International Journal of Bifurcation and Chaos, Vol. 9, No. 1 (1999) 155–213

SPREAD SPECTRUM COMMUNICATION VIA CHAOS

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Received October 10, 1997; Revised May 21, 1998

A new scheme is proposed for spread spectrum communication which transmits both analog and binary data via chaotic carriers. The proposed systems have some standard properties of spread spectrum communication. Some computer simulations and performance analysis are given to examine the validity of this scheme.

1. Introduction

There are two principal types of spread spectrum (SS) systems–frequency hopping (FH) and direct sequence (DS) [Magill *et al.*, 1994; Flikkéma, 1997]. The direct-sequence signaling is accomplished by phase modulating the data signal with a pseudorandom sequence. The frequency-hopping spread spectrum divides the available bandwidth into N channels and hops between these channels according to pseudorandom code known to both the modulator and demodulator. A key problem for spread spectrum communication based on pseudorandom spreading sequences is synchronization. This is typically accomplished by means of a correlator which evaluates the autocorrelation function and looks for the isolated maximum corresponding to synchronization.

A pseudorandom sequence generator used in FH and DS is considered to be a special case of a chaotic system, the principal difference being that the chaotic system has an infinite number of states, while the pseudorandom generator has a finite number. A pseudorandom sequence is produced by visiting each state of the system once in a deterministic manner; with a finite number of states to visit, the output sequence is necessarily periodic. By contrast, a chaotic generator can visit an infinite number of states in a deterministic manner and therefore produce an output sequence which never repeats itself. The inherent periodicity of pseudorandom sequence compromises the overall security. The greater the length of pseudorandom sequence, the higher is the level security.

Recently, there has been much interest in utilizing chaotic signals for spread spectrum communication because nonperiodic synchronizable chaos-based systems offer potential advantage over conventional pseudorandom-based systems in terms of security and synchronization. The problem of synchronizing chaotic systems was solved for a class of so-called drive-response system [Pecora & Carroll, 1990]. Some interesting schemes are proposed, which implement spread spectrum communication (of binary data) utilizing these synchronized chaotic systems or one common chaotic system at both the modulator and demodulator [Heidari-Bateni & McGillem, 1994; Parlitz & Ergezinger, 1994; Lipton & Dabke, 1996; Milanović & Zaghloul, 1996; Milanović *et al.*, 1997; Schweizer & Hasler, 1996].

Over past ten years, four new chaos-based spreading techniques have been developed: chaotic masking, chaotic modulation, chaos shift keying, and predictive Poincaré control modulation. The first two techniques spread analog information data by chaotic signals, and the remainings spread

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binary information data. In chaotic masking, the analog information signal $s(t)$ is spread by adding it to the output $x(t)$ of a chaotic system. The resulting signal $s(t) + x(t)$ is modulated and transmitted [Kocarev *et al.*, 1992]. In chaotic modulation, the analog information signal $s(t)$ is injected into a chaotic circuit. This addition modifies the dynamics of the chaotic circuit, and so the information signal $s(t)$ is modulated [Halle *et al.*, 1993; Itoh & Murakami, 1995]. In chaos shift keying, a binary information signal is encoded by transmitting one chaotic signal when a binary "1" needs to be transmitted, and another chaotic signal when a binary "0" needs to be transmitted. The two chaotic signals come from two different systems (or the same systems with different parameters) [Parlitz *et al.*, 1992; Dedieu *et al.*, 1993]. In predictive Poincaré control modulation, symbolic analysis of a chaotic system is used to encode and decode the information. In a suitable Poincaré section of the analog chaotic system in the transmitter, two or more disjoint regions are identified and assigned values for coding information (for example "0" and "1"). By an appropriate control method, the state space trajectory of the transmitter system is successively steered through the desired regions of the Poincaré section such that the resulting succession of assigned values corresponds to the information signal that is to be transmitted [Schweizer & Kennedy, 1994].

Since most information is digital today, the pseudorandom-based SS techniques are mainly used to spread binary data sequences. On the other hand, the chaos-based SS techniques are dealing with both analog and binary information data. It is due to the following reasons:

(a) A chaotic signal (as a carrier wave) is essentially analog, and so chaos can deal with real numbers (analog data) directly.
(b) If analog information data can be spread directly, then binary data can be also spread by the same method. Furthermore, additional information data, besides binary data, can be transmitted simultaneously by using more complicated waveforms of the analog-based DS and FH. Then, we can use frequency resources more effectively.

In this paper, a new scheme for transmitting and encoding analog data by using chaotic carriers is proposed. The basic DS and FH techniques are employed to encode and spread analog data sequences. The modulators consist of multiplication of chaotic carriers by the information signals. The decoding is performed by dividing or multiplying the transmitted signals by the chaotic carriers and after that averaging (or calculating correlation functions). That is, the binary data and pseudorandom sequences in DS and FH are replaced by the analog data and chaotic sequences, respectively. Hence, the pseudorandom-based system is considered to be a special case of the chaos-based system. Furthermore, some spread spectrum communication systems are unified by this scheme. The proposed systems have the following standard properties of DS and FH [Magill *et al.*, 1994; Flikkéma, 1997; Pickholtz *et al.*, 1982]:

(1) Multiple user random access communication.
(2) Resistance to multi-user interference from other transmitters in the network.
(3) Interference rejection.
(4) Antijamming.

Finally, in order to examine the validity of this scheme, some computer simulations and performance analysis will be given.

2. Chaos-Based Direct Sequence

Consider a discrete-time chaotic dynamical system

$$\mathbf{x}[n+1] = \mathbf{f}(\mathbf{x}[n])\,, \tag{1}$$

where $\mathbf{x}[n] = (x_1[n], x_2[n], \ldots, x_M[n])$ is the state, and $\mathbf{f} = (f_1, f_2, \ldots, f_M)$ maps the state $\mathbf{x}[n]$ to the next state, $\mathbf{x}[n+1]$. Starting with an initial condition $\mathbf{x}[0]$, repeated applications of the map $\mathbf{f}$ give rise to the sequence of the points $\{\mathbf{x}[n]\}$. A new sequence $\{y[n]\}$ is obtained by mapping $\mathbf{x}[n]$ by a smooth function $g(\cdot)$, that is, $y[n] = g(\mathbf{x}[n])$. The function g is known as a coding function, which plays an important role in the decoding process. We discuss it in the next subsection.

The direct sequence signaling is accomplished by multiplying the data signal by a chaotic sequence. Thus, the modulation method consists of multiplication of the carrier, $y[n]$, by the information signal, $s[n]$, representing analog data. Assume that $|s[n]| \leq 1$. The transmitted signal is given by

$$p[n] = y[n]s[n] = g(\mathbf{x}[n])s[n]\,. \tag{2}$$

The decoding is done by dividing $p[n]$ by $y[n]$. The recovered signal $q[n]$ is given by

$$q[n] = p[n]y[n]^{-1} = s[n]\,, \quad \text{for } y[n] \neq 0\,. \tag{3}$$

In case of $y[n] = 0$, the information signal cannot be recovered. However, the measure of the set $\{m | \, y[m] = 0\}$ is expected to be zero.

2.1. *Multiple user access*

Consider the case where there are a number of users. The modulator consists of multiplication of the delayed carriers $y[n - l_j]$ (l_j are positive integers) by the information signals $s_j[n]$. Therefore, the transmitted signal for $K (\geq 2)$ users is given by

$$p[n] = \sum_{j=1}^{K} y[n - l_j] s_j[n] \,. \tag{4}$$

The decoding is done by dividing $p[n]$ by $y[n - l_k]$

$$q_k[n] \stackrel{\triangle}{=} p[n] y[n - l_k]^{-1} \tag{5}$$

$$= \sum_{j=1}^{K} \frac{y[n - l_j]}{y[n - l_k]} s_j[n] \tag{6}$$

$$= s_k[n] + e_k[n] \,, \tag{7}$$

where

$$e_k[n] = \sum_{j=1,\, j\neq k}^{K} \frac{y[n - l_j]}{y[n - l_k]} s_j[n] \,. \tag{8}$$

If $e_k[n]$ is sufficiently small, then the information signal $s_k[n]$ can be recovered from $q_k[n]$. However, it is not always true. For example, if $y[n - l_k]$ is sufficiently small compared with $y[n - l_j]$, then the term $y[n - l_j]/y[n - l_k]$ takes sufficiently large number, which makes a large impulse in the sequence $\{q_k[n]\}$.[1] To remove this impulse, we choose the coding function g of $x[n]$ such that

$$y[n] = g(x[n]) = x[n]^r \,, \quad (r : \text{integer}) \,. \tag{9}$$

Then, for sufficiently large r, there exists small $\varepsilon > 0$ such that

$$\frac{y[n - l_j]}{y[n - l_k]} < \varepsilon \quad \text{for} \quad \frac{x[n - l_j]}{x[n - l_k]} < 1 \,, \tag{10}$$

$$\frac{y[n - l_j]}{y[n - l_k]} > \frac{1}{\varepsilon} \quad \text{for} \quad \frac{x[n - l_j]}{x[n - l_k]} > 1 \,. \tag{11}$$

That is, if $x[n - l_j]/x[n - l_k] \neq 1$, then the coding function g divides the terms $y[n - l_j]/y[n - l_k]$ into two groups: One is greater than $1/\varepsilon$, the other is less than ε. Therefore, if we can remove the terms $(y[n - l_j]/y[n - l_k])/s_j[n]$ which is greater than $1/\varepsilon$ from e_k, then $|e_k| \leq K$. It is approximately done by using an amplitude limiter, that is, by replacing

$$q_k[n] \in \{q_k[m] \quad | \quad |q_k[m]| \geq 1\} \,, \tag{12}$$

with the nearest

$$q_k[n - l] \in \{q_k[m] \quad | \quad |q_k[m]| \leq 1\} \,, \quad (l > 0) \,, \tag{13}$$

or holding the previous $q_k[n - 1]$ (m is an integer satisfying the relation $m \leq n$). This is due to the following reason:

$$\begin{aligned} &\text{if} && |q_k[n]| = |s_k[n] + e_k[n]| \leq 1 \\ &\text{and} && |s_k[n]| \leq 1 \,, \quad \text{then} \quad |e_k[n]| \leq 2 \leq K \,. \end{aligned} \tag{14}$$

Let $\{e'_k[n]\}$ and $\{q'_k[n]\}$ be the modified sequence of $\{e_k[n]\}$ and $\{q_k[n]\}$, respectively, and therefore $e_k[n]$ is replaced by $e_k[n-l]$ or $e_k[n-1]$ if $|q_k[n]| \geq 1$.

By averaging this modified sequence $\{q'_k[n]\}$ over n, we get

$$\overline{q}'_k \approx \overline{s}_k + \overline{e}'_k \,, \tag{15}$$

where the symbol $\approx$ means approximately equal and

$$\overline{q}'_k = \frac{1}{N} \sum_{m=n}^{n+N-1} q'_k[m] \,, \tag{16}$$

$$\overline{s}_k = \frac{1}{N} \sum_{m=n}^{n+N-1} s_k[m] \,, \tag{17}$$

$$\overline{e}'_k = \frac{1}{N} \sum_{m=n}^{n+N-1} e'_k[m] \,. \tag{18}$$

Here, N is some constant.

Next, we claim the quick decay of $\overline{e}'_k$ for large N and k, since $y[n]$ and $y[n - k]$ are uncorrelated for large k. Indeed, the autocorrelation function

[1]The division operation was used to spread the information signals of the secure communication systems [Halle *et al.*, 1993]. In this system, the recovered signals had the saturated waveforms. However, the error which due to saturation was short in time, and so it was negligible. For more details, see [Halle *et al.*, 1993].

$c(y)$ satisfies

$$c(y) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} y[n]y[n-k] \to 0 \quad \text{for } k \to \infty\,, \tag{19}$$

if the mean value of $y[n]$ is chosen to be zero.

Then, we can expect that $\overline{e}_k \approx 0$ for sufficiently large N and k, and therefore the information signal is recovered by

$$\overline{q}'_k \approx \overline{s}_k \approx s_k[n]\,. \tag{20}$$

Here, we assumed that s_k is slowly varying, that is,

$$\begin{aligned} s_k[n] &\approx s_k[n+j] \\ &\approx \overline{s}_k\,, \quad (j = 1, 2, \ldots, N-1)\,. \end{aligned} \tag{21}$$

Thus, this scheme offers the chaos-based code-division multiple access (CDMA).

2.2. *Synchronization requirements*

We note that the recovery of the information signal requires the receiver's own copy of the chaotic spreading sequence be synchronized with the transmitter's one. To see this, let the receiver's local sequence $\hat{y}[n]$, instead of being exactly time-aligned (or in phase), be offset in time, $\hat{y}[n] = y[n+i] (i \neq 0)$. Then the recovered signal of the kth user will be

$$q_k[n] = p[n]\hat{y}[n-l_k]^{-1} = p[n]y[n-l_k+i]^{-1} \tag{22}$$

$$= \sum_{j=1}^{K} \frac{y[n-l_j]}{y[n-l_k+i]} s_j[n] \tag{23}$$

$$= \frac{y[n-l_k]}{y[n-l_k+i]} s_k[n] + \sum_{j\neq k}^{K} \frac{y[n-l_j]}{y[n-l_k+i]} s_j[n]\,. \tag{24}$$

Therefore, the information signal $s_k[n]$ is spread again by $y[n-l_k]/y[n-l_k+i]$ and lost, since we cannot expect that $y[n-l_k] = y[n-l_k+i]$ for all n.

Similar results hold for the cases where the sequence $\hat{y}[n]$ is corrupted by noise or the synchronization errors occur. Then, the sequence $\hat{y}[n]$ has the form

$$\hat{y}[n] = y[n] + w[n]\,, \tag{25}$$

where $w[n]$ indicates noise or an error. The recovered signal of the kth user will be

$$q_k[n] = p[n]\hat{y}[n-l_k]^{-1} = p[n](y[n-l_k] + w[n-l_k])^{-1} \tag{26}$$

$$= \sum_{j=1}^{K} \frac{y[n-l_j]}{y[n-l_k]+w[n-l_k]} s_j[n] \tag{27}$$

$$= \frac{y[n-l_k]}{y[n-l_k]+w[n-l_k]} s_k[n] + \sum_{j\neq k}^{K} \frac{y[n-l_j]}{y[n-l_k]+w[n-l_k]} s_j[n] \tag{28}$$

$$= \frac{1}{1+\dfrac{w[n-l_k]}{y[n-l_k]}} s_k[n] + \sum_{j\neq k}^{K} \frac{y[n-l_j]}{y[n-l_k]+w[n-l_k]} s_j[n] \quad \text{for } y[n-l_k] \neq 0\,, \tag{29}$$

and therefore the information $s_k[n]$ is lost when $w[n-l_k]$ becomes large compared to $y[n-l_k]$. That is, the error of the recovered signal becomes large when the sequence $y[n-l_k]$ is sufficiently small compared to $w[n-l_k]$. Furthermore, we note that chaos has a strong and sensitive dependence on initial conditions. Tiny differences or errors in the initial conditions lead quickly to large differences. Thus, once chaotic synchronization is broken, the difference $w[n]$ becomes large quickly. Therefore, synchronization is a key requirement in chaos-based spread spectrum system design. In the sequel, to maintain focus on the spread-spectrum concept, we assume perfect synchronization.

2.3. *Interference suppression and channel noise*

Suppose that the channel contains an interferer: An unknown periodic signal, $I[n]$, is added to the received signal. The received sequence is

$$q[n] = p[n] + I[n]\,. \tag{30}$$

Dividing this equation by $y[n-l_k]$ and using an amplitude limiter, we get

$$\overline{q}'_k \approx \overline{s}_k + \overline{e}'_k + \overline{d}'_k\,. \tag{31}$$

Here, the term $d_k = I[n]y[m-l_k]^{-1}$ is modified by the amplitude limiter, and we described it by d'_k. Thus,

$$\overline{d}'_k = \frac{1}{N}\sum_{m=n}^{n+N-1} d'_k\,. \tag{32}$$

Since $y[n]$ and $I[n]$ are uncorrelated (i.e. $I[n]$ is spread by $y[n]$), we can expect that $\overline{d}'_k \approx 0$ for sufficiently large N. Thus, we obtain

$$\overline{q}_k \approx \overline{s}_k \approx s_k[n]\,. \tag{33}$$

Next, assume that the transmitted signal is corrupted by white Gaussian noise $w[n]$. Then the received sequence is given by

$$q[n] = p[n] + w[n]\,, \tag{34}$$

and the recovered signal has the form

$$\overline{q}'_k \approx \overline{s}_k + \overline{e}'_k + \overline{d}'_k\,. \tag{35}$$

where $\{d'_k\}$ is the modified sequence of $\{d_k\}(=\{w[n]y[m-l_k]^{-1}\})$ by the amplitude limiter. The term d'_k and e'_k remains as noise. Thus, if the channel noise increases, the recovered signals are corrupted by noise.

2.4. *Interference from other transmitters*

Now we consider the case where there is multi-user interference from other transmitters in the network, that is, the K signals simultaneously share a channel. Assuming time synchronization among the signals, the received signals is

$$q[n] = \sum_{j=1}^{K} y^{(j)}[n-l_{k^{(j)}}]s^{(j)}[n]\,, \tag{36}$$

where $y^{(j)}[n-l_{k^{(j)}}]$ and $s^{(j)}[n]$ are a chaotic sequence and an information signal of the jth transmitter, respectively. It follows that the averaging procedure and the amplitude limiter for signal $k=1$ generate

$$\overline{q}'^{(1)} \approx \overline{s}^{(1)} + \overline{e}'^{(1)}\,, \tag{37}$$

where $\{e'^{(1)}\}$ is the modified sequence, that is, we modified

$$e^{(1)} = \sum_{j=2}^{K} y^{(j)}[n-l_{k^{(j)}}]y^{(1)}[n-l_{k^{(1)}}]^{-1}s^{(j)}[n]\,, \tag{38}$$

by using the amplitude limiter. We can expect that $\overline{e}'^{(1)} \approx 0$ for large N, since the cross-correlation of chaotic signals satisfies

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} y^{(j)}[n-l_{k^{(j)}}]y^{(i)}[n-l_{k^{(i)}}] = 0\,, \quad (i\neq j)\,. \tag{39}$$

Thus, the recovered signal is given by

$$\overline{q}^{(1)} \approx \overline{s}^{(1)} \approx s^{(1)}[n]\,. \tag{40}$$

2.5. *Other encoding and decoding techniques*

In the encoding process, the delayed carriers $y[n-l_k]$ are used. A number of different sequences are available too, if they satisfy the quick decay of the correlation.

Next, we assume that the information sequences $\{s_k[n]\}$ are given by binary data (± 1), and for each information bit, instead of one pulse, a sequence of N pulses is transmitted, that is, $s_j[n] = s_j[n+1] = \cdots = s_j[n+N-1]\,(=\pm 1;\ n = 0,\ N,\ 2N, \ldots)$. Then, we can recover the signal by multiplying $p[n]$ by $y[n-l_k]$, instead of $y[n-l_k]^{-1}$,

$$q_k[n] = p[n]y[n-l_k] \tag{41}$$

$$= \sum_{j=1}^{K} y[n-l_j]y[n-l_k]s_j[n]\,. \tag{42}$$

The receiver makes the decision as follows

$$1 \quad \text{was sent for} \quad \overline{q}_k \geq 0\,, \tag{43}$$

$$-1 \quad \text{was sent for} \quad \overline{q}_k < 0\,, \tag{44}$$

where $\overline{q}_k = (1/N)\sum_{m=n}^{n+N-1} q_k[m]$ (for more details, see [Parlitz & Ergezinger, 1994]).

2.6. *Continuous-time version of DS systems*

In this section, we discuss the continuous-time version of chaos-based direct sequence. Consider a chaotic dynamical system

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}) \in R^n\,, \tag{45}$$

where $\mathbf{x}(t) = (x_1(t),\ x_2(t), \ldots,\ x_n(t))$ is the state vector and $\mathbf{F}$ is a smooth map on R^n. A chaotic carrier is given by

$$y(t) = g(\mathbf{x}(t))\,, \tag{46}$$

where $g(\cdot)$ is a coding function.

The modulator consists of multiplication of delayed carriers $y(t-\tau_j)$ (τ_j are real numbers) by information signals $s_j[t]$,

$$p(t) = \sum_{j=1}^{K} y(t-\tau_j) s_j(t) \,. \tag{47}$$

The decoding is done by multiplying $p(t)$ by $y(t-\tau_k)^{-1}$

$$\begin{aligned} q_k(t) &= p(t) y(t-\tau_k)^{-1} && (48)\\ &= \sum_{j=1}^{K} \frac{y(t-\tau_j)}{y(t-\tau_k)} s_j(t) && (49)\\ &= s_k(t) + \sum_{j=1,\, j\neq k}^{K} \frac{y(t-\tau_j)}{y(t-\tau_k)} s_j(t) && (50)\\ &= s_k(t) + e_k(t)\,, && (51) \end{aligned}$$

where

$$e_k(t) = \sum_{j=1,\, j\neq k}^{K} \frac{y(t-\tau_j)}{y(t-\tau_k)} s_j(t) \,. \tag{52}$$

Next, we replace

$$q_k(t) \in \{ q_k(t) \quad | \quad |q_k(t)| \geq 1 \} \,, \tag{53}$$

with the nearest

$$q_k(t-\tau') \in \{ q_k(t-\tau) \quad | \quad |q_k(t-\tau)| \leq 1 \} \,, \tag{54}$$

and so $e_k(t)$ is replaced with $e_k(t-\tau')$ too. Here, τ is a positive real number.

Let $q'_k(t)$ and $e'_k(t)$ be the modified sequence of $q_k(t)$ and $e_k(t)$, respectively. Averaging this modified $q'_k(t)$ over t, we get

$$\overline{q}'_k \approx \overline{s}_k + \overline{e}'_k \,, \tag{55}$$

where

$$\overline{q}'_k = \frac{1}{T} \int_t^{t+T} q'_k(t) dt \,, \tag{56}$$

$$\overline{s}_k = \frac{1}{T} \int_t^{t+T} s_k(t) dt \,, \tag{57}$$

$$\overline{e}'_k = \frac{1}{T} \int_t^{t+T} e'_k(t) dt \,, \tag{58}$$

where T is some constant.

Here, we assume that $s_k(t)$ are slowly varying and T is sufficiently large. Furthermore, we claim the quick delay of $\overline{e}'_k$ for large τ, since the autocorrelation function of chaotic signals satisfies

$$\lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} y(t) y(t-\tau) dt \to 0 \,, \quad \text{for } \tau \to \infty \,. \tag{59}$$

Then, we can expect that $\overline{e}'_k \approx 0$, and so the information signal is recovered by

$$\overline{q}'_k(t) \approx \overline{s}_k(t) \approx s_k(t) \,. \tag{60}$$

Therefore, all the discussions are as same as that for the discrete-time communication systems.

3. Pseudorandom-Based Direct Sequence

In the case of a pseudorandom sequence, the number N becomes a period of a sequence. The series $\{y[n]\}$ are chosen to be a binary sequence satisfying the following relations

$$\begin{aligned} &\frac{1}{N} \sum_{n=1}^{N} y[n-l_j] y[n-l_k] = -\frac{1}{N} \,, \quad &&\text{for } j \neq k \,,\\ &\frac{1}{N} \sum_{n=1}^{N} y[n-l_j] y[n-l_k] = 1 \,, &&\text{for } j = k \,. \end{aligned} \tag{61}$$

where $y[n] = \pm 1$. Such a sequence is realized by the maximum length code (sequence) [Flikkéma, 1997; Pickholtz *et al.*, 1982]. Without loss of generality, the formal inverse of $y[n-l_k]$ is defined as follows

$$y[n-l_k]^{-1} \triangleq y[n-l_k] \,, \quad (n = 1, 2, \ldots, N) \,. \tag{62}$$

Furthermore, we assume that for each information bit, instead of one pulse, a sequence of N pulses is transmitted, that is, $s_j[n] = s_j[n+1] = \cdots = s_j[n+N-1]$ ($= \pm 1$; $n = 0, N, 2N, \ldots$). Then, all the discussion for the pseudorandom-based communication system is as same as that for the chaos-based communication system. That is, the transmitted signal is given by

$$p[n] = \sum_{j=1}^{K} y[n-l_j] s_j[n] \,, \tag{63}$$

where $s_j[n]$ is a message signal representing a binary signal (± 1).

The decoding is done by multiplying $p[n]$ by $y[n-l_k]$

$$q_k[n] = p[n]y[n-l_k] \tag{64}$$

$$= \sum_{j=1}^{K} y[n-l_j]y[n-l_k]s_j[n]\,. \tag{65}$$

Averaging $q_k[n]$ over one period, we obtain

$$\overline{q}_k = \overline{s}_k + \overline{e}_k\,, \tag{66}$$

where

$$\overline{q}_k = \frac{1}{N}\sum_{m=n}^{n+N-1} q_k[m]\,, \tag{67}$$

$$\overline{s}_k = \frac{s_k[n]}{N}\sum_{m=n}^{n+N-1} y[m-l_k]^2 = s_k[n]\,, \tag{68}$$

$$\overline{e}_k = \frac{1}{N}\sum_{j=1,\,k\neq j}^{K}\left(\sum_{m=n}^{n+N-1} y[m-l_j]y[m-l_k]s_j[m]\right)$$

$$= -\frac{1}{N}\sum_{j=1}^{K} s_j[n]\,. \tag{69}$$

Therefore, assuming that N is sufficiently large, we can expect that $\overline{e}_k \approx 0$, and therefore we get

$$\overline{q}_k \approx \overline{s}_k \approx s_k[n]\,. \tag{70}$$

4. Chaos-Based Frequency Hopping

The FH spread spectrum system divide the available bandwidth into N channels and hops between those channels according to the pseudorandom sequence. The chaos based FH hops to any frequency in the available band according to chaotic sequence $\{x[n]\}$.

The carriers for FH are given by

$$y^{(j)}[n] = \sin(n\Omega^{(j)}) \tag{71}$$

$$\Omega^{(j)} = \omega^{(j)} + g(x[n-l_j])\,, \tag{72}$$

where $\omega^{(j)}$ and l_j are constants and $g(\cdot)$ is a coding function.[2] The modulation consists of multiplication of $y^{(j)}[n]$ by the information signal $s_j[n]$

$$p[n] = \sum_{j=1}^{K} y^{(j)}[n]s^{(j)}[n]\,. \tag{74}$$

The decoding is done by multiplying $p[n]$ by $y^{(k)}[n]$

$$q_k[n] = p[n]y^{(k)}[n] \tag{75}$$

$$= \sum_{j=1}^{K} y^{(j)}[n]y^{(k)}[n]s^{(j)}[n] \tag{76}$$

$$= s^{(k)}[n]y^{(k)}[n]^2 + \sum_{j=1,\,j\neq k}^{K} y^{(j)}[n]y^{(k)}[n]s^{(j)}[n] \tag{77}$$

$$= \frac{s_k[n]}{2} - \frac{s_k[n]\,\cos(2n\Omega^{(k)})}{2} + \sum_{j=1,\,j\neq k}^{K} y^{(j)}[n]y^{(k)}[n]s^{(j)}[n] \tag{78}$$

$$= \frac{s_k[n]}{2} + e_s + e_k\,, \tag{79}$$

where

$$e_s[n] = -\frac{s_k[n]\,\cos(2n\Omega^{(k)})}{2}\,, \tag{80}$$

$$e_k[n] = \sum_{j=1,\,j\neq k}^{K} y^{(j)}[n]y^{(k)}[n]s^{(j)}[n]\,, \tag{81}$$

and $e_s[n]$ can be removed by averaging $q_k[n]$ or using a lowpass filter. Therefore, by averaging $q_k[n]$, we get the equation

$$\overline{q}_k = \frac{\overline{s}_k}{2} + \overline{e}_s + \overline{e}_k\,, \tag{82}$$

[2]The carriers $y^{(j)}[n]$ can be written in the form:

$$y^{(j)}[n] = h(x[n-l_j])\,, \tag{73}$$

where $h(z) = \sin(n(\omega^{(j)} + g(z)))$ is a coding function and $g(x)$ is a function of x. In the direct sequence signaling, $h(z)$ is given by a polynomial: $h(z) = z^r$ (r: integer). Therefore, the coding function makes the difference between DS and FH spread spectrum systems.

where

$$\overline{q}_k = \frac{1}{N} \sum_{m=n}^{n+N-1} q_k[m]\,, \tag{83}$$

$$\overline{s}_k = \frac{1}{N} \sum_{m=n}^{n+N-1} s_k[m]\,, \tag{84}$$

$$\overline{e}_s = \frac{1}{N} \sum_{m=n}^{n+N-1} e_s[m]\,, \tag{85}$$

$$\overline{e}_k = \frac{1}{N} \sum_{m=n}^{n+N-1} e_k[m]\,. \tag{86}$$

We claim the quick decay of and $\overline{e}_k$, since $y^{(j)}[n]$ and $y^{(k)}[n]$ are uncorrelated. Thus, we obtain the relation

$$\overline{q}_k[n] \approx \frac{\overline{s}_k[n]}{2}\,. \tag{87}$$

Accordingly, the chaos based FH has the standard property of FH: Interference rejection, multiple user access, and so on. Furthermore, if we replace $\{x[n]\}$ with any pseudo random sequence, then the standard FH system is obtained. We can derive the continuous-time version of chaos-based FH, but only a slight variation is needed.

5. Synchronization Problems and Multiplexing

One of the common features of the spread spectrum scheme is that the transmitter and receiver synchronize. In case where the receiver can use the transmitter's chaotic generator, no synchronization mechanism is needed [Lipton & Dabke, 1996]. In this section, we discuss two methods for achieving synchronization between two systems, that is, self-synchronization and adaptive synchronization.

5.1. *Self-synchronization*

The synchronization method we consider here is unidirectional; the master system (the transmitter) sends a signal to the slave system (the receiver); the output of the slave system approaches that of the master asymptotically in time. In this way, the master influences the slave system but the slave system does not influence the master. Unlike a pseudorandom-based system, where a correlator is required to perform synchronization, self-synchronization chaos-based schemes do not require separate synchronization and tracing hardware.

This scheme decomposes a chaotic system $\mathbf{x}[n+1] = \mathbf{f}(\mathbf{x}[n])$ into

$$\left.\begin{aligned} x_1[n+1] &= f_1(x_1[n],\, x_2[n]),\\ x_2[n+1] &= f_2(x_1[n],\, x_2[n]), \end{aligned}\right\} \quad \text{master system} \tag{88}$$

$$z_2[n+1] = f_2(x_1[n],\, z_2[n])\,, \quad \text{slave system} \tag{89}$$

where $\mathbf{x}[n] = (x_1[n],\, x_2[n])$ and $\mathbf{f} = (f_1,\, f_2)$. Here, the master system impose one of its state, $x_1[n]$, directly onto the slave system. If synchronization occurs, the remaining state $z_2[n]$ of the slave system asymptotically follow their counter parts $x_2[n]$ in the master system [Pecora & Carroll, 1990].

The communication systems proposed in this paper require fundamentally two communication channels; one is used for transmitting the synchronization signal $x_1[n]$ and the other for transmitting the modulated signal $p[n] = y[n]s[n] = g(\mathbf{x}[n])s(n)$. In order to transmit multiple user signals with one communication channel, two kinds of techniques are available: Impulsive synchronization [Stojanovski *et al.*, 1997; Yang & Chua, 1997] and chaotic multiplexing [Itoh & Chua, 1997a, 1997b].

5.1.1. *Impulsive synchronization*

In order to transmit multiple user signals by using one communication channel, we can use impulsive synchronization method, which was proposed in [Stojanovski *et al.*, 1997; Yang & Chua, 1997].

In this method, the transmitted signal consists of a sequence of frames. Every frame has a length of T and consists of two regions. The first region of the frame is a synchronization region consisting of synchronization pulses. The synchronization pulses are used to impulsively synchronize the chaotic systems in both transmitter and receiver. That is, the self-synchronization method is impulsively applied to the receiving system. The second region is the spread spectrum signal region, which is used to transmit a modulated signal of DS or FH. Within every time frame, the synchronization region has a length of Q and the remaining time interval T–Q is the modulated signal region. Since Q is usually very small compared with T, the lost of time for packing an information signal is negligible. The stability of the impulsive synchronization and the robustness to additive channel noise and parameter mismatch are guaranteed by the results in [Yang & Chua, 1997].

5.1.2. *Chaotic multiplexing*

The other technique for transmitting multiple user signals with one communication channel is chaotic multiplexing. First, we modify the master–slave system, and define the transmitting system as follows

$$\begin{aligned} x_1[n+1] &= f_1(x_1[n],\, x_2[n]) + \varepsilon p[n]\,, \\ x_2[n+1] &= f_2(x_1[n],\, x_2[n])\,, \end{aligned} \tag{90}$$

where

$$p[n] = \sum_{j=1}^{K} y[n - l_j] s_j[n]\,, \tag{91}$$

$$y[n] = g(x_1[n],\, x_2[n])\,, \tag{92}$$

g is a smooth functions, $s_j[n]$ are information signals, and ε is sufficiently small. The transmitted signal is given by $x_1[n]$. The receiving system has the form

$$z_2[n+1] = f_2(x_1[n],\, z_2[n])\,, \tag{93}$$

$$q[n] = \varepsilon^{-1}(x_1[n+1] - f_1(x_1[n],\, z_2[n]))\,. \tag{94}$$

When the master and slave systems are synchronized, that is, $|x_2[n] - z_2[n]| \to 0$, we get

$$q[n] = p[n] = \sum_{j=1}^{K} y[n - l_j] s_j[n]\,. \tag{95}$$

Thus, multiplying $q[n]$ by $y[n - l_k]^{-1}$ and averaging the modified $q'[n]$, we can recover the signal

$$\overline{q}'_k \approx \overline{s}_k \approx s_k[n]\,. \tag{96}$$

For more details, see [Itoh & Chua, 1997a, 1997b]. In this scheme, the properties of antijamming, interference rejection, and resistance to multiple user interference are lost, which the spread spectrum communication systems inherently have.

5.2. *Adaptive synchronization and multiplexing*

The adaptive controller can synchronize the two systems and make the driven system's parameters converge towards the driving system's parameters even though the parameters of the two systems differ. Consider a discrete-time chaotic dynamical system

$$\mathbf{x}[n+1] = \mathbf{f}(\mathbf{x}[n],\, \mathbf{a})\,, \tag{97}$$

where $\mathbf{a} = (a_1,\, a_2, \ldots,\, a_m)$ is system parameters. In [Huberman & Lumer, 1990], the following adaptive controller is proposed

$$\mathbf{z}[n+1] = \mathbf{f}(\mathbf{z}[n],\, \mathbf{b}[n])\,, \tag{98}$$

$$\mathbf{b}[n+1] = \mathbf{b}[n] - \mathbf{G}\left(\mathbf{z}[n] - \mathbf{x}[n],\, \frac{\partial \mathbf{f}}{\partial \mathbf{b}}\right)\,, \tag{99}$$

where $\mathbf{b}[n] = (b_1[n],\, b_2[n], \ldots,\, b_m[n])$ are the control parameters and $\mathbf{G}$ is a continuous map. The controller adjusts $\mathbf{b}[n]$ such that two systems will synchronize. That is, the adaptive controller maintain synchronization by continuously tracking the change in the modulated parameter $\mathbf{a}$. If the parameters $\mathbf{a} = (a_1,\, a_2, \ldots,\, a_m)$ are modulated by multiple signals: $\mathbf{s}[n] = (s_1[n],\, s_2[n], \ldots,\, s_m[n])$ at the transmitter, then the adaptive controller can recover the parameters $\mathbf{a}$, and therefore the multiple signals $\mathbf{s}[n]$ can be decoded (it is known as a parameter modulation; for more details see [Wu *et al.*, 1996]).

6. Computer Simulations

In this section, the multiple user access and the interference rejection are examined by using computer simulations.

6.1. *Direct sequence*

6.1.1. *Discrete-time dynamical systems*

Consider the chaotic dynamical system (Ikeda map [Ikeda, 1979])

$$u[n+1] = a + b(u[n] \cos w[n] - v[n] \sin w[n])\,, \tag{100}$$

$$v[n+1] = b(u[n] \sin w[n] + v[n] \cos w[n])\,, \tag{101}$$

where $w[n] = 0.4 - 6/u[n]^2 + v[n]^2$, $a = 0.84$, and $b = 0.95$. Assume that three users are sharing a channel. The coding function $g(\cdot)$ is given by a polynomial of $u[n]$

$$g(u[n]) = (u[n] - 0.5)^r \quad (r = 11)\,, \tag{102}$$

and so the sequence $\{y[n] = g(u[n])\}$ is used as a carrier. The delay l_j and the average number N are given by

$$l_1 = 0\,, \quad l_2 = 60\,, \quad l_3 = 100\,, \quad N = 60\,. \tag{103}$$

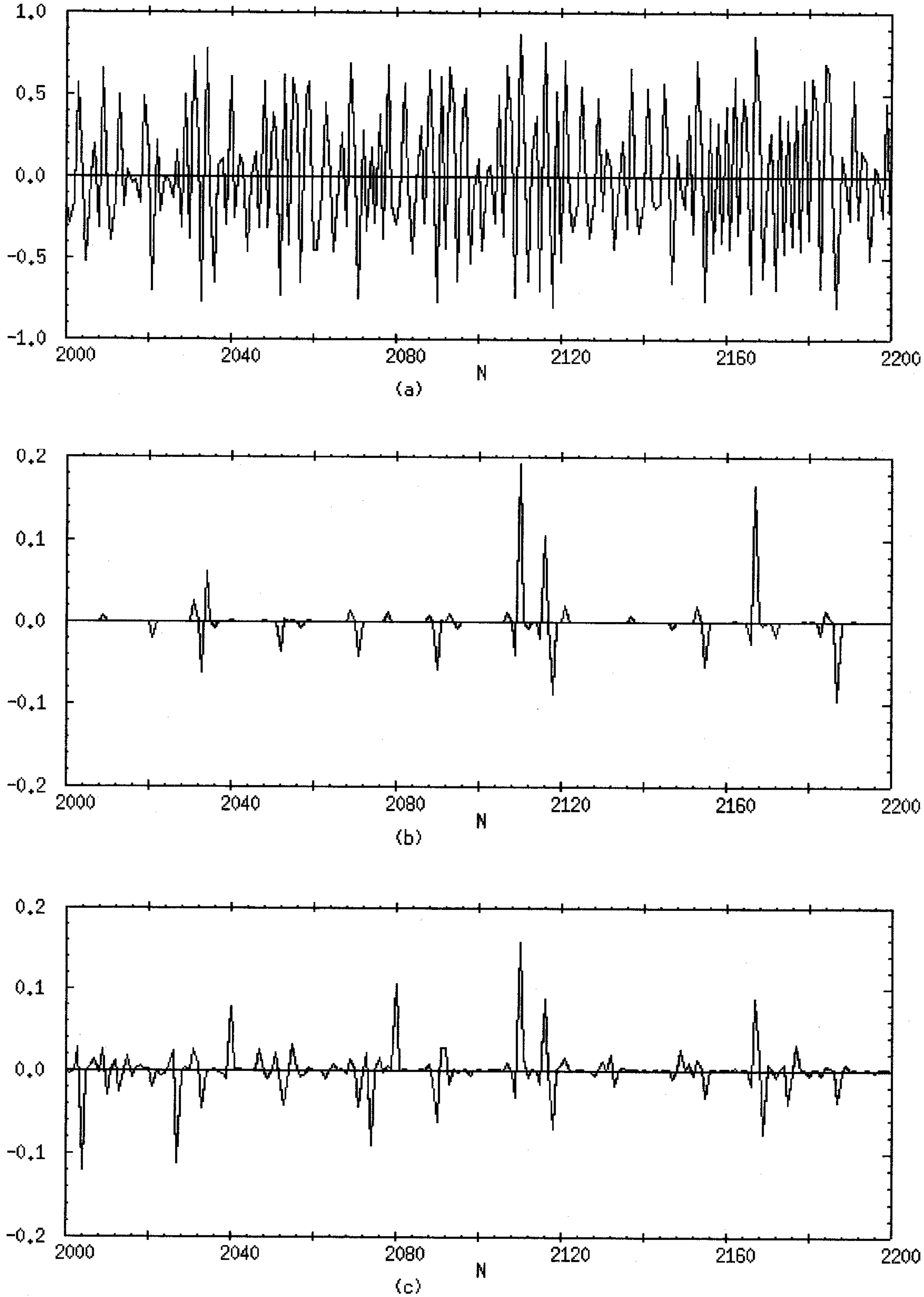


Fig. 1. Sequence of a chaotic carrier. (a) Chaotic sequence $\{x[n]\}$ of Ikeda map, (b) chaotic carrier $\{y[n]\}$, (c) transmitted signal $\{p[n]\}$.

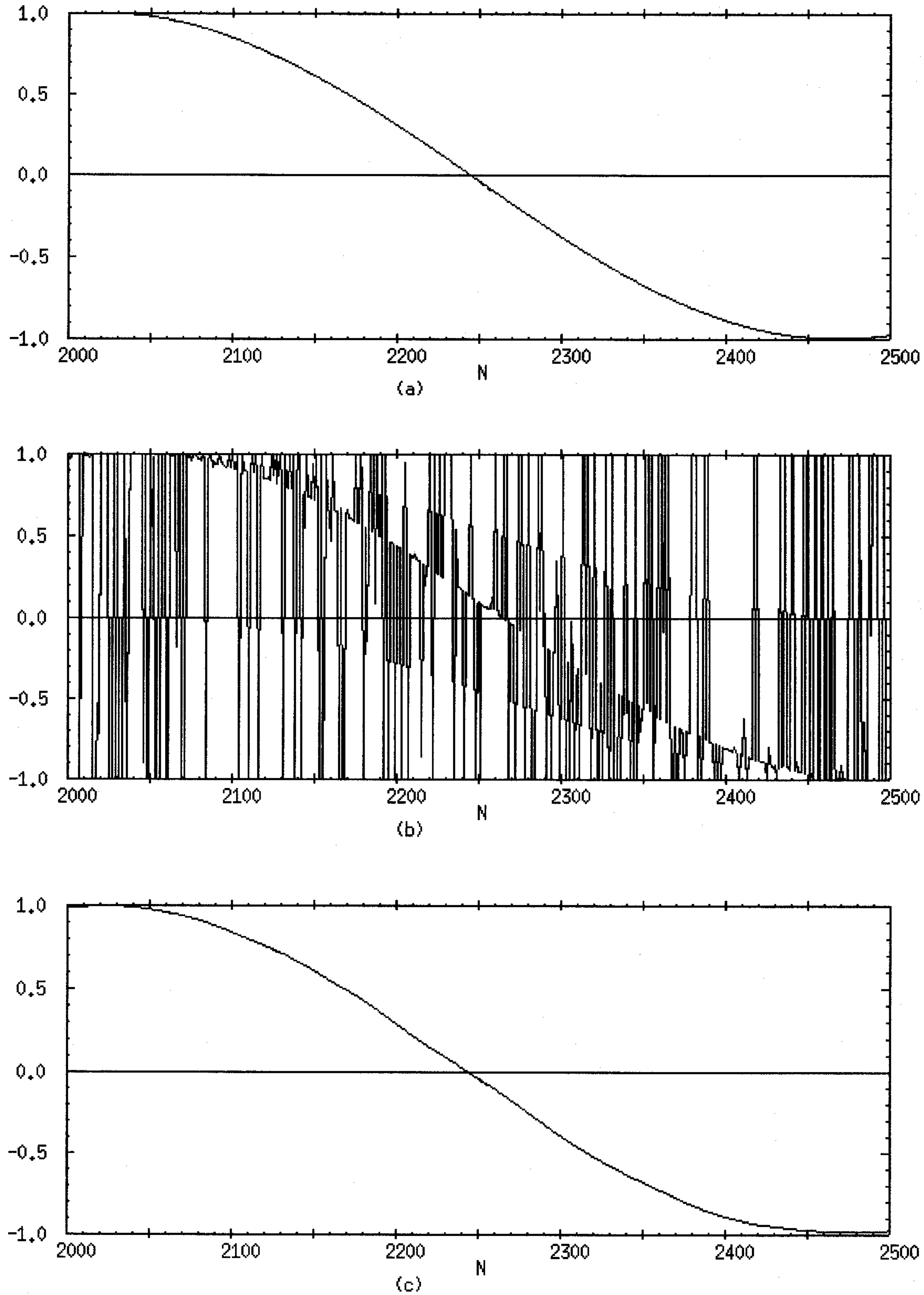


Fig. 2. Recovering process. (a) Information signal $s_1[n] = \sin(0.007n)$, (b) recovered signal $q_1[n]$ (before averaging), (c) recovered signal $\overline{q}_1[n]$ (after averaging).

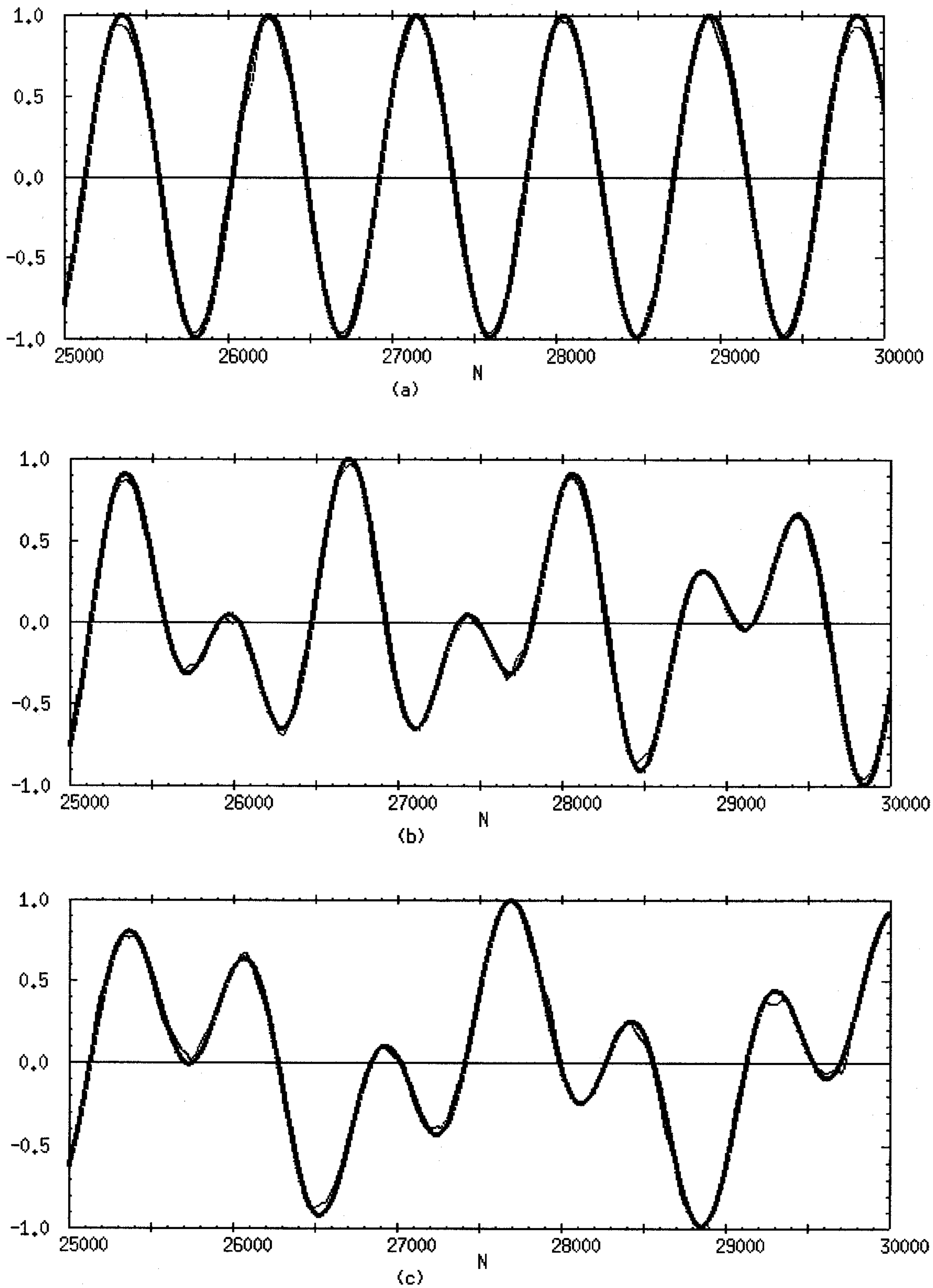


Fig. 3. Recovered signals $\overline{q}_j[n]$ (thin solid lines) and information signals $s_j[n]$ (thick solid lines) when the communication channel is shared by three users. (a) Information signal: $s_1[n] = \sin(0.007n)$, (b) $s_2[n] = 0.5(\sin(0.005n) + \sin(0.009n))$, (c) $s_3[n] = 0.5(\sin(0.003n) + \sin(0.008n))$.

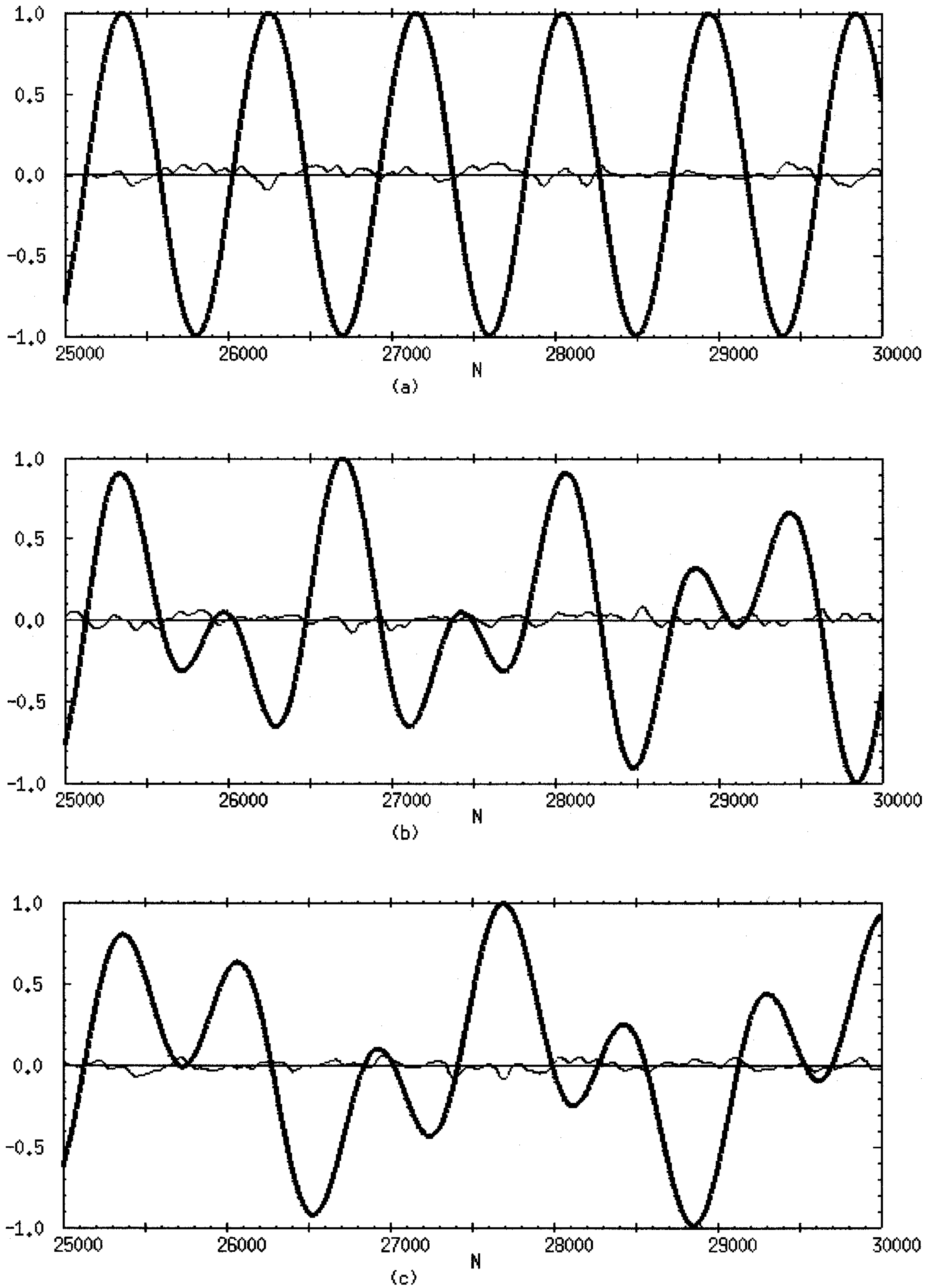


Fig. 4. Recovered signal with carrier mismatch; decoding is done by using the mismatched carrier $y[n-l_j-1]$ (or $y[n-l_j+1]$) instead of $y[n-l_j]$; only one time-step is mismatched.

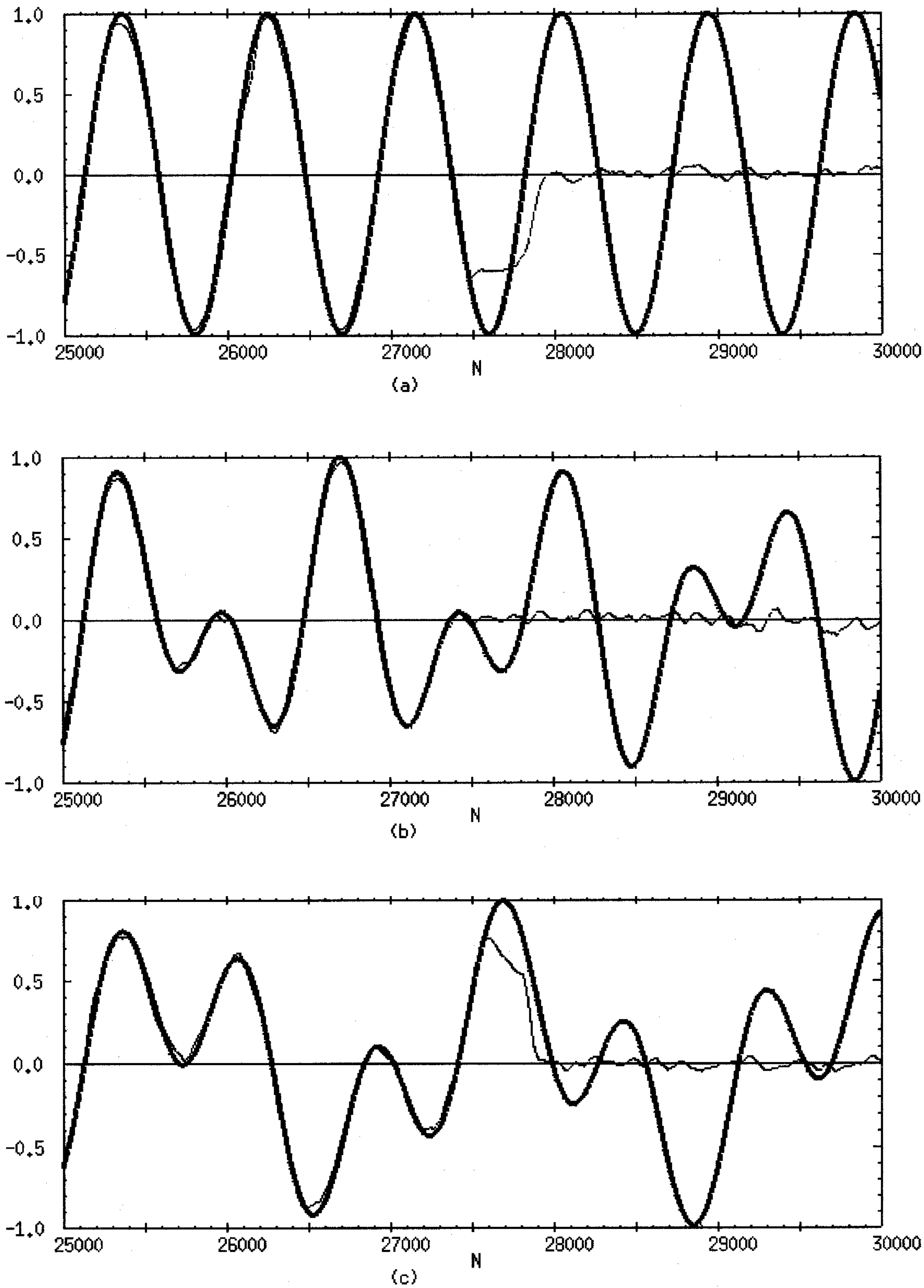


Fig. 5. Recovered signal with synchronization errors. The synchronization is broken by the impulsive noise at $n = 27\,500$.

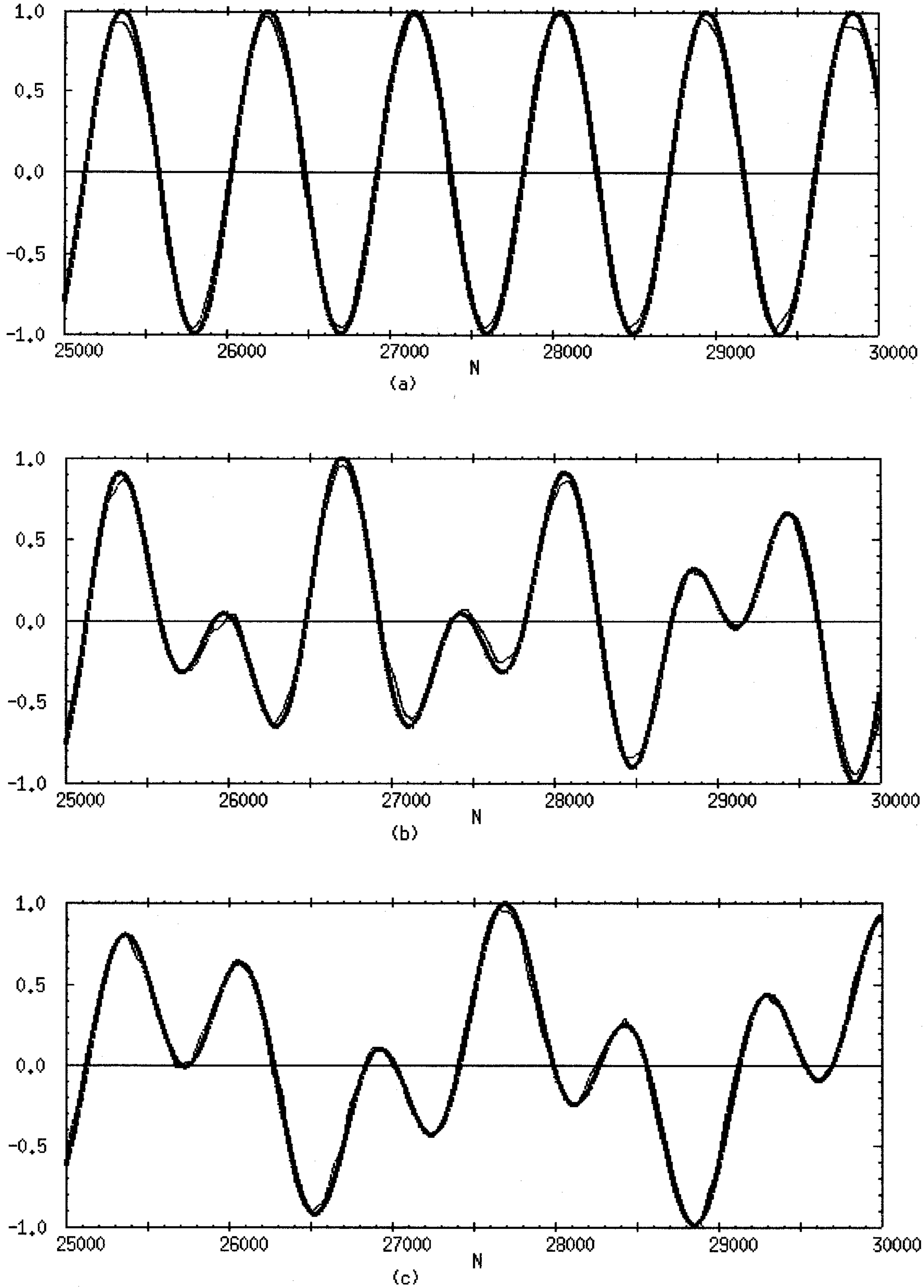


Fig. 6. Interference rejection. (a) and (b) Ikeda map is used for generating chaotic carriers. (c) Hénon map is used for generating a chaotic carrier. Recovered signals are shown in thin solid lines.

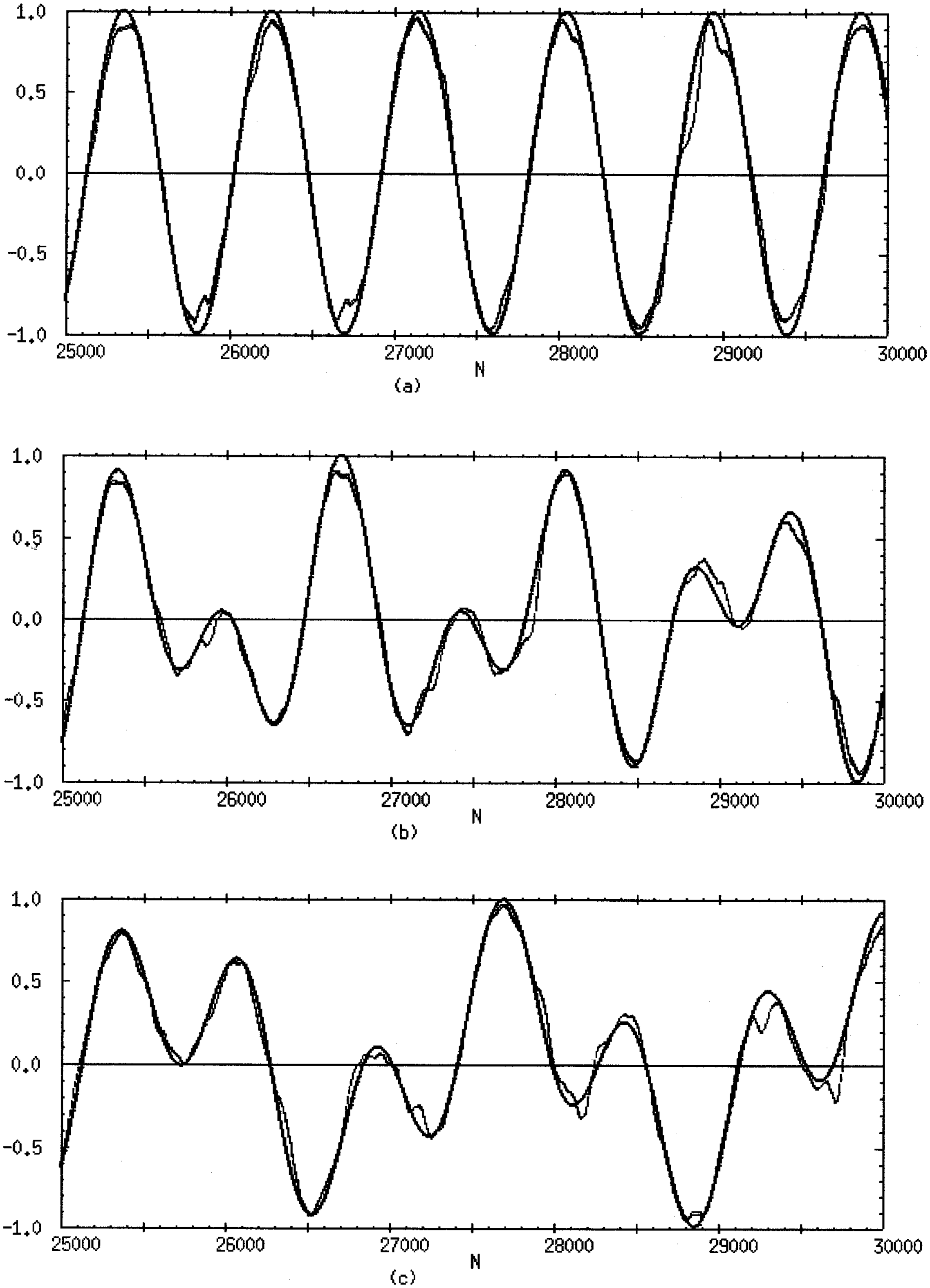


Fig. 7. Interference rejection. A periodic signal $I[n] = \sin(0.0065n)$ is injected to the channel. Recovered signals are shown in thin solid lines.

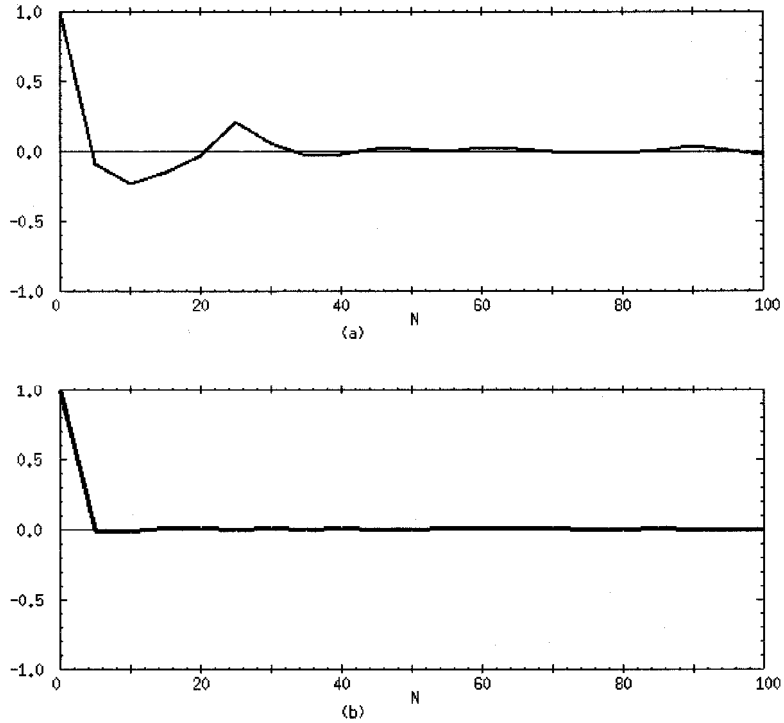


Fig. 8. Quick decay of the autocorrelation functions for the sequence (a) $\{x[n]\}$ and (b) $\{y[n]\}$.

The computer results are given in Figs. 1–8. The encoded and decoded signals are shown in Figs. 1–3. Some distortions still remain in the recovered signals. It is due to the following reason: The delays of the chaotic carriers are chosen to be finite, and so the cross correlation of the delayed carriers does not vanish completely. Thus, some errors (distortions) remain. Furthermore, the recovered signals are corrupted near the extremum points, which satisfies $ds_k(t)/dt = 0$. At these points, $s_k(t)$ does not increase nor decrease, and after that it starts moving rapidly (except the extremum points, the information signals $s_k(t)$ are monotone increasing or decreasing). Therefore, the recovered signal cannot trace the information signal (rapid change) quickly. However, if the information signal is slowly varying, then the errors are small as shown in Fig. 2.

Figure 4 shows the recovered signals with the mismatched carriers (one time-step is mismatched). The information signals cannot be recovered at all. Figure 5 shows the recovered signals when the synchronization errors occur in the receiver. The information signals cannot be recovered at all when the synchronization is broken.

Next, we verify the interference rejection from the other transmitter. In this case, two users are communicating with Ikeda map and the other is using Hénon map, which becomes the interference for the users communicating with Ikeda map. The dynamics of Hénon map is given by

$$u[n+1] = 1 + v[n] - cu[n]^2 \,, \tag{104}$$

$$v[n+1] = du[n] \,, \tag{105}$$

where $c = 1.4$ and $d = 0.3$. The decoded signals are slightly distorted as shown in Fig. 6. The interference suppression for the periodic signal: $\sin(0.0065n)$ is shown in Fig. 7. In this case, the averaged powers of the transmitted signal and the interference signal are adjusted to be on the same level. Finally, we show the quick decay of the autocorrelation functions in Fig. 8.

6.1.2. *Continuous-time dynamical systems*

Consider the Lorenz system [Lorenz, 1963]

$$\frac{du}{dt} = 10(v-u) \,, \tag{106}$$

$$\frac{dv}{dt} = -uw + 60u - v \,, \tag{107}$$

$$\frac{dw}{dt} = uv - \frac{8}{3}w \,. \tag{108}$$

This system has the flat power spectrum [Lipton & Dabke, 1996]. Assume that the channel is shared by three users. The coding function

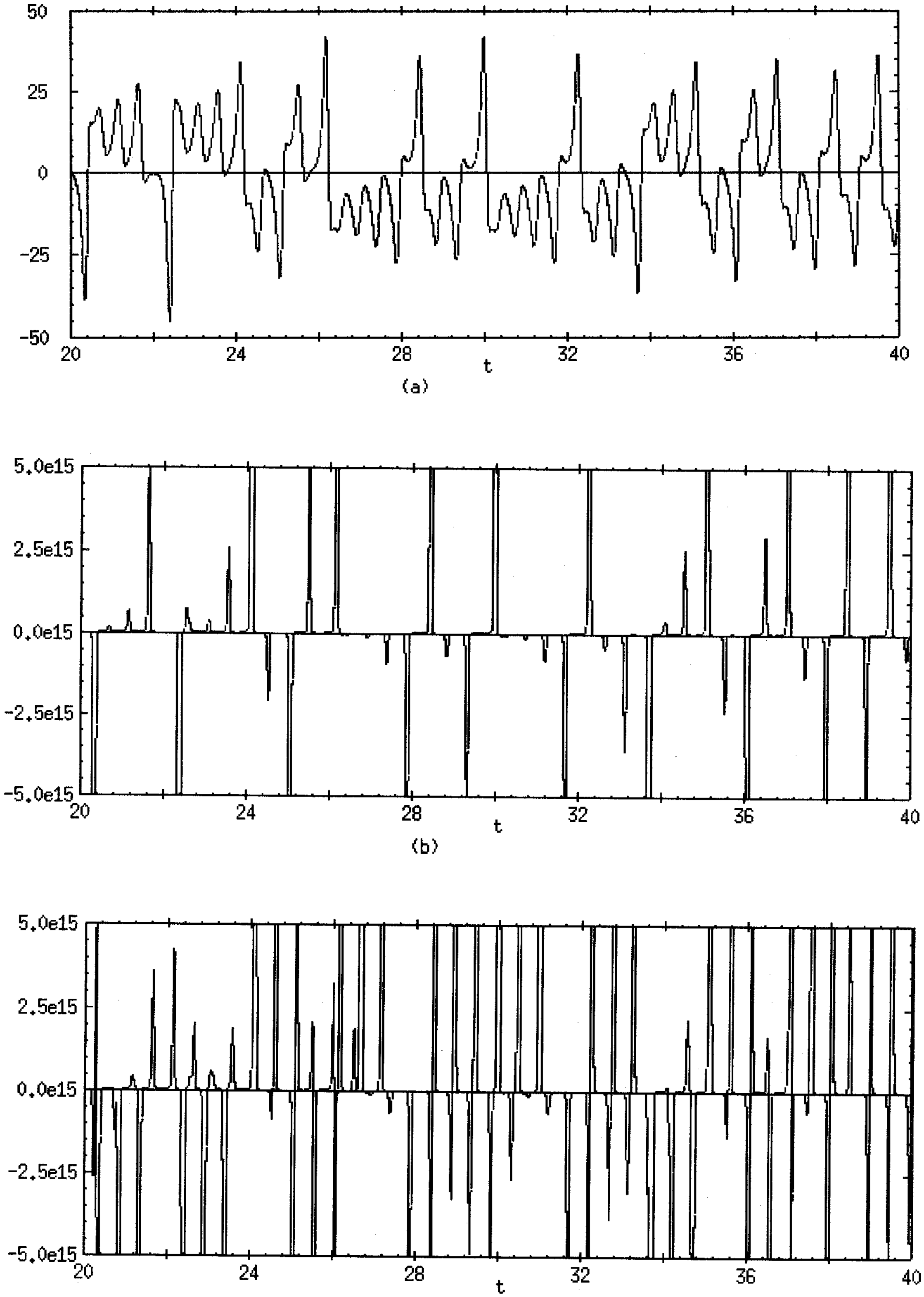


Fig. 9. Sequence of a chaotic carrier. (a) Chaotic waveform $v(t)$ of Lorenz system, (b) chaotic carrier $y(t)$, (c) transmitted signal $p(t)$. ($5.0e15$ means 5.0×10^{-15}.)

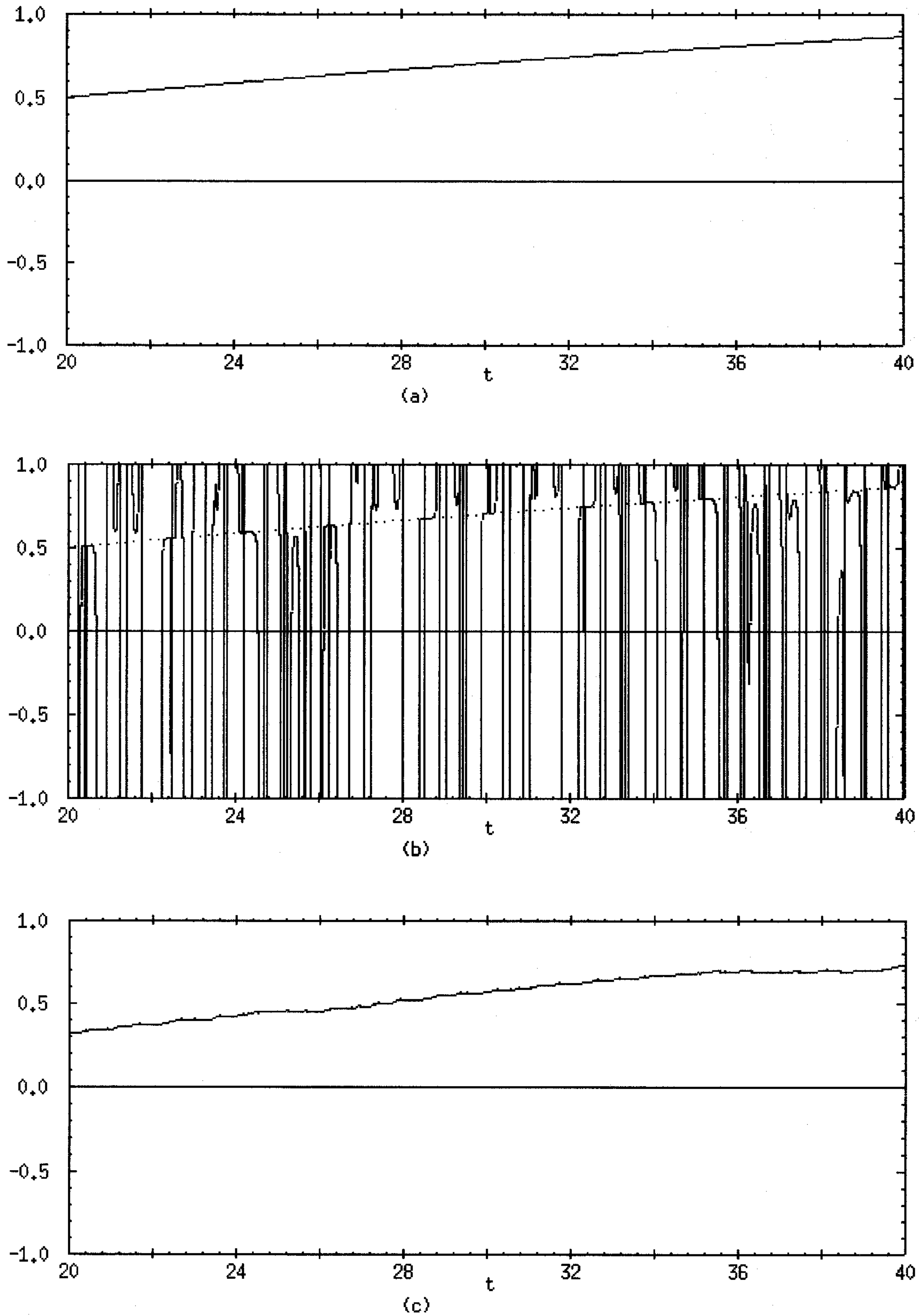


Fig. 10. Recovering process. (a) Information signal $s_1 = \sin(0.026t)$, (b) recovered signal $q_1(t)$ (before averaging), (c) recovered signal $\overline{q}_1(t)$ (after averaging).

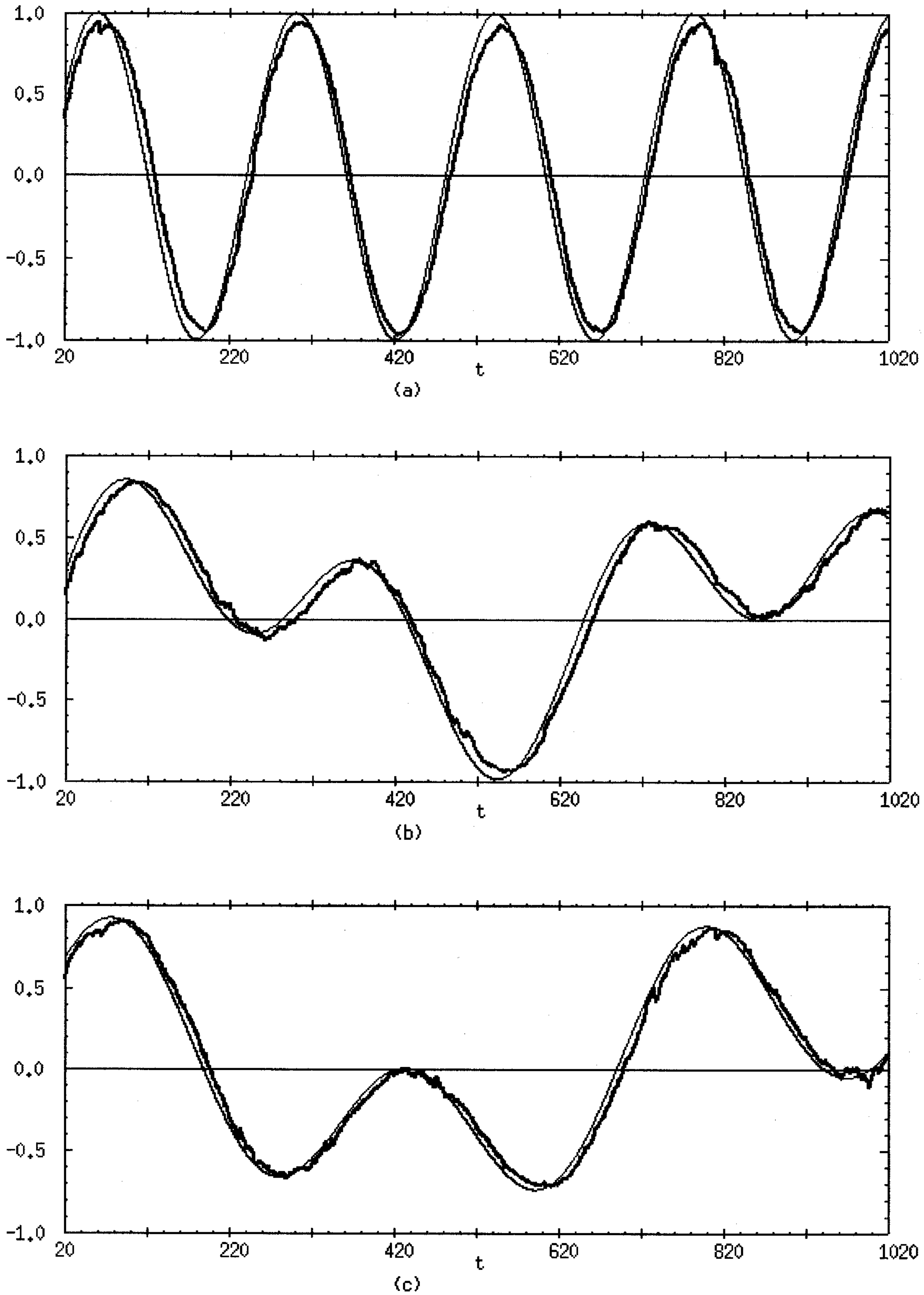


Fig. 11. Recovered signals $\overline{q}_j(t)$ (thin solid lines) and information signals $s_j(t)$ (thick solid lines) when the communication channel is shared by three users. (a) Information signal: $s_1(t) = \sin(0.026t)$, (b) $s_2(n) = 0.5(\sin(0.02t) + \sin(0.009t))$, (c) $s_3[n] = 0.5(\sin(0.018t) + \cos(0.007t))$.

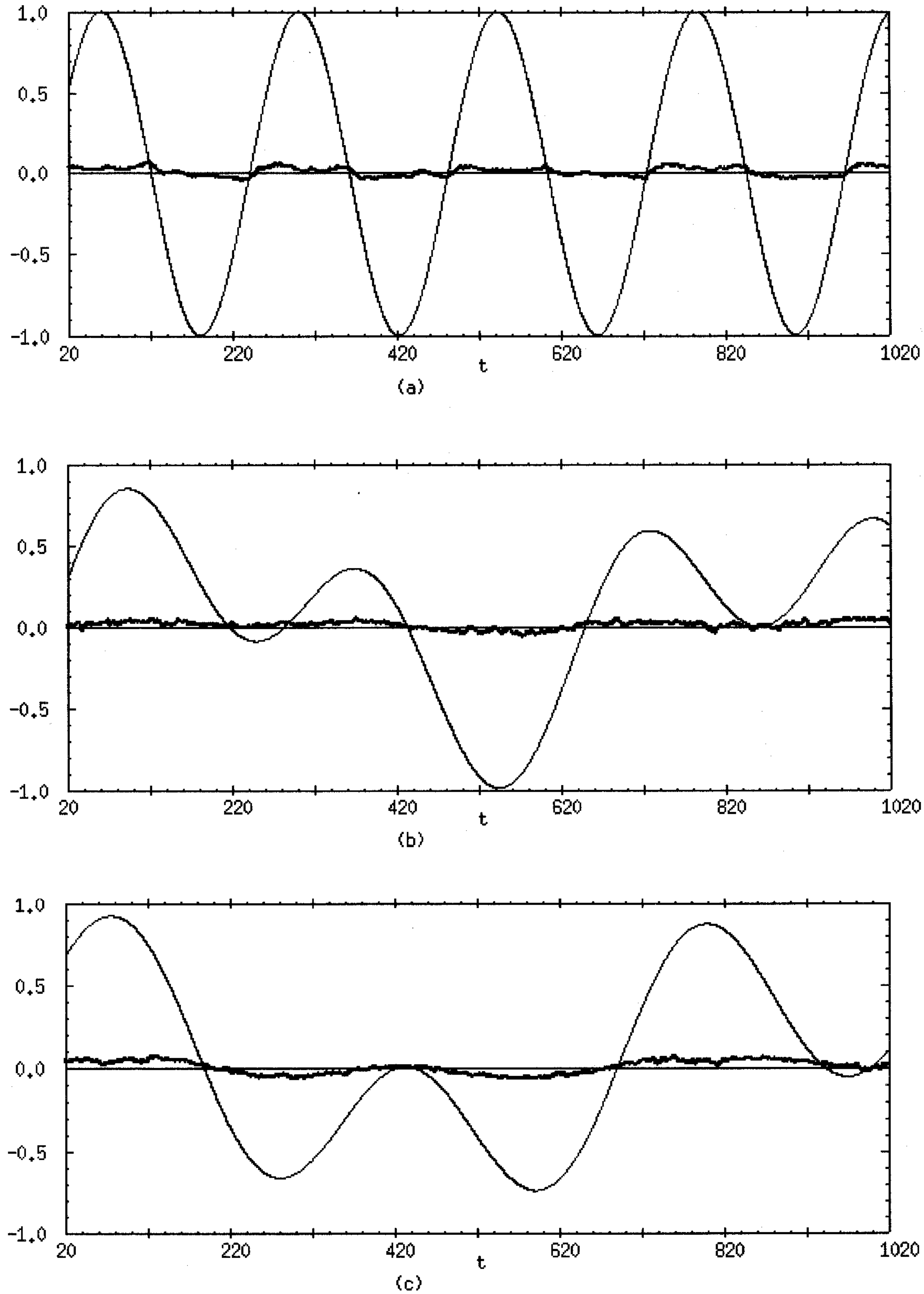


Fig. 12. Recovered signal with carrier mismatch; decoding is done by using the mismatched carrier $y(t - \tau_j')$ instead of $y(t - \tau_j)$, where $|\tau_j' - \tau_j| = 0.1$.

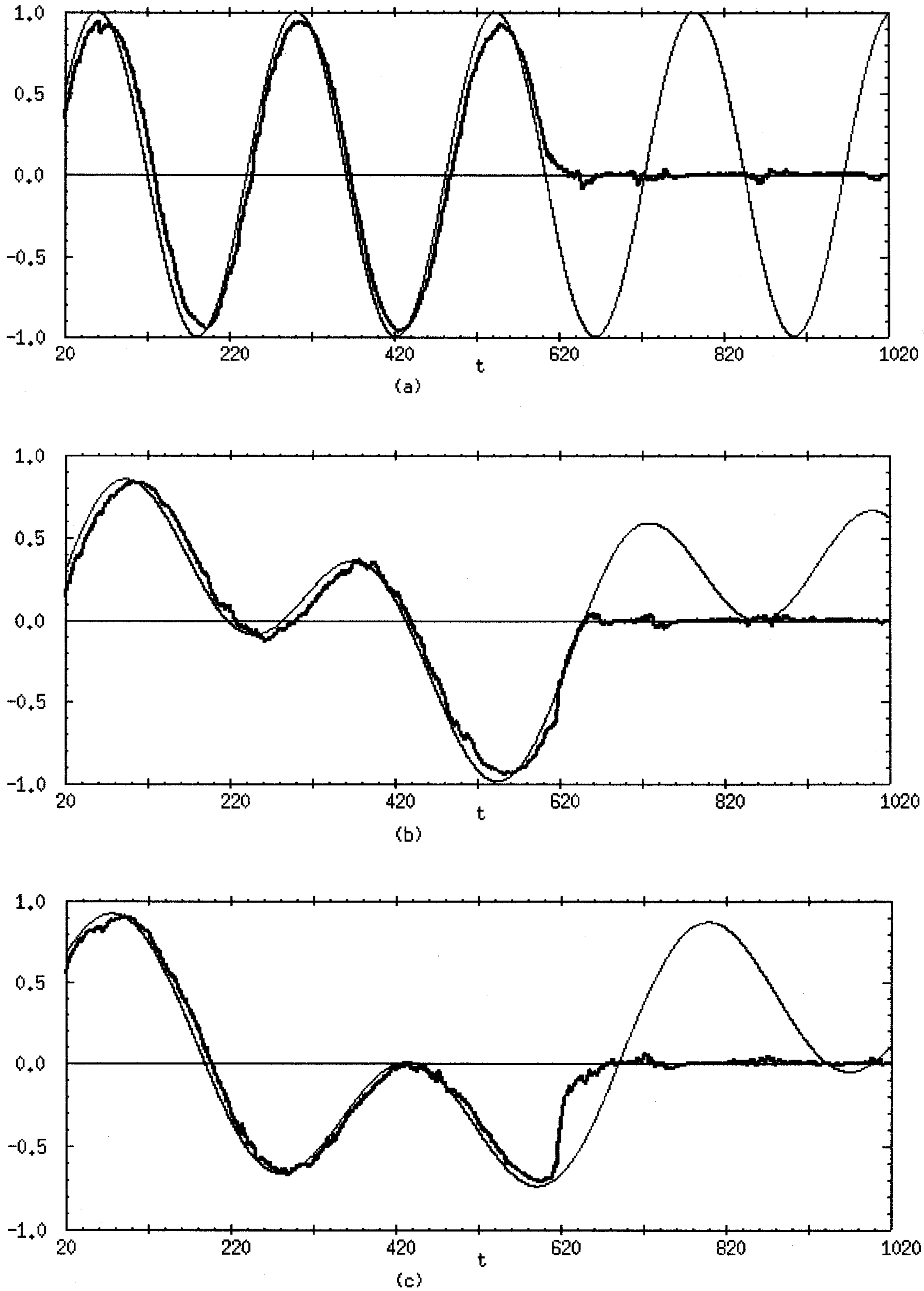


Fig. 13. Recovered signal with synchronization errors. The synchronization is broken by the impulsive noise at $t = 600$.

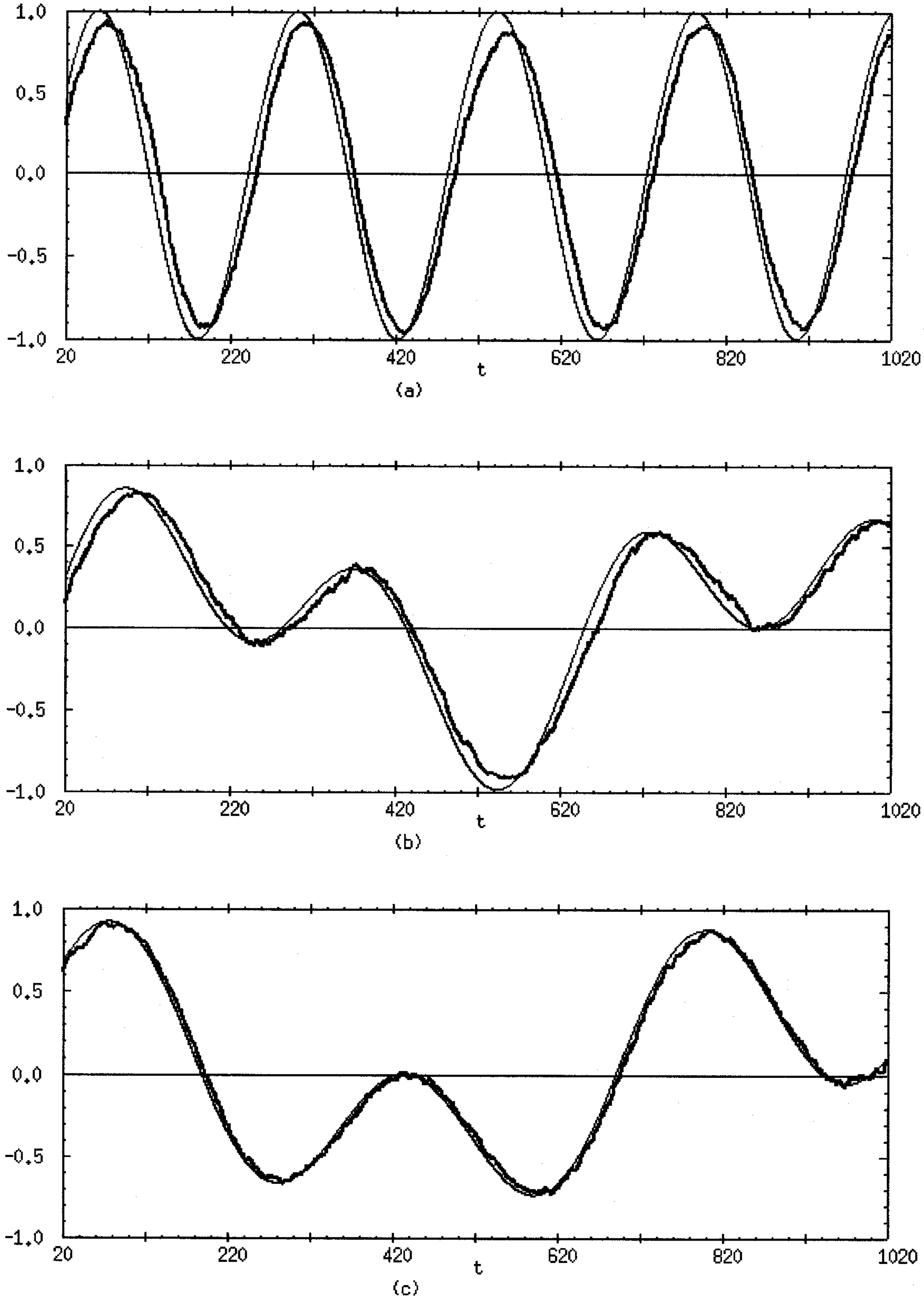


Fig. 14. Interference rejection. (a) and (b) Lorenz system is used for generating chaotic carriers. (c) Chua's oscillator is used for generating a chaotic carrier. Recovered signals are shown in thick solid lines.

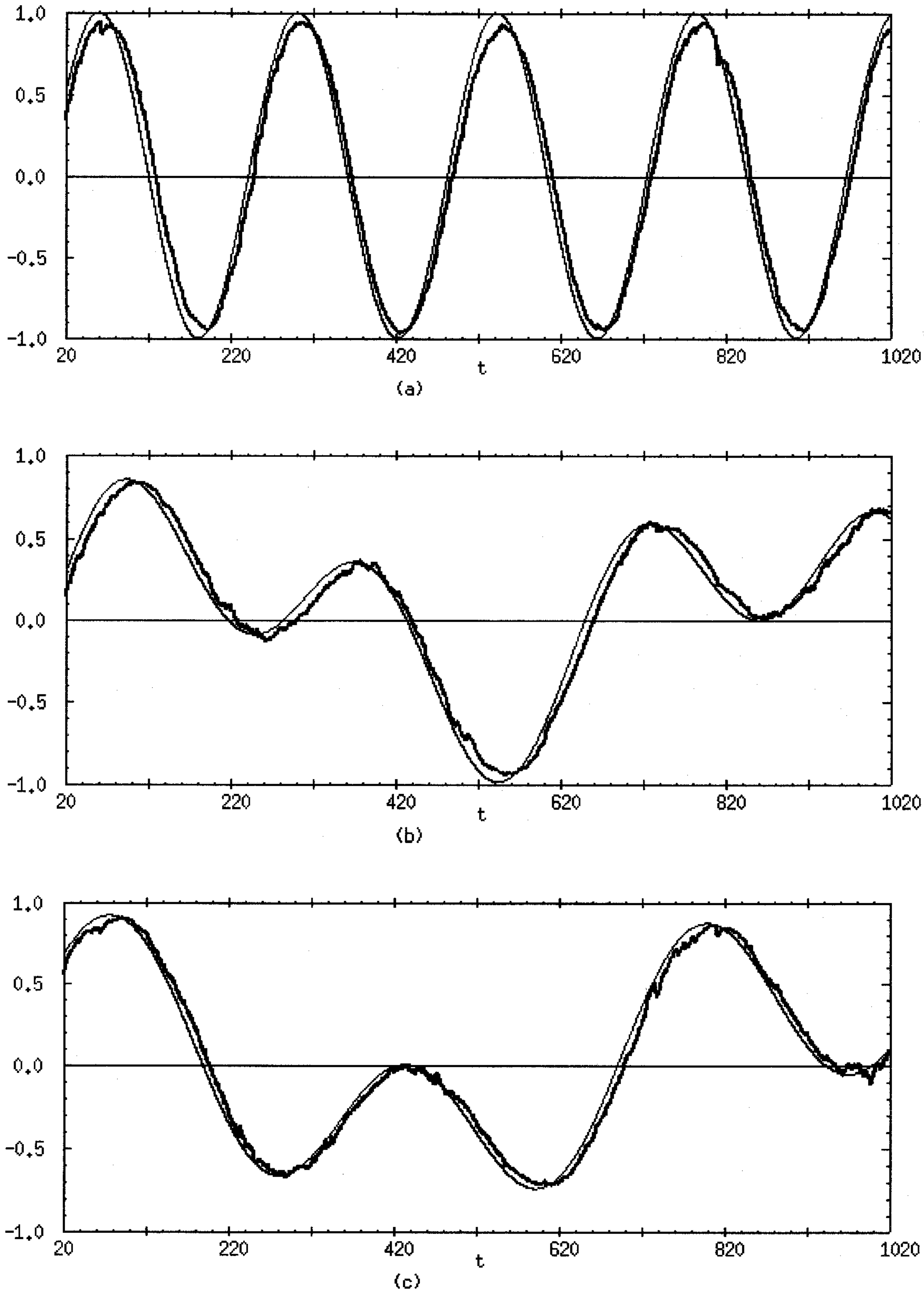


Fig. 15. Interference rejection. A periodic signal $I(t) = \sin(0.03t)$ is injected to the channel. Recovered signals are shown in thick solid lines.

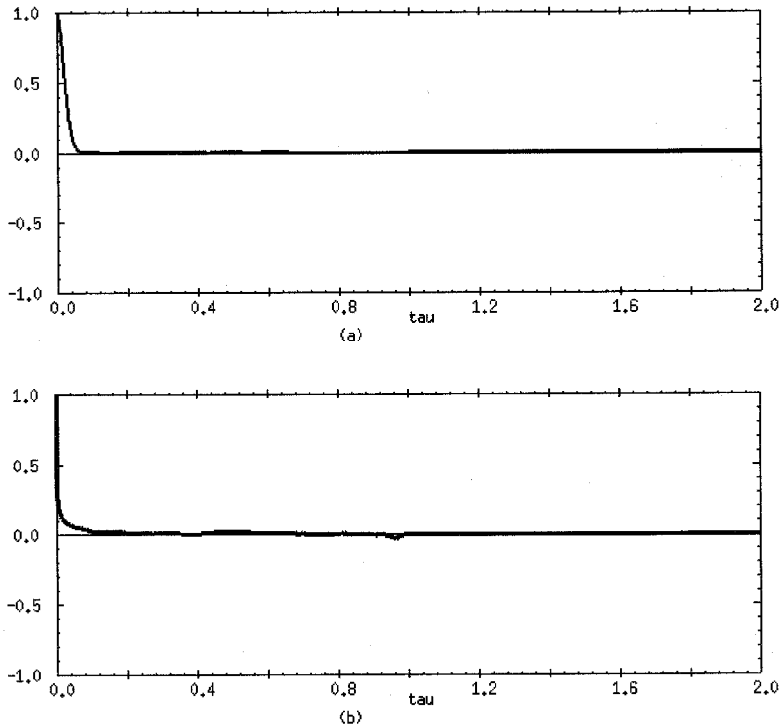


Fig. 16. Quick decay of autocorrelation functions for (a) $x(t)$ and (b) $y(t)$.

$g(\cdot)$ is given by a polynomial of $v(t)$

$$g(v(t)) = (v(t))^r \quad (r = 11)\,. \tag{109}$$

The sequence $\{y(t) = g(v(t))\}$ is used as a carrier. The delay τ_j and the period T are given by

$$\tau_1 = 0\,, \quad \tau_2 = 0.5\,, \quad \tau_3 = 1\,, \quad T = 0.07\,. \tag{110}$$

The computer results are shown in Figs. 9–16. The encoded signals and decoded signals are shown in Figs. 9–11. Small phase lag is observed in the recovered signals because of the averaging procedure [Carlson, 1975] (we can easily compensate the phase lag). Figures 12 and 13 show the recovered signals with the mismatched carriers and the synchronization errors, respectively. The information signals cannot be recovered. Next, the interference rejection is examined. In this case, two users are communicating with Lorenz map, and the other is using Chua's oscillator [Madan, 1993]. The dynamics of Chua's oscillator in a dimensionless form is given by

$$\frac{dx}{d\tau} = \alpha(y - x - f(x))\,, \tag{111}$$

$$\frac{dy}{d\tau} = x - y + z\,, \tag{112}$$

$$\frac{dz}{d\tau} = -\beta y - \gamma z\,, \tag{113}$$

where $f(x) = bx + 0.5(a - b)[|x + 1| - |x - 1|]$. The following parameters are used in our computer experiments:

$$\begin{gathered} \alpha = 10\,, \quad \beta = 15\,, \quad \gamma = 0.1\,, \\ a = -1.27\,, \quad b = -0.68\,. \end{gathered} \tag{114}$$

The decoded signals are slightly distorted as shown in Fig. 14. The interference suppression for a periodic signal $\sin(0.03t)$ is shown in Fig. 15. The quick decay of the autocorrelation functions is shown in Fig. 16.

6.2. *Frequency hopping*

6.2.1. *Discrete-time dynamical systems*

Consider the Ikeda map as a chaotic generator. Assume that three users are sharing a channel. The parameters used in the computer simulations are given by

$$\begin{gathered} l_1 = 0\,, \quad l_2 = 250\,, \quad l_3 = 400\,, \quad N = 110\,, \\ \Omega^{(j)} = 0.1 + 0.01x[n]\,, \quad g(x) = 0.01x\,. \end{gathered} \tag{115}$$

We made the same computer simulation as those for the DS. Their results are shown in Figs. 17–24.

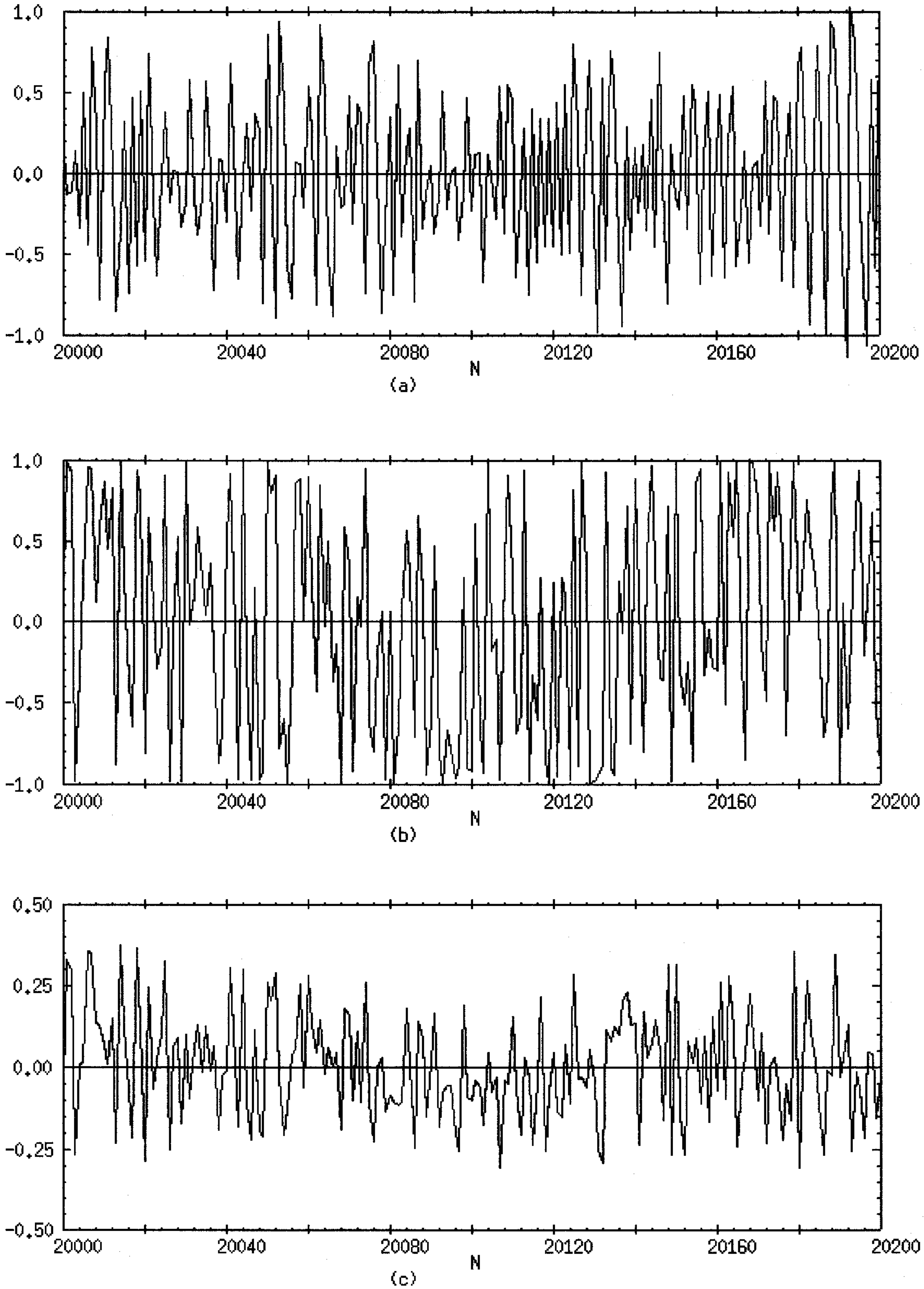


Fig. 17. Sequence of a chaotic carrier. (a) Chaotic sequence $\{x[n]\}$ of Ikeda map, (b) chaotic carrier $\{y[n]\}$, (c) transmitted signal $\{p[n]\}$.

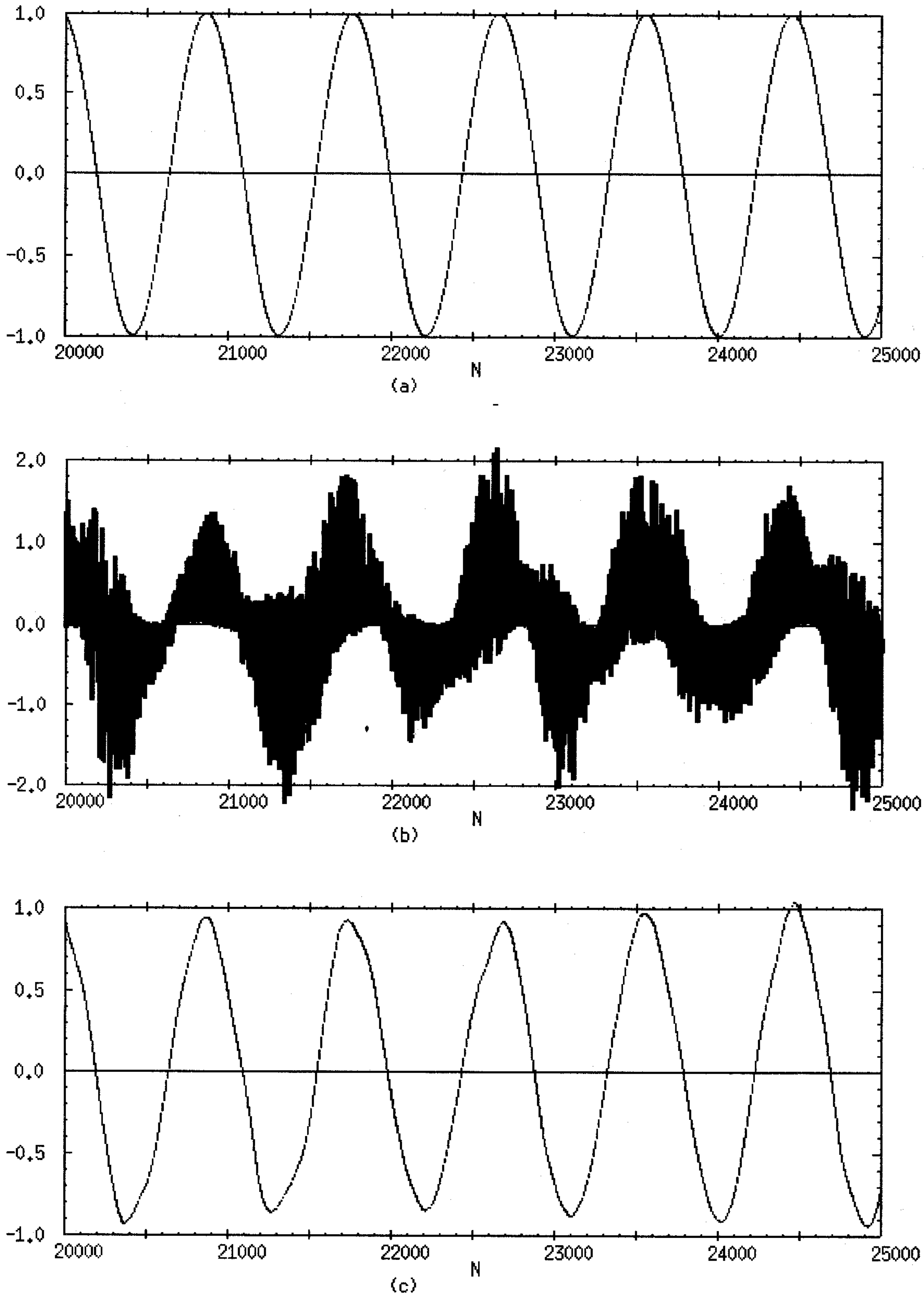


Fig. 18. Recovering process. (a) Information signal $s_1[n] = \sin(0.007n)$, (b) recovered signal $q_1[n]$ (before averaging), (c) recovered signal $\overline{q_1}[n]$ (after averaging).

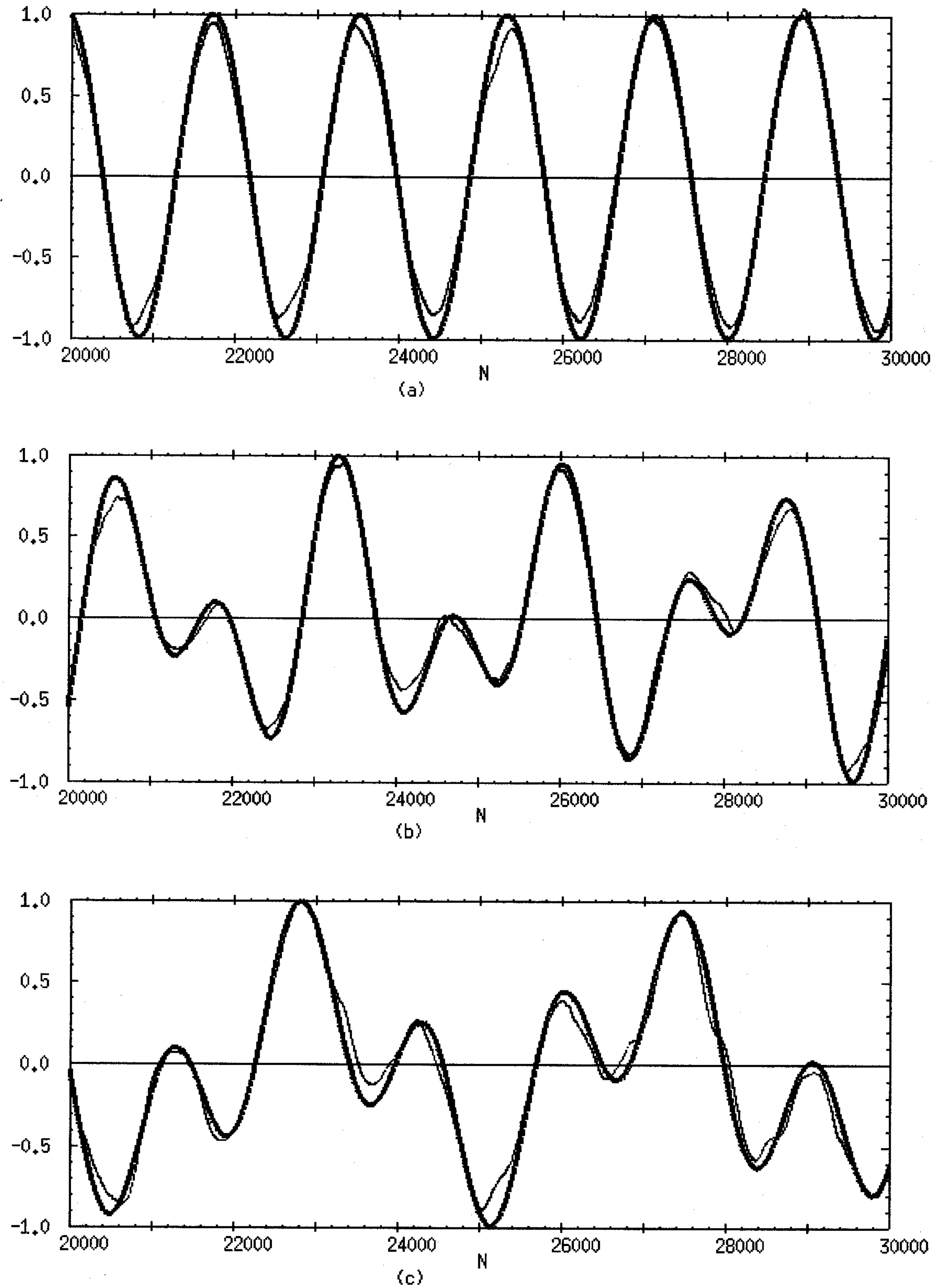


Fig. 19. Recovered signals $\overline{q_j}[n]$ (thin solid lines) and information signals $s_j[n]$ (thick solid lines) when the communication channel is shared by three users. (a) Information signal: $s_1[n] = \sin(0.007n)$, (b) $s_2[n] = 0.5(\sin(0.005n) + \sin(0.009n))$, (c) $s_3[n] = 0.5(\sin(0.003n) + \sin(0.008n))$.

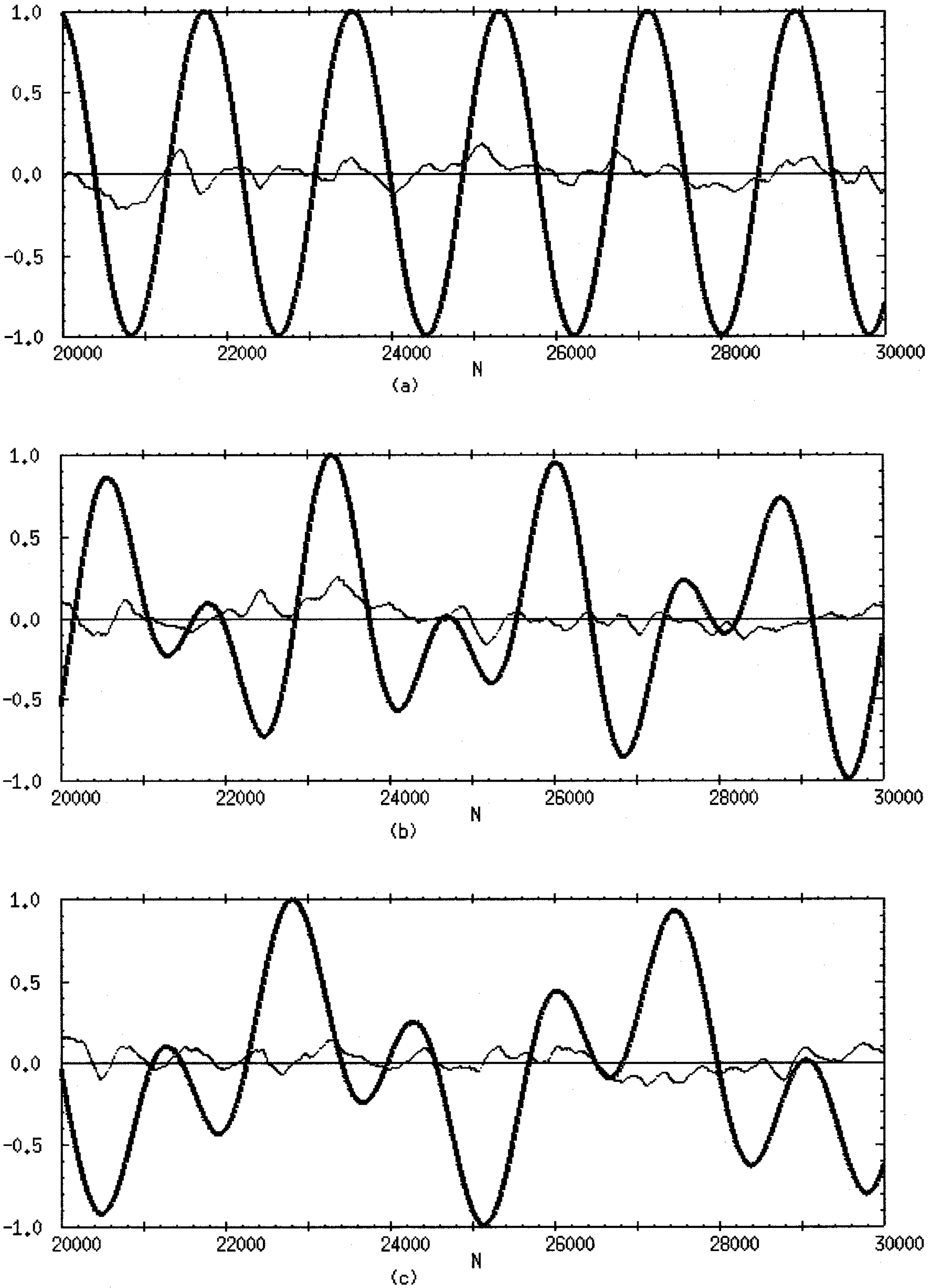


Fig. 20. Recovered signal with carrier mismatch; decoding is done by using the mismatched signal $x[n-l_j-1]$ (or $x[n-l_j+1]$) instead of $x[n-l_j]$; only one time-step is mismatched.

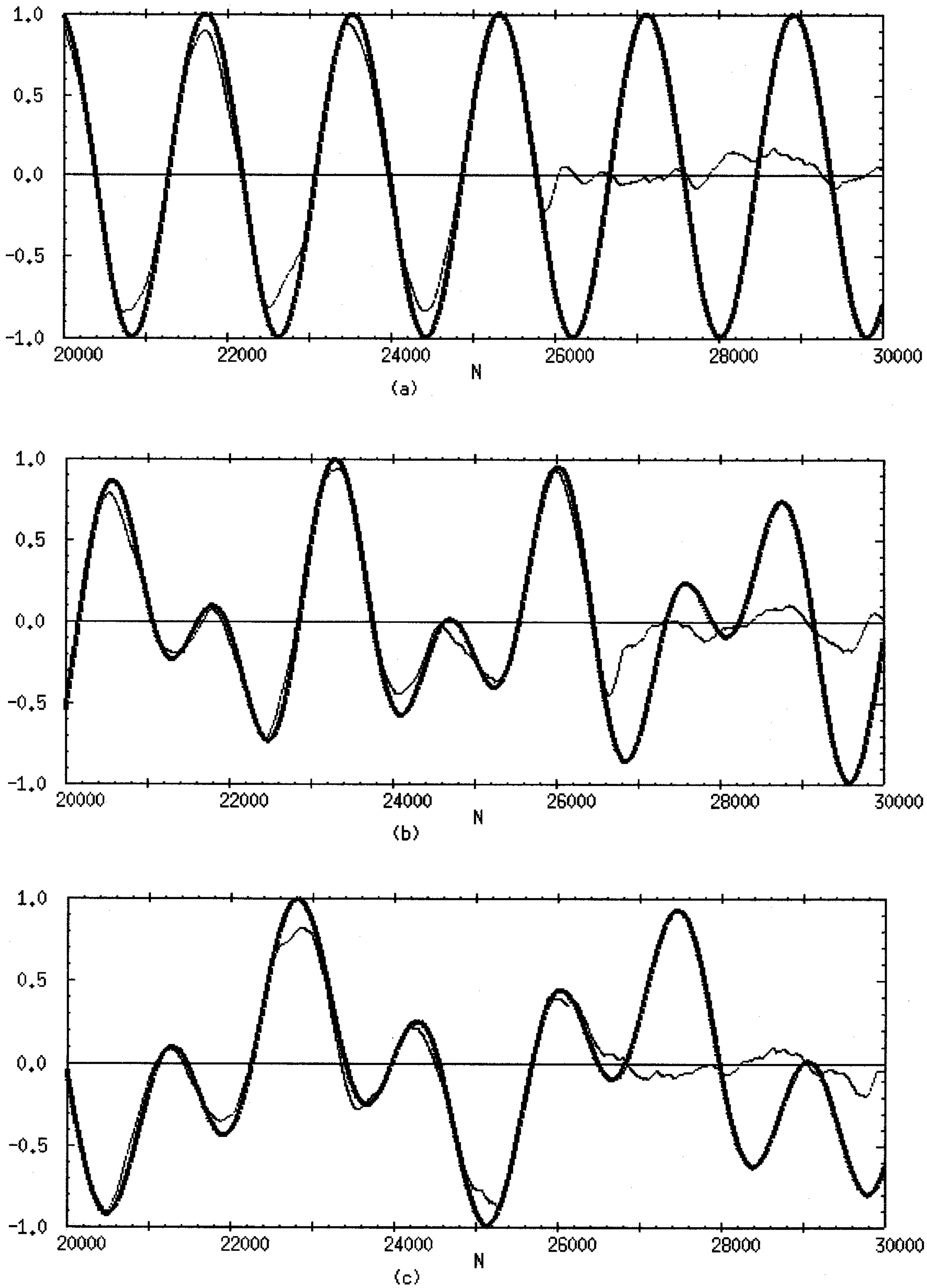


Fig. 21. Recovered signal with synchronization errors. The synchronization is broken by the impulsive noise at $n = 26\,000$.

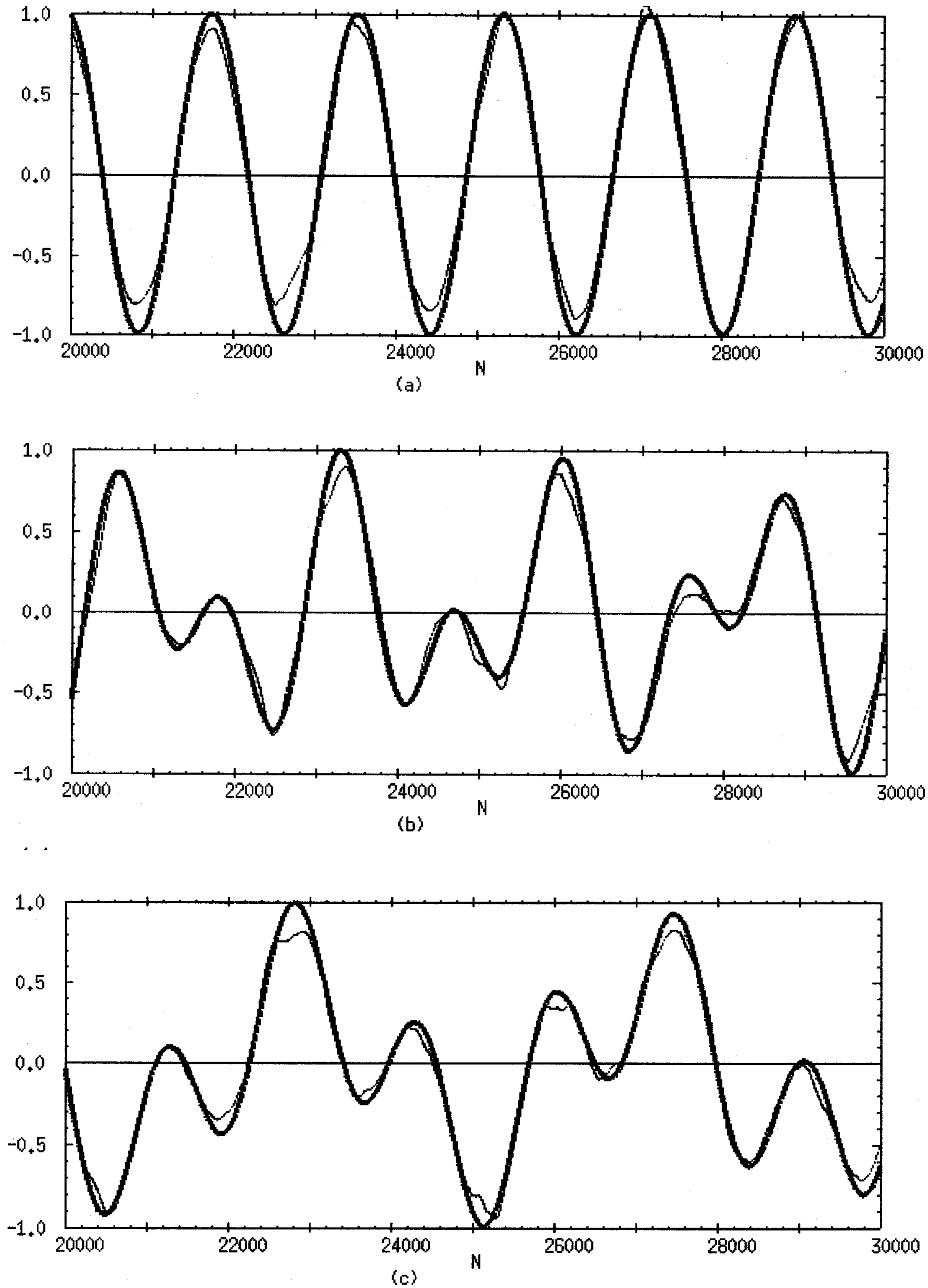


Fig. 22. Interference rejection. (a) and (b) Ikeda map is used for generating chaotic carriers. (c) Hénon map is used for generating a chaotic carrier. Recovered signals are shown in thin solid lines.

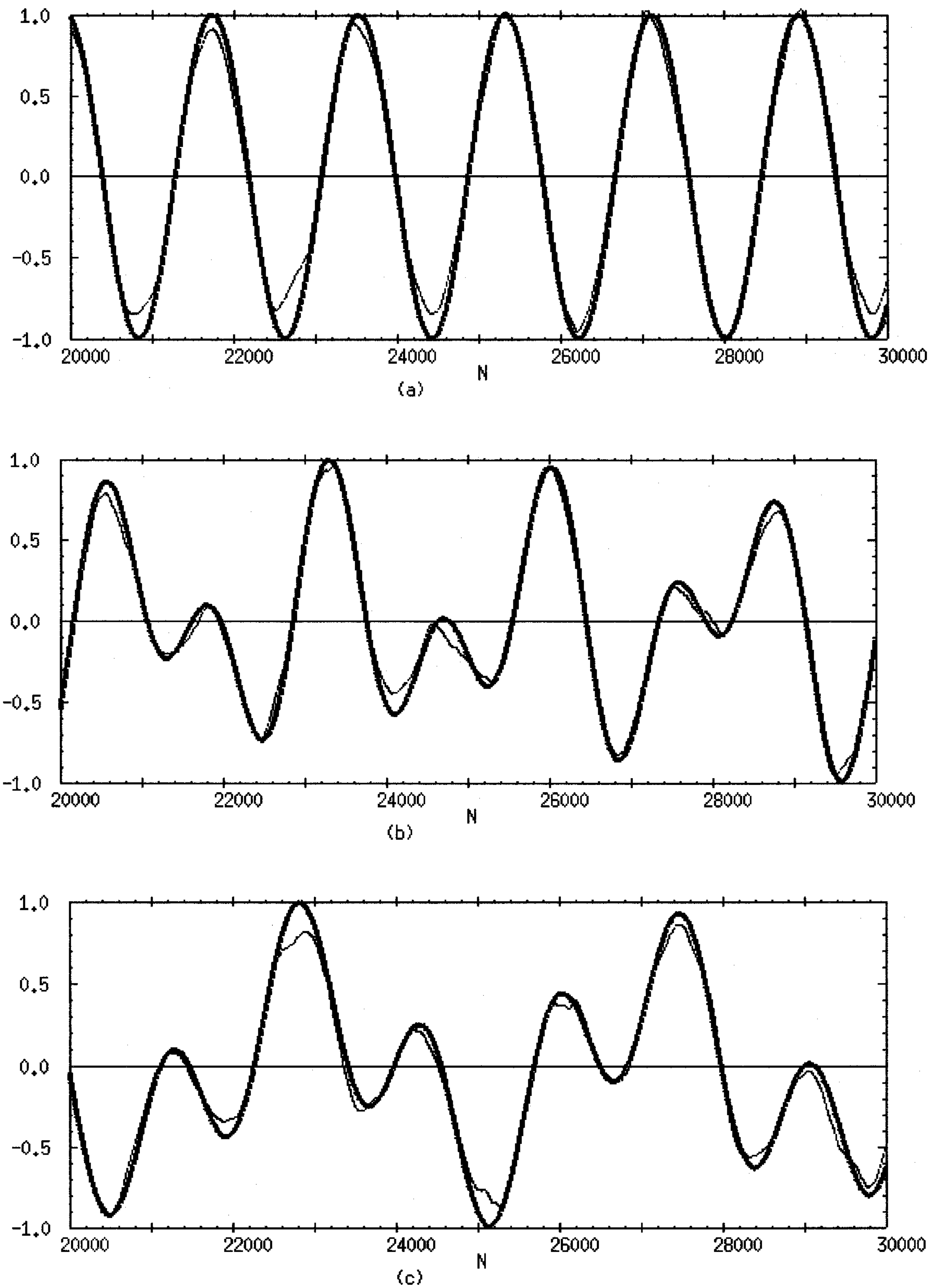


Fig. 23. Interference rejection. A periodic signal $I[n] = \sin(0.0065n)$ is injected to the channel. Recovered signals are shown in thin solid lines.

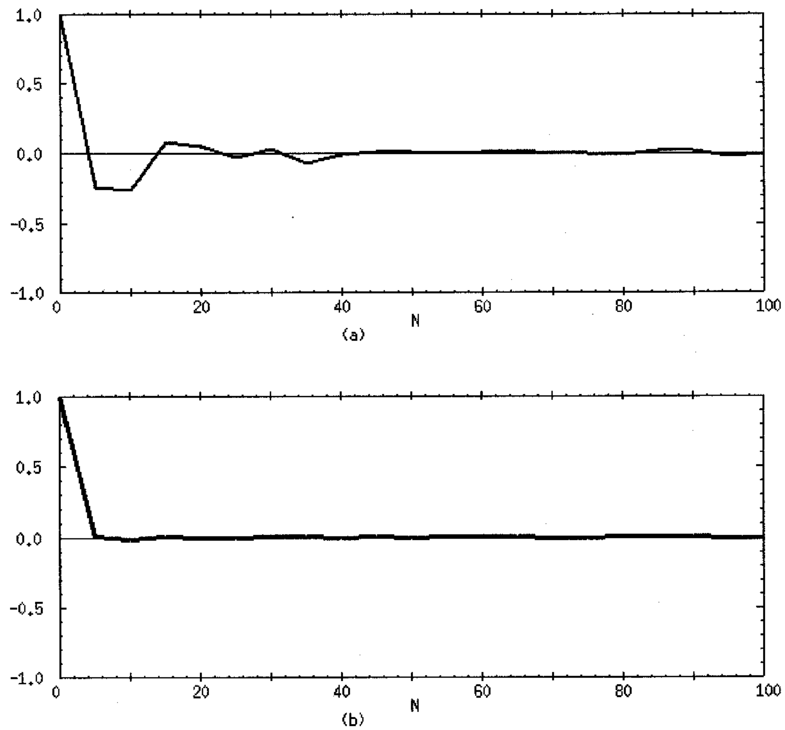


Fig. 24. Autocorrelation functions for (a) $x[n]$ and (b) $y[n]$.

6.2.2. *Continuous-time dynamical systems*

Consider the Lorenz map as a chaotic generator. Assume that the channel is shared by three users. The parameters are given by

$$\begin{gathered}\tau_1 = 0\,, \quad \tau_2 = 0.5\,, \quad \tau_3 = 1\,, \quad T = 0.15\,, \\ \Omega^{(j)} = 4 + \frac{v(t)}{30}\,, \quad g(x) = \frac{x}{30}\,.\end{gathered} \tag{116}$$

The computer results are shown in Figs. 25–32.

7. Performance Analysis of DS and FH

In this section, we evaluate the following quantities to analyze the performance of the proposed systems

(1) SNRs (Signal to Noise Ratios) for analog information data sequences,
(2) BERs (Bit Error Rates) for binary information data sequences,

as a function of multiple users by using white Gaussian noise and burst impulsive noise. We also examined the impulsive synchronization method to transmit signals through some noisy channels. Referring the report of [Kennedy & Kolumban, 1997], we found that a BER of less than 10^{-3} is required for digital communication; otherwise the communication link is broken at a system level.[3]

7.1. *Discrete-time dynamical systems*

7.1.1. *Analog information data*

We use the signal to noise ratio (SNR) to examine the error rates of recovered signals. The SNR of the kth user is defined as

$$\mathrm{SNR}(k) = 10\,\log_{10} \frac{\sum_{n=1}^{M} s_k[n]^2}{\sum_{n=1}^{M} e_k[n]^2}\,, \tag{117}$$

where $e_k[n] = r_k[n] - s_k[n]$ is an error (i.e. "noise" generated by the recovering process). Here, $r_k[n]$ and $s_k[n]$ are the recovered signal and the

[3] Their system required a SNR of at least 30 dB to achieve a BER of 10^{-2}. If the SNR fell below 30 dB, their system failed to operate. For more details, see [Kennedy & Kolumban, 1997].

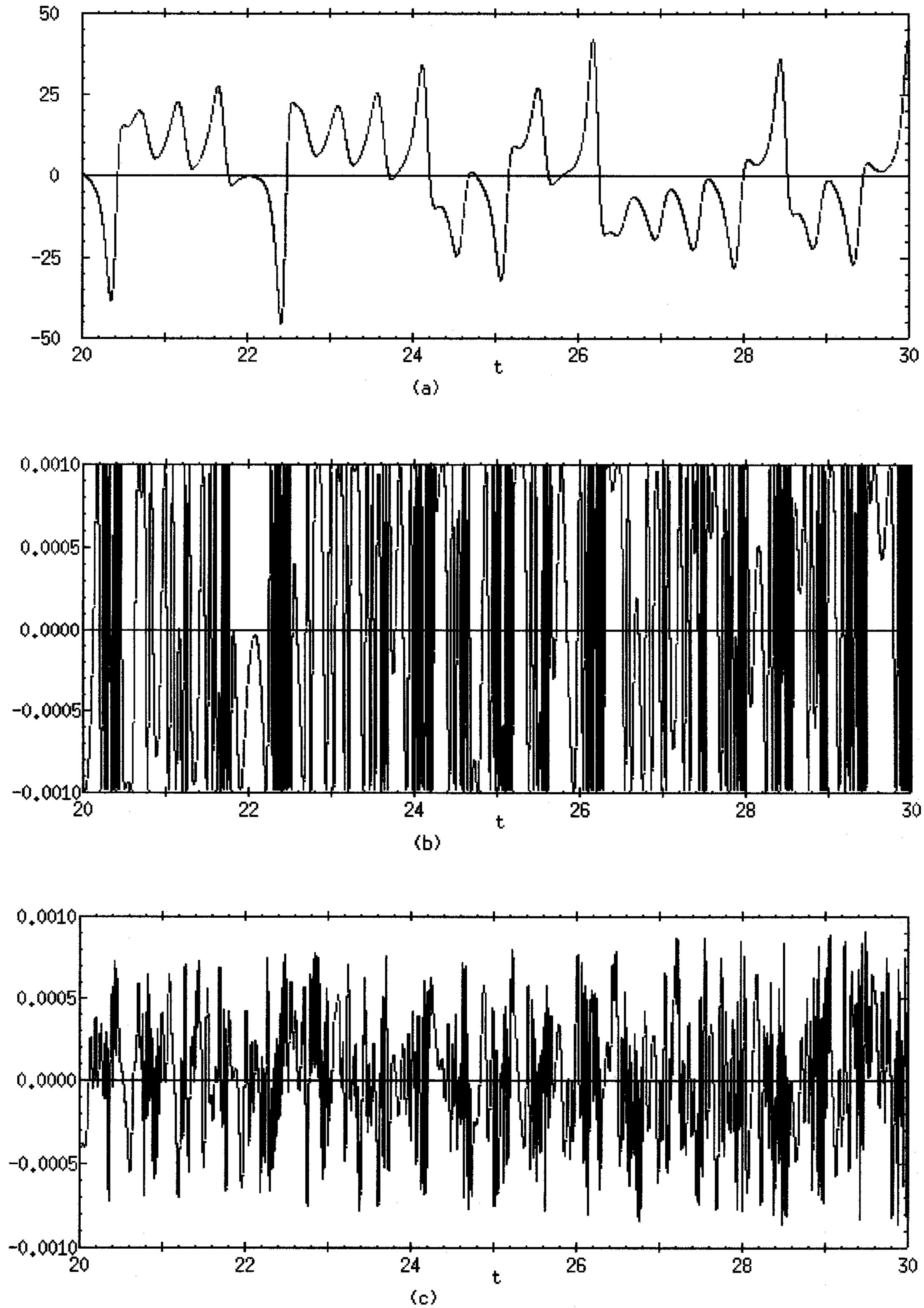


Fig. 25. Sequence of a chaotic carrier. (a) Chaotic waveform $v(t)$ of Lorenz system, (b) chaotic carrier $y(t)$, (c) transmitted signal $p(t)$.

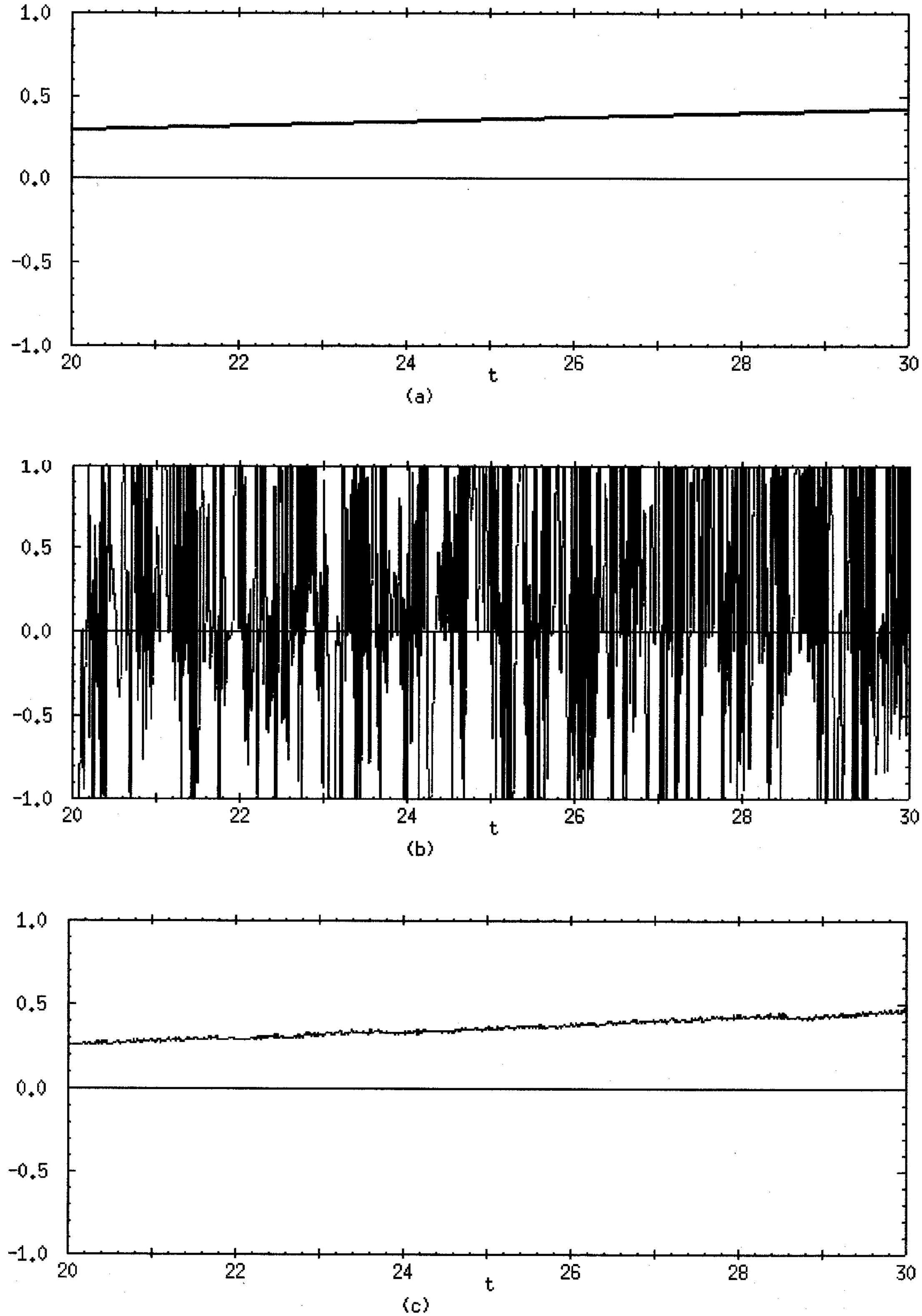


Fig. 26. Recovering process. (a) Information signal $s_1(t) = \sin(0.026t)$, (b) recovered signal $q_1(t)$ (before averaging), (c) recovered signal $\overline{q}_1(t)$ (after averaging).

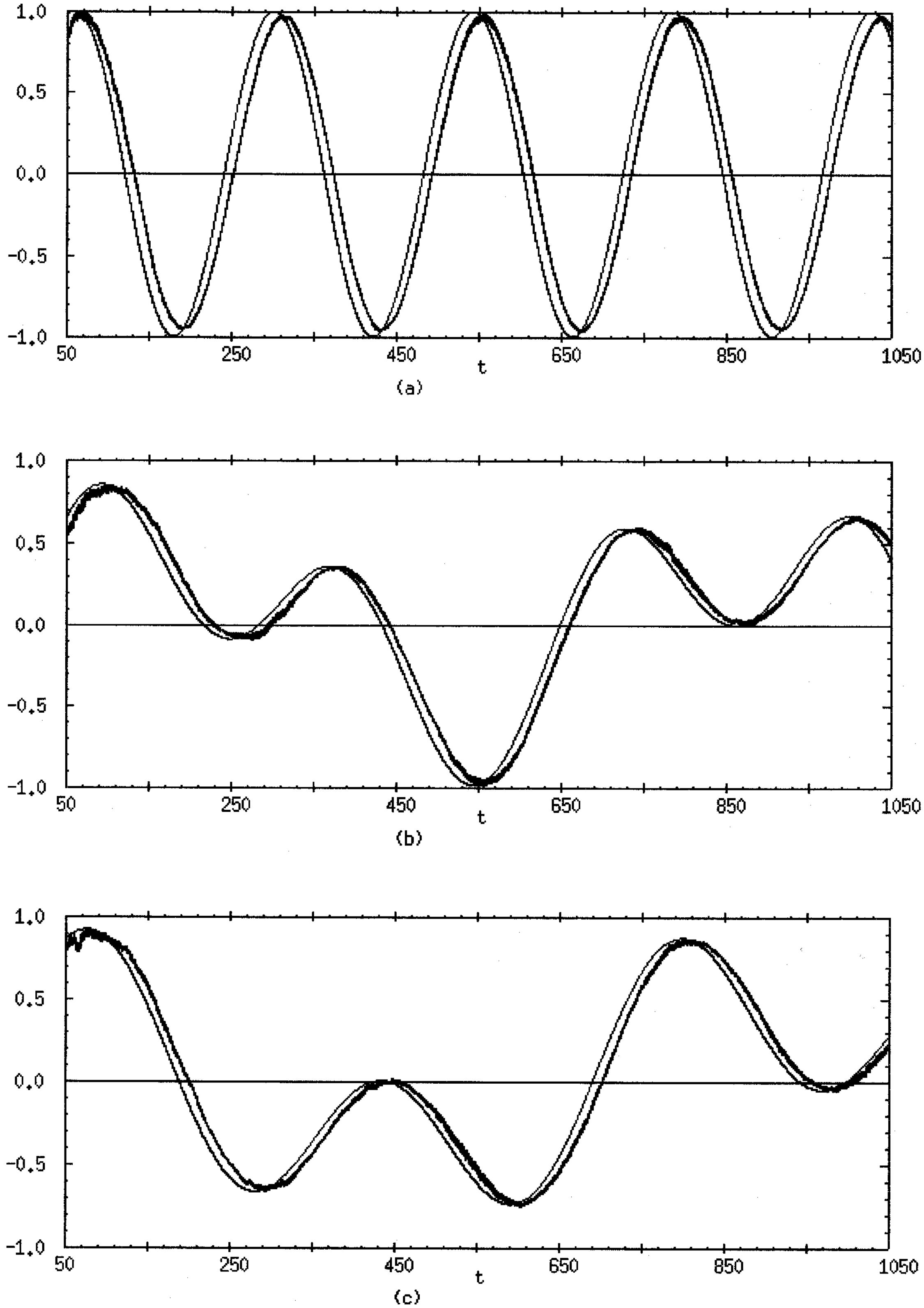


Fig. 27. Recovered signals $\overline{q}_j(t)$ (thin solid lines) and information signals $s_j(t)$ (thick solid lines) when the communication channel is shared by three users. (a) Information signal $s_1(t) = \sin(0.026t)$, (b) $s_2(n) = 0.5(\sin(0.02t) + \sin(0.009t))$, (c) $s_3[n] = 0.5(\sin(0.018t) + \cos(0.007t))$.

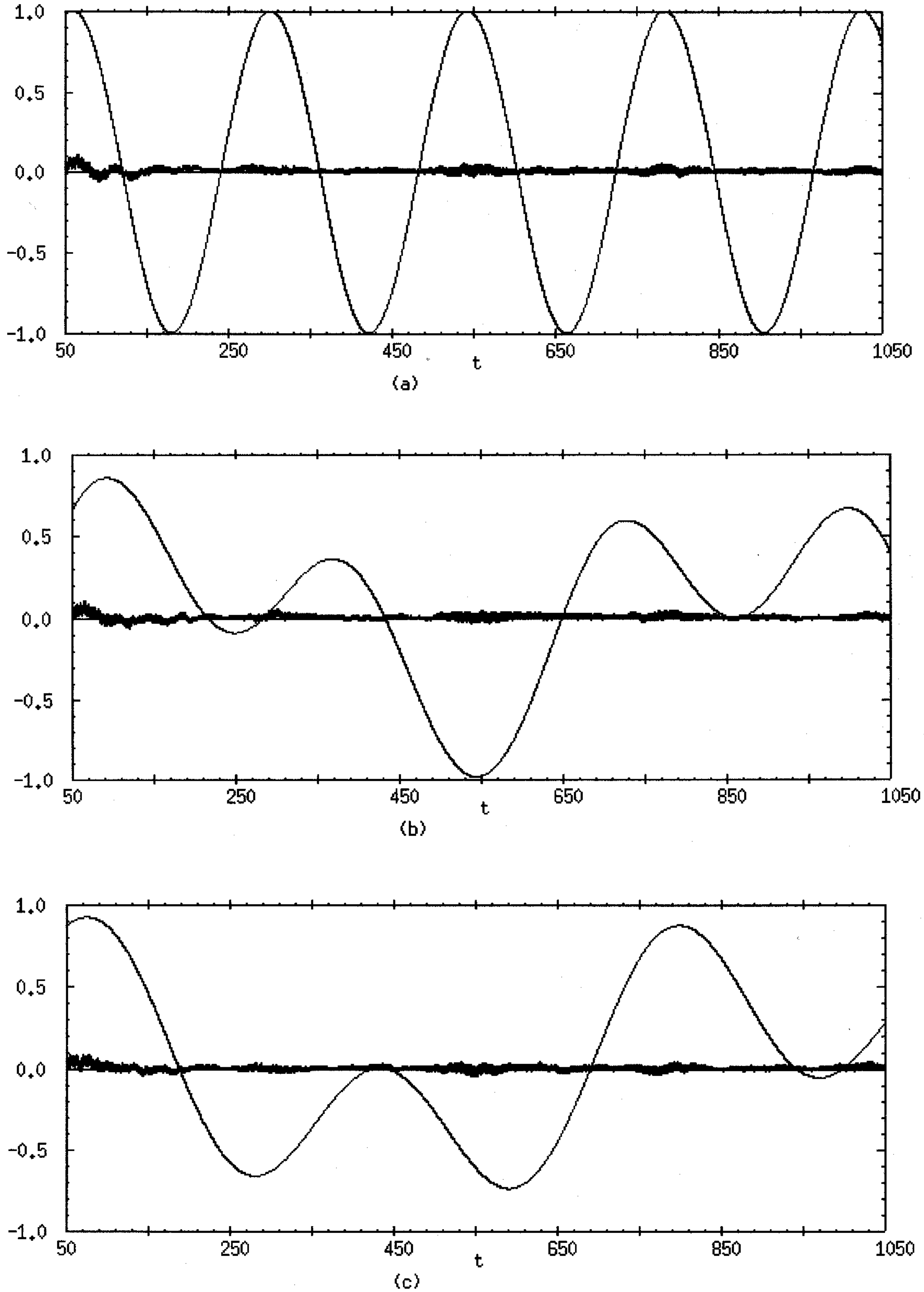


Fig. 28. Recovered signal with carrier mismatch; decoding is done by using the mismatched signal $x(t-\tau_j')$ instead of $x(t-\tau_j)$, where $|\tau_j' - \tau_j| = 0.05$.

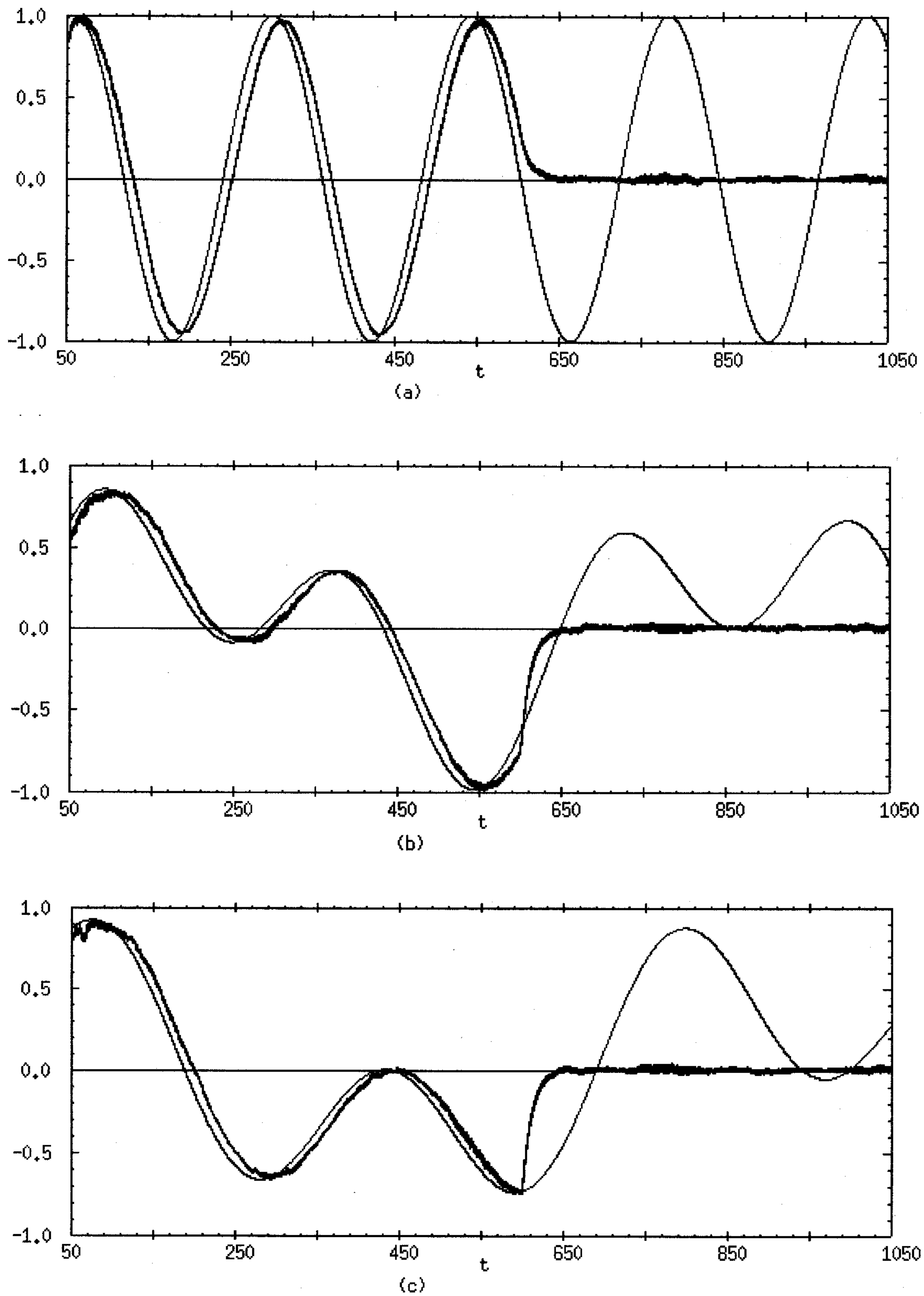


Fig. 29. Recovered signal with synchronization errors. The synchronization is broken by the impulsive noise at $t = 600$.

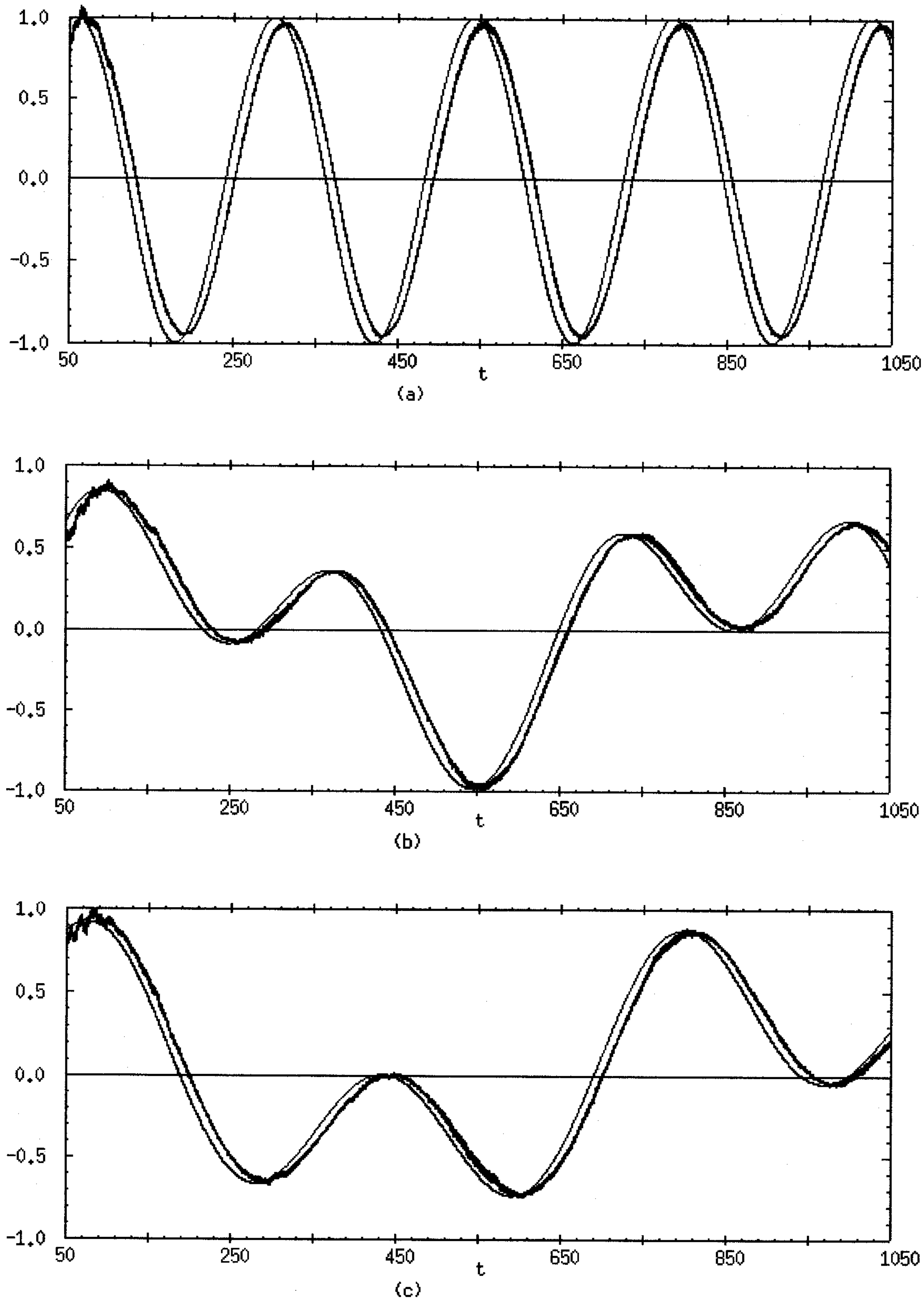


Fig. 30. Interference rejection. (a) and (b) Lorenz system is used for generating chaotic carriers. (c) Chua's oscillator is used for generating a chaotic carrier. Recovered signals are shown in thick solid lines.

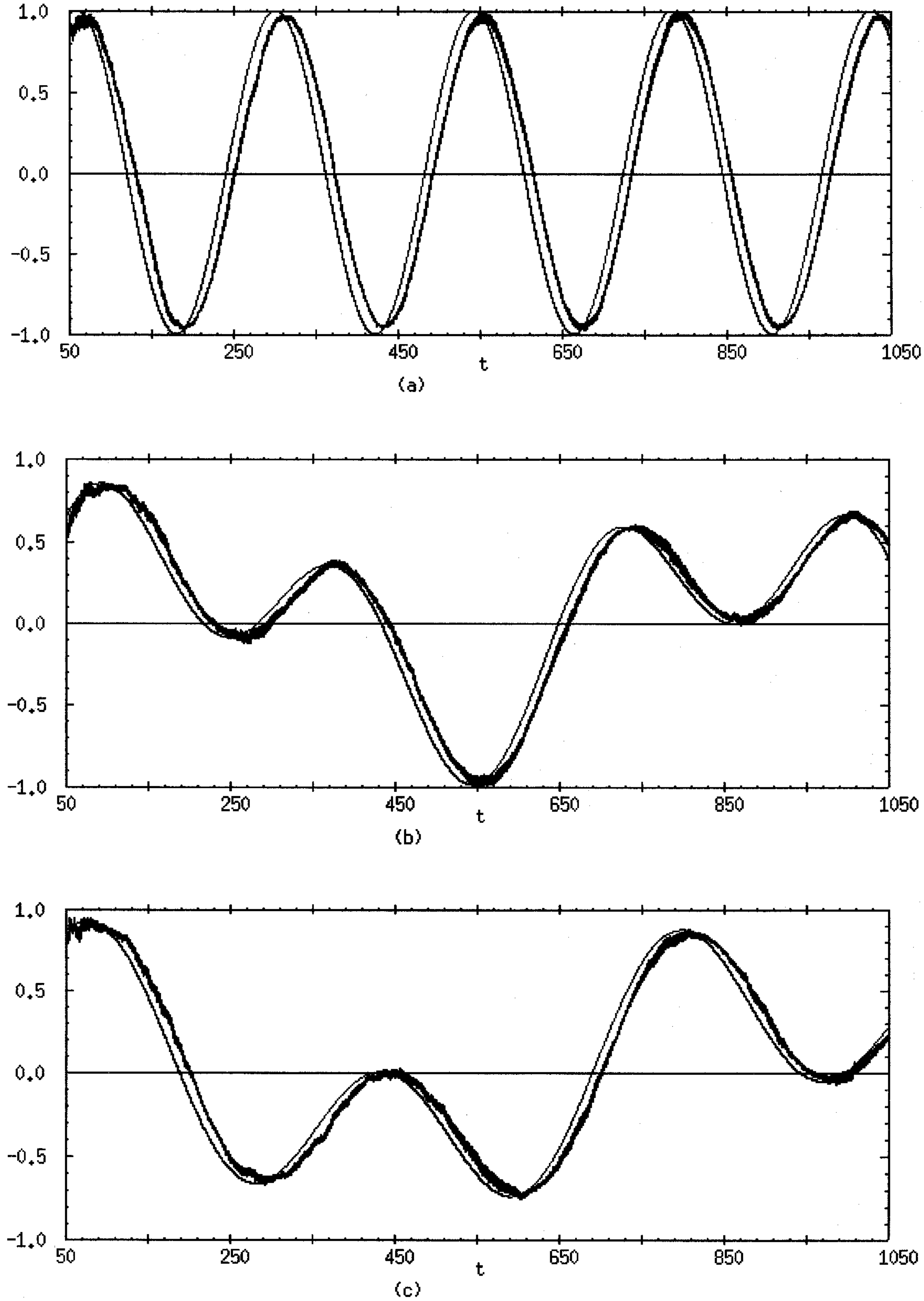


Fig. 31. Interference rejection. A periodic signal $I(t) = \sin(0.03t)$ is injected to the channel. Recovered signals are shown in thick solid lines.

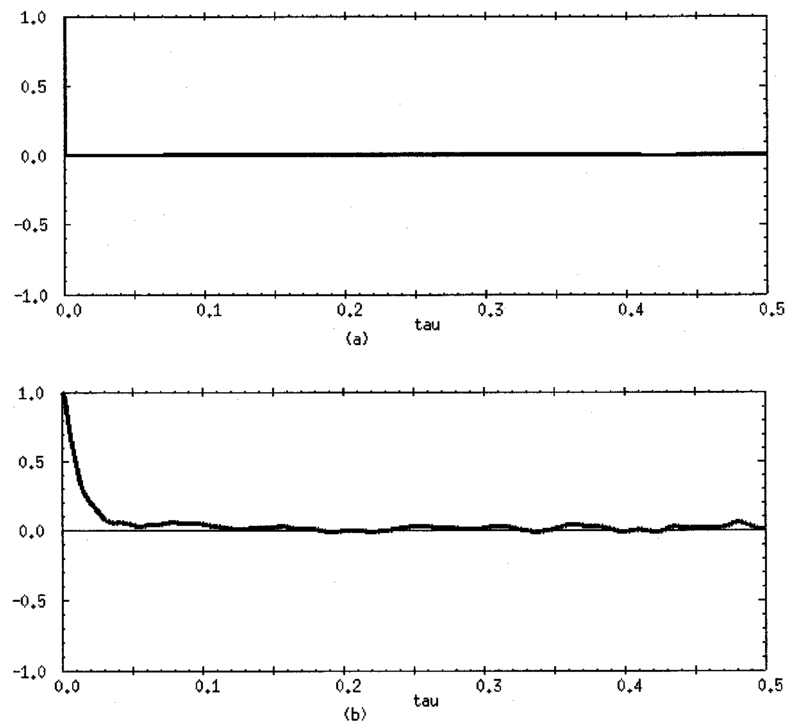


Fig. 32. Autocorrelation functions for (a) $x(t)$ and (b) $y(t)$.

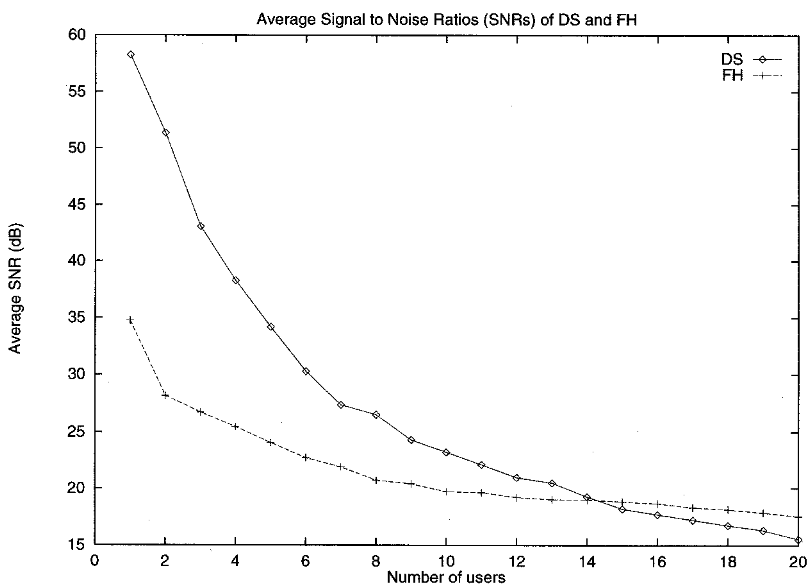


Fig. 33. Average signal to noise ratios (SNRs) of DS and FH.

information signal of the kth user, respectively. The mean values of $s[n]$ is chosen to be zero. The average SNR is given by

$$\text{Average SNR} = \frac{\sum_{k=1}^{K} \text{SNR}(k)}{K}, \qquad (118)$$

where K is the number of users.

We now show the SNRs of DS and FH as a function of multiple users in Fig. 33. The sine curves are chosen as the information signals. Furthermore, the delayed information signals are used to obtain the SNRs, since the recovered signals have some phase lag as stated before. The DS has a good performance for a few users, but the average SNR of DS decreases quickly as the number of users increases.

Next, we verify the SNRs by using a white Gaussian channel. The transmitted signal to noise ratio is defined as

$$10 \log_{10} \frac{\sum_{n=1}^{M} p[n]^2}{\sum_{n=1}^{M} w[n]^2}, \tag{119}$$

where M is the total length of the transmitted signal, and $p[n]$ and $w[n]$ are the transmitted signal and white Gaussian noise, respectively. The mean values of $p[n]$ and $w[n]$ are chosen to be zero. By sending synchronization data, we made the chaotic system in the receiver to synchronize. Then, we added white Gaussian noise to a channel. Therefore, in this case, the synchronization was not disturbed by noise. The experimental results are shown in Figs. 34 and 35, which indicate that the DS is weak in white Gaussian noise.

We also examined the impulsive synchronization method to transmit signals through noisy

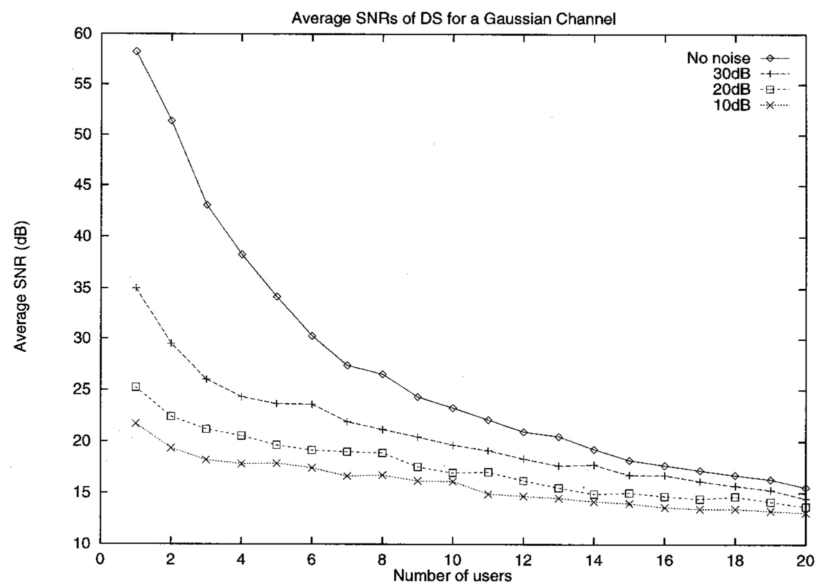


Fig. 34. Average signal to noise rations (SNRs) of DS. White Gaussian Noise is added to the channel.

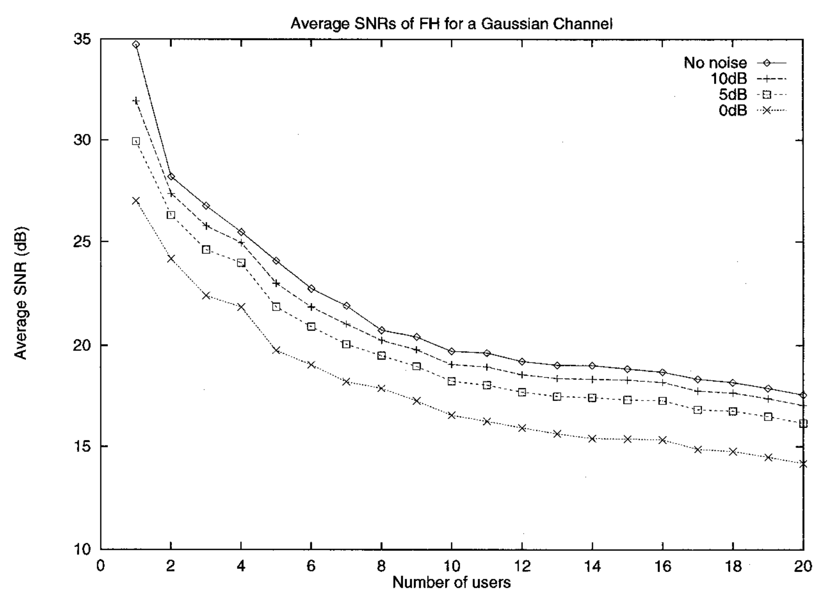
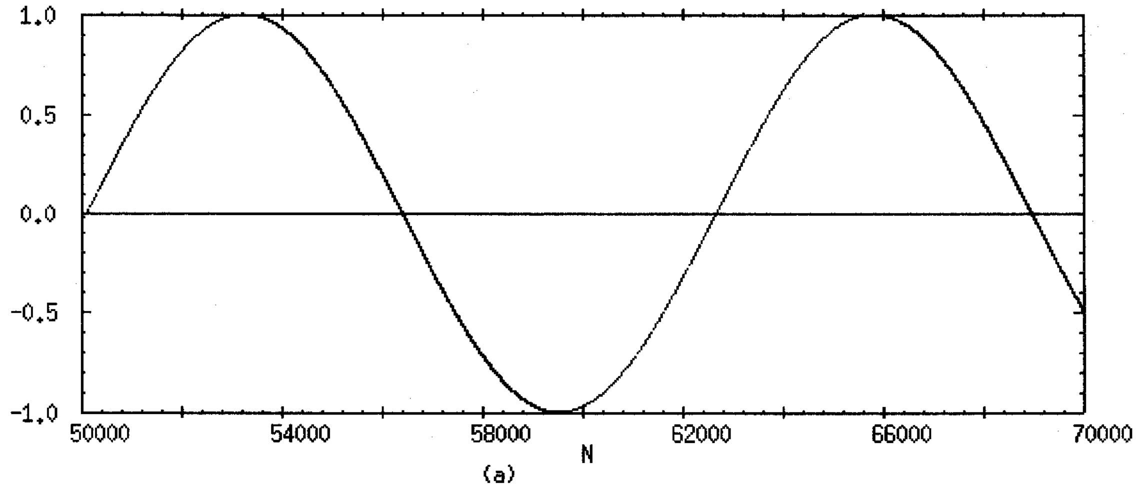
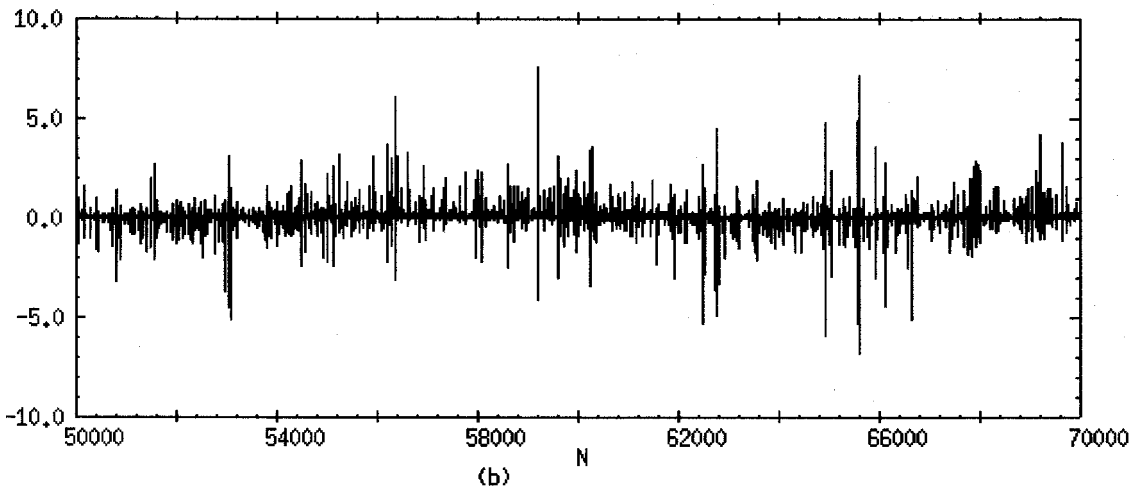


Fig. 35. Average signal to noise rations (SNRs) of FH. White Gaussian Noise is added to the channel.





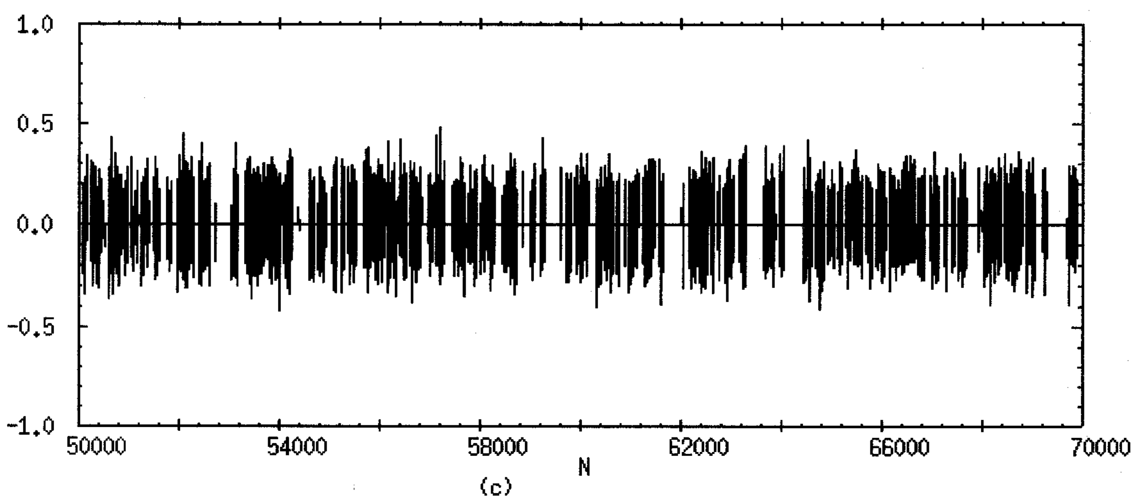


Fig. 36. Waveform of burst impulsive noise. (a) Information signal $s_1(t) = \sin(0.0005n)$, (b) modulated signal $p[n]$ of DS, (c) burst impulsive noise $w[n]$.

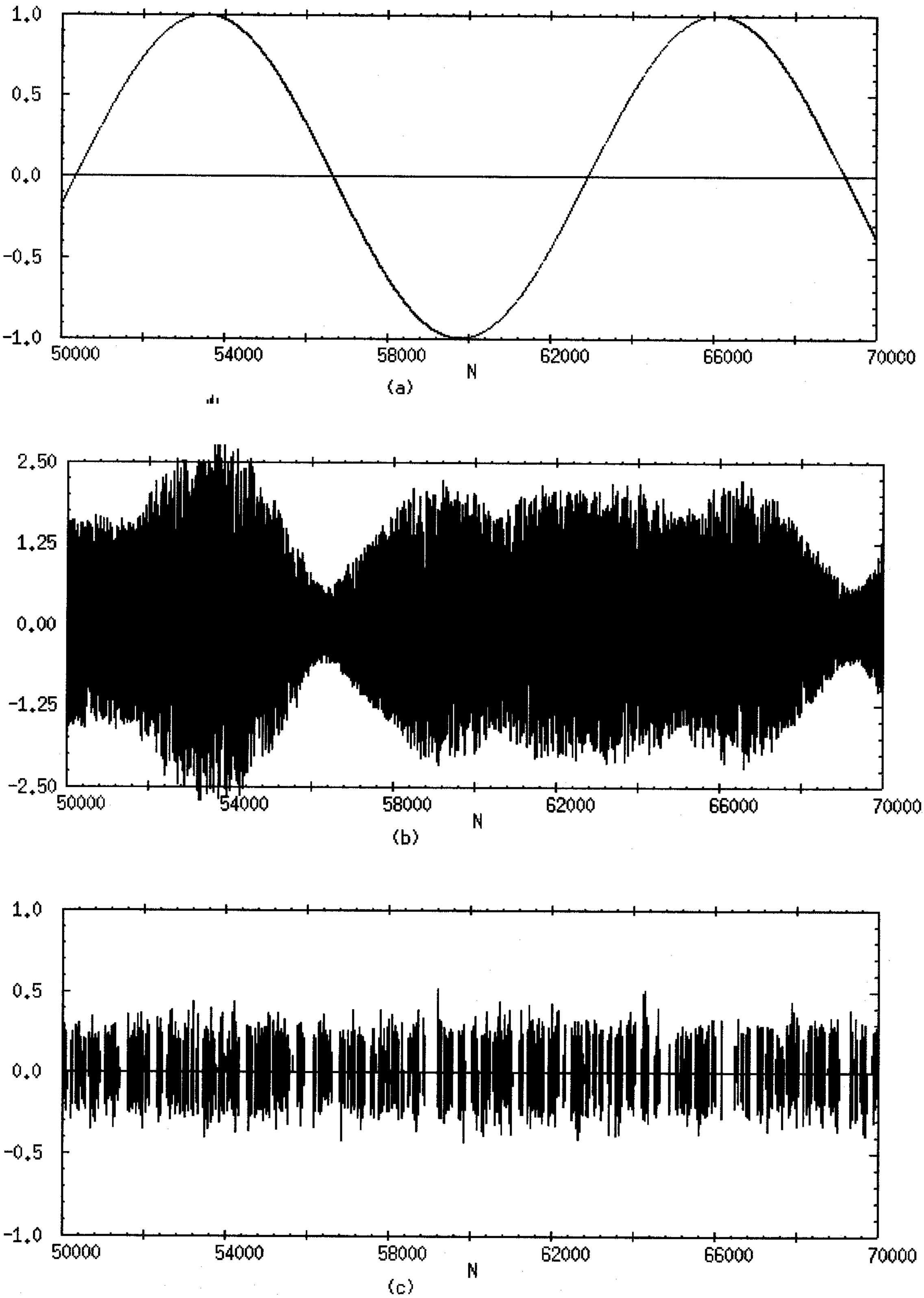


Fig. 37. Waveform of burst impulsive noise. (a) Information signal $s_1(t) = \sin(0.0005n)$, (b) modulated signal $p[n]$ of FH, (c) burst impulsive noise $w[n]$.

channels. It is known that the impulsive synchronization is robust to additive channel noise. In the case of the discrete-time dynamical systems, the synchronization error increases gradually as the iteration increases. This is because the synchronization impulses are continuously corrupted by white Gaussian noise. Furthermore, chaos has a strong and sensitive dependence on initial conditions. In short, tiny differences or errors in the initial conditions lead quickly to large differences in the discrete states. Hence, we need to study the noise suppression property under the other situations. In this paper, we verified it by using burst impulsive noise (see Figs. 36 and 37). In the absence of noise, there is a chance to make the chaotic system in the receiver to synchronize by using impulsive synchronization. Then, we can evaluate the SNRs of chaos-based DS and FH. The noise suppression properties are shown in Figs. 38 and 39. Their performances are almost as same as those for white Gaussian noise.

7.1.2. *Binary information data*

As stated before, we can transmit binary data by using the analog-based DS and FH. In this case, for each information bit, instead of one pulse, a sequence of N pulses are sent. Hence, N discrete-time pulses are sent to present the bit of information. The duration of data bit is partitioned into subintervals. The sub-bits are known as chips. The decision by majority was used to determine the bit information ($= \pm 1$) from N pulses.

Figures 40–43 show the transmission properties of the binary data. The bit errors increase as the number of users increases. Figure 44 shows the BERs of DS and FH when a single bit consists of 100 chips. Furthermore, we study the cases where a single bit consists of 200 and 300 chips. The FH has a good performance for the binary data transmission, and the BERs decrease quickly as the number of users increases as shown in Figs. 45 and 46. We examined the impulsive synchronization method to transmit signals through noisy channels. Figures 47 and 48 show the waveforms of the burst impulsive noises. Figures 49 and 50 show the suppression property for burst impulsive noise. The BERs of DS and FH decrease as the number of users increases.

7.2. *Continuous-time dynamical systems*

7.2.1. *Analog information data*

The average SNRs of DS and FH as a function of multiple users are shown in Fig. 51. Next, we examine the SNRs by using white Gaussian channel.

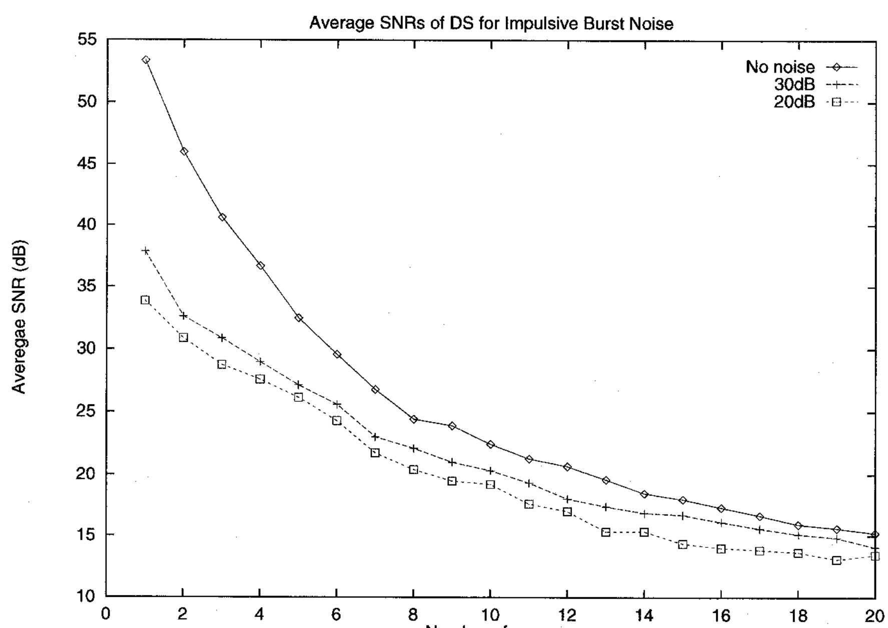


Fig. 38. Average signal to noise ratios of DS. The impulsive burst noise is added to the channel. The impulsive synchronization technique is used to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 1000$ and $Q = 100$, respectively.

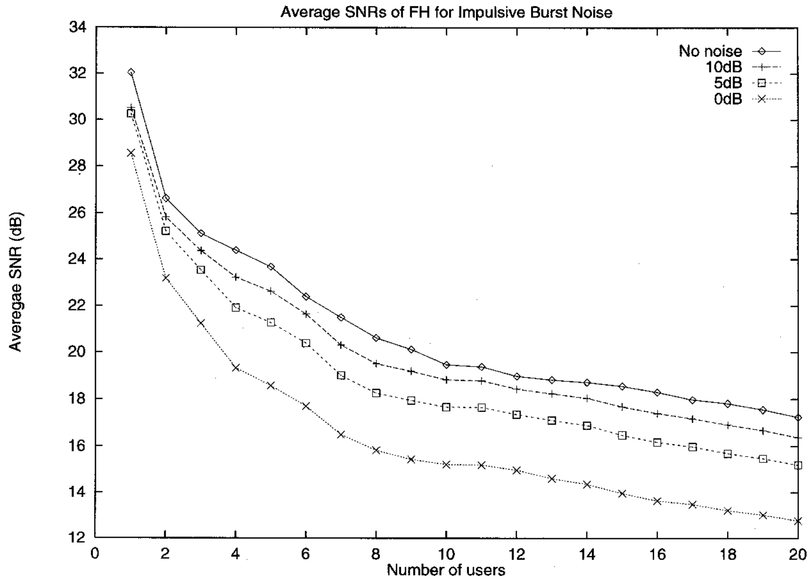


Fig. 39. Average signal to noise rations of FH. The impulsive burst noise is added to the channel. The impulsive synchronization technique is used to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 1000$ and $Q = 100$, respectively.

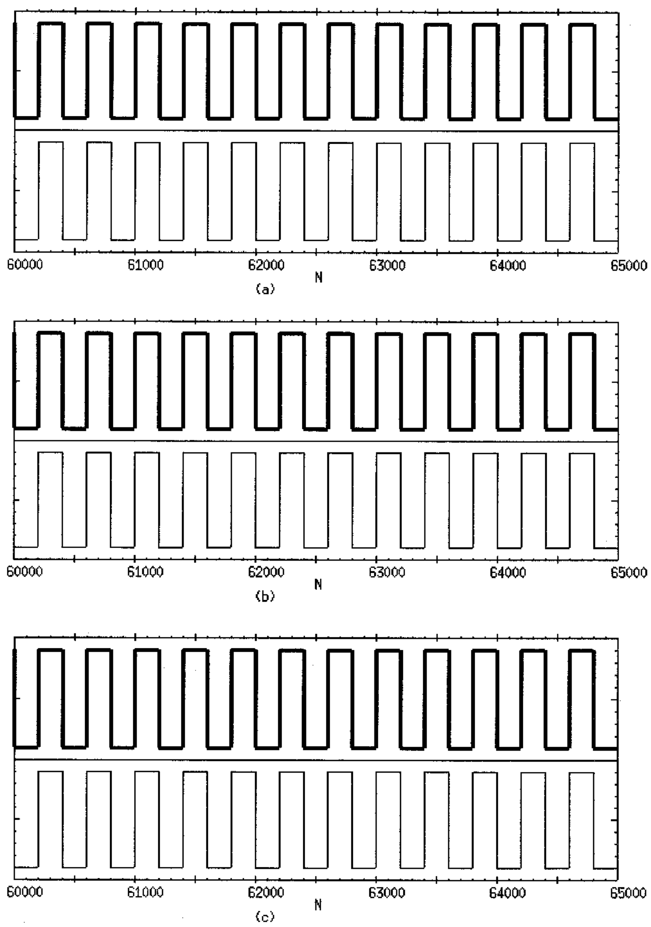


Fig. 40. Binary information signals (thick solid lines) and recovered signals of DS (thin solid lines). A single bit consists of 200 chips and five users are sharing a communication channel. The information signals are completely recovered.

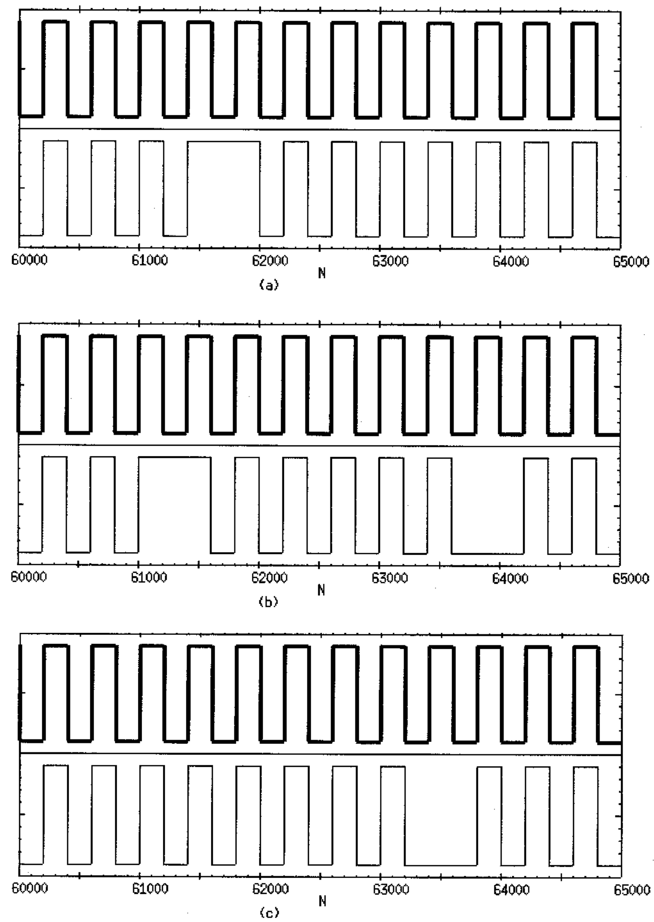


Fig. 41. Binary information signals (thick solid lines) and recovered signals of DS (thin solid lines). A single bit consists of 200 chips and eight users are sharing a communication channel. Some errors occurred in the recovered signals.

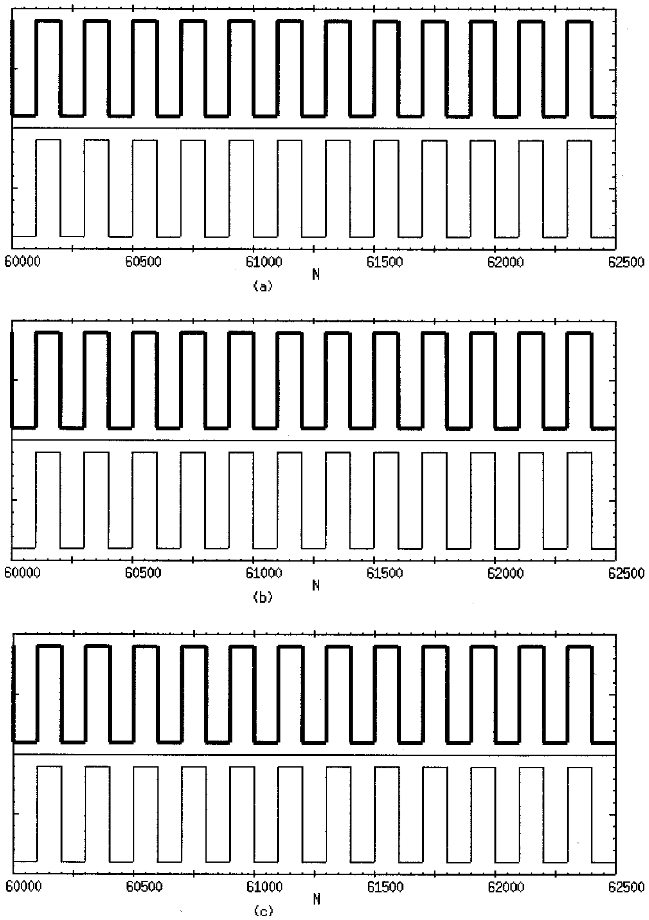


Fig. 42. Binary information signals (thick solid lines) and recovered signals of DS (thin solid lines). A single bit consists of 100 chips and five users are sharing a communication channel. The information signals are completely recovered.

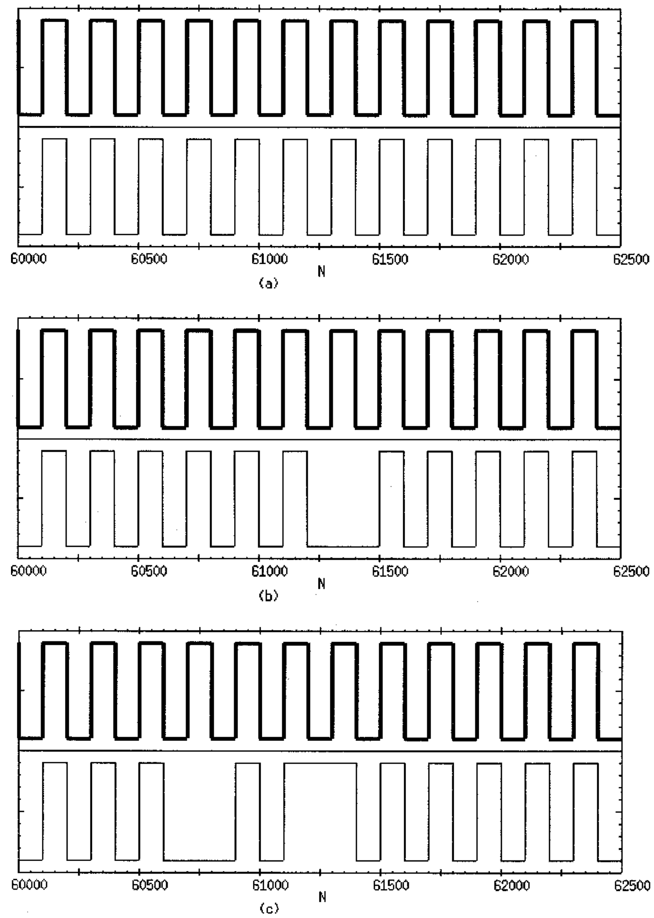


Fig. 43. Binary information signals (thick solid lines) and recovered signals of DS (thin solid lines). A single bit consists of 100 chips and fifteen users are sharing a communication channel. Some errors occurred in the recovered signals.

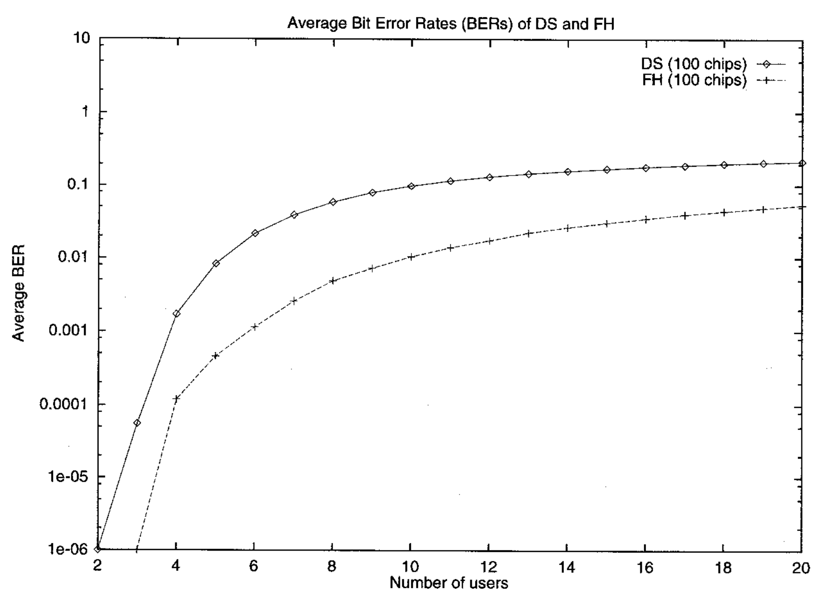


Fig. 44. Average bit error rates (BERs) of DS and FH.

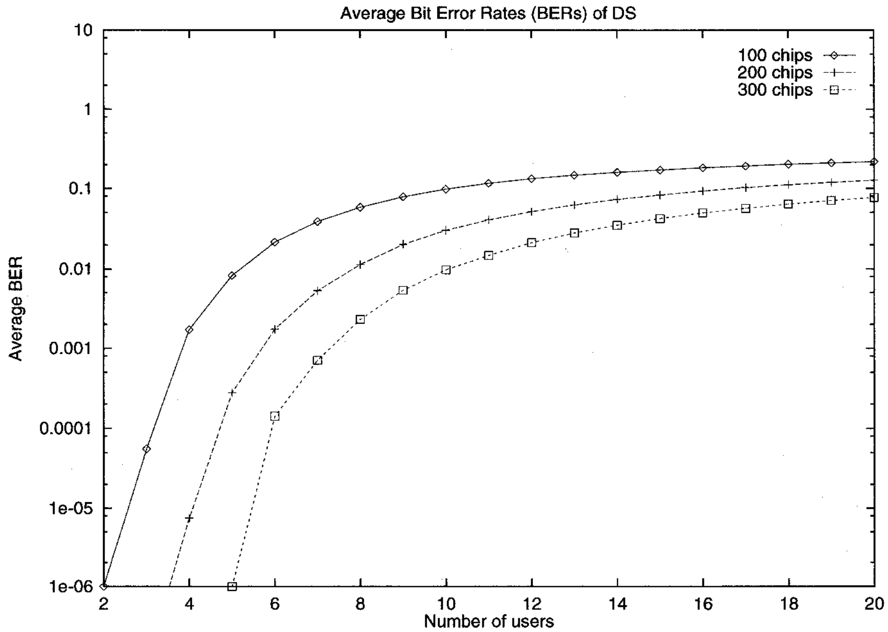


Fig. 45. Average bit error rates (BERs) of DS.

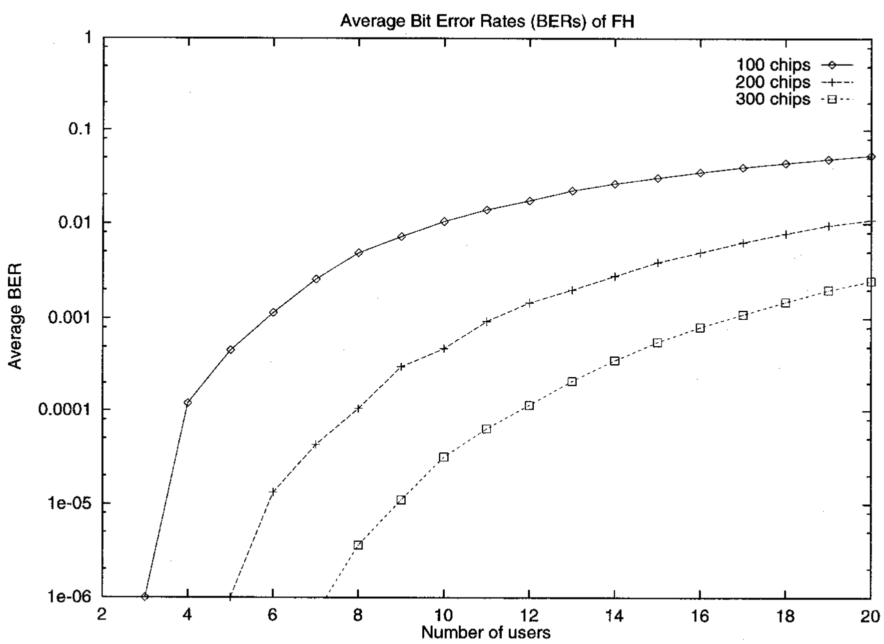


Fig. 46. Average bit error rates (BERs) of FH.

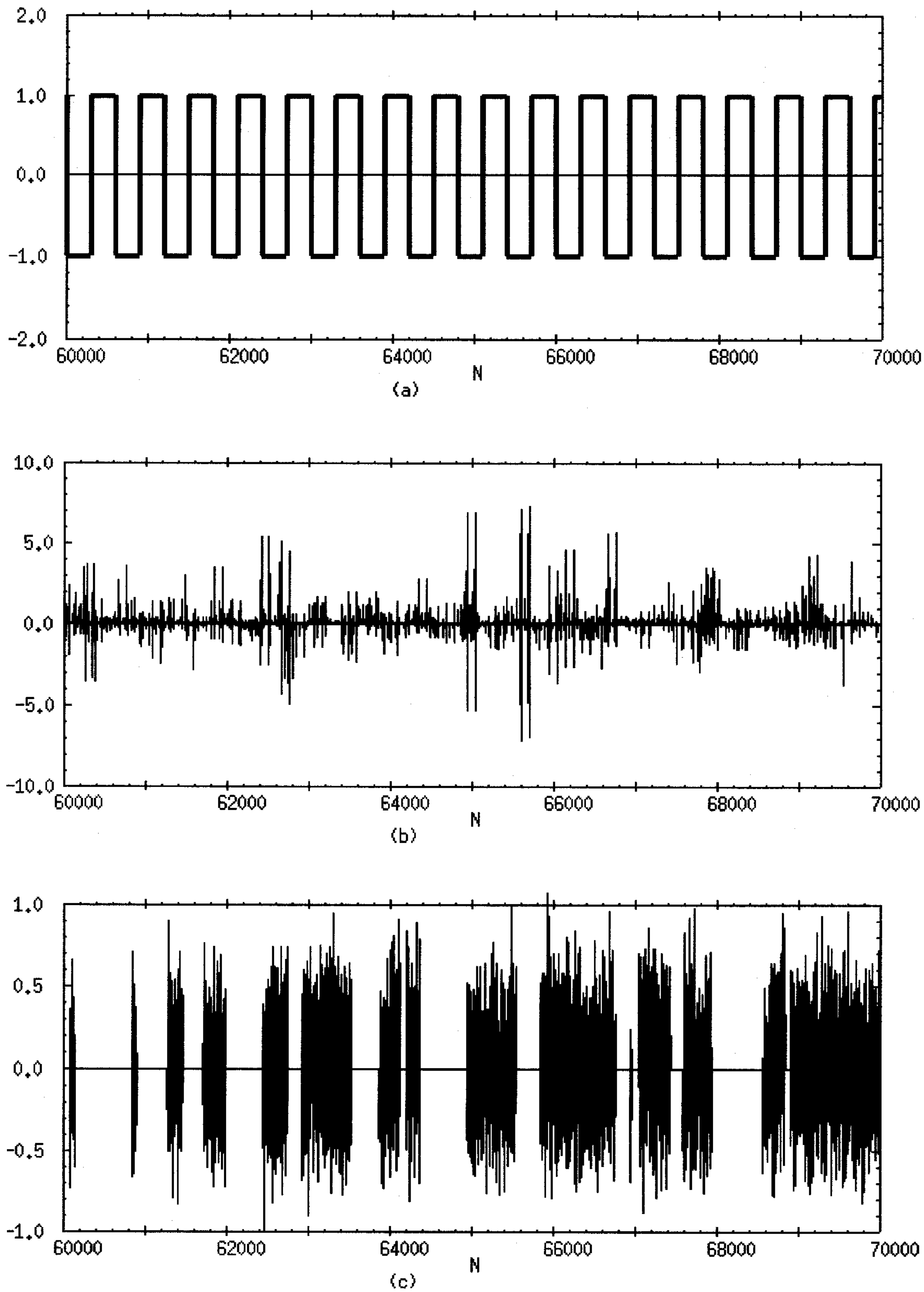


Fig. 47. Waveform of the burst impulsive noise of the channel. (a) Binary information signal $s_1[n]$, (b) modulated signal $p[n]$ of DS, (c) burst impulsive noise $w[n]$.

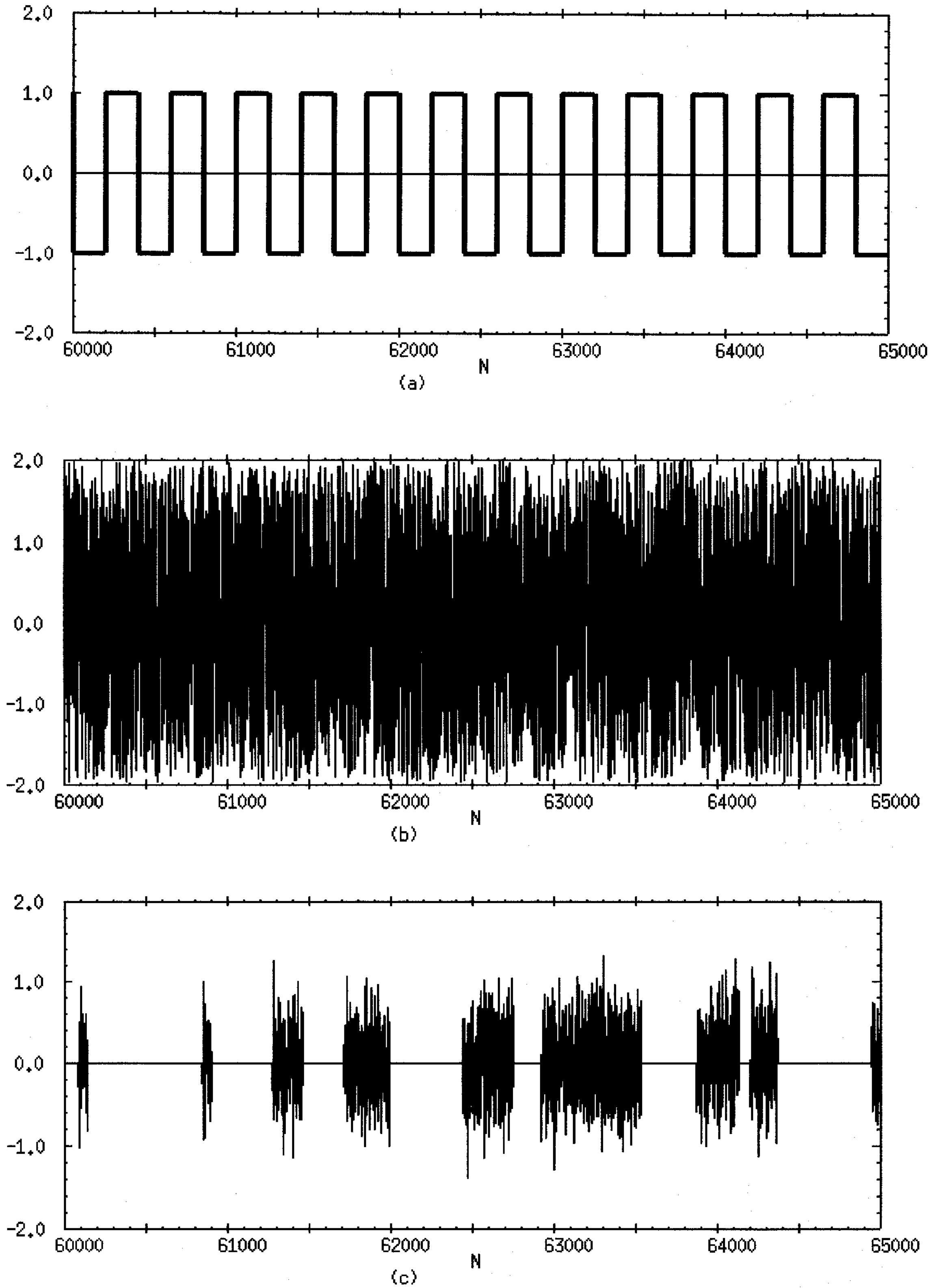


Fig. 48. Waveform of the burst impulsive noise of the channel. (a) Binary information signal $s_1[n]$, (b) modulated signal $p[n]$ of FH, (c) burst impulsive noise $w[n]$.

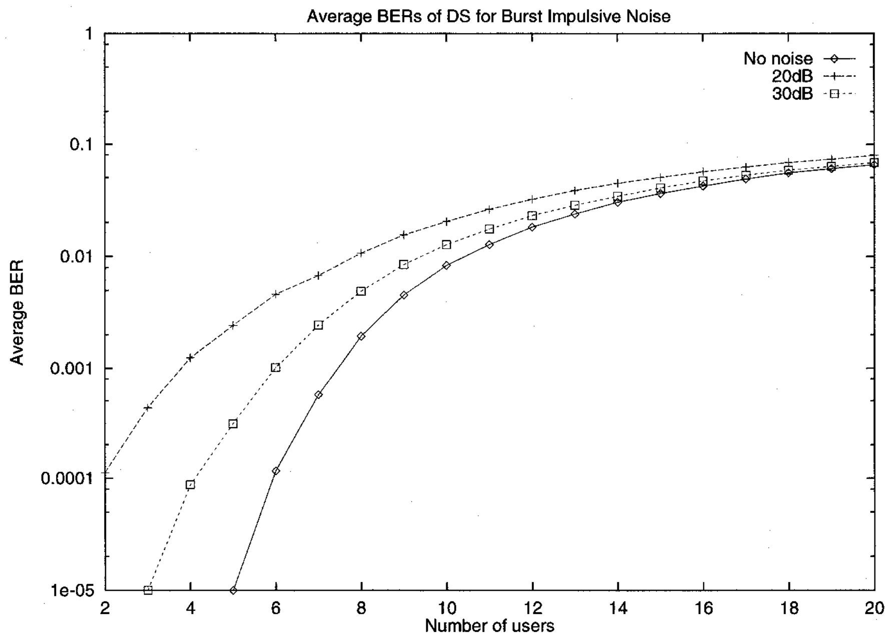


Fig. 49. Average bit error rates (BERs) of DS. The impulsive burst noise is added to the channel. We used the impulsive synchronization technique to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 300$ and $Q = 2$, respectively. (A single bit consists of 300 chips.)

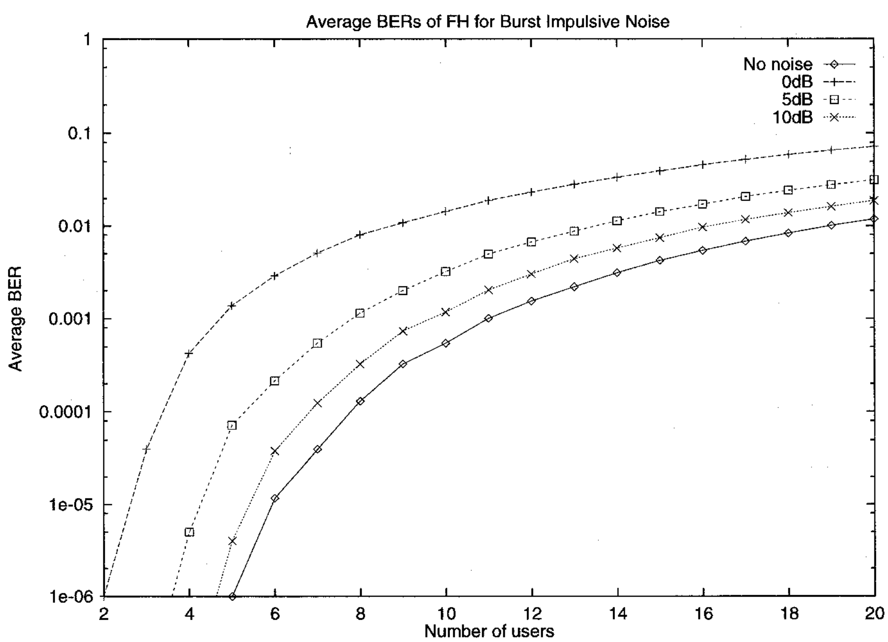


Fig. 50. Average bit error rates (BERs) of FH. The impulsive burst noise is added to the channel. We used the impulsive synchronization technique to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 200$ and $Q = 2$, respectively. (A single bit consists of 200 chips.)

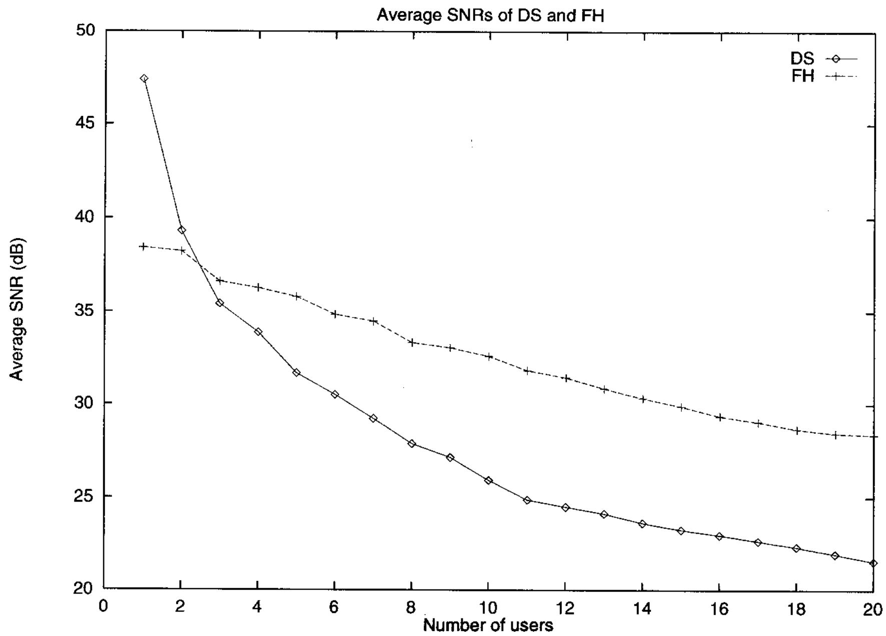


Fig. 51. Average signal to noise ratios (SNRs) of DS and FH.

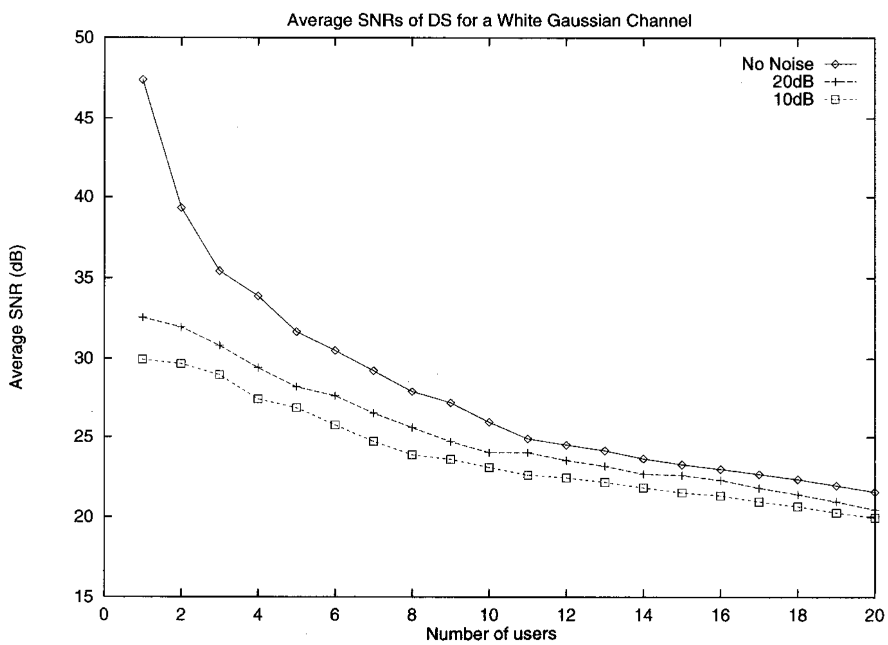


Fig. 52. Average signal to noise rations (SNRs) of DS. White Gaussian noise is added to the channel.

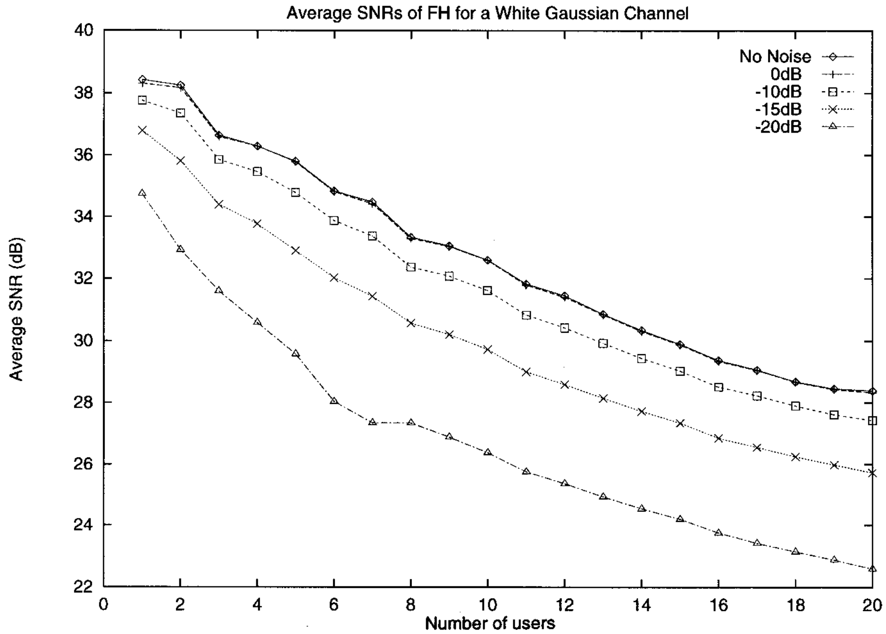


Fig. 53. Average signal to noise rations (SNRs) of FH. White Gaussian noise is added to the channel.

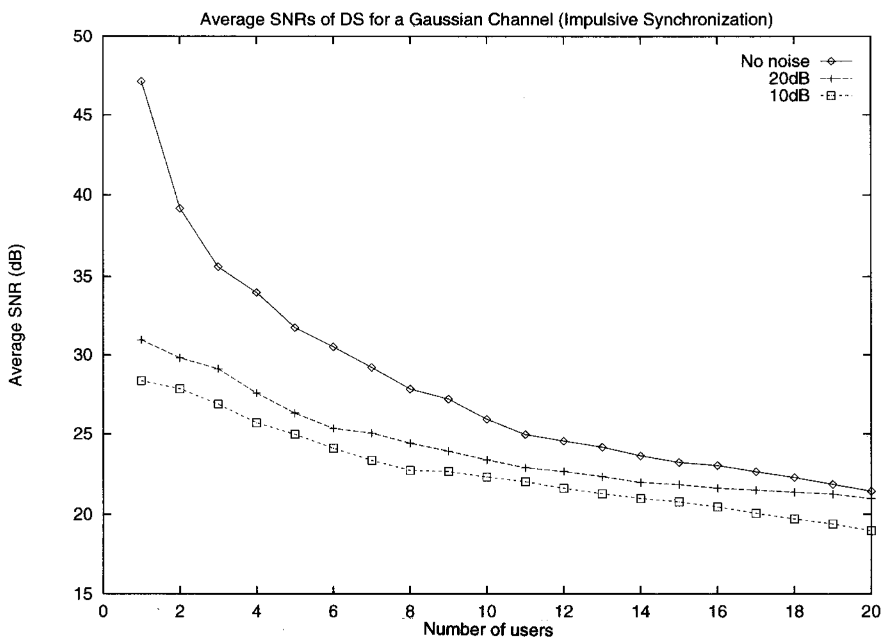


Fig. 54. Average signal to noise rations (SNRs) of DS. White Gaussian noise is added to the channel. We used the impulsive synchronization technique to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 1$ and $Q = 0.02$, respectively.

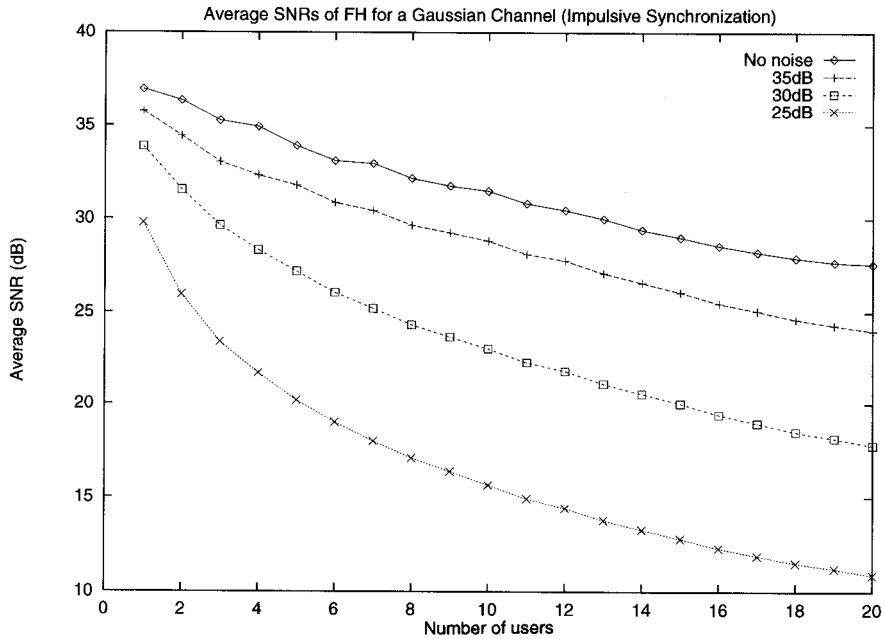


Fig. 55. Average signal to noise rations (SNRs) of FH. White Gaussian noise is added to the channel. We used the impulsive synchronization technique to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 1$ and $Q = 0.02$, respectively.

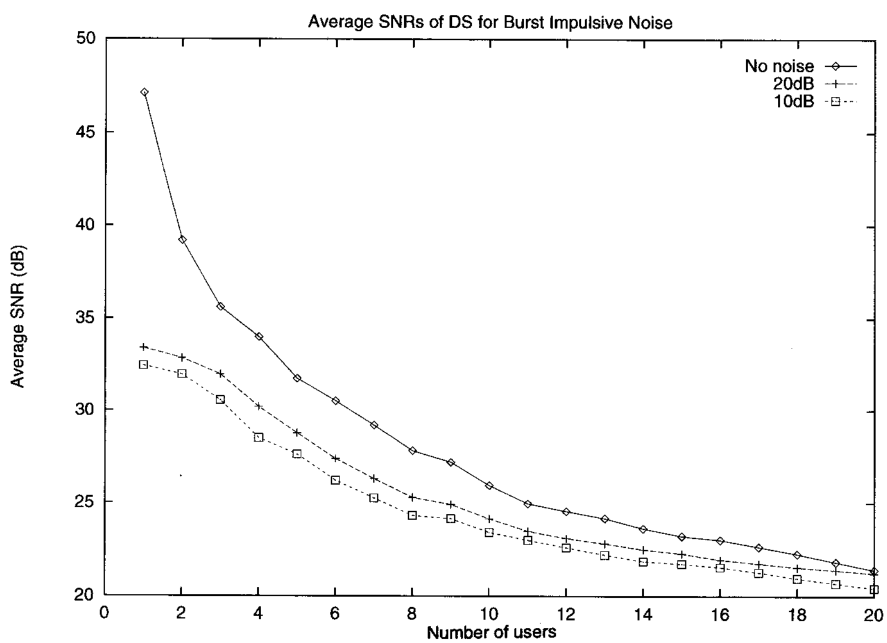


Fig. 56. Average signal to noise ratios (SNRs) of DS. The impulsive burst noise is added to the channel. We used the impulsive synchronization technique to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 1$ and $Q = 0.02$, respectively.

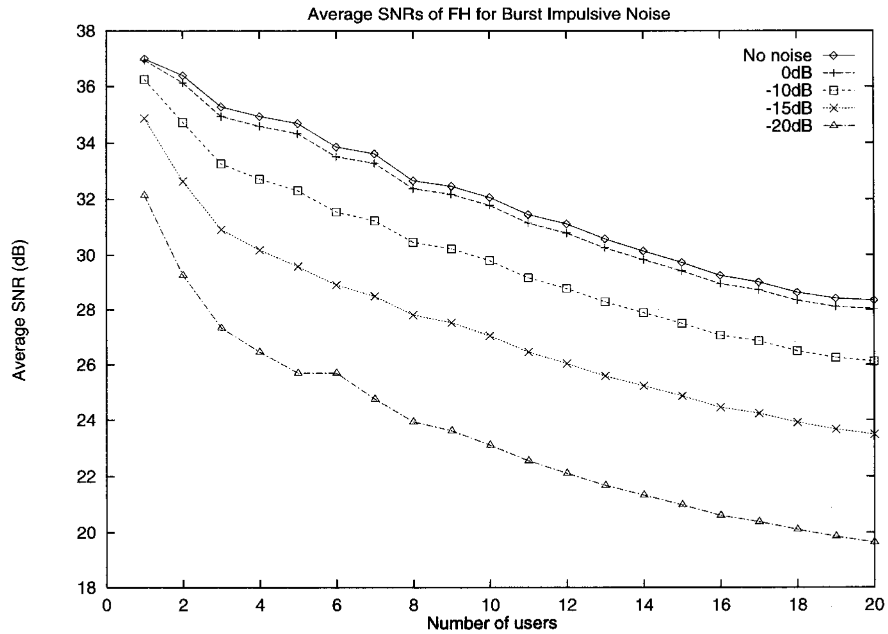


Fig. 57. Average signal to noise ratios (SNRs) of FH. The impulsive burst noise is added to the channel. We used the impulsive synchronization technique to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 1$ and $Q = 0.02$, respectively.

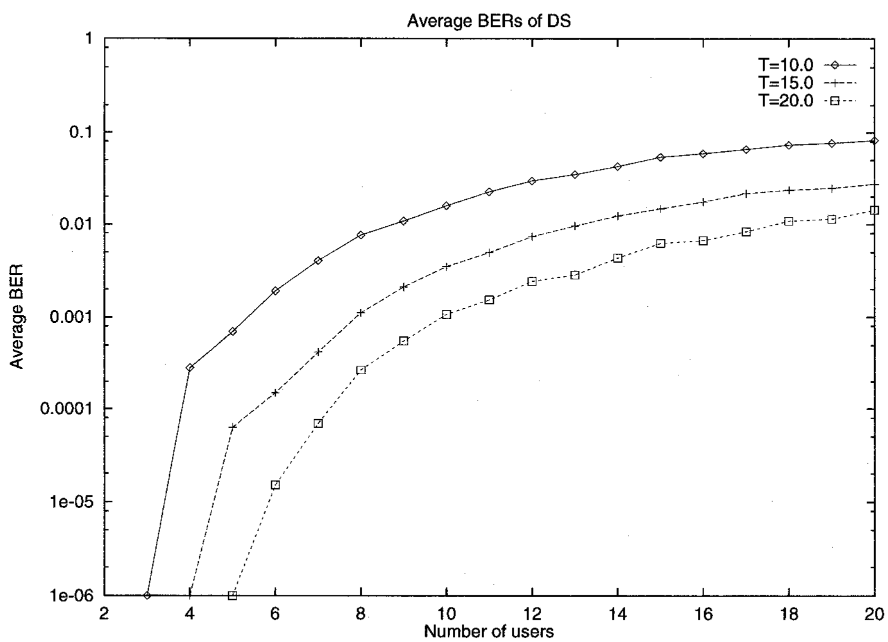


Fig. 58. Average bit error rates (BERs) of DS.

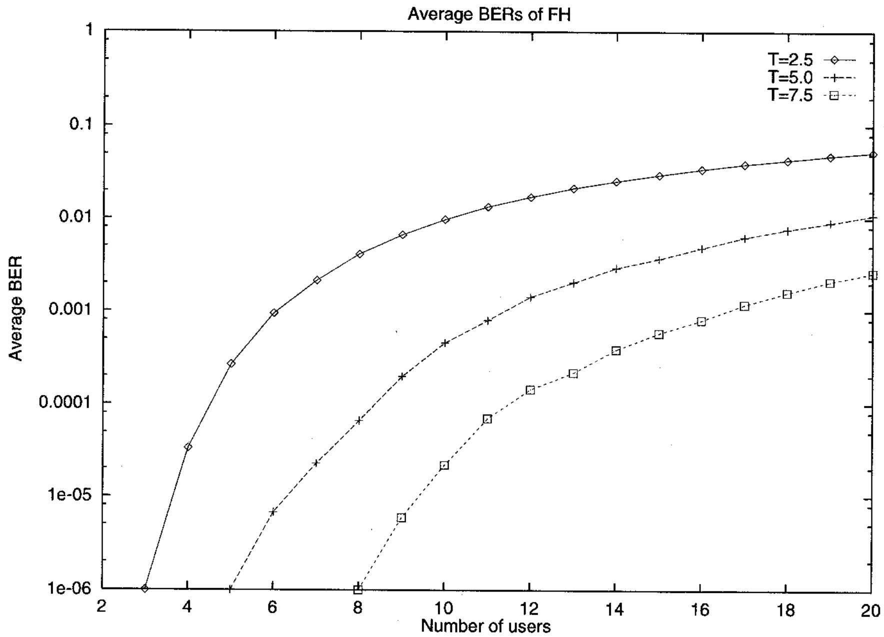


Fig. 59. Average bit error rates (BERs) of FH.

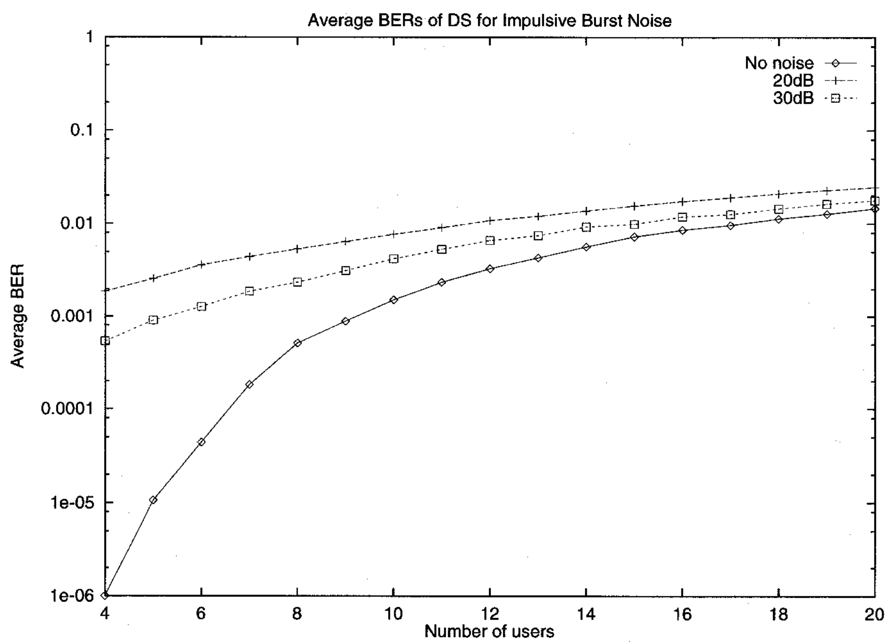


Fig. 60. Average bit error rates (BERs) of FH. The impulsive burst noise is added to the channel. We used the impulsive synchronization technique to make the system in the receiver to synchronize with that in the transmitter through a noisy channel.

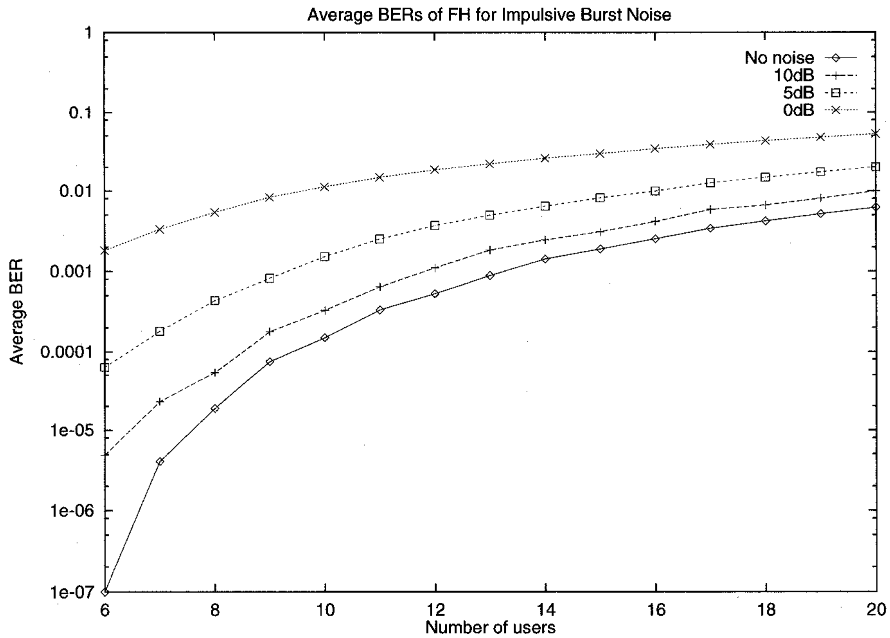


Fig. 61. Average bit error rates (BERs) of FH. The impulsive burst noise is added to the channel. We used the impulsive synchronization technique to make the system in the receiver to synchronize with that in the transmitter through a noisy channel. The frame length and the synchronization region are chosen as $T = 10$ and $Q = 4$, respectively. (The pulse duration is chosen as $T = 10$.)

First, by sending synchronization data, we made the chaotic system in the receiver to synchronize. Then, we added white Gaussian noise to the channel. Therefore, in this case, the synchronization was not disturbed by noise. The experimental results are shown in Figs. 52 and 53. We also examined the impulsive synchronization method to transmit signals through noisy channels. In the case of the continuous-time dynamical systems, the impulsive synchronization is robust to additive channel noise. However, the noise level is supposed to be sufficiently low to satisfy the high SNRs. The average SNRs for both white Gaussian noise and burst impulsive noise are illustrated in Figs. 54–57. The DS is weak for a burst impulsive noise. On the contrary, the FH is weak for a white Gaussian noise.

7.2.2. *Binary information data*

Figures 58 and 59 show the average BERs of DS and FH as a function of multiple users. In our computer study, three kinds of pulse durations are examined, that is, durations 10, 15, and 20 are examined for DS, and 2.5, 5.0, and 7.5 for FH. Furthermore, we verified the average BERs by using a burst impulsive noise. The computer results are given in Figs. 60 and 61.

8. Chaos-Based Time-Division and Frequency-Division Multiplexing

In addition to the code-division multiplexing (the direct sequence and the frequency hopping), there are two basic multiplexing techniques: One is frequency-division multiplexing (FDM), and the other is time-division multiplexing (TDM) [Carlson, 1975]. Recently, new chaos-based FDM and TDM techniques are proposed in [Itoh & Chua, 1997a]. In this section, these multiplexing techniques are discussed from the viewpoint of the chaos-based spread spectrum communication systems (chaos-based DS and FH systems).

8.1. *Chaos-based TDM*

Time-division multiplexing is a technique for transmitting several messages on one facility by dividing the time domain into slots, one slot for each message. The principle of TDM is described as follows [Carlson, 1975]: Several input signals, all bandlimited to W by lowpass filters, are sequentially sampled at the transmitter by a rotary switch. The switch makes one complete revolution in $T_s \leq 1/2W$, extracting one sample from each input. If there are M inputs, the pulse to pulse spacing is T_s/M. At the receiving side a similar rotary switch separates the samples and distributes them to a bank of lowpass filters, which in turn reconstruct the original message.

A key problem for TDM is identical revolution of rotary switches at both transmitting and receiving sides. Since chaotic dynamical systems inherently generate broadband spectrum signals or spread spectrum signals, the high frequency signal for rotating the switch is extracted by applying the chaotic sequences $\{x[n]\}$ to some highpass or bandpass filters (digital filters) [Itoh & Chua, 1997b]. These filters have the form of the difference equation

$$y[n] = \sum_{i=0}^{r} L_i x[n-i] - \sum_{i=1}^{m} K_i y[n-i]\,, \quad (120)$$

where the sequence $\{y[n]\}$ is the output of the filter and L_i, K_i are some constants. That is, the delayed chaotic sequence is used to create the revolution signals. Since the output sequence $\{y[n]\}$ is chaotic, the switching interval is also chaotic. However, if we use an identical revolution at the transmitting and receiving sides, that is, if we can use transmitter's chaotic sequences, the demodulation is possible. There are a number of ways to rotate the switches. For example, if the K users share the channel, then the revolution sequence is given by

$$p[n] = s_k[n]\,, \quad (121)$$

$$k = \left(\sum_{m=0}^{n} \frac{y[m] + |y[m]|}{2|y[m]|} + 1\right) (\mathrm{mod}\ K) + 1\,, \quad (122)$$

where $s_k[n]$ is the information signal of the kth user. Decoding is done by calculating the number k from $y[n]$ and sending $p[n]$ to the kth output.

If we replace $\{y[n]\}$ with the sequence $\{\sin n\omega |\, \omega = \text{const.}\}$, then the standard TDM system is obtained.

8.2. *Chaos-based FDM*

The principle of FDM is described as follows [Carlson, 1975]: Several bandlimited input messages individually modulate the subcarriers f_1, f_2, etc. The modulated signals are then summed to produce the baseband signal $x_b(t)$. The baseband signal may then be transmitted directly or used to modulate a transmitted carrier frequency f. The demodulation of FDM is accomplished in three steps. First, the carrier demodulator reproduces the baseband signal $x_b(t)$. Then the modulated subcarriers are separated by a bank of bandpass filters in parallel, and the messages are individually detected.

In FDM, several input messages individually modulate the subcarriers. By applying the chaotic sequence $\{x[n]\}$ to some highpass filters or bandpass filters,

$$y^{(j)}[n] = \sum_{i=0}^{r} L_i^{(j)} x[n-i] - \sum_{i=1}^{m} K_i^{(j)} y^{(j)}[n-i]\,, \quad (123)$$

we get the chaotic subcarriers $y^{(j)}[n]$ which are separable in the frequency domain [Itoh & Chua, 1997b]. The modulator consists of multiplication of subcarriers $y^{(j)}[n]$ by $s_j[n]$

$$p[n] = \sum_{j=i}^{K} y^{(j)}[n] s_j[n]\,. \quad (124)$$

Decoding is done by averaging $q[n] = p[n] y^{(j)}[n]^{-1}$ or applying $q[n]$ with lowpass filters. The information signal can be recovered, if some identical subcarriers are obtained at the receiving side. Considering the property of the chaos-based FH and DS, we don't need the completely separable carriers in the frequency domain. Hence, the chaos-based FDM is considered to be a special case of the chaos-based FH. Furthermore, if we replace $\{y^{(j)}[n]\}$ with the sequence $\{\sin n\omega^{(j)} |\, \omega^{(j)} = \text{const.}\}$, then the standard FDM system is obtained.

8.3. *Continuous-time version of chaos-based TDM and FDM*

In case of continuous-time dynamical systems, the chaotic carriers $y^{(j)}(t)$ for TDM and FDM are extracted from a chaotic signal $x(t)$ by using the following differential equations:

$$\frac{d^m y^{(j)}}{dt^m} = \sum_{i=0}^{r} A_i^{(j)} \frac{d^i x}{dt^i} - \sum_{i=0}^{m-1} B_i^{(j)} \frac{d^i y^{(j)}}{dt^i}\,, \quad (125)$$

where $A_i^{(j)}$ and $B_i^{(j)}$ are constants. That is, the lowpass filters and bandpass filters are realized by using these equations. Therefore, we can derive the continuous-time version of chaos-based TDM and FDM easily. Their details and computer simulations are given in [Itoh & Chua, 1997a, 1997b].

9. Conclusion

We have proposed a new scheme for spread spectrum communication which transmits both analog and binary data by using chaotic carriers. Many spread spectrum communication systems are using the delayed chaotic sequences to create carriers. Our computer studies show that some systems have a SNR of at least 20 dB and a BER of less than 0.001 for ten users.

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