

# Spontaneous Formation of Space-Time Structures and Criticality

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# Spontaneous Formation of Space-Time Structures and Criticality

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## PREFACE

This volume contains the proceedings of a NATO Advanced Study Institute held at Geilo, Norway between 2 - 12 april 1991. This institute was the eleventh in a series held biannually at Geilo on the subject of phase transitions. It was intended to capture the latest ideas on selforganized patterns and criticality.

The Institute brought together many lecturers, students and active researchers in the field from a wide range of NATO and non-NATO countries. The main financial support came from the NATO Scientific Affairs Division, but additional support was obtained from the Norwegian Research Council for Science and the Humanities (NAVF) and Institutt for energiteknikk. The organizers would like to thank all these contributors for their help in promoting an exciting and rewarding meeting, and in doing so are confident that they echo the appreciation of all the participants.

In cooperative, equilibrium systems, physical states are described by spatio-temporal correlation functions. The intimate connection between space and time correlations is especially apparent at the critical point, the second order phase transition, where the spatial range and the decay time of the correlation function both become infinite. The salient features of critical phenomena and the history of the development of this field of science are treated in the first chapter of this book.

The characteristics of the correlation function referred to above imply self-similarity, i.e. lack of characteristic scales in space and time. In later years it has been realized that many of the patterns and phenomena that can be observed display self-similarity. Fractal patterns, which are self similar, are widespread in nature, be it coastal lines, rocks or clouds. It has also been realized that temporal fluctuations or statistics of characteristic events of river flow, earthquakes, electrical resistors etc., can be expressed by power law time-series or  $1/f$  power spectra. This led Per Bak and coworkers to suggest that such spatial and temporal behaviours are manifestations of a self-organized critical state. In contrast to the critical state discussed in the first lecture, which can only be observed after fine-tuning of an external control parameter such as the temperature, they suggest that extended dissipative systems may automatically go to a (self-organized) critical state. The second paper in this book introduces this concept. Some of the later articles expand on these ideas and on the properties of time series, and deal with self similar phenomena and self-organized patterns in hydrodynamics, condensed matter physics, geophysics and chemical reactions.

Much attention is paid to the development and spatio-temporal struc-

tures of turbulence. For a long time, the success of the theory of non-linear dynamics in explaining the development of temporally chaotic flows in liquids distracted the attention from the spatial structure. A study of the relationship between the spatial and temporal structures in extended systems is now considered a necessary prerequisite for understanding the development from regular to turbulent patterns of flow. Several papers in this book deal with turbulence. On the theoretical side coupled-maps and other statistical methods are applied. Experimental results, some of which produced the beautiful coloured figures on the next following pages, challenged some of the theoretical definitions of the turbulent state.

The aim of the Study Institute was to create a learning environment and a forum for discussion on the topics stated above. The invited lecturers set the scene for this, and were supplemented by contributed seminar papers and posters. Most of the seminars and some of the most relevant posters are included in these proceedings.

The description of space-time correlations and criticality in driven systems far from equilibrium is still in its infancy, compared with criticality at phase transitions in more conventional condensed matter physics. It has, however, provided excitement and challenges for research for many years to come. We hope that the papers presented in this volume will also encourage readers, who did not participate in the Geilo school to enter into this fascinating field.

We are most grateful to Jens Feder, Mogens Høgh Jensen, Roger Pynn, Arne Skjeltorp and Harry Thomas on the Programme Committee who helped us to plan the programme and contact many of the lecturers.

Finally, we would like to express our deep gratitude to Gerd Jarrett of the Institutt for energiteknikk, Kjeller, Norway, who did all the practical organization before, during and after the school, including the preparation of these proceedings, with incredible efficiency.

June 1991

Tormod Riste  
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## SCALING AND UNIVERSALITY IN STATISTICAL PHYSICS

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ABSTRACT. The development of the concepts of scaling and universality in the description of critical behaviour at phase transitions is reviewed. First, the classical theory of phase transitions is presented which is based on the molecular-field concept of van der Waals and Weiss and the order-parameter concept of Landau. Further, the soft-mode picture of dynamic behaviour near phase transitions is discussed. Next, the discrepancy between the results of classical theory and experimentally observed critical behaviour is demonstrated, and the reason for its failure is identified. Finally, it is shown that the scaling idea and the renormalization concept provide a suitable frame for understanding the roots of critical behaviour.

### 1. Normal States and Critical Points

#### 1.1. NORMAL EQUILIBRIUM STATES, PHASE COEXISTENCE

Normal equilibrium states of a thermodynamic system possess particular stability properties: They show a finite response to perturbing forces; the response to a perturbation localized in space usually decays exponentially on an atomic length scale  $\xi$ ; and the response to a pulse usually decays (possibly in an oscillatory manner) on a finite time scale  $\tau$ . Exceptions to this rule occurring in the case of long-range interactions, e.g. for elastic deformations, are well understood. The way in which this stability is reduced at critical points of phase transitions is the subject of the present review.

Elementary experience shows that the same substance may exist in different "phases", and that several phases may coexist on certain submanifolds in the space of thermodynamic equilibrium states. The number of coexisting phases is related to the codimension of the coexistence manifold by Gibbs' phase rule: In the case of a single-component liquid/gas system, two phases may coexist along a line, and three phases in a point of  $(p,T)$  space.

In fact, the difference between different phases was originally perceived as more significant than the difference between different substances: The "four elements" certainly characterize phases rather than substances. In the case of a single-component liquid/gas system, according to Gibbs' phase rule two phases may coexist along a line, and three phases in a point of  $(p,T)$  space.

## 1.2. THE CRITICAL POINT

The discovery by T. Andrews in the 1860's,

T. Andrews: Phil. Trans. Roy. Soc. London 159 (1869) 575; 166 (1876) 421;  
Poggendorfs Annalen, Ergänzungsband V (1871) 64,

that the coexistence line of a liquid/gas system ended in a "critical point", and that the two phases were connected by a continuous path circumnavigating the critical point (see Fig. 1), found widespread attention. In fact, the sudden disappearance at the critical point of the meniscus separating the liquid and the gaseous phase certainly represents one of the most fascinating phenomena of classical physics.

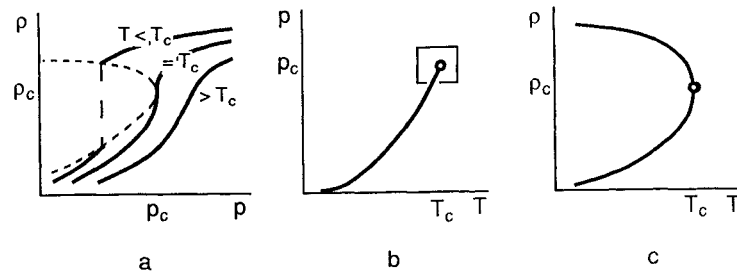


Figure 1. Equation of state of a liquid/gas system ( $\rho$  = density,  $p$  = pressure,  $T$  = temperature). a. Isotherms in the  $(p, \rho)$  plane. b. Coexistence line with critical point in the  $(T, p)$  plane. c. Coexistence region in the  $(T, \rho)$  plane.

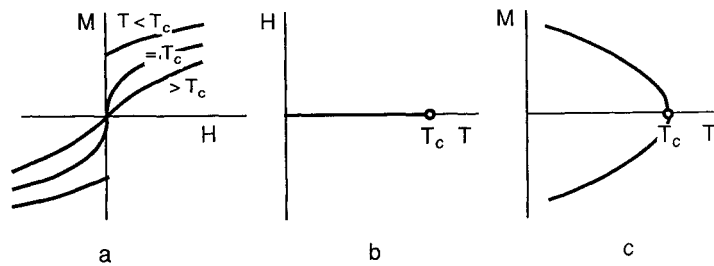


Figure 2. Magnetic equation of state of a ferromagnet ( $H$  = magnetic field,  $M$  = magnetization,  $T$  = temperature). a. Isotherms in the  $(H, M)$  plane. b. Coexistence line with Curie point in the  $(T, H)$  plane. c. Coexistence region in the  $(T, M)$  plane.

A similar behaviour was discovered for the magnetic equation of state of a ferromagnet by J. Hopkinson,

J. Hopkinson: Phil. Trans. Roy. Soc. (1889) 443,

and by P. Curie, who in 1895 published an extensive account of his magnetic

studies, immediately before he turned his attention to the radioactive phenomena,

P. Curie: Propriétés Magnétiques des Corps à Diverses Températures. Ann. Chim. Phys. (7) 5 (1895) 298  
[Oeuvres de Pierre Curie. Gauthier- Villars, Paris 1908, pp. 232-334].

In this paper, he emphasized the distinct similarity of the two equations of state (Fig. 2), and thus essentially anticipated the concept of universality.

In the course of time, critical points were discovered in a variety of other physical systems, e.g. in binary liquid mixtures, binary alloys, superconductors, antiferromagnets, ferroelectrics, and in crystals with structural phase transitions. The proceedings of the first of this series of schools,

Structural Phase Transitions and Soft Modes, Proceedings of the NATO Advanced Study Institute at Geilo, Norway, 13th - 20th April 1971, Edited by E. J. Samuelsen, E. Andersen and J. Feder. Universitetsforlaget Oslo 1971,

give a vivid account of the state of progress in the latter field at that time.

## 2. Classical Theory of Phase Transitions

### 2.1. THE MOLECULAR-FIELD CONCEPT

The existence of a critical point and of a continuous connection between the liquid and the gaseous phase presented a major challenge to theoretical understanding. It was therefore a significant progress when in 1873 Johannes Diderik van der Waals in his Dutch PhD thesis

J. D. van der Waals: Over de Continuïteit van den Gas en Vloeistof-toestand. Academisch Proefschrift, Leiden 1873

derived from statistical-mechanical considerations his famous equation of state

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT, \quad (2.1)$$

which gives the pressure  $p$  in both phases as parts of a single analytic function of density  $\rho = 1/v$  and temperature  $T$ . The attractive forces between the particles give rise to an internal pressure

$$p^{mf} = a\rho^2, \quad (2.2)$$

such that the external pressure  $p$  in the equation of state of an ideal gas is replaced by an effective pressure

$$p^{eff} = p + p^{mf}. \quad (2.3)$$

In addition to the effect of the attractive forces, the van der Waals equation of state contains also the effect of hard-core repulsion represented by the parameter  $b$ , which is outside of our present interest.

The importance of van der Waals' work was immediately noted by J. C. Maxwell:

J. C. Maxwell: Van der Waals on the Continuity of the Gaseous and Liquid States. *Nature* 10 (1874); On the Dynamical Evidence of the Molecular Constitution of Matter. *Nature* 11 (1874) 357  
[The Scientific Papers of James Clerk Maxwell. Ed. by W. D. Niven, University Press, Cambridge 1890, Vol. II, pp. 407-415, 418-438].

The second of these notes contains the well-known "Maxwell construction" for the vapour pressure in the coexistence regime.

The similarity of the equations of state of ferromagnets and of liquid/gas systems, which had been stressed by P. Curie, lead P. Weiss in 1907 to the development of the equally famous Weiss equation of state for ferromagnets, in close analogy to the van der Waals equation:

P. Weiss: L'Hypothèse du Champ Moléculaire et la Propriété Ferromagnétique. *J. de Phys.* (4) 6 (1907) 661-690;  
Molekulares Feld und Ferromagnetismus. *Physik. Z.* 9 (1908) 358-367.

The interactions between the atomic moments are assumed to give rise to a molecular field proportional to the magnetization  $M$ ,

$$H^{mf} = \Lambda M, \quad (2.4)$$

such that the external magnetic field  $H$  in the equation of state of an ideal paramagnet is replaced by an effective field

$$H^{eff} = H + H^{mf}. \quad (2.5)$$

The occurrence of the square of the density in (2.2) as compared to the first power of the magnetization in (2.5) is due to the fact that the force conjugate to the density is  $p/\rho$  rather than  $p$ , i.e. (2.2) should properly be read as  $p^{mf}/\rho = ap$ .

The molecular-field concept was reinvented several times, in particular as "Bragg-Williams approximation" for the description of order-disorder transitions in binary alloys:

W. L. Bragg, E. J. Williams: The Effect of Thermal Agitation on Atomic Arrangement in Alloys. *Proc. Roy. Soc. (London) A* 145 (1934) 699-730, 151 (1935) 540-566.

The similarity of the equations of state of ferromagnets and of liquid/gas systems has its correspondence in the equivalence of two simple microscopic models for these systems, the Ising model,

E. Ising: Beitrag zur Theorie des Ferromagnetismus. *Z. Physik* 31 (1925) 253-258

and the lattice gas model,

F. Cernuschi, H. Eyring: An Elementary Theory of Condensation .  
*J. Chem. Phys.* 7 (1939) 547-551.