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Spontaneous symmetry breaking in discretized light-cone field theory

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The problem of spontaneous symmetry breaking in scalar field theories quantized on the light cone is considered. Within the framework of “discretized” light-cone field theory, a constrained zero mode of the scalar field, which is necessary for obtaining a consistent dynamics, is responsible for supporting nonzero vacuum expectation values classically. This basic structure is shown to carry over to the quantum theory as well, and the consistency of the formalism is checked in an explicit perturbative calculation in $(1+1)$ -dimensional ϕ^4 theory.

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I. INTRODUCTION

There has recently been a renewal of interest in developing a practical Hamiltonian approach to relativistic field theory, based on light-cone quantization [1]. Many of the technical difficulties that plagued the original scheme of Tamm [2] and Dancoff [3] seem to disappear, or are at least rendered more tractable, when a relativistic system is quantized at equal “light-cone time” $x^+ \equiv x^0 + x^3$. The most striking aspect of quantum field theories formulated in this way is surely that in some cases the bare Fock vacuum is the full physical vacuum state of the theory. There is a simple (that is, naive) kinematical argument for why this is the case: the light-cone Hamiltonian conserves light-cone momentum p^+ , so that the bare vacuum can only mix with other states of $p^+ = 0$. But in many theories, in particular those with only massive excitations, there are no such states; p^+ is strictly positive for states containing quanta.¹ Thus the Fock vacuum is an eigenstate of the full interacting Hamiltonian.²

The occurrence of nontrivial vacuum structure is therefore somewhat mysterious in the light-cone framework [7]. At present phenomena such as spontaneous symmetry breaking (SSB) and the formation of condensates are poorly understood. It is clearly important to study how effects such as these might be incorporated into the formalism, since the QCD vacuum, for example, is believed to be quite complex.

Hornbostel has recently given an illuminating analysis of these issues [6], using a quantization surface that inter-

polates between $t=0$ and $x^+=0$. In many cases he finds that the physical vacuum state does indeed approach the bare Fock vacuum as the quantization surface approaches the light cone. Nonzero vacuum expectation values can be supported in the limit, however, by singularities in the field operators near $p^+=0$.

In this paper I consider the problem of SSB in scalar field theories quantized *ab initio* on the light cone. So that everything is well defined, I shall work within the framework of “discretized” light-cone (DLC) quantization, a popular method of setting up actual numerical calculations. In the present context, the nice feature of DLCQ is that it provides a systematic regularization of the small- p^+ singularities that are endemic to light-cone field theory. The aim is to see whether, e.g., vacuum expectation values and multiple vacua can arise in this formalism.

The basic formulation of DLC field theory has recently been reexamined by several authors [8–10], who have pointed out the generic need for a constrained $p^+=0$ mode in theories with bosonic fields. In light of the kinematical argument described above, if SSB can be accommodated in DLC field theory, it may be expected to be closely connected to the properties of this zero mode. This connection was worked out for classical scalar field theory in Ref. [9]; in the present paper I shall verify that the same basic structure obtains in the quantum theory, and check the consistency of the formalism in perturbative calculations.

Other work extending the results of Ref. [9] to the quantum theory of self-interacting scalar fields has recently been described in Refs. [11,12].

II. DLC QUANTIZATION AND THE CONSTRAINED ZERO MODE

The formulation of DLC field theory has been described in many places (see, for example, Ref. [13] and references therein); here I shall be quite brief. I shall focus on scalar fields in $1+1$ dimensions for simplicity. Transverse coordinates may be included without any real difficulty.

The basic point is that the dynamical fields are re-

¹Particles with $p^3 = -\infty$ do carry vanishing p^+ ; however, these constitute a set of measure zero in the space of states and so can usually be neglected.

²This argument obviously fails if there are massless particles in the theory; for example, gauge particles. In this case there are states with $p^+=0$ that can mix with the bare vacuum, so that nontrivial vacuum structure can in principle be supported. Experience with theoretical models suggests, however, that even in these cases the vacuum structure on the light cone is far simpler than in the equal-time representation [4–6].

quired to satisfy certain boundary conditions on the initial value surface $x^+ = 0$, leading to a discrete set of allowed Fourier modes.³ Bosonic fields are traditionally taken to be periodic in x^- , with period $2L$. (This is because the fermion bilinears to which they typically couple are necessarily periodic.) Thus we may write

$$\phi(x^-) = \phi_0 + \varphi(x^-) \quad (2.1)$$

with ϕ_0 independent of x^- and φ a sum of periodic oscillators:

$$\varphi(x^-) = \frac{1}{\sqrt{2\pi}} \sum_{q=2,4,\dots} \frac{1}{\sqrt{q}} \left[a_q \exp \left[-i \frac{q\pi}{2L} x^- \right] + a_q^\dagger \exp \left[i \frac{q\pi}{2L} x^- \right] \right]. \quad (2.2)$$

This is just a general expansion in periodic functions, with the zero-mode piece explicitly separated out. The a_q are Fourier coefficients which, in the quantum theory, satisfy the standard commutation relations:

$$[a_q, a_p^\dagger] = \delta_{qp}. \quad (2.3)$$

The Fock space is generated by acting with a_q^\dagger on the bare vacuum state $|0\rangle$. Note that the integer $q=2,4,\dots$ is associated with a (conserved) light-cone momentum $p^+ = q\pi/L$. It therefore follows quite generally that $|0\rangle$ is an eigenstate of the full interacting DLC Hamiltonian, since it is the only state in the theory with $p^+ = 0$.

Now, the conjugate momentum of a scalar field is

$$\Pi_\phi = 2\partial_- \phi, \quad (2.4)$$

so that the zero mode ϕ_0 is not a true dynamical degree of freedom. However, it is *not* legitimate to set $\phi_0 \equiv 0$, as is often done. Instead there is a constraint relation that determines ϕ_0 in terms of the dynamical fields of the theory [8–10,14]. This occurs quite generally in DLC theories of bosonic fields, including vector fields. The constraint relation is most easily obtained from the scalar field equation of motion. For definiteness, consider a scalar coupled to a fermion current, with the equation of motion

$$(4\partial_+ \partial_- + \mu^2)\phi = -g\bar{\psi}\psi. \quad (2.5)$$

If ϕ is assumed to be periodic in x^- for all x^+ , then integration of Eq. (2.5) in x^- from $-L$ to L results in

$$\mu^2 \phi_0 = -\frac{g}{2L} \int_{-L}^L dx^- \bar{\psi}\psi. \quad (2.6)$$

Thus ϕ_0 is a constrained (operator) functional of the dynamical fields in the theory. In general, this constraint is very complicated. For example, ϕ_0 actually appears in the current $\bar{\psi}\psi$ as well, so that Eq. (2.6) really defines ϕ_0 implicitly [10]. Further examples of these constraint re-

lations will be discussed in detail below.

It should perhaps be emphasized that the need for the constrained zero mode arises from demanding only that the field operators satisfy the correct equations of motion everywhere. If it is discarded, we simply do not have a representation of the desired dynamics. The equation defining ϕ_0 can also be obtained via the Dirac-Bergmann algorithm for quantizing constrained systems [9]. In this approach, it follows from requiring that the vanishing of the momentum conjugate to ϕ_0 be preserved in (light-cone) time.

III. (1+1)-DIMENSIONAL EXAMPLES

Let us now study how the constrained zero mode affects SSB in DLC field theory. An extremely simple example, which nevertheless contains most of the relevant features, is that of a massive scalar field in 1+1 dimensions coupled to a constant source. The Lagrangian density is

$$\mathcal{L} = 2\partial_+ \phi \partial_- \phi - \frac{1}{2}\mu^2 \phi^2 - \mu^2 v \phi \quad (3.1)$$

with v a c -number constant. We can regard the last two terms as (the negative of) a potential with minimum at $\phi = -v$, so that after solving the theory we expect to find $\langle \Omega | \phi | \Omega \rangle = -v$.

The equation of motion derived from Eq. (3.1) is

$$(4\partial_+ \partial_- + \mu^2)\phi = -\mu^2 v. \quad (3.2)$$

In this case the relation defining the constrained zero mode is trivial; integration in x^- from $-L$ to L gives immediately

$$\phi_0 = -v. \quad (3.3)$$

Using this result, the DLC Hamiltonian

$$P^- = \frac{\mu^2}{2} \int_{-L}^L dx^- [:\phi^2(x^-): + 2v\phi] \quad (3.4)$$

becomes

$$P^- = \sum_q \left[\frac{\mu^2 L}{q\pi} \right] a_q^\dagger a_q - \frac{1}{2}\mu^2 v^2 (2L), \quad (3.5)$$

where I have made use of $\int_{-L}^L dx^- \varphi = 0$. From Eq. (3.5) we see that the Fock vacuum is in fact the physical vacuum state, with

$$P^- |0\rangle = -\frac{1}{2}\mu^2 v^2 (2L) |0\rangle. \quad (3.6)$$

Note that the energy of this state is just the value of the classical potential-energy density at the minimum times the volume of space. Furthermore, the expectation value of ϕ in the physical vacuum is

$$\langle \Omega | \phi | \Omega \rangle = \langle 0 | (\phi_0 + \varphi) | 0 \rangle = -v. \quad (3.7)$$

Thus our expectations regarding this simple system are realized. Note, however, that the language used to describe these results is somewhat unconventional. The vacuum state is simple in this theory. The constraint relation, however, forces a particular condition on the *field* itself, namely, that it have a c -number zero-mode piece,

³I define light-cone coordinates $x^\pm \equiv x^0 \pm x^3$, and corresponding derivatives $\partial_\pm \equiv \partial/\partial x^\pm$. As is conventional, x^+ will here play the role of time.

and this reproduces the correct vacuum expectation value (VEV) and vacuum energy.

Let us now turn to the more complicated problem of the spontaneous breaking of reflection symmetry in $(\phi^4)_2$ theory. We take for the Lagrangian density

$$\mathcal{L} = 2\partial_+ \phi \partial_- \phi + \frac{1}{2} \mu_0^2 \phi^2 - \frac{\lambda}{4} \phi^4 \quad (3.8)$$

with $\mu_0^2 > 0$, so that at tree level the potential has degenerate minima at $\phi = \pm \mu_0 / \sqrt{\lambda}$. The resulting Euler-Lagrange equation is

$$(4\partial_+ \partial_- - \mu_0^2) \phi = -\lambda \phi^3, \quad (3.9)$$

from which we derive

$$\mu_0^2 \phi_0 = \lambda \phi_0^3 + \frac{\lambda}{2L} \int_{-L}^L dx^- (3\phi_0 \varphi^2 + \varphi^3) \quad (3.10)$$

as the equation defining ϕ_0 . This is obviously more complicated than our previous relation. First of all, ϕ_0 is an explicit functional of φ , that is, it is an operator, not a c number. Furthermore, it will not in general commute with φ , so that the ordering of factors on the right-hand side of (3.10) must be prescribed. A general solution of Eq. (3.10) in the quantum theory thus seems difficult to achieve.

If φ is a c -number field, however, then Eq. (3.10) is simply a cubic equation for ϕ_0 . Anticipating the use of perturbation theory, let us begin by rewriting Eq. (3.10) in terms of the dimensionless coupling $g \equiv \lambda / \mu_0^2$:

$$\phi_0 = g \phi_0^3 + g \phi_0 A + g B, \quad (3.11)$$

where for notational simplicity I have defined

$$A \equiv \frac{3}{2L} \int_{-L}^L dx^- \varphi^2, \quad (3.12)$$

$$B \equiv \frac{1}{2L} \int_{-L}^L dx^- \varphi^3. \quad (3.13)$$

Equation (3.11) has three solutions, which I shall denote $\phi_0^{(i)}$, $i=1,2,3$. They are given by

$$\phi_0^{(1)} = -\frac{(2/3)^{1/3}}{\sqrt{g}} \frac{-1+gA}{C} + \frac{C}{18^{1/3}\sqrt{g}}, \quad (3.14)$$

$$\phi_0^{(2)} = \frac{(1+\sqrt{3}i)}{12^{1/3}\sqrt{g}} \frac{-1+gA}{C} - \frac{1}{2} \frac{1-\sqrt{3}i}{18^{1/3}\sqrt{g}} C, \quad (3.15)$$

$$\phi_0^{(3)} = \frac{(1-\sqrt{3}i)}{12^{1/3}\sqrt{g}} \frac{-1+gA}{C} - \frac{1}{2} \frac{1+\sqrt{3}i}{18^{1/3}\sqrt{g}} C, \quad (3.16)$$

where

$$C \equiv [-9g^{3/2}B + \sqrt{3}\sqrt{4(-1+gA)^3 + 27g^3B^2}]^{1/3}. \quad (3.17)$$

In the quantum theory the presence of complicated inverses of roots of the operators A and B renders these expressions intractable.

We can expand the $\phi_0^{(i)}$ in powers of g , however, resulting in approximate solutions that are polynomial in A and B . It is straightforward to show that

$$\phi_0^{(1)} = \frac{1}{\sqrt{g}} \left[1 - \frac{1}{2} g A - \frac{1}{2} g^{3/2} B + O(g^2) \right], \quad (3.18)$$

$$\phi_0^{(2)} = \frac{1}{\sqrt{g}} \left[-1 + \frac{1}{2} g A - \frac{1}{2} g^{3/2} B + O(g^2) \right], \quad (3.19)$$

$$\phi_0^{(3)} = gB + g^2 AB + O(g^3). \quad (3.20)$$

The first two solutions contain a c -number background field $\pm 1/\sqrt{g} = \pm \mu_0 / \sqrt{\lambda}$ at lowest order. For these, therefore, we have the tree-level result

$$\langle 0 | \phi | 0 \rangle = \pm \frac{\mu_0}{\sqrt{\lambda}}. \quad (3.21)$$

The third contains no c -number piece, and furthermore, since B is trilinear in φ we have $\langle 0 | B | 0 \rangle = 0$. Thus $\langle 0 | \phi | 0 \rangle = 0$ through $O(g)$ for $\phi_0^{(3)}$. It is clear that these solutions correspond, in the usual language, to versions of the theory expanded about one of the three extrema of the classical potential. Here, however, we speak of a choice of solution of the constraint relation (3.10), rather than a choice of vacuum state, characterizing the theory.

If we let $\mu_0^2 \rightarrow -\mu_0^2$, corresponding to $g \rightarrow -g$, we then obtain the $\lambda\phi^4$ theory without symmetry breaking. Solutions $\phi_0^{(1)}$ and $\phi_0^{(2)}$ become imaginary in this case, and must therefore be rejected. Thus there is only one physically acceptable solution to the constraint, namely, $\phi_0^{(3)}$ with $g \rightarrow -g$, and hence only one phase of the theory.

The solution $\phi_0^{(3)}$ is slightly pathological in the broken-symmetry case, however. It turns out that, although the Fock vacuum is an eigenstate of the full P^- (for the general reasons discussed in Sec II), it is *not* the state with the lowest eigenvalue of P^- . Thus the true vacuum state, that with the lowest light-cone energy, is complicated. This is in accordance with the standard equal-time analysis: were we foolish enough to write the Lagrangian in terms of the unshifted field we would encounter spurious instabilities and ‘‘tachyonic’’ modes. We anticipate that P^- is in fact bounded from below, based on our experience at equal times, but to see this would require going beyond perturbation theory. For the remainder of this paper I shall focus on the better-behaved solution $\phi_0^{(1)}$.

In order to check that this formalism makes sense, let us now calculate the lowest-order correction to $\langle 0 | \phi | 0 \rangle$, due to the A term in Eq. (3.18). Using the expansion (2.2) we have

$$\begin{aligned} -\frac{\sqrt{g}}{2} A &= -\frac{3\sqrt{g}}{4L} \int_{-L}^L dx^- \varphi^2 \\ &= -\frac{3\sqrt{g}}{4\pi} \sum_{q=2,4,\dots}^{\infty} \frac{1}{q} (a_q^\dagger a_q + a_q a_q^\dagger) \\ &= -\frac{3\sqrt{g}}{4\pi} \sum_q \frac{1}{q} (2a_q^\dagger a_q + 1). \end{aligned} \quad (3.22)$$

The correction to $\langle 0 | \phi | 0 \rangle$ is therefore

$$-\left\langle 0 \left| \frac{\sqrt{g}}{2} A \right| 0 \right\rangle = -\frac{3\sqrt{g}}{4\pi} \sum_q \frac{1}{q}, \quad (3.23)$$

which is logarithmically divergent. For our purposes we can simply imagine that the sum is regulated with a cutoff. Notice that this correction is completely independent of L , and so survives in the $L \rightarrow \infty$ limit.

We must also renormalize the boson mass at this order. The bare mass μ_0 is connected to the renormalized mass μ via

$$\mu_0^2 = \mu^2 + \delta\mu^2, \quad (3.24)$$

where $\delta\mu^2/\mu^2 = O(g)$. In terms of μ , then, the c -number background field part of $\phi_0^{(1)}$ is

$$\frac{\mu_0}{\sqrt{\lambda}} = \frac{\mu}{\sqrt{\lambda}} \left[1 + \frac{\delta\mu^2}{2\mu^2} + \dots \right], \quad (3.25)$$

giving a contribution to the VEV,

$$\delta\langle 0|\phi|0\rangle = \frac{\delta\mu^2}{2\mu\sqrt{\lambda}}, \quad (3.26)$$

for the solution $\phi_0^{(1)}$.

Let us compute $\delta\mu^2$. The Hamiltonian is given by

$$P^- = \int_{-L}^L dx \left[\frac{\lambda}{4} \phi^4 - \frac{\mu_0^2}{2} \phi^2 \right]. \quad (3.27)$$

Plugging in $\phi = \phi_0 + \varphi$, with ϕ_0 given by the first two terms in Eq. (3.18), this becomes

$$P^- = \mu^2 \int_{-L}^L dx \left[\varphi^2 \left[1 + \frac{\delta\mu^2}{\mu^2} \right] + \sqrt{g} \varphi^3 + \frac{g}{4} \varphi^4 - \frac{g}{4} A^2 + O(g^{3/2}) \right], \quad (3.28)$$

where I have thrown away c -number terms. Inserting into Eq. (3.28) the expansion (2.2) for φ and the expression (3.22) for A , we obtain

$$P^- = P_0^- + \sqrt{g} P_{(3)}^- + g P_{(4)}^- + g P_{\text{ZM}}^-, \quad (3.29)$$

where

$$P_0^- = \frac{2\mu^2 L}{\pi} \left[1 + \frac{\delta\mu^2}{\mu^2} \right] \sum_q \frac{1}{q} a_q^\dagger a_q, \quad (3.30)$$

$$P_{(3)}^- = \frac{6\mu^2 L}{(2\pi)^{3/2}} \sum_{p,q,r} \frac{1}{\sqrt{pqr}} (a_p^\dagger a_q a_r \delta_{p-q-r} + a_p^\dagger a_q^\dagger a_r \delta_{p+q-r}), \quad (3.31)$$

$$P_{(4)}^- = \frac{2\mu^2 L}{(2\pi)^2} \sum_{p,q,r,s} \frac{1}{\sqrt{pqrs}} (a_p^\dagger a_q a_r a_s \delta_{p-q-r-s} + \frac{3}{2} a_p^\dagger a_q^\dagger a_r a_s \delta_{p+q-r-s} + a_p^\dagger a_q^\dagger a_r^\dagger a_s \delta_{p+q+r-s}) + \frac{6\mu^2 L}{(2\pi)^2} \sum_{p,q} \frac{1}{pq} a_p^\dagger a_p, \quad (3.32)$$

$$P_{\text{ZM}}^- = -\frac{18\mu^2 L}{(2\pi)^2} \left[\sum_{p,q} \frac{1}{pq} (a_p^\dagger a_q^\dagger a_p a_q + a_p^\dagger a_p) + \sum_p \frac{1}{p^2} a_p^\dagger a_p \right]. \quad (3.33)$$

P_{ZM}^- is the piece coming from the A^2 term in Eq. (3.28). Note that I have written everything in normal order, discarding leftover c -number constants but nothing else. Thus I explicitly retain the self-induced inertia terms in Eq. (3.32).

Note also that the free Hamiltonian P_0^- has come out with the correct sign so that the zeroth-order eigenvalues of P^- for states containing particles are strictly positive. Since perturbative corrections to these are by assumption small, we conclude that, at least in perturbation theory, the Fock vacuum $|0\rangle$ will be the eigenstate of lowest P^- , and hence the physical vacuum state of the theory. (We would obtain the same result were we to use the solution $\phi_0^{(2)}$ to define the theory.) That the vacuum is simple in the light-cone representation is therefore seen to be true even in the presence of symmetry breaking. In the equal-time representation this state is of course very complicated, containing an infinite number of bare quanta.

We can now calculate the $O(\lambda)$ correction to the energy of the one-boson state $a_p^\dagger|0\rangle$. As we are interested in the mass counterterm, we retain only on the divergent part of this quantity. The contribution which proceeds through a two-boson intermediate state via the trilinear coupling turns out to be finite. From the self-induced inertia term in Eq. (3.32) we obtain a contribution

$$\frac{6g\mu^2 L}{(2\pi)^2} \frac{1}{p} \sum_q \frac{1}{q}. \quad (3.34)$$

And, finally, the terms arising from the zero mode give a contribution

$$-\frac{18g\mu^2 L}{(2\pi)^2} \frac{1}{p} \sum_q \frac{1}{q} + (\text{finite}). \quad (3.35)$$

Adding these together, and comparing the result to Eq. (3.30), we find

$$\delta\mu^2 = +\frac{12g\mu^2}{8\pi} \sum_q \frac{1}{q}. \quad (3.36)$$

Equation (3.26) then gives the corresponding correction to the VEV:

$$\delta\langle 0|\phi|0\rangle = +\frac{3\sqrt{g}}{4\pi} \sum_q \frac{1}{q}. \quad (3.37)$$

Thus the divergent parts of $\langle 0|\phi|0\rangle$ cancel, and we obtain the sensible result

$$\langle 0|\phi|0\rangle = \frac{\mu}{\sqrt{\lambda}} + O(\lambda^{3/2}), \quad (3.38)$$

that is, the tree-level result (3.21) with μ_0 replaced by the renormalized mass μ .

IV. DISCUSSION

We have seen that spontaneous breaking of a discrete symmetry can be accommodated in an apparently consistent way within the framework of DLCQ. The language used to describe SSB is slightly unconventional, however. In the standard treatment of this type of system, we speak of a choice of vacuum state upon which to construct the theory. The degenerate vacua (and the excitations built up from them) are disconnected, in the sense that there are no operators that transform one into the other. Thus either results in a perfectly valid version of the theory. In the DLC formalism, on the other hand, defining the theory requires choosing a particular solution ϕ_0 of the constraint equation. The different ϕ_0 's now characterize the possible phases of the theory. For the two-dimensional ϕ^4 [$(\phi^4)_2$] theory with symmetry breaking there are two simple solutions, for which $\langle 0|\phi|0\rangle = \pm\mu_0/\sqrt{\lambda}$ at the tree level. For either of these we have a *simple* physical vacuum state, the bare Fock vacuum, corresponding to one or the other of the complicated vacua in the equal-time representation.

This is reminiscent of the results of Hornbostel [6], in that VEV's are associated with properties of the field operators themselves. In the framework of his interpolating quantization surface these were singularities near $p^+ = 0$, which pick out the leading corrections to the bare vacuum state as the light cone is approached. In DLCQ the $p^+ = 0$ singularity is regulated by the boundary conditions, but the information it contains survives, in a sense, encoded into the constrained zero mode.

It is straightforward to extend these results to higher dimensions and continuous symmetries [9,10]. An important feature is worth noting, however: the constrained zero mode will have a nontrivial dependence on any transverse coordinates x in general—it is only completely independent of x^- . See Ref. [10] for an explicit example of this in a (3+1)-dimensional Yukawa theory.

It is perhaps not surprising that a part of the price paid for a simple vacuum structure is the appearance in the

theory of other complexities; in this case, complicated operator constraints. It will be of great importance to the DLCQ program to find ways of going beyond the perturbative type of solution of the constraint relation describe here. A “mean-field” ansatz has been proposed in Ref. [11], and used to study the phase transition in the $(\phi^4)_2$ model [15]. (The transition is of course invisible to a perturbative analysis.) This leads to a correct prediction of the second-order nature of the transition, along with a reasonable estimate of the critical coupling g_c . In the context of a Tamm-Dancoff truncation, this type of approach might prove very useful. A general ansatz for ϕ_0 would have to include all zero-momentum Fock operators not forbidden by symmetry considerations. Within the truncated Fock space, however, one has only a limited number of possibilities, so that a solution for the “truncated” zero mode may become feasible.

Finally, it should be noted that while inclusion of the constrained zero mode is a necessary condition for the equivalence of DLCQ to the equal-time formulation, it may not be sufficient. Its presence certainly insures that the degrees of freedom that are included obey the correct dynamics. It may be, however, that additional degrees of freedom are required to construct a theory that is fully equivalent to the corresponding equal-time theory [10,16,17]. This is certainly true when massless fields are present [16]. Work on understanding the role of these “boundary” degrees of freedom in the massive case is currently in progress [18].

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- [1] A good introduction to this work is S. J. Brodsky, G. McCartor, H.-C. Pauli, and S. Pinsky, “The Challenge of Light-Cone Quantization of Gauge Field Theory,” Report No. SLAC-PUB-5811 (unpublished). For an exhaustive list of references, see A. Harindranath, “Sources for Light-Front Physics,” available via anonymous file transfer protocol (ftp) from public.mps.ohio-state.edu (in the subdirectory tmp/infolight).
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