Spontaneous Construction of higher order Voronoi Diagram

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Abstract—**The higher order Voronoi diagrams are difficult to construct because of their complicated structures. In traditional algorithm, production process was extremely complex. While spontaneous algorithm is only concerned with positions of generators, so it is effective for constructing Voronoi diagrams with complicated shapes of Voronoi polygons. It can be applied to higher order Voronoi diagram with any generators, and can get over most shortcomings of traditional algorithm. So it is more useful and effective. Model is constructed with spontaneous algorithm. And the application example shows that the algorithm is both simple and useful, and it is of high potential value in practice.**

Keywords- Voronoi diagram; generator; spontaneous; higher order

I. INTRODUCTION

As a branch of Computational Geometry, Voronoi diagram has been quickly developed on account of the development of theory and the need of application. We have already noted that the concept of the Voronoi diagram is used extensively in a variety of disciplines and has independent roots in many of them. Voronoi diagram was appeared in meteorology, biology discipline and so on. [1-3] Higher order Voronoi diagram is an important concept of it. And it can be used for spatial interpolation , which is applied in statistical estimation as well as in cartography. The multiplicatively weighted higher order Voronoi diagram has been used for retail trade area analysis. More and more people pay attention to the algorithm that can construct higher order Voronoi diagram fast and effectively[4-6]. We investigate a spontaneous method for constructing, and it proved to be satisfactory by experiment.

II. DEFINITIONS

A. *Definition 1 （A Planar Ordinary Voronoi Diagram）[7]*

Given a finite number of distinct points in the Euclidean plane

$P=\{p_1,p_2,\cdots,p_n\}\subset R^2$, where $2<n<+\infty$, $x_i\neq x_j$, for $i\neq j, i,j\in I_n$.

We call the region given by

$$V(p_i)=\left\{x\middle|\|x-x_i\|\leq\|x-x_j\|\ for\ j\neq i, j\in I_n\right\} \quad (1)$$

the planar ordinary Voronoi polygon associated with p_i , and the set given by

$$\mathrm{V}=\{V(p_1),V(p_2),\cdots,V(p_n)\} \quad (2)$$

the planar ordinary generated by P (or the Voronoi diagram of P) [8]. We call p_i of $V(p_i)$ the generator point or generator of the ith Voronoi, and the set $P=\{p_1,p_2,\cdots,p_n\}$ the generator set of the Voronoi diagram (in the literature, a generator point is sometimes referred to as a site), as shown in Figure 1.

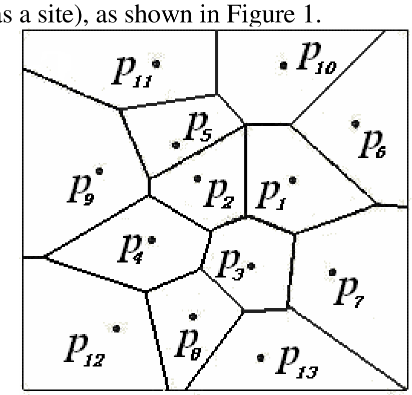


Figure 1. A planar ordinary Voronoi diagram, There are 13 points in the plane, which are generator..

B. *Definition 2 （Higher order Voronoi Diagram）[9]*

In the ordinary Voronoi diagram, a generator is a point p_i , or a generator set $P=\{p_1,p_2,\cdots,p_n\}$ of points, we consider the family of generalized Voronoi diagrams generated by a set of all possible subsets consisting of k points out of P , i.e.

$$A^{(k)}(P)=\{\{p_{11},\ldots,p_{1k}\},\ldots,\{p_{l1},\ldots,p_{lk}\}\} \quad (3)$$

Here $p_{ij}\in P$, $l=n!/(k!(n-k)!)$. We call this family the higher-order Voronoi diagram.

Here the ‘order’ means the number of points constituting a generator and ‘higher’ means more than one point. Note that ‘higher’ does not refer to the dimension of a space [10].

Let $A^{(k)}(P)$ be the set of all possible subsets consisting of k points out of P, i.e. We call the set given by

$$V\left(P_i^{(k)}\right) = \left\{ p \,\middle|\, \max_{p_h} \left\{ d(p,p_h) \middle| p_h \in P_i^{(k)} \right\} \le \min_{p_h} \left\{ d(p,p_j) \middle| p_j \in P \setminus P_i^{(k)} \right\} \right\} \quad (4)$$

The higher order Voronoi polygon associated with $P_i^{(k)}$, and the set of higher order Voronoi polygons, $V\left(A^{(k)}(P), d, R^m\right)$= V(k)=$\left\{V(P_1^{(k)}), \ldots, V(P_n^{(k)})\right\}$, the higher order Voronoi diagram generated by P, as shown in Figure 2.

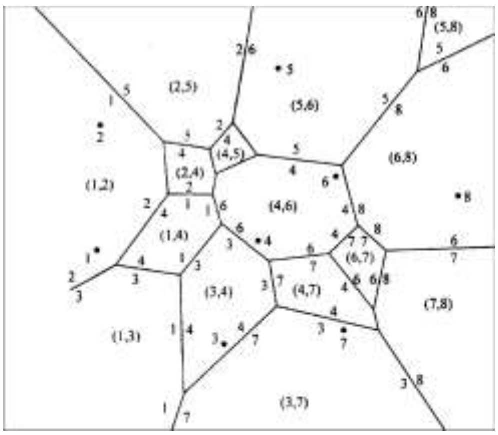


Figure 2. An order-2 Voronoi diagram in which the region with the symbol indicates the order-2 Voronoi polygon of .

III. SPONTANEOUS CONSTRUCTION OF HIGHER ORDER VORONOI DIAGRAM

A. Outline of Spontaneous Algorithm

Suppose that there are generator points in the Euclidean plane, and we will construct higher order voronoi diagram. First, we divide colors range of computer into parts, and assign different colors within first part for different generator points. Then use spontaneous algorithm constructing Voronoi diagram. In the process of spreading out, every time before assign a color value to a pixel, we should make a judgment: assign a pixel the color of the generator point if it is background color; assign it the color of the th part if it is the color of the th part; and stop spreading out if it is the color of the th part. The procedure end when all points on screen are marked color. This time, we get the Voronoi diagram.

B. Algorithm

Input: $p_1, p_2, \cdots, p_n, k$. Here , p_i is generator, k is the order.

Output: order- k Voronoi diagrams generated by those generators.

Step 1: suppose the colors range of possibilities is from 1 to N, an divide it into K parts: $1 \sim N_1$, $N_1+1 \sim N_2$, …, $N_{k-1}+1 \sim N_k$.

Here, $N_K \le N$,

$N_1 - N_0 = N_2 - N_1 = \cdots = N_K - N_{K-1}$, $n \le N_1 - 1$.

Assign generator points p_i with color i, $i = 1,2,\ldots,n$; (2.3)

Step 2: Built linked lists, that holds generators’ data including: abscissa ‘x’, ordinate ‘y’, color ‘i’;

Step 3: Initialize screen as white color. define a pointer p to linked lists, $j = 1$;

Step 4: Generate data sheet of Δx, Δy, and r2

Step 5: When p is not empty, do loop:

{ (1) Read data of row k :Δx, Δy, and r2;

(2) read p->x, p->y, p->color;

if p->color=0

then {SetPixel (p->x±Δx, p->y±Δy, p->color=N_1+p->color);

else if p->color=the color of the ith part

then { SetPixel (p->x±Δx, p->y±Δy, N_1+p->color); }

else { record the point;

if points form a closed circumference

then delete the node which ‘p’ pointed to from list;

if ‘p’ point to the end of

then k++, let ‘p’ point to the first node of list;

else p++;}

}

Step 6: Do landscape and portrait scanning for screen separately. When color of one pixel point is different with the next one, assign black color to it;

Step 7: Do landscape and portrait scanning for screen separately. Let pixel point white color if it is not black;

C. Judgment of Region of Higher Order Voronoi Diagram

In the generation of process, we need always judge how many times a pixel be deal with. The method is as the following:

We suppose that there are n generator points. First, Assign color value c_i ($i = 1, 2, \ldots, n, c_i \in [a,b]$) to every generator point, and the color of other pixel is white (value is 0). Given: $s > \sum c_i$. If a pixel was assigned a color, the color of the pixel is the sum of value in present and s . And it is denoted as ss . Now if a pixel was deal with d times, $\lfloor ss/s \rfloor = d$.

When the color value of a pixel is k , it belongs to a region of higher order Voronoi diagram generated by that generator point that extend to it. This time, we can do deal with next pixel. Otherwise, let the color value of the pixel is the sum of value in present, value of generator point.

D. 4 Practical Application.

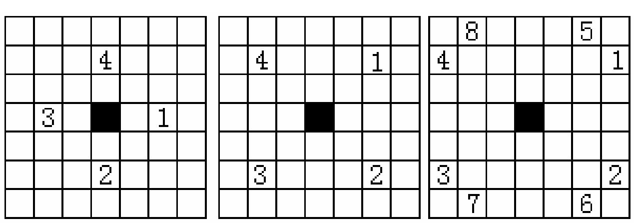
First, we need to manufacture a table of distance (See Table 1.).

order	1 2 3 4 5 6 7 8 9 …
Δx	1 1 2 2 2 3 3 3 4 …
Δy	0 1 0 1 2 0 1 2 0 …
r^2	1 2 4 5 8 9 10 13 16 …

Table 1. Table of distance

The table of distance consists of Δx , Δy and r^2 : Δx and Δy are both natural numbers ($\Delta x \geq \Delta y$, $r^2 = (\Delta x)^2 + (\Delta y)^2$), and is arranged in sequence of r^2 from small to big. When we draw a circle around the generator point $p(x, y)$, take out a set of data of $(\Delta x, \Delta y)$ in the order shown. And process is as follows:

1. if $\Delta x \neq 0, \Delta y = 0$, do: $(x+\Delta x, y)$, $(x, y-\Delta y)$, $(x-\Delta x, y)$, $(x, y+\Delta y)$ ($\Delta x = 2$ and $\Delta y = 0$, for example, see Figure 3(a).);
2. if $\Delta x \neq 0, \Delta y \neq 0$ and $\Delta x = \Delta y$, do: $(x+\Delta x, y+\Delta y)$, $(x+\Delta x, y-\Delta y)$, $(x-\Delta x, y-\Delta y)$, $(x-\Delta x, y+\Delta y)$ ($\Delta x = 2$ and $\Delta y = 2$, for example, see Figure 3(b).);
3. if $\Delta x \neq 0, \Delta y \neq 0$ and $\Delta x \neq \Delta y$, do: $(x+\Delta x, y+\Delta y)$, $(x+\Delta x, y-\Delta y)$, $(x-\Delta x, y-\Delta y)$, $(x-\Delta x, y+\Delta y)$, $(x+\Delta y, y+\Delta x)$, $(x+\Delta y, y-\Delta x)$, $(x-\Delta y, y-\Delta x)$, $(x-\Delta y, y+\Delta x)$ ($\Delta x = 3$ and $\Delta y = 2$, for example, see Figure 3(c).);



(a) (b) (c)

Figure 3. When draw a circle around a generator point, different conditions should be discussed.

Now we take 3 generator points as the example, and construct higher order Voronoi diagram using spontaneous algorithm.

We construct the higher order Voronoi diagram with 100 generator points and 200 generator points respectively by VC++6.0. As shown in Figure 4 and Figure 5.

IV. CONCLUSIONS

In this paper, we give a spontaneous algorithm of constructing higher order Voronoi diagram. The method is simple, practical and has strong universality and remarkable effect. The experimental results indicate that it has unique advantage in the construction of higher order Voronoi diagrams, and it is of high potential value in practice.

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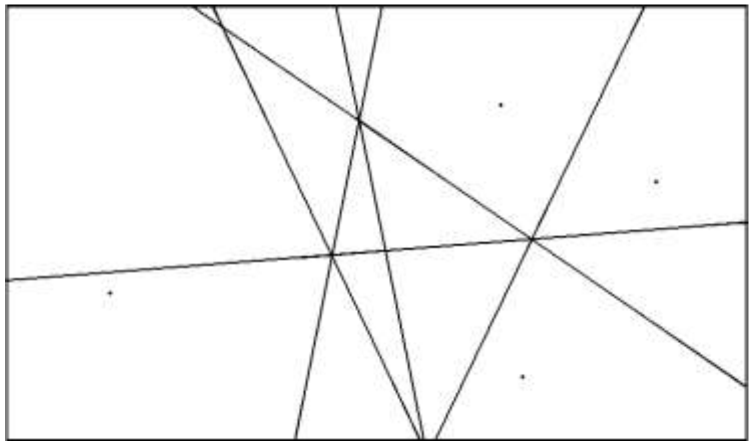

Figure 4. The order-3 Voronoi diagram with 4 generator points constructed with spontaneous algorithms

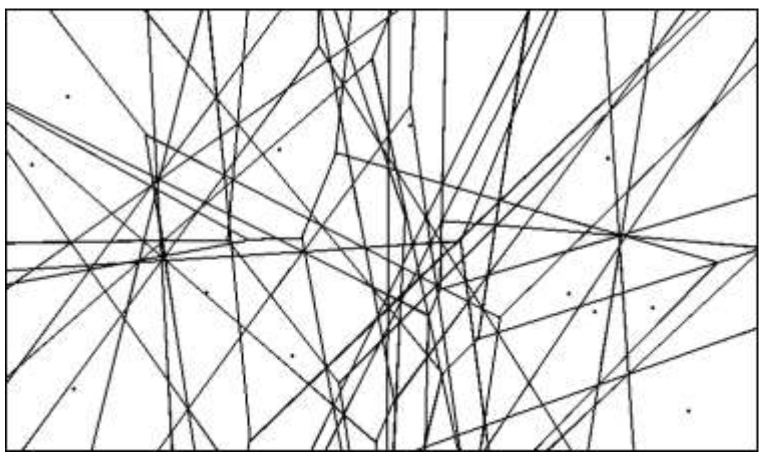

Figure 5. The order-5 Voronoi diagram with 12 generator points constructed with spontaneous algorithms