

Splitting trees stopped when the first clock rings and Vervaat's transformation

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Abstract

We consider a branching population where individuals have i.i.d. life lengths (not necessarily exponential) and constant birth rate. We let N_t denote the population size at time t. We further assume that all individuals, at birth time, are equipped with independent exponential clocks with parameter δ . We are interested in the genealogical tree stopped at the first time T when one of those clocks rings. This question has applications in epidemiology, in population genetics, in ecology and in queuing theory.

We show that conditional on $\{T < \infty\}$, the joint law of $(N_T, T, X^{(T)})$, where $X^{(T)}$ is the jumping contour process of the tree truncated at time T, is equal to that of $(M, -I_M, Y_M')$ conditional on $\{M \neq 0\}$, where : M+1 is the number of visits of 0, before some single independent exponential clock \mathbf{e} with parameter δ rings, by some specified Lévy process Y without negative jumps reflected below its supremum; I_M is the infimum of the path Y_M defined as Y killed at its last 0 before \mathbf{e} ; Y_M' is the Vervaat transform of Y_M .

This identity yields an explanation for the geometric distribution of N_T and has numerous other applications. In particular, conditional on $\{N_T = n\}$, and also on $\{N_T = n, T < a\}$, the ages and residual lifetimes of the n alive individuals at time T are i.i.d. and independent of n. We provide explicit formulae for this distribution and give a more general application to outbreaks of antibiotic-resistant bacteria in the hospital.