Squeal vibrations, glass sounds and the stick-slip effect

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Abstract: The origin of the squeal acoustic emissions when a chalk is rubbed on a blackboard or better on a ceramic plate, and those when a wet finger is rubbed on a smooth surface, such as a glass surface, is sought in the stick-slip effect between the rubbing surfaces. The elastic agency is sought in a shear band between the two surfaces characterized by very low shear modulus. In the case of the squealing chalk, it can be argued that the shear band is a layer of chalk powder, about 0.3 mm thick, forced to slide over the ceramic plate surface. In the case of the wet finger on a glass surface, it can be argued that the shear band is the layer of soft tissue between the epidermis and the finger bone, and that the water layer simply facilitates the stick-slip effect.

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1. **Introduction**

In a previous study of the acoustic emissions from a certain type of silica gel grains, placed in a small container, it is shown that a light touch by a brass pestle rounded to a 9 mm diameter sphere at the end, can produce a very pleasant sound with dominant frequency, $f_d \approx 250$ Hz, Patitsas [1]. Such grains are used for humidity control purposes and are characterized by; average size about 0.4 mm, large size distribution, extreme angularity and the appearance of being more flat than rounded. With an estimated impact velocity of about 10 cm/s, there was no visible evidence of grains bouncing off other grains on the surface grain layer, implying that this is also the case in the layers below the surface. Thus, it is safe to assume that the grain motion is mainly characterized by grains sliding over one another. It is argued in [1] that a singing grain mass at rest, due to some yet unknown grain surface effect, is characterized by a relatively high level of rigidity. Thus, when the pestle is driven into the grain mass, singing sand or silica gel, the stress level between the rubbing grains builds up to the level where the surface state at the contact areas is drastically altered (liquidized). This results in a well defined shear band (boundary layer), comprising several grain layers, around the leading front of the pestle, characterized by a very low modulus of rigidity (shear modulus). The recent report by Braeck and Podladchikov [2] supports such a hypothesis. The authors argue that a thermal runaway in viscoelastic materials can lead to a highly localized shear band. Moreover, it is argued in [1] that the acoustic and seismic emissions are due to shear modes of vibration in such a boundary layer. Consequently, the question arises as to whether the chalk squeal emissions, or the wet finger on glass musical emissions, could be due similarly to vibrations in a shear band at the contact area. However, there has to be an agency that would result in the excitation of such modes of vibration, and the only such agency appears to be that of the stick-slip effect. Such an effect implies that the friction coefficient between the shear band and the substrate decreases with increasing relative velocity between the two surfaces. The example that follows, based on the spring-block system on a moving platform, appears to best elucidate the stick-slip effect.

2. The stick-slip effect; The platform driven block

Figure 1a depicts block, B, with mass m sliding over the moving platform. It is attached to a fixed wall through a weightless spring with spring constant k. Its position is specified by the distance y from the equilibrium point O, where the spring force is zero when the platform is at rest. The relative velocity, v_r, between the platform and the block is, $v_r = V_P - \dot{y}$, where V_P is the constant velocity of the platform to the right. The net force acting on the block can be written as, $-ky - r\dot{y} + \mu mg$, and thus the acceleration of the block can be written as, $\dot{y} = -yk/m - r/m\dot{y} + \mu g$, where $-r\dot{y}$ represents a dashpot damping effect according to McMillan [3]. It is assumed that the relative velocity, v_r, remains always greater than zero, *i.e.*, the platform always pulls the block to the right. Unlike the studies by Heckl and Abrahams [4], Heckl [5], for example, there is no attempt here to examine the conditions that would or would not lead to squeal emissions. The condition that the relative velocity, $v_r = V_p - \dot{y}$, remains always positive, *i.e.*, the block velocity amplitude never exceeds the platform velocity, V_p, appears to be consistent with the plots in Fig. 8 in [4], where the disc velocity is plotted versus time. The crabbing velocity of the disc is interpreted to be equivalent to the platform velocity here.

Fig. 1b depicts the kinetic coefficient of friction, μ, as a function of v_r. Both diagrams in Fig. 1 are based on those in the paper by Rabinowicz [6]. The relatively simple dependence of μ on v_r and the absence of any hysteresis effect considerations may be justified on the basis of the relative softness of the shear band of the chalk powder rubbed on a ceramic plate. Furthermore, according to McMillan [3], the Coulomb law with $F_p = mg\mu$ ought to apply when v_r is not too small. It is assumed that μ varies linearly with v_r in the interval $v_B - v_F$, *i.e.*,

$$\mu = -av_r + \gamma = -a(V_P - \dot{y}) + \gamma = \gamma - aV_P + a\dot{y} \tag{1}$$

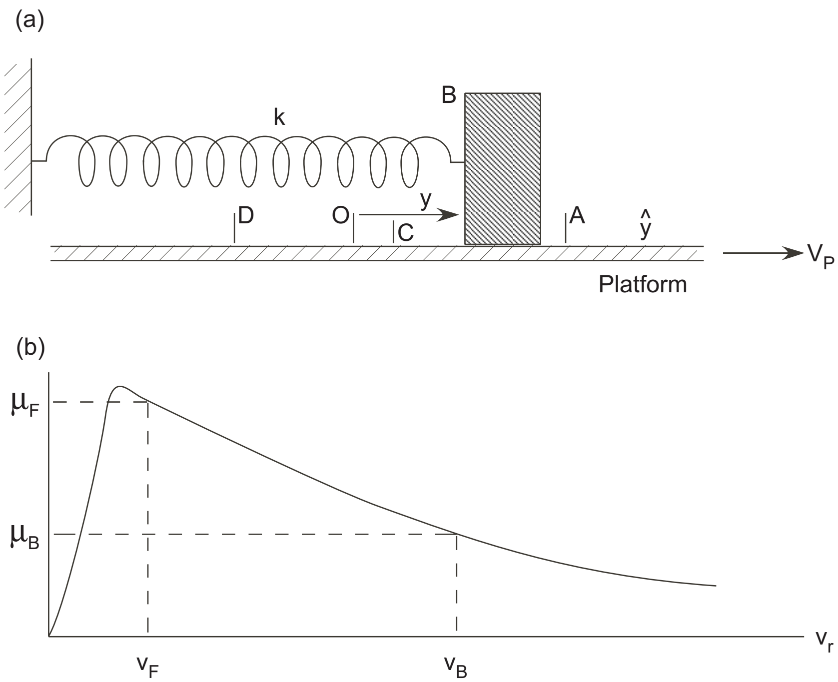
Thus, the equation of motion becomes, $\ddot{y} - ga\dot{y} + r\dot{y} + yk/m = g\gamma - gaV_P$, having the solution, $y = y_c + y_p$, where the complimentary and particular solutions are,

$$y_c = G\mathrm{e}^{((ga-r)/2)t}\cos(\omega t + \theta), \qquad y_p = (mg/k)(\gamma - aV_P) \tag{2}$$

where

$$\omega^2 = k/m - ((ga - r)/2)^2 \tag{3}$$

Fig. 1. (a) The platform moves to the right with constant velocity V_P. The block B is attached to the wall through the weightless spring, with spring constant k, and oscillates between points D and A, equidistant from point C. At point O, the spring force on the block is zero and the block position is specified by the coordinate y.
(b) Variation of the kinetic coefficient of friction, μ, between the block and the moving platform as function of the relative velocity, $v_r = V_P - \dot{y}$. The extreme relative velocities, when the block moves to the right and to the left, are labeled as, v_F and v_B.



In Fig. 1a, $y_p = OC$ and the amplitude of oscillation, $G\mathrm{e}^{((ga-r)/2)t}$, is equal to CA. The effect of the slope, a, in (1) is to reduce the elastic strength of the spring, *i.e.*, the spring constant k is replaced by the effective spring constant, $k_s = k - m((ga-r)/2)^2$. It will be seen below that there can be no oscillatory motion when a=0, since all the power transfered to the block by the platform is dissipated as heat. The stick stage occurs when the block moves to the right in the neighborhood of the point C and the slip stage occurs when the block moves to the left in the same neighborhood. More exactly, if the left side of the block is at C and moves to the right at $t = 0$, then, $y_c = G\mathrm{e}^{((\mathrm{ga}-\mathrm{r})/2)\mathrm{t}}\sin(\omega\mathrm{t})$. Then,

$$\dot{y} = G\mathrm{e}^{((ga-r)/2)t}\omega\sqrt{1+((ga-r)/2\omega)^2}\cos(\omega t - \phi) \tag{4}$$

where $\cos\phi = (1 + ((ga - r)/2\omega)^2)^{-1/2}$. Thus, the maximum value of $\dot{y}$, that corresponds to the value v_F in Fig. 1, occurs when the block has just passed the center point C on its way towards the point A. Similarly, the maximum value of $\dot{y}$ in the negative direction, that corresponds to the value v_B in Fig. 1a, occurs when the block has just passed the point C. An estimate of the dependence of ω on the platform velocity V_P can be obtained as follows; From Fig. 1b, the slope a is equal to, $(\mu_F - \mu_B)/(v_B - v_F)$. Since v_F could be close to zero and v_B could be close to $2V_P$, it is reasonable to write, $v_B - v_F \approx V_P$. Therefore, ω increases weakly with increasing V_P.

The force exerted by the platform on the block is, $F_p = mg\mu$. Then, from (1, 4), F_p can be written as,

$$F_p = mg(d - aV_P) + mgaG\mathrm{e}^{((ga-r)/2)t}\omega\sqrt{1 + ((ga - r)/2\omega)^2}\cos(\omega t - \phi) \tag{5}$$

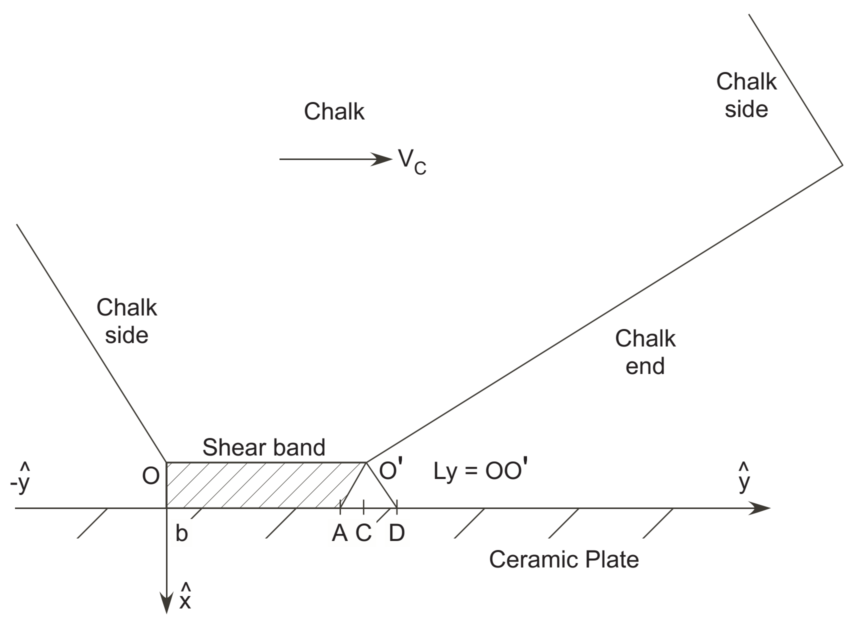
The power delivered to the block by the platform is, $P_B = F_pV_P = mg\mu V_P$ and the power dissipated as heat is, $P_H = F_pv_r = mg\mu V_P - mg\mu\dot{y} = P_B - P_O$, where $P_O = mg\mu\dot{y}$ is the power delivered to the oscillatory motion. It follows that the average value of P_O is zero, implying no oscillatory motion if μ is constant, *i.e.*, if the slope $a = 0$. The second term in (5) implies the presence of a *resonance effect* where the block is driven by a sinusoidal force with frequency equal to the natural frequency of oscillation of the spring-mass system. In other words, the mode of vibration of the spring-mass system anchors the stick-slip effect. In more complex systems, it could be argued that the most stable mode anchors the same effect. Such a mode is usually the fundamental, but in some cases, as can be seen at the end of Section 5, the first harmonic can play such a role. In the literature, such an effect is described as "self-excited vibration" [3].

3. Squeal sounds

3.1. The squealing chalk

Squeal sounds can be produced fairly easily when an ordinary chalk, length $L_c \approx$80 mm, diameter, D_c=9 mm, is forced to rub on a ceramic (clay) plate. For best results, the chalk is held about 30^o from the vertical direction when the plate lies on a horizontal plane, Fig. 2. Invariably, the chalk has to be rubbed several times, until a faint coat of chalk powder is visible on the plate surface, before the squeal sound can be produced. The contact area, when the squeal is produced, amounts to about 10 mm^2 and its shape appears to be irrelevant.

Fig. 2. The chalk, diameter 9 mm, is held at an angle of about 30^o from the vertical direction and is pushed to the right along $\hat{y}$ with velocity V_C against the ceramic plate on the horizontal yz plane. The shear band, responsible for the acoustic emission, has dimensions; $L_y \approx 3$ mm, $L_z \approx 4$ mm and estimated thickness $b \approx 0.3$ mm. The end of the shear band oscillates between the lines $O'A$ and $O'D$ during the acoustic emission in the same sense the block oscillates between points A and D in Fig. 1. $b \ll OO'$ and $AD \ll b$.



One of the most sinusoidal microphone recorded squeal signals is shown in Fig. 3. It was produced by rubbing a 56 mm long chalk on a relatively small irregularly shaped ceramic plate held lightly by the other hand on a soft foam. This plate, measuring roughly 30×25 mm by 5 mm thick, resulted from the breaking up of the base of a flower pot and it comprises part of the base rim with thickness close to 10 mm. Thus, it is highly unlikely that the sharp peak at about 2050 Hz and its second faint harmonic could be due to vibrations in such a plate. There is no explanation as to why the mode corresponding to the first harmonic was not excited. When the plate was tapped with a 12 mm glass bead, there was no sifnificant frequency content above 1500 Hz. An ordinary pin microphone was utilized a few cm from the contact area. The signal was processed using the NI USB-6210 analogue to digital converter and analyzed using the Labview Signal Express of National Instruments. Furthermore, similar acoustic emissions could be obtained by rubbing a chalk on larger ceramic plates. Such a geophone recorded signal can be seen in Fig. 4 where an 80 mm chalk was rubbed on the inner side of a relatively large flower pot. The microphone recorded signal, shown in Fig. 5, was produced when a full chalk was rubbed on a fairly flat area on an irregularly shaped rock with average diameter equal to about 50 mm. These and other emissions from various other plates provide sufficient evidence that the vibrations responsible for such squeal emissions are not to be found in the objects on which the chalks were rubbed. Consequently, there remains to establish whether such vibrations are to be found in the sheared chalk.

No squeal emission could be produced when the chalk length was reduced to about 40 mm or less. This could be due to excessive damping by the skin when the chalk piece was totally surrounded by the fingers. When chalk pieces shorter than 40 mm were held by a pair of ordinary pliers, the squeal sound was readily produced (Fig. 6), even when the short piece was obtained by splitting the chalk along its length (Fig. 7). It is noteworthy that f_d is only 1100 Hz in Fig. 6, *i.e.*, nearly half that in the three previous figures. In Fig. 7, $f_d \approx 1650$ Hz and there is also low frequency content. From what follows in Section 4, it can be argued that the lower values of f_d are mainly due to larger shear band thickness (Fig. 2) and also due to larger loading of the shear band. The frequency spectra in both these figures support the hypothesis that the acoustic emissions are not due to vibrations in the chalk but rather due to vibrations in a chalk layer, the **shear band**, as depicted in Fig. 2. Such a conclusion is contrary to the established convention found in the literature. For example, in the case of a train wheel rounding a curved track, the squeal emission is sought in the modes of vibration of the wheel [4]. Most likely, the very intense acoustic emission is due to the excitation of one or more modes of vibration in the wheel. However, the excitation of such a mode could originate with the excitation of a mode of vibration in a very thin shear band on the sheared surface of the track. A bronze bowl, diameter about 20 cm at the rim, has to be rubbed with a special brush along its exterior top surface in order for intense vibrations to be induced in the bowl. Most likely, the special brush best facilitates the stick-slip effect that leads directly to the excitation of a given mode in the bowl, without the intermediary excitation of a shear mode in the brush.

Fig. 3. Frequency spectrum and the microphone recorded squeal signal when a 56 mm long chalk was rubbed on a small irregularly shaped ceramic plate, roughly 30×25 mm by 5 mm thick. $f_d \approx$2050 Hz.

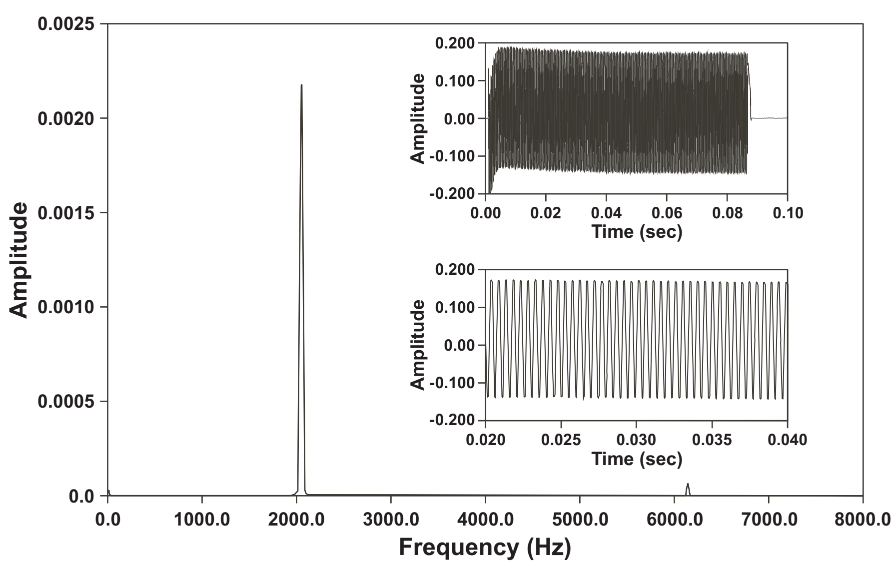


Fig. 4. Frequency spectrum and the geophone recorded signal when an 80 mm chalk was rubbed on the inner upper wall of a large flower ceramic pot, upper diameter 22 cm and depth 9 cm.

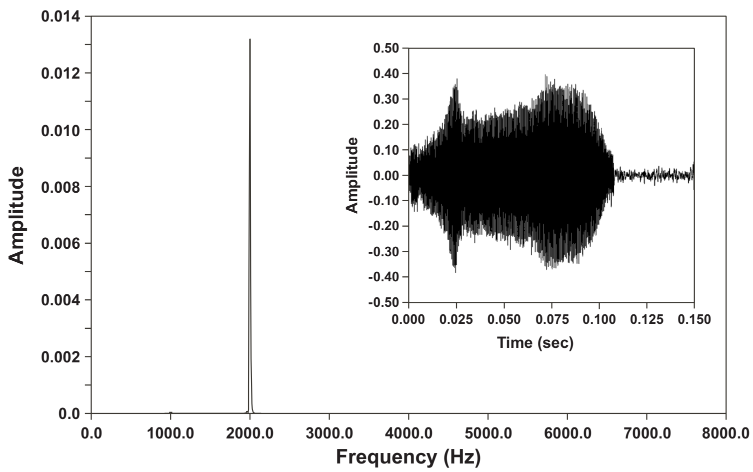


Fig. 5. Frequency spectrum and the microphone recorded signal when an 80 mm chalk was rubbed on the surface of an irregularly shaped rock with an overall diameter about 50 mm. The squeal sound was emitted after the chalk was rubbed several times and a thick coat of chalk powder had formed on the rock surface. $f_d \approx$ 1900 Hz. There is the presence of the harmonic at $3f_d$ and the small side peak at about 2100 Hz.

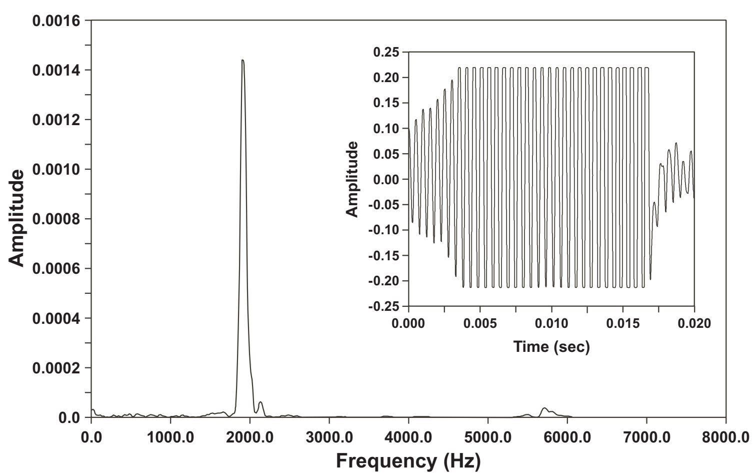


Fig. 6. Frequency spectrum and the microphone recorded signal when a 30 mm chalk, held by a relatively small pair of pliers, was rubbed on a ceramic plate obtained by breaking a medium size pot, 15 cm rim diameter by 3 cm depth, into three pieces. Dominant frequency, $f_d \approx 1100$ Hz.

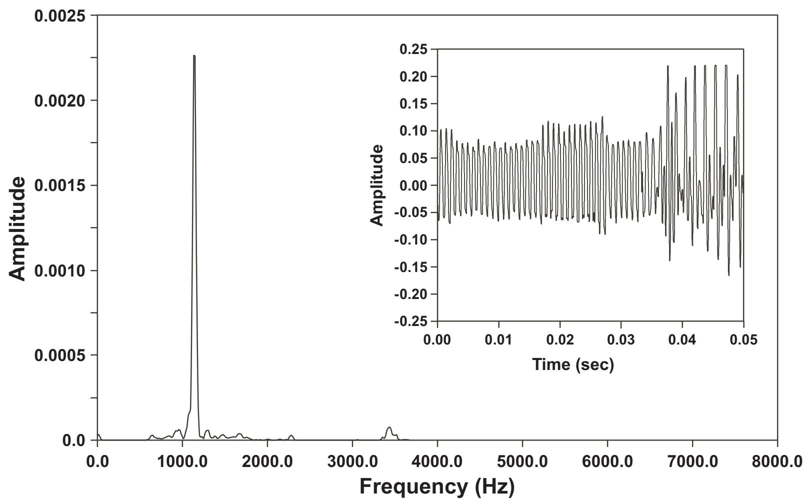
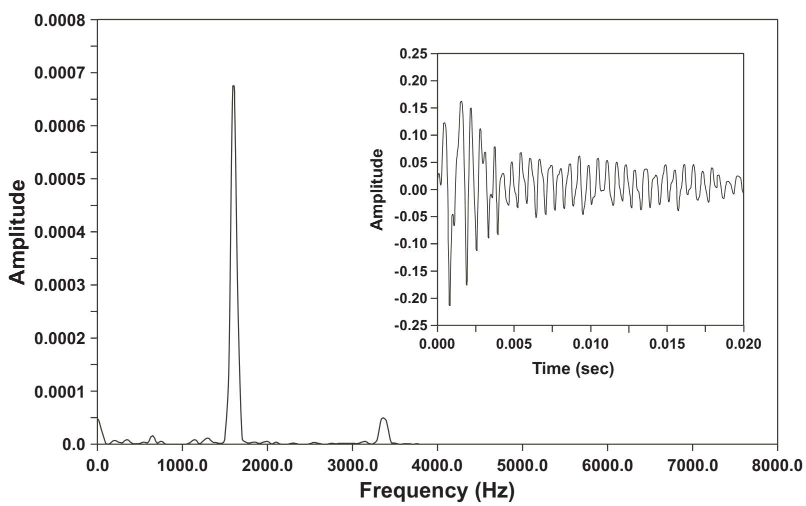


Fig. 7. Same as in Fig. 6 but the 30 mm chalk was replaced by a 20 mm piece splintered from a 20 mm chalk along its length. $f_d \approx$1650 Hz.



3.2. The squealing pencil

Fig. 8. Frequency spectrum and the geophone recorded signal when the back-end (no eraser) of an ordinary wood pencil, length = 80 mm, diameter = 7 mm, was rubbed on a ceramic plate with a thin coat of chalk powder on it. The squeal occured after the pencil was rubbed several times, *i.e.*, after a layer (shear band) of chalk powder had accumulated on the rubbing edge of the pencil. $f_d \approx$1320 Hz.

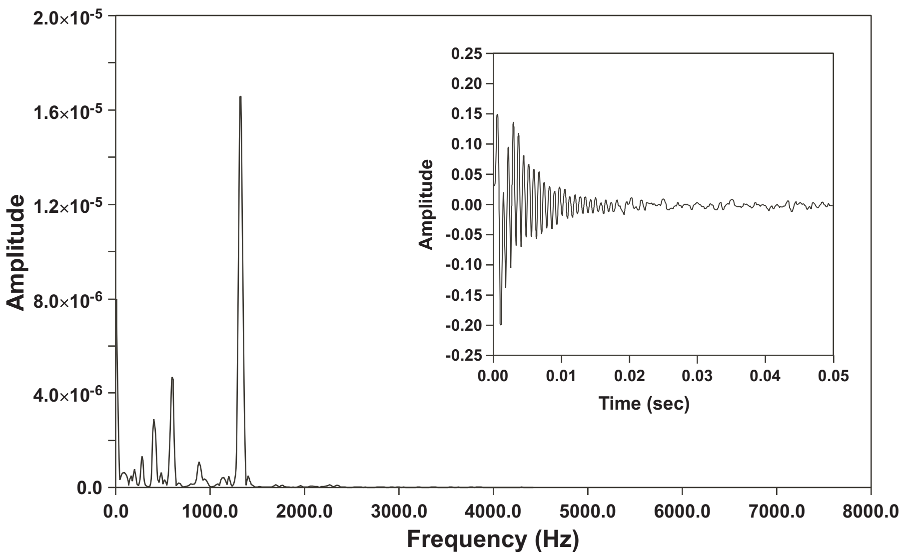
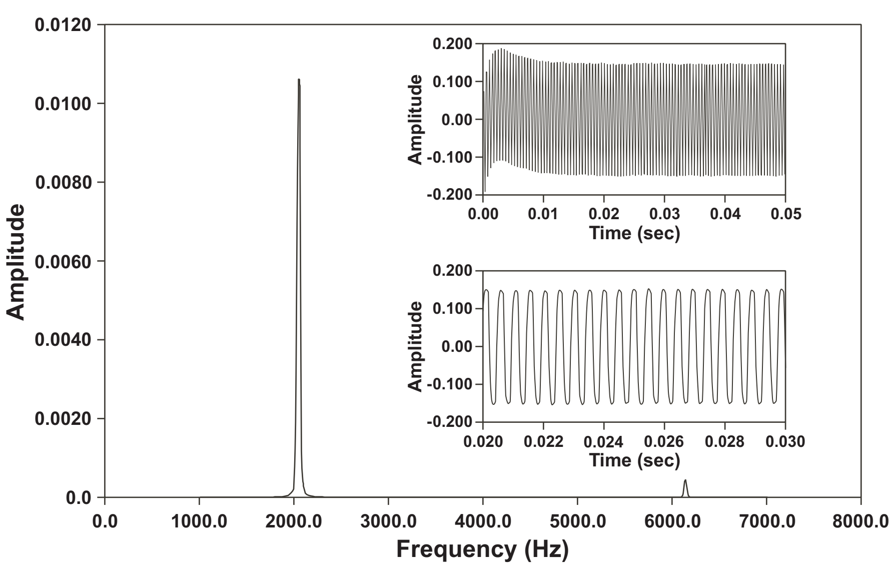


Fig. 9. Same as in Fig. 8, but 1 to 2 sec later. $f_d \approx$2050 Hz.



The contents in Figs. 8, 9, and the process used to obtain these Figures establish nearly conclusively the validity of the above hypothesis. When the bare-back-end (no eraser) of an ordinary pencil, 80 mm in length and 7 mm in diameter, was rubbed on the clean surface of a plate, there was no squeal sound. However, when the same pencil, held at an angle of about 30^o from the vertical, was rubbed several times on the same plate covered with a thin coat of chalk powder, the squeal sound was produced. It was observed that a chalk powder layer had accumulated on the rubbing edge of the pencil with dimensions of about 4 mm along the edge and 2mm along the direction of motion. The layer thickness was estimated at about 0.3 mm. In the plot shown in Fig. 8, the signal is not well developed and there is strong low frequency content at around 500 Hz. The signal shown in Fig. 9 was recorded about 1 second after that in Fig. 8. Seemingly, after two or three rubs the shear band was well formed, resulting in the usual strong peak at about 2050 Hz.

Fig. 10. Same as in Fig. 8, but in absence of any chalk powder on the plate.

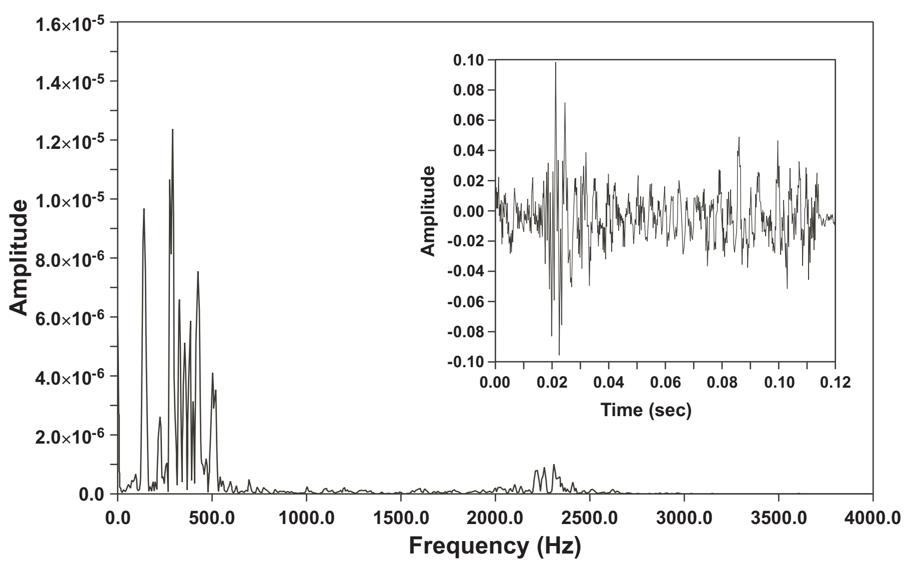
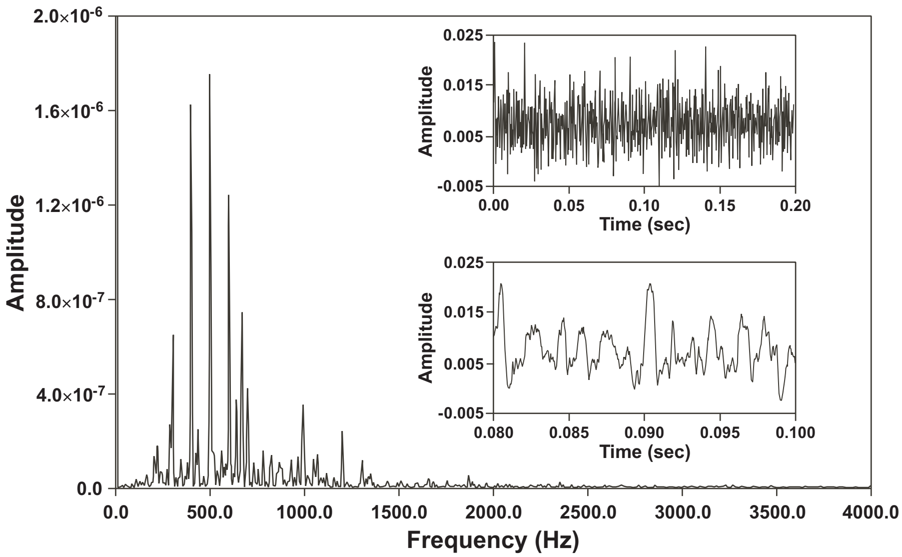


Fig. 11. Same as in Fig. 3, *i.e.*, the 56 mm chalk was rubbed on the small ceramic plate. Evidently, in this 200 ms interval the shear band was not developed sufficiently to result in the stick-slip resonance effect.



The results of rubbing the bare-back-end of the same pencil on the bottom of a 15 cm ceramic pot, in absence of any chalk powder, are shown in Fig. 10. Only the low frequency content is present at around 250 Hz with some relatively high frequency content at around 2300 Hz, likely due to vibrations in the pot. Roughly, the same spectrum was observed when the same pencil-end was rubbed on a regular dinner dish. It could be argued that, without a chalk powder layer at the pencil edge, the slope, a, in (1) was not sufficiently large to allow for the stick-slip resonance effect to become coupled to one of the shear modes in the shear band at the pencil edge. Furthermore, the relatively high stiffness at the pencil edge could result in very high natural frequencies of vibration in the shear band, and in particle velocity amplitudes well above the velocity of the pencil over the plate surface, preventing the onset of the stick-slip effect. If the shear band thickness, b, in Fig. 2, is about 0.3 mm, the particle displacement amplitude has to be considerably lower, say 0.003 mm. Then, for frequency, f=2000 Hz, the particle velocity amplitude is $2\pi f$ times the displacement amplitude, *i.e.*, 3.8 cm/s, a value lower than 10 cm/s, which is deemed to be the approximate velocity of the chalk over the plate. If the spring constant in Fig. 1, or equivalently the shear band stiffness in Fig. 2, becomes too large, then the velocity amplitude could exceed the platform velocity, V_P, resulting, possibly, in no squeal emission. In the report by Heslot et al. [7], it is recognized that for stick-slip to take place as opposed to steady sliding, the loading system has to be soft enough.

Similar results to those in Fig. 10, except for the low frequency content occuring at around 400 Hz, were obtained when the led-end of the same pencil was rubbed on the same pot. The results of rubbing the 56 mm chalk on the same plate as in Fig. 3, without any squeal emission, are shown in Fig. 11. Evidently, the shear band was not developed sufficiently to result in the stick-slip resonance effect. In the next section, an attempt will be made to account for the presence of the various peaks in this frequency spectrum with an average frequency about 4 times lower than the usual $f_d \approx 2000$ Hz.

In order to further establish that the thin chalk powder layer at the edge of the pencil is the vibrating shear band anchoring the stick-slip effect, as opposed to simply facilitating such an effect, the following experiments were effected; Two irregularly shaped pieces were broken from a regular chalk. One, with approximate dimensions ($10 \times 8 \times 5$ mm) was glued to the tip of a nail with length, L=9 cm, and diameter, D=3.6 mm, while the other with nearly similar dimensions was glued to the tip of a nail with dimensions, L=9 cm and D=4.4 mm. Squeals were readily produced, more readily than when using the full chalk, by holding a given nail and rubbing the chalk piece on the small irregularly shaped ceramic plate with approximate dimensions 30×25 mm. In the former case, the dominant peak occured at f_d=3240 Hz and in the latter at f_d=3160 Hz. The proximity of the values of these frequencies tends to rule out the possibility that the modes of vibration that anchor the stick-slip effect are to be found in the nails. The corresponding frequency using the full chalk was at f_d= 2600 Hz, ruling out the possibility that the modes of vibration are to be found in the broken chalk pieces.

Furthermore, a small piece, 12 mm in length, was cut from the end of an ordinary pencil, split in half along its length, and one such piece was glued to the tip of a smaller nail, L=6 cm, D=2.7 mm. A rub on the same ceramic plate produced a fairly wide frequency peak, with several side peaks, centered at f_d=2250 Hz and a second wide peak at 3700 Hz. It may be noted that 3700 is less than 2×2250. Such inequality is consistent with the transcendental equation (9) with roots, $r_1, r_2, r_3..$ where $r_3 < 2r_2$.

In Fig. 9, f_d= 2050 Hz, leading to the conclusion that the modes of vibration are not to be found in the pencil.

4. A theoretical treatment

The shear band, shown in Fig. 2, is characterized by shear modulus, μ_e, and mass density, ρ, and thus by shear phase velocity, $c_s = \sqrt{\mu_e/\rho}$. The particle displacement, ξ_s, is written as, $\xi_{\mathbf{s}} = \nabla \times \mathbf{A}$, where $\mathbf{A}$ satisfies the vector wave equation with phase velocity c_s. As in the report by Patitsas [1], $\mathbf{A}$ is chosen to lie along $\hat{z}$ resulting in,

$$A_z = [A_1\cos\alpha x + B_1\sin\alpha x][A_2\cos\beta y + B_2\sin\beta y]e^{j\omega t} \tag{6}$$

and this in turn results in, $\xi_x = [A_1\cos\alpha x + B_1\sin\alpha x]\beta[-A_2\sin\beta y + B_2\cos\beta y]$ and, $\xi_y = \alpha[A_1\sin\alpha x - B_1\cos\alpha x][A_2 cos\beta y + B_2 sin\beta y]$, where the factor $e^{j\omega t}$ is understood to be included and α, β are in the neighborhood of π/b and π/L_y respectively, as will be seen below. The wave number, $k_s = \omega/c_s$, is given as, $k_s^2 = \alpha^2 + \beta^2 \approx \alpha^2$ since $b << L_y$ resulting in $\beta << \alpha$. It is mathematically convenient, but not necessary, to assume that $\xi_y = 0$ at $y = 0$. The inclined position of the chalk or pencil in Fig. 2 is not in contradiction with such an assumption. Then, A_2=0 and since the end at $y = L_y$ is free, $\beta L_y = m\pi/2$, $m = 1, 3, 5..$ Thus,

$$\xi_x = \beta[A_1\cos\alpha x + B_1\sin\alpha x]\cos(\frac{m\pi y}{2L_y}) \tag{7}$$

and,

$$\xi_y = \alpha[A_1\sin\alpha x - B_1\cos\alpha x]\sin(\frac{m\pi y}{2L_y}) \tag{8}$$

Since $\beta << \alpha$, ξ_y is overall much greater than ξ_x. The boundary condition, ξ_x=0 at $x = b$, results in , $B_1 = -A_1\cot\alpha b$ and this implies that $\partial\xi_y/\partial x$=0 at $x = b$, *i.e.*, ξ_y has an antinode at $x = b$. The normal stress along $\hat{x}$ can be written as, $\sigma_{xx} = (\lambda_e + 2\mu_e)\partial\xi_x/\partial x \approx \lambda_e\partial\xi_x/\partial x$, since $\mu_e \ll \lambda_e$, where, λ_e is the Lame' constant that determines the compression phase velocity, *i.e.*, $c_p = \sqrt{(\lambda_e + 2\mu_e)/\rho}$. At x=0, the equation of motion can be written as, $\sigma_{xx}S = M\partial^2\xi_x/\partial t^2 = -M\omega^2\xi_x$ and this results in the transcendental equation,

$$cot(\alpha b) = \frac{M}{\rho S b}(\frac{c_s}{c_p})^2(\alpha b) = L_f(\alpha b) \tag{9}$$

where, M is the equivalent load mass on the shear band, $S = L_y L_z$ is the contact area and L_f is the overall load factor. The familiar procedure in determining the roots of (9) consists in plotting $\cot(\alpha b)$ as function of αb, draw the straight line $L_f(\alpha b)$ and look for the intersection points. The first root, r_1, lies in the interval, $0 < \alpha b < \pi/2$, the second in the interval, $\pi < \alpha b < 3\pi/2$ etc. For a given root r_n, the angular frequency is given as, $\omega_{on} = k_s c_s = \alpha_n c_s = (r_n/b)c_s$.

If the load factor, L_f, is sufficiently large, then $r_2 = \pi + \epsilon$, where ϵ is a small number. Therefore, r_2 is rather insensitive to variations in L_f. It is highly likely that such variations occur as the hand pushes the chalk or the pencil. It is also very likely that the mass of the hand renders the load factor relatively large.

The relatively large values of ξ_x and ξ_y when $\alpha b = r_2 \approx \pi$, the presence of the antinode of ξ_y at the rubbing interface and the insensitivity of the root r_2 to changes in the load factor lead to the conclusion that the corresponding mode is responsible for the sharp peak at about 2000 Hz seen in the above plots. Then, the low frequency content in the above plots must correspond to the root, r_1, which is fairly sensitive to changes in the load factor. Thus, the various peaks in Fig. 11 could be, in part, due to such changes during a given rub. Furthermore, the several peaks could be due to several shear bands developing simultaneously. If $r_1 = \pi/4$, then $r_2/r_1 \approx 4$, corresponding to the ratio of 2000/500 between the dominant squeal frequency and the low frequency content seen in Fig. 11. However, when the 30×25 mm ceramic plate was tapped lightly by a 12 mm glass bead or the sharp edge of another small ceramic plate, the frequency spectra of the resulting low intensity signals exhibited a wide frequency content, with many peaks, centered at around 500 Hz. Therefore, the low frequency content in Fig. 11, for example, could also be noise generated by the air acceleration as the chalk or pencil is drawn over the plate.

The straight lines $O'A$, $O'D$ in Fig. 2 correspond only to the mode with $r_1 = \pi/2$. Then, $B_1 << A_1$ and $\xi_y = \alpha A_1 \sin(\pi x/2b)\sin(m\pi y/2L_y)$. For the mode with $r_2 = \pi+\epsilon$, $A_1 << B_1$ and thus, $\xi_y = -\alpha B_1 \cos((\pi + \epsilon)x/b)\sin(m\pi y/2L_y)$, implying a node at $x \approx b/2$ and an antinode at $x \approx b$.

As in the case of (3), the effect of the slope, a, in (1) is to reduce the natural frequencies of vibration, ω_{on}. Towards this end, the equation of motion is written as,

$$\rho\frac{\partial^2\xi_y}{\partial t^2} - \mu_e\nabla^2\xi_y = \frac{\mu Mg}{bS} \tag{10}$$

where bS is the shear band volume. Thus, the equation analogous to (3) is,

$$\omega_n^2 = \omega_{on}^2 - (\frac{Mga}{2\rho bS})^2, \quad \omega_{on} = \frac{r_n}{b}c_s \tag{11}$$

5. Wet skin on glass surface

Figure 12 depicts the microphone recorded signal and its frequency spectrum when the small finger was rubbed, flat, on the surface of an irregularly shaped glass plate; area about 60 cm^2, thickness variable between 4 and 2 mm. The curvature suggests that it originated from a jar with volume about 1 liter. There was an excess layer of water on the concave surface when the finger was pushed-pulled with a speed of about 10 cm/s. The sound was rumbling-like, as suggested by the lack of signal smoothness, that was likely due to water moving between the fingerprint pattern, *i.e.*, the crevices in the epidermis. However, after a few rubs the water layer thickness was reduced and the sound became musical as seen in Fig. 13. It has been observed that when the contact area is relatively large, a flat thumb for example, several major peaks can emerge presumably due to various sub-areas acting independently. A few minutes later, the outer side of the same small finger was rubbed lightly on the same glass area and resulted in a more musical sound and at a higher frequency seen in Fig. 14. The irregularity of the geometry of the glass plate and the overall similarity of the signals from various other glass or dish plates suggest that the origin of the emitted sound is not to be found in vibrations in the glass plate. Therefore, such an origin has to lie in a **shear band** comprising the skin and the water layer. The skin from other parts of the body can also be used similarly to produce musical sounds. In order to confirm the assumption that these signals could not originate with vibrations in the glass plate, the plate was tapped by a 12 mm glass bead. It was determined that there was no frequency content below 1000Hz in the emitted signal.

Fig. 12. Frequency spectrum and the microphone recorded signal when the small finger, flat, was rubbed on an irregularly shaped glass plate, area about 60 cm^2, variable thickness. At this early stage, there was a relatively thick layer of water on the concave plate surface.

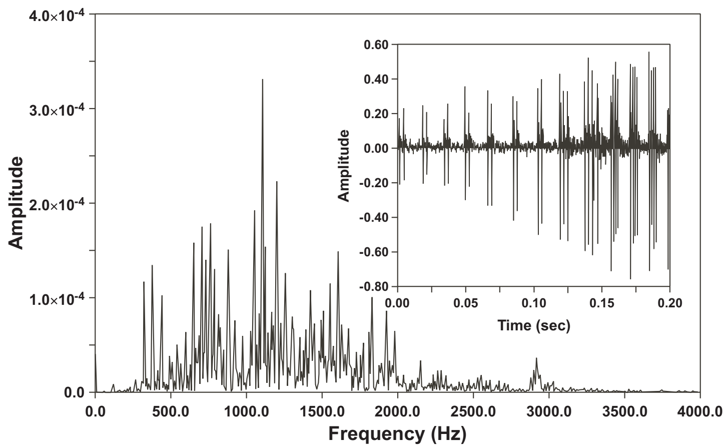


Fig. 13. Same as in Fig. 12, except that after a few rubs the thickness of the water layer had diminished. $f_d \approx 450$ Hz. The second large peak corresponds to $f_2 \approx 2f_d$. There is faint presence of harmonics at 3, 4, and 6 times f_d.

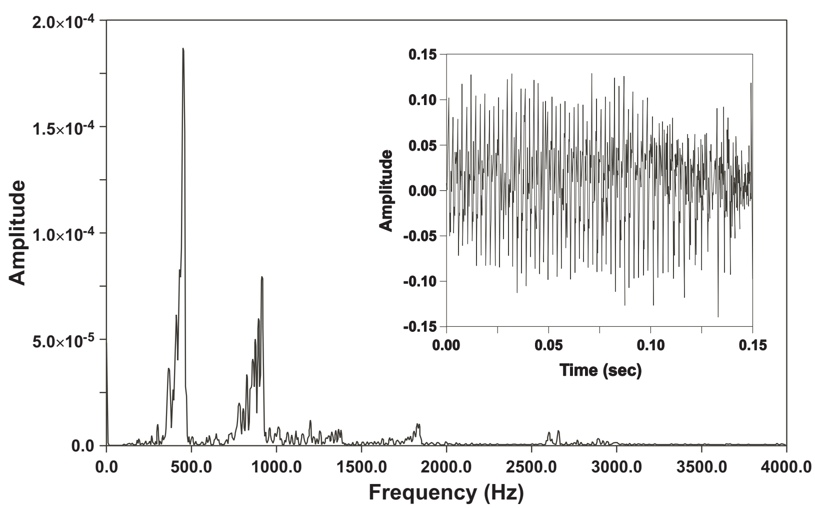
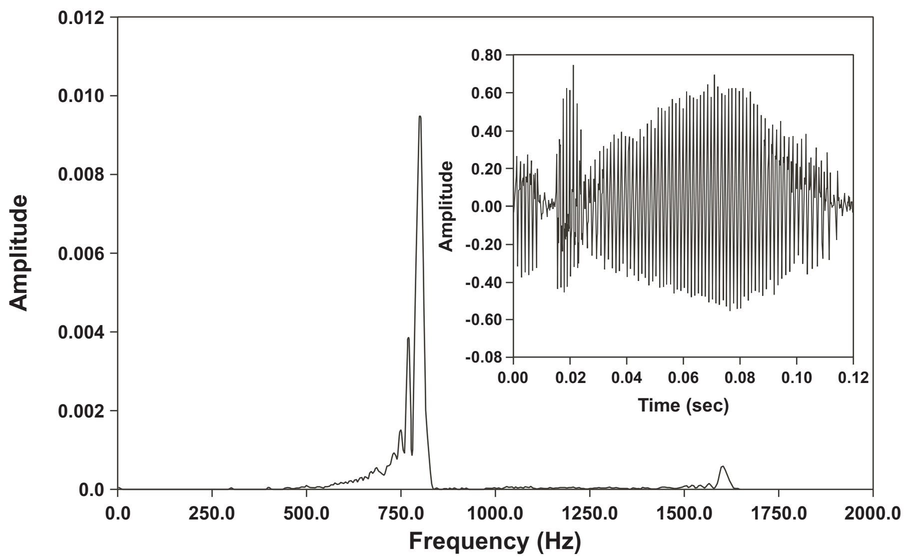


Fig. 14. Same as in Fig. 12, but a few minutes later and with a light rub by the outer side of the same small finger.



It can be argued that the water layer, under turbulent conditions, could aquire sufficient viscosity to support a minute shear modulus. Furthermore, the friction coefficient between the water layer and the glass surface must decrease with relative velocity as in (1). A bundle of wet soft paper towel can produce the musical sounds, but with a bit more difficulty, when rubbed on a glass surface. More recently, pleasant sounds could be heard while washing a glass window, area approximately 1 m^2, with a cotton cloth. The water to dish washing liquid soap ratio was about 20 to 1. Similarly, squeal sounds can be heard when the rubber edge of the washer is drawn over the windshield of a car covered with washing fluid. Any smooth surface can serve in this capacity, that of a regular dinner dish or a varnished wood for example. The shift in frequency between Fig. 13 and Fig. 14 could be due to reduced shear band thickness and to reduced load. More study is required in order to establish the exact role of the thin water layer on the smooth surface, apart from facilitating the stick-slip effect. The elastic agency appears to be the soft matter between the epidermis and the finger bone. If the thickness of the soft tissue is about 5 mm, resulting in wave length, $\lambda \approx 5$ mm, and if f_d=1000Hz, then the shear velocity, c_s=5 m/s. If, on the other hand, the elastic agency is to be found in the thin water layer, thickness about 0.1 mm, then, c_s=0.1 m/s.

In order to examine the more probable scenario, *i.e.*, that the modes of vibration are to be found in the soft tissue between the epidermis and the finger bone, the following procedure was effected; A relatively soft cloth, mostly cotton, was cut into 12 pieces about 5×5 cm in size. All pieces were dipped into a bath of water with a small amount of dish washing fluid and then squeezed until there was no dripping. When the middle finger was rubbed on the concave side of the irregularly shaped glass plate, area about 60 cm^2, the frequency spectrum comprised the strongest peak at 410 Hz, that would correspond to the root r_2 in (9), an almost equally strong peak at nearly twice 410 Hz and also lesser peaks at 3×410 Hz etc. Evidently, the load factor L_f was sufficiently large so as $r_3 \approx 2 \times r_2$ etc. The pressure exerted on the glass plate by the finger exceeded by far the pressure exerted by the chalk on the ceramic plate. When 4 cloth pieces were interposed between the same finger and the glass plate, the strongest frequency peak was at 280 Hz and a lesser one at nearly 2×280 Hz. When 8 cloth pieces were interposed the first peak occured at 170 Hz, but it was considerably weaker than that at 2×170 Hz. Thus, in this case, it was the third mode of vibration, corresponding to the root r_3, that anchored the stick-slip effect. When 12 pieces were interposed, there was no appreciable change. The apparent reduction in the frequency of vibration with increased thickness tends to confirm the assumption that the modes of vibration are to be found in the soft tissue between the epidermis and the finger bone. Thus, the water layer facilitates the stick-slip effect by allowing for an easy slide and by providing for a friction coefficient that decreases with increasing relative velocity between the surfaces.

The side peak at 770 Hz in Fig. 14 could be due to a minor shear band along the contact area with slightly lower shear modulus and or slightly higher thickness. In the case of the rubbing chalk or pencil, where the shear band is limited to a relatively narrow contact area, there are no side peaks, Fig. 9 for example. However, in Fig. 5, where the rock surface is relatively rough, there is a small side peak at 2100 Hz. In Figs. 4 and 5 in [1], where the silica gel grains were impacted (sheared) lightly by a brass pestle, there are several side peaks, more so in Fig. 5 where the impact was more forceful. Pronounced side peaks have been observed more recently when the singing sand from Lake Michigan, USA, was impacted by a 16 mm diameter wood rod. Furthermore, the frequency spectra of the signals emitted, when the so-called *frog sand* was shaken back and forth, comprise very pronounced side peaks, as can be seen in Figs. 8 and 9 in [8]. Such a sand consists of about 100 cm^3 of quartz sand and water, in equal parts, sealed inside a plastic cylindrical cell 6 cm in diameter by 12 cm in length, and can be obtained from the *Sand Museum* in Nima, Japan. The role of the stick-slip effect in the generation of acoustic and seismic emissions from sheared granular media is the subject of a forthcoming paper.

6. Conclusions

The analysis of the oscillatory motion of a block on a moving platform shows that the decrease of the friction coefficient, between the block and the platform, results in a stick-slip resonance effect that drives the block oscillation. The period of the stick-slip cycle depends principally on the elastic properties of the spring-block system and somewhat on the rate of decrease of the friction coefficient with relative velocity. It is fairly well established that the squeal sound, dominant frequency, $f_d \approx 2000$ Hz, emitted when a chalk is rubbed on a ceramic plate, and occasionally when rubbed on a blackboard, is due to vibrations in a thin shear band at the contact area. The band thickness amounts to about 0.3 mm and the geometry of its surface area appears to be irrelevant. The band thickness can differ from time to time resulting in appreciable difference in the dominant frequencies f_d. Regarding the nature of the vibrations, the only viable option appears to be that of shear modes in the shear band characterized by very low modulus of rigidity, resulting in shear phase velocity of about 1 m/s.

The more musical sound, $f_d \approx 700$ Hz, emitted when a wet finger is rubbed on a smooth surface, is most likely due to a shear band comprising the tissue between the epidermis and the finger bone, while the thin water layer facilitates the stick-slip effect. The established convention that squeal emissions originate exclusively with the excitation of modes of vibration in one or both bodies rubbing one on another needs re-evaluation. Whereas, in many cases such modes are responsible for the squeal emissions, their excitation could originate with the excitation of modes, with similar frequencies, in a thin shear band at the rubbing interface. In the cases of the chalk on a small ceramic plate and the wet finger on a small glass plate, the squeals are due solely to vibrations in the shear bands. Finally, it ought to be argued that seismic tremors that occur when two plates slide over one another could also be due to shear modes of vibration in a layer of rock powder interposed between the plates.

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