n_{i-1}$, $n_{i+}=n_{i+1} \ldots n_k$

Formulas

- Variations of DFT(8)

$$F_8 = (F_2 \otimes I_4) T^8_4 (I_2 \otimes F_2) L^8_2$$

$$F_8 = (F_2 \otimes I_4) T^8_4 (I_2 \otimes ((F_2 \otimes I_2) T^4_2 (I_2 \otimes F_2) L^4_2)) L^8_2$$

$$F_8 = (F_2 \otimes I_4) T^8_4 (I_2 \otimes F_2 \otimes I_2)(I_2 \otimes T^4_2)(I_4 \otimes F_2) R_8$$

The SPL Language

- Domain-specific programming language for describing matrix factorizations

$$F_4 = (F_2 \otimes I_2) T_2^4 (I_2 \otimes F_2) L_2^4$$

```
(compose
     (tensor (F 2)(I 2))
     (T 4 2)
     (tensor (I 2)(F 2))
     (L 4 2)
```

matrix operations

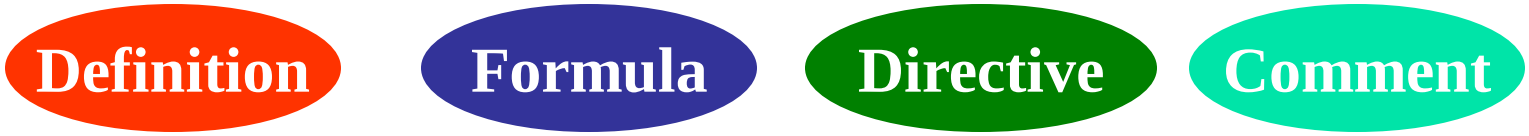
primitives: parameterized special matrices

SPL In A Nut-shell

- SPL expressions
  - General matrices
    - (matrix (a_{11}…a_{1n}) … (a_{m1} … a_{mn}))
    - (diagonal (a_{11}…a_{nn}))
    - (sparse (i_1 j_1 a_1) … (i_k j_k a_k))
  - Parameterized special matrices
    - (I n), (L mn n), (T mn n), (F n)
  - Matrix operations
    - (compose A_1 … A_k)
    - (tensor A_1 … A_k)
    - (direct_sum A_1 … A_k)
- Others: definitions, directives, template, comments

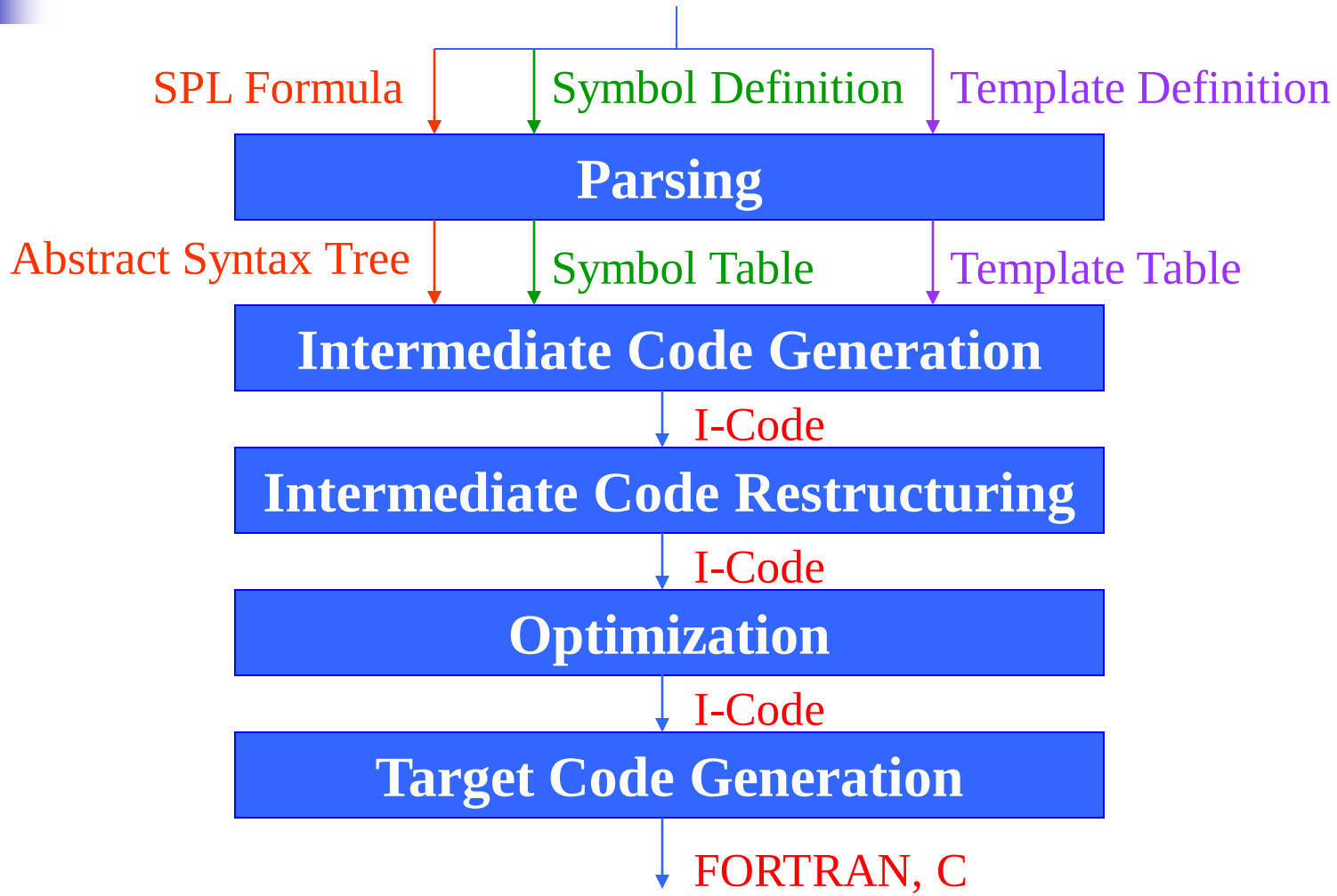
$A \oplus B = diag(A,B)$

A Simple SPL Program



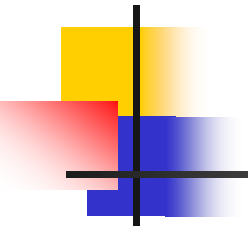

```
; This is a simple SPL program
(define A (matrix(1 2)(2 1)))
(define B (diagonal(3 3))
#subname simple
(tensor (I 2)(compose A B))
;; This is an invisible comment
```

The SPL Compiler



Template Based Intermediate Code Generation

- Why use template?
  - User-defined semantics
  - Language extension
  - Compiler extension without modifying the compiler
  - Be integrated into the search space
- Structure of a template
  - Pattern, condition, code
- Template match
  - Generate I-code from matching template
  - Template matching is a recursive procedure



I-Code

- I-code is the intermediate code of the SPL compiler
- Internally I-code is four-tuples
  - <op, src1, src2, dest>
- The external representation of I-code
  - Fortran-like
  - Used in template

Template

```
(template
   (F n)[ n >= 1 ]
   ( do i=0,n-1
        y(i)=0
        do j=0,n-1
          y(i)=y(i)+W(n,i*j)*x(j)
        end
     end ))
```

Pattern

Condition

I-code

Code Generation and Template Matching

(F 2) matches pattern **(F n)** and assigns 2 to n.
Because n=2 satisfies the condition n>=1,
the following i-code is generated from the template:

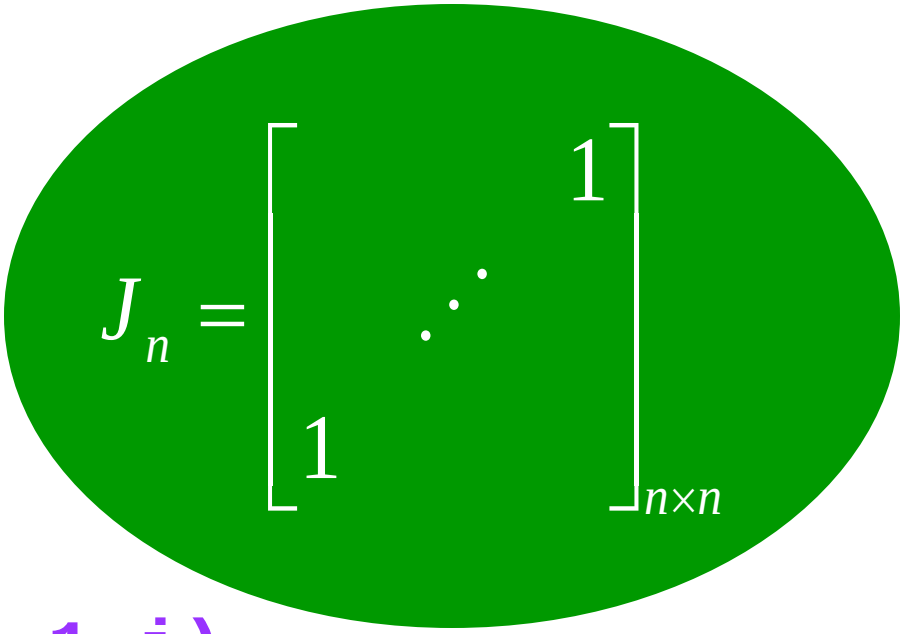
```
do i = 0,1
  y(i) = 0
  do j = 0,1
    y(i) = y(i)+W(2,i*j)*x(j)
  end
end
```

```
Y(0)=x(0)+x(1)
y(1)=x(0)-x(1)
```

Unrolling & Optimization

Define A Primitive

```
(primitive J)
(template
   (J n)
   [ n >= 1 ]
   ( do i=0,n-1
       y(i) = x(n-1-i)
     end ))
```

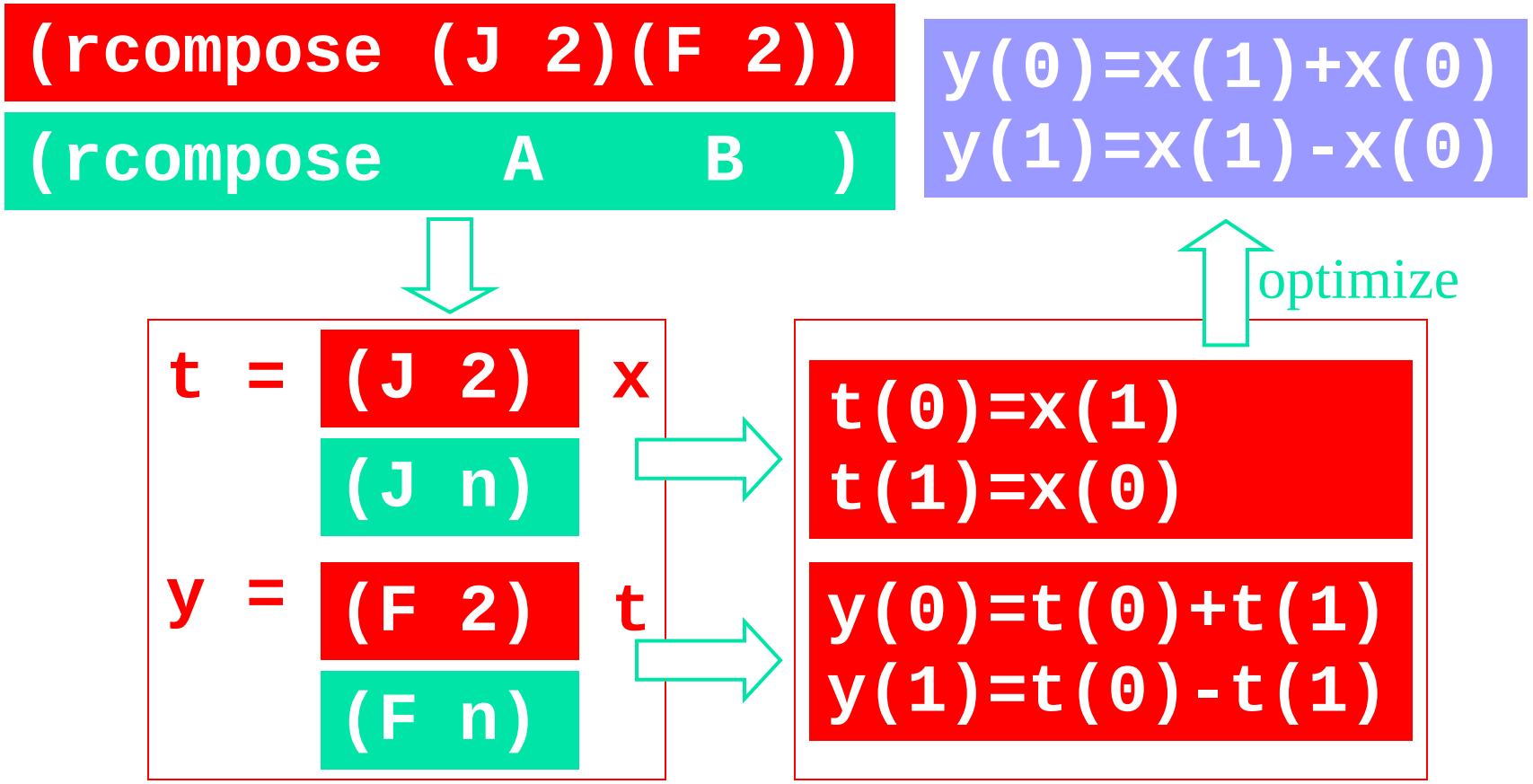


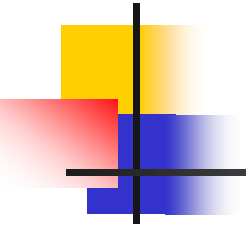
Define An Operation

```
(operation rcompose)
(template
  (rcompose A B)
  [ B.nx == A.ny ]
  ( t = A(x)
    y = B(t)))
```

$$y = (A \circ B)x$$
$$\equiv \; t = Ax$$
$$y = Bt$$

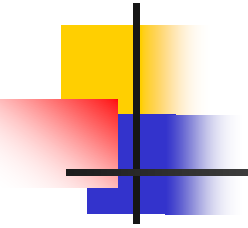
Compound Template Matching





Intermediate Code Restructuring

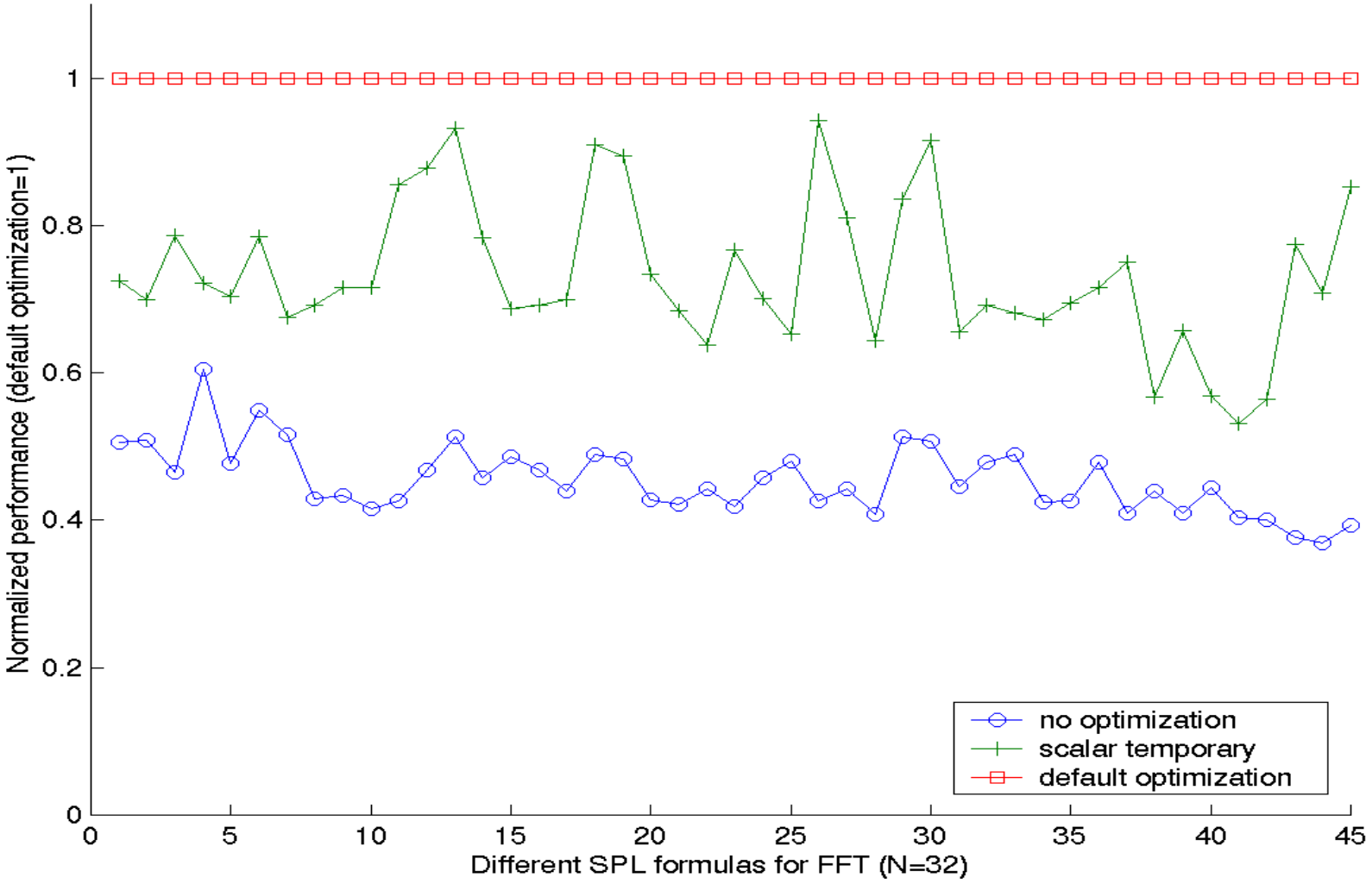
- Loop unrolling
  - Degree of unrolling can be controlled globally or case by case
- Scalar function evaluation
  - Replace scalar functions with constant value or array access
- Type conversion
  - Type of input data: real or complex
  - Type of arithmetic: real or complex
  - Same SPL formula, different C/Fortran programs



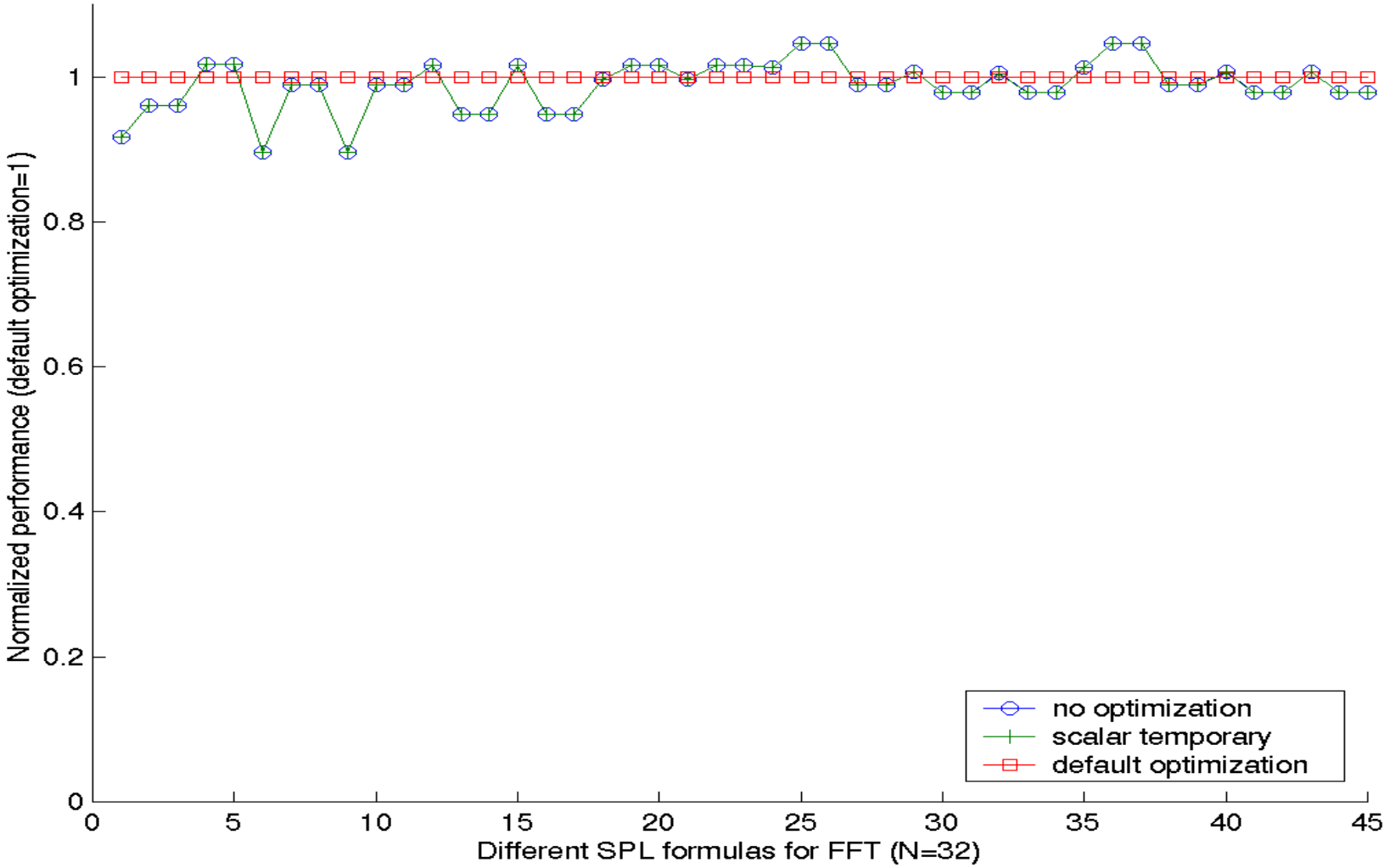
Optimizations

- Low-level optimizations:
  - Instruction scheduling, register allocation, instruction selection, …
  - Leave them to the native compiler
- Basic high-level optimizations:
  - Constant folding, copy propagation, CSE, dead code elimination,…
  - The native compiler is supposed to do the dirty work, but not enough.
- High-level scheduling, loop transformations:
  - Formula transformation
  - Integrated into the search space

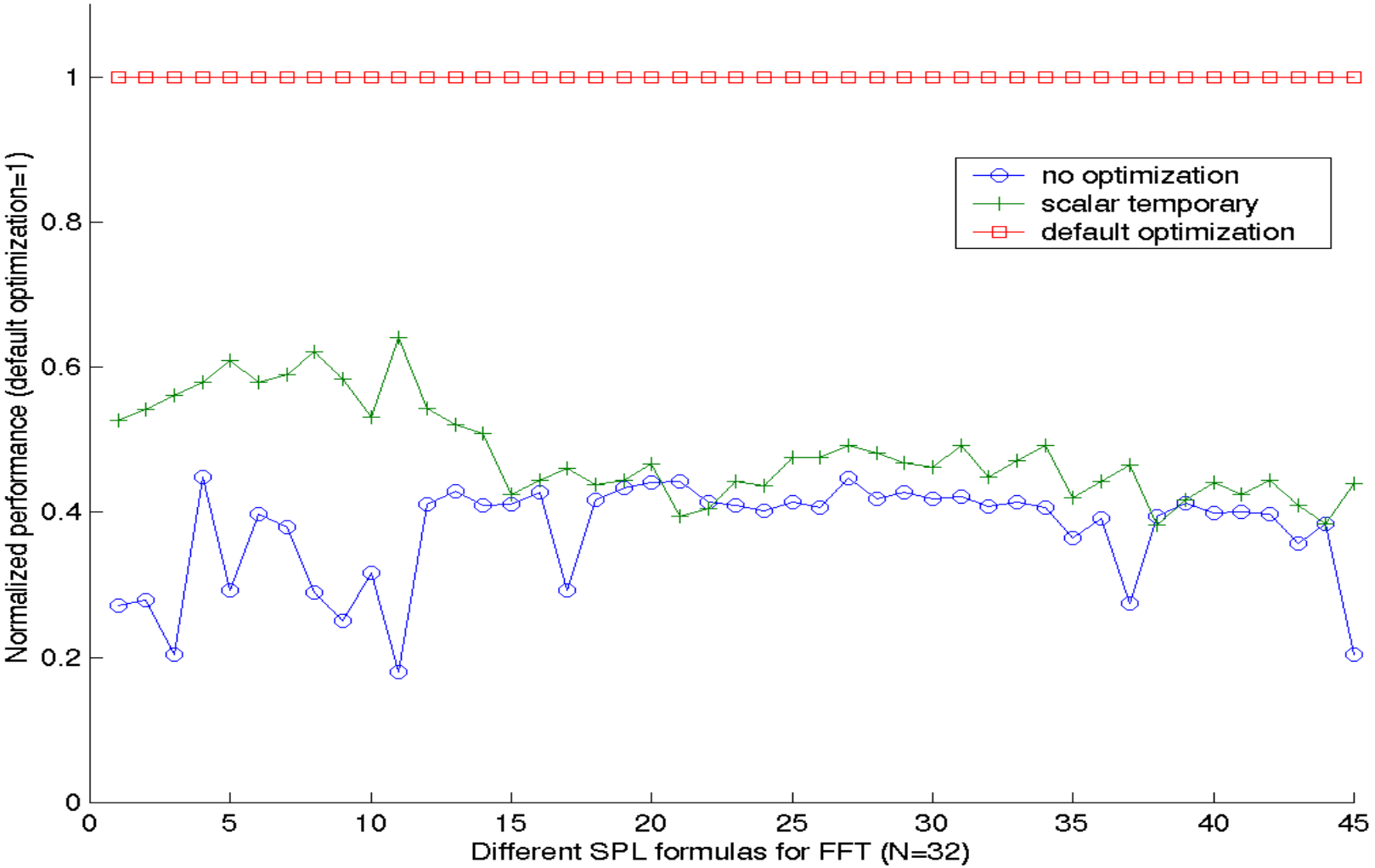
Basic Optimizations(FFT,N=2^5,Ultra5)



Basic Optimizations(FFT,N=2^5,Origin200)

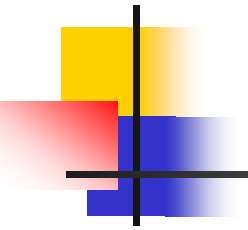


Basic Optimizations(FFT,N=2^5,PC)



Performance Evaluation

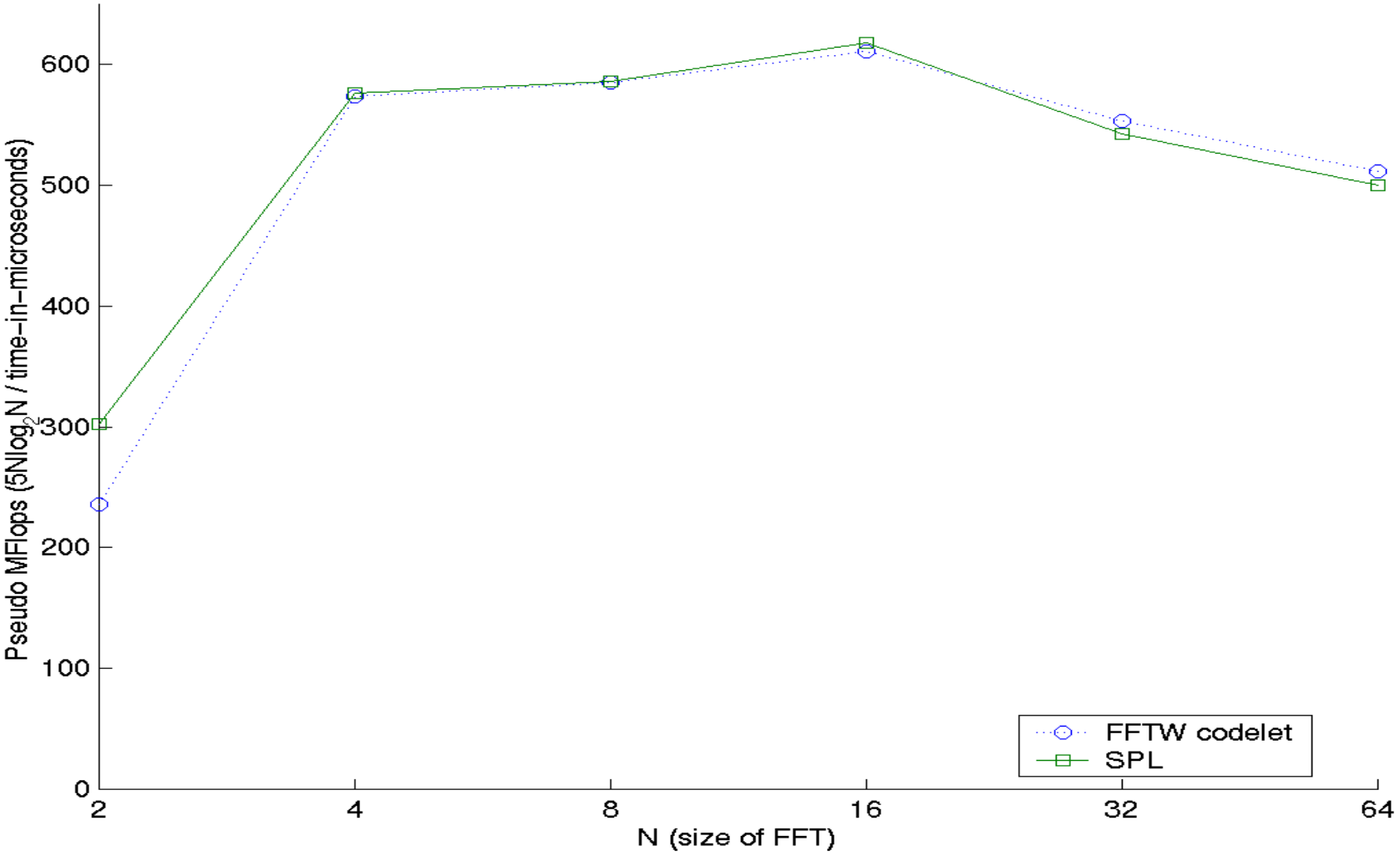
- Platforms: Ultra5, Origin 200, PC
- Small-size FFT (2^1 to 2^6)
  - Straight-line code
  - K-way factorization
  - Dynamic programming
- Large-size FFT (2^7 to 2^{20})
  - Loop code
  - Binary right-most factorization
  - Dynamic programming
- Accuracy, memory requirement



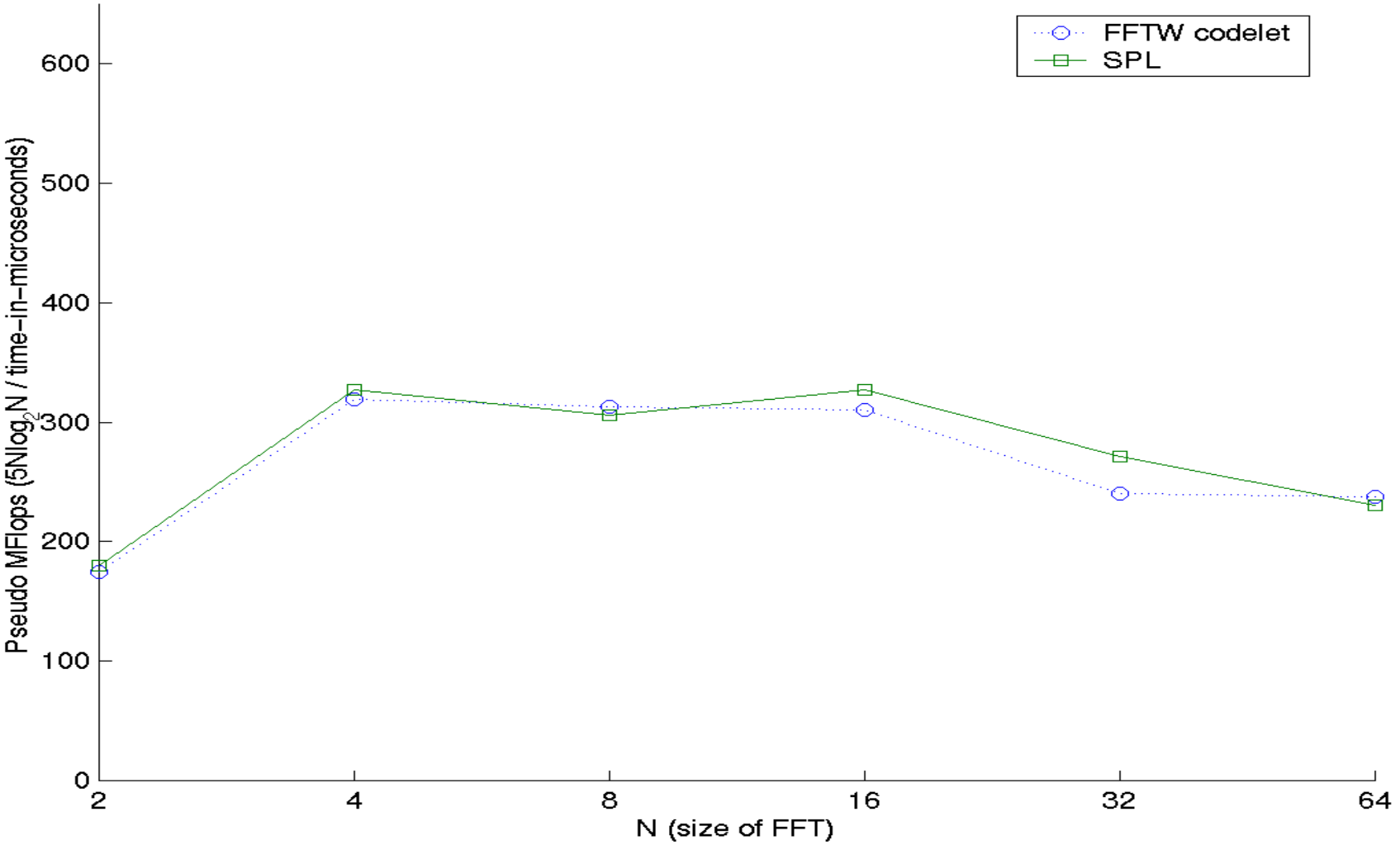
FFTW

- A FFT package
  - Codelet: optimized straight-line code for small-size FFTs
  - Plan: factorization tree
  - Use dynamic programming to find the plan
  - Make recursive function calls to the codelet according to the plan
- Measure and estimate

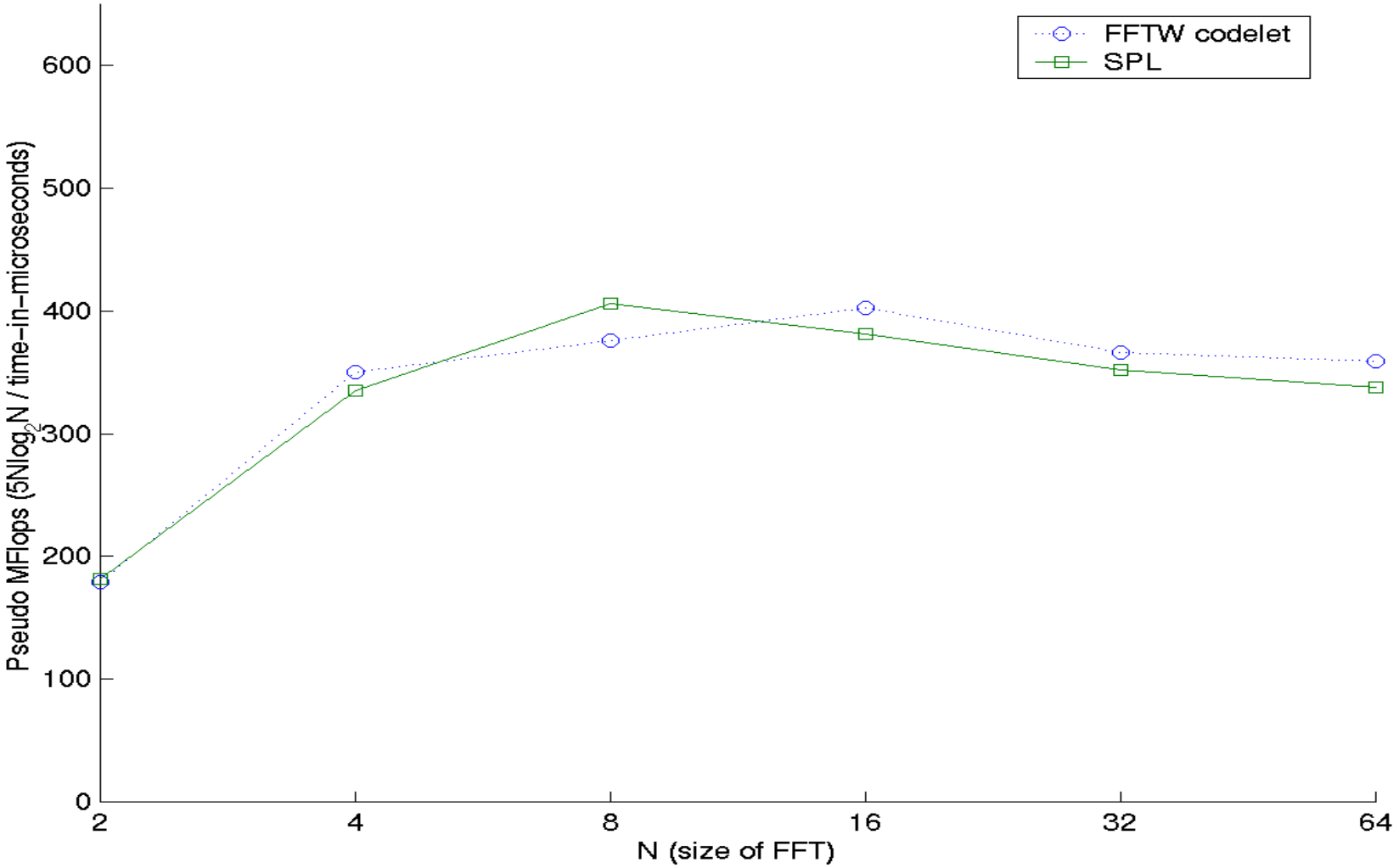
FFT Performance (N=2^1 to 2^6,Ultra5)



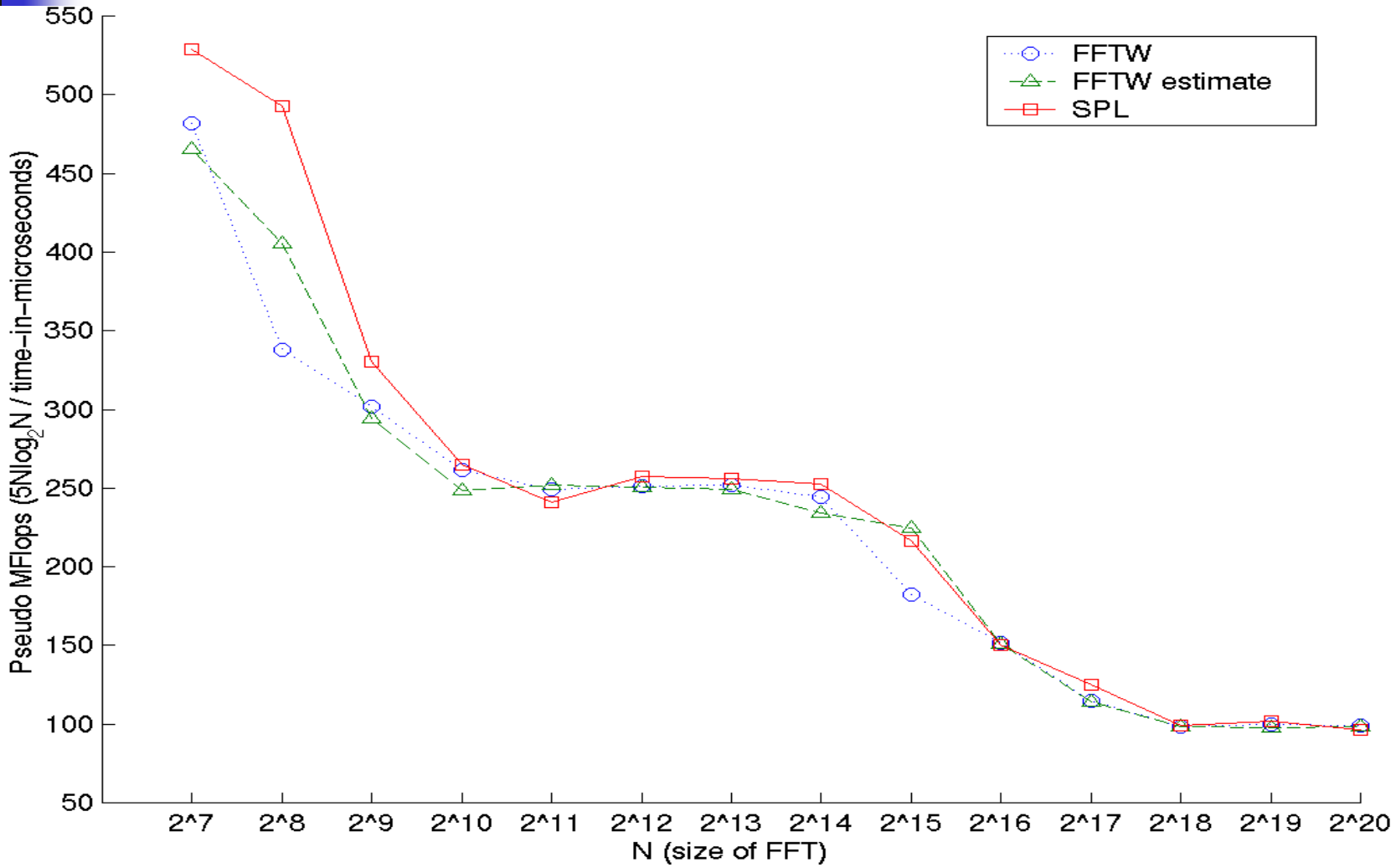
FFT Performance (N=2^1 to 2^6,Origin200)



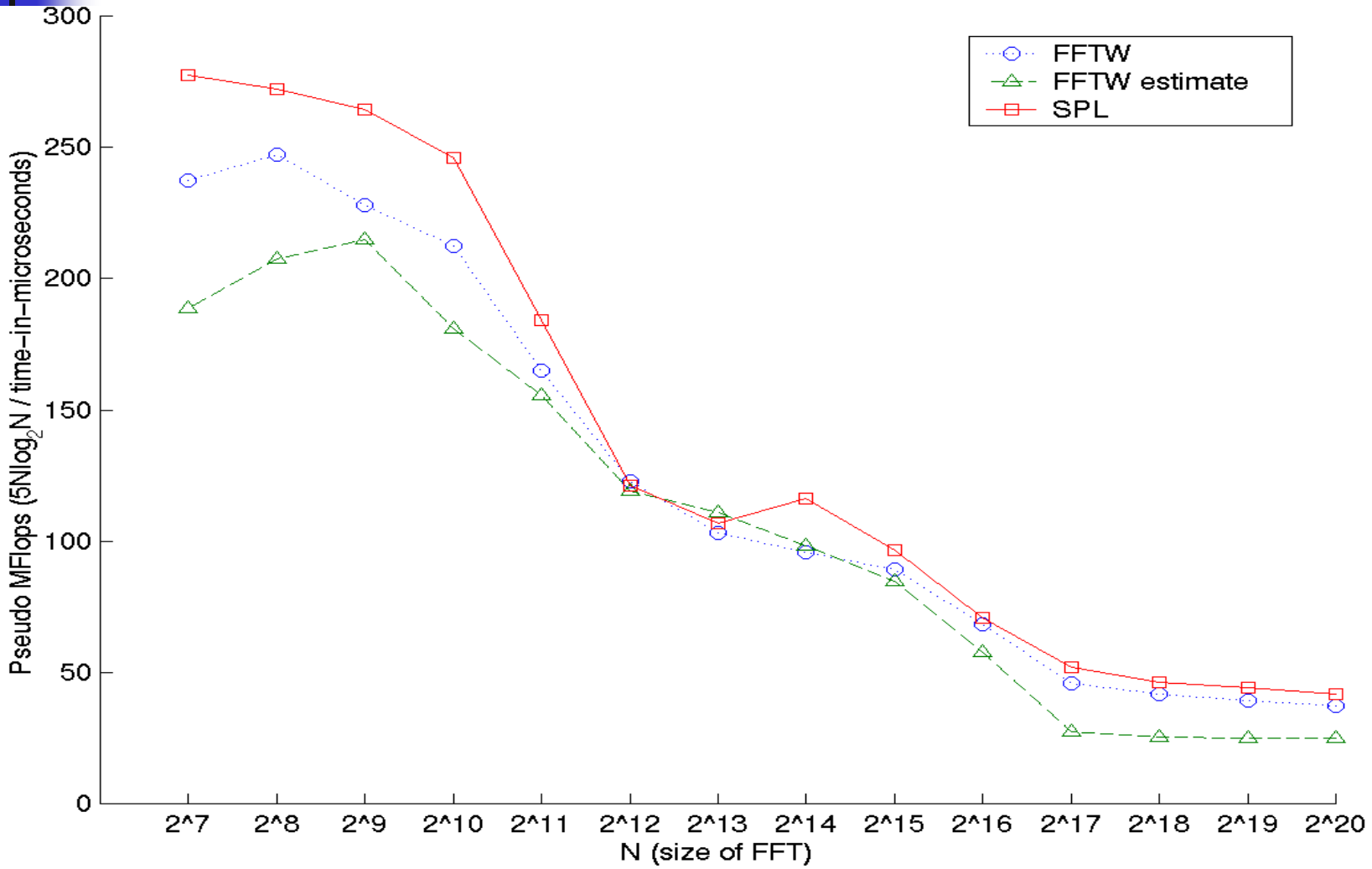
FFT Performance (N=2^1 to 2^6,PC)



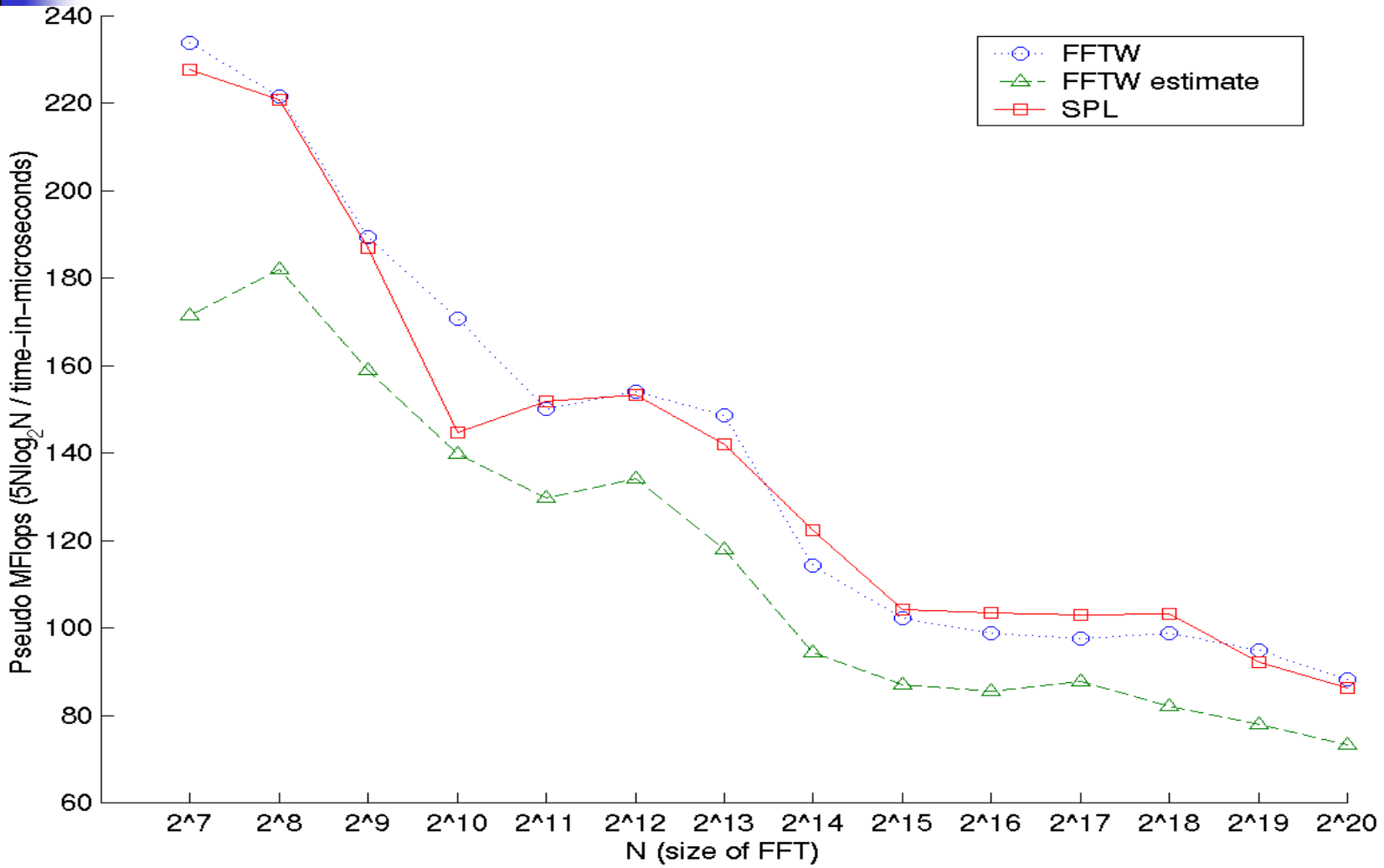
FFT Performance (N=2^7 to 2^{20},Ultra5)



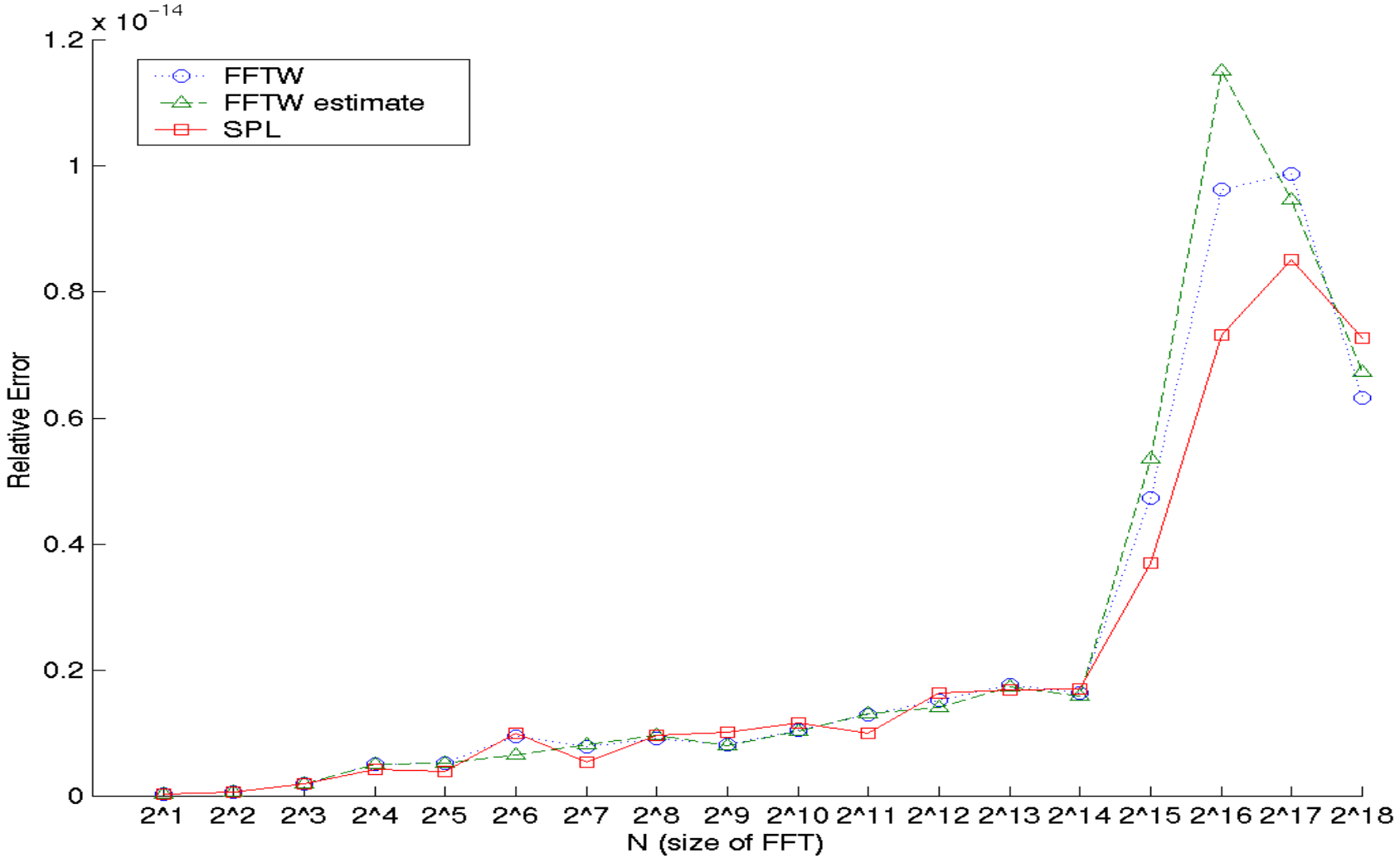
FFT Performance (N=2^7 to 2^{20},Origin200)



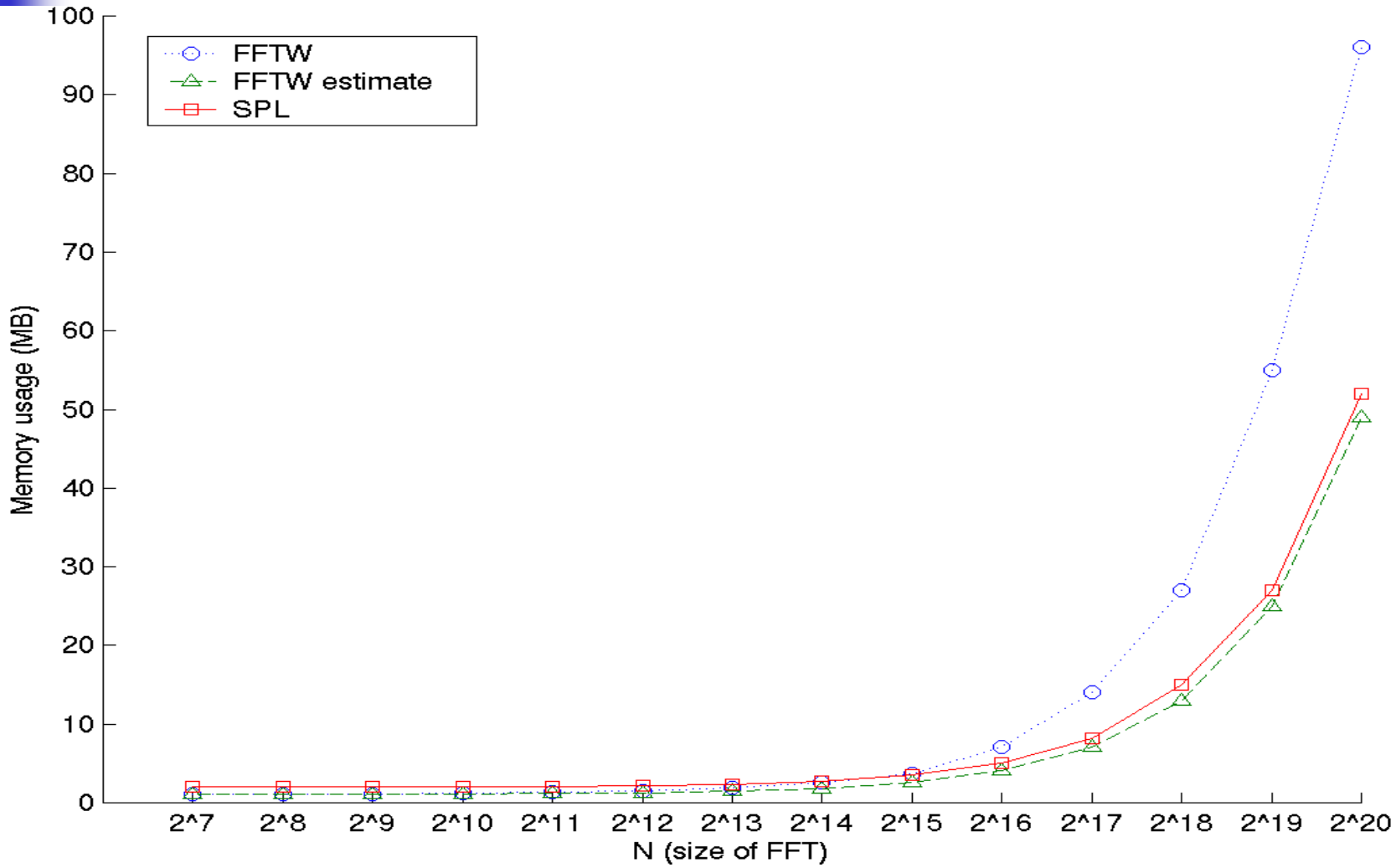
FFT Performance (N=2^7 to 2^{20},PC)

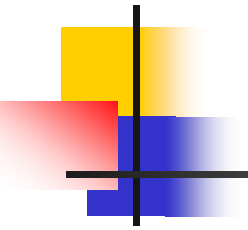


FFT Accuracy (N=2^1 to 2^{18})



FFT Memory Utilization (N=2^7 to 2^{20})





Conclusion

- The SPL compiler is capable of producing efficient code on a variety of platforms.
- The standard optimizations carried out by the SPL compiler are necessary to get good performance.
- The template mechanism makes the SPL language and the SPL compiler highly extensible

Related Work

	Domain	**Code Generator**	**Tuning**
FFTW	FFT	Fix algorithms	DP
WHT Package	WHT	Built-in	DP, GA
EXTENT	Block recursive	Built-in	Manual
ATLAS	BLAS	Hand coded, Blocking, unrolling	Search
PHiPAC	BLAS	Hand coded	Search
Iterative Compilation	Compiler option	N/A	Search

Performance Evaluation: Platforms

- Ultra5
  - Solaris 7, Sun Workshop 5.0
  - 333MHz UltraSPARC Iii, 128MB, 16KB/16KB/2MB
- Origin 200
  - IRIX64 6.5, MIPSpro 7.3.1.1m
  - 180MHz MIPS R10000, 384MB, 32KB/32KB/1MB
- PC
  - Linux kernel 2.2.18, egcs 1.1.2
  - 400MHz Pentium II, 256MB, 16K/16K/512KB