Squeezed-State Generation by the Normal Modes of a Coupled System

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A new regime for the generation of squeezed states of the electromagnetic field is described for a collection of atoms within a high-finesse cavity. The process responsible for squeezing is a coupling-induced splitting in the normal-mode structure of the atom-field system. A theoretical analysis is presented that predicts large degrees of squeezing for modest operating conditions. An experimental investigation of this regime has produced noise reductions of 30% relative to the vacuum noise level.

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In recent years there has developed a rapidly growing interest in the manifestly quantum or nonclassical features of the electromagnetic field. Squeezed states are one example of nonclassical states and are characterized by a phase-dependent redistribution of the quantum fluctuations of the field such that the variance in one of two orthogonal quadrature operators drops below the level of fluctations set by the vacuum state of the field (the zero-point level). While a diverse set of processes has been identified and utilized for squeezed-state generation,[1] it has proven to be somewhat difficult to find experimental systems that actually fulfill the potential indicated by many model calculations. In this Letter we describe a promising new regime for squeezed-state generation involving a collection of two-level atoms coupled to a single mode of a high-finesse optical resonator.[2] We demonstrate both experimentally and theoretically that significant degrees of squeezing can be achieved by employment of the normal-mode structure of this coupled system in a domain in which an oscillatory exchange of excitation between cavity mode and atoms in the cavity provides a mechanism for phase-sensitive amplification and deamplification of quantum fluctuations. Although the experimental configuration is not yet optimum with respect to the region of best squeezing indicated by our theoretical analysis, we have nonetheless observed noise reductions of 30% relative to the vacuum noise level.

To gain qualitative insight into the nature of the processes responsible for squeezed-state generation in this new regime, we begin with an analysis of the interaction of a collection of N two-level atoms with a single mode of a high-finesse interferometer. The model Hamiltonian $\hat{H}$ for this system is taken to be of the form[3]

$$\hat{H}=\hat{H}_0+\hat{H}_a+\hat{H}_c,$$

$$\hat{H}_0=(\hbar\omega_a/2)\hat{J}_z+\hbar\omega_c\hat{a}^\dagger\hat{a}+\hbar g[i\hat{J}_-\hat{a}^\dagger+\text{H.c.}]. \quad (1)$$

The coherent coupling of the atomic polarization to the cavity field is described by $\hat{H}_0$. $\{\hat{J}_z,\hat{J}_\pm\}$ are collective atomic operators for the N atoms of transition frequency ω_a, and $\{\hat{a},\hat{a}^\dagger\}$ are the annihilation and creation operators for the single cavity mode of resonant frequency ω_c. The atoms and field mode are coupled through an assumed dipole interaction with coupling coefficient $g=(\omega_c\mu^2/2\hbar\epsilon_0 V)^{1/2}$. The decay rate of the atomic inversion is γ, while the atomic polarization decays at rate $\gamma_\perp$. Both decay processes are described by $\hat{H}_a$. The field amplitude decays at a rate κ via coupling at the cavity mirrors to a set of continuum input-output modes as described by $\hat{H}_c$, which also includes the possibility of excitation by an external field of frequency ω_L.

While an incredibly diverse set of phenomena is described by Eq. (1), we wish to focus on a feature that is well known within the context of cavity QED with Rydberg atoms[4-7] but which is often overlooked in optical physics.[8] Consider a linearization about steady state of the Maxwell-Bloch equations that result from Eq. (1), which for the sake of brevity is restricted to the case $2\gamma_\perp=\gamma$ with $\mu=\kappa/\gamma$. The eigenvalue spectrum consists of five eigenvalues, which in general comprise a set of one real value λ_0 and two pairs of complex conjugates (λ_1,λ_2). Figure 1 shows the dependence of the imaginary parts (Ω_1,Ω_2) of these eigenvalues on intracavity field $x=(\langle\hat{a}^\dagger\hat{a}\rangle/n_0)^{1/2}$, $n_0=\gamma^2/8g^2$, for fixed values of the cavity detuning $\theta=(\omega_c-\omega_L)/\kappa$, atomic detuning $\Delta=(\omega_a-\omega_L)/\gamma_\perp$, and atomic cooperativity parameter $C=Ng^2/2\kappa\gamma_\perp$. The limiting values of (Ω_1,Ω_2) as $x\to 0$ in Fig. 1 represent a mode splitting and correspond to the "vacuum-field Rabi" splitting,[5] here generalized to include damping, $N>1$ atoms, and nonzero detunings.[7,8] For $\theta=0=\Delta$, the pair of complex eigenvalues at $x=0$ is degenerate,

$$\lambda_1^\pm=\lambda_2^\pm=-\tfrac{1}{2}(\gamma_\perp+\kappa)\pm[\tfrac{1}{4}(\gamma_\perp-\kappa)^2-g^2N]^{1/2}.$$

We see that λ contains an imaginary part only for $\frac{1}{2}|\gamma_\perp-\kappa|<g\sqrt{N}$. That is, since the atoms and the cavity field are independently coupled to separate reservoirs, a periodic exchange of excitation at frequency $\Omega\simeq\Omega_0=gN^{1/2}=\gamma(\mu C)^{1/2}$ occurs only if the decay rates of these two "oscillators" are not too dissimilar or if the coupling is sufficiently large. It is this exchange that is crucial to our analysis[2,8-11] and which is lost in investigations of squeezed-state generation in optical cavities that perform an adiabatic elimination of either atomic or field variables ($\mu\to 0$ or $\mu\to\infty$ for fixed C).[12-14] More

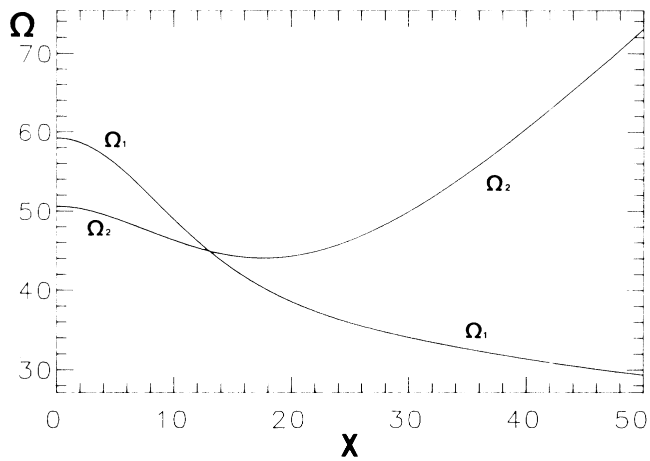

FIG. 1. Imaginary parts (Ω_1, Ω_2) of the eigenvalues resulting from a linearization of the Maxwell-Bloch equations vs intracavity field x. The frequency Ω is in units of $\gamma_\perp$. Atomic detuning $\Delta = -14.6$, cavity detuning $\theta = 0.76$, cooperativity parameter $C = 52$, and the ratio $\mu = 13.5$. Note that for $x \gg 1$, Ω_2 approaches the usual free-space Rabi frequency $\delta^2 = 2x^2 + \Delta^2$ (with damping given by $\gamma_\perp$) and Ω_1 the cavity ringing frequency $2\mu\theta$ (with damping given by κ).

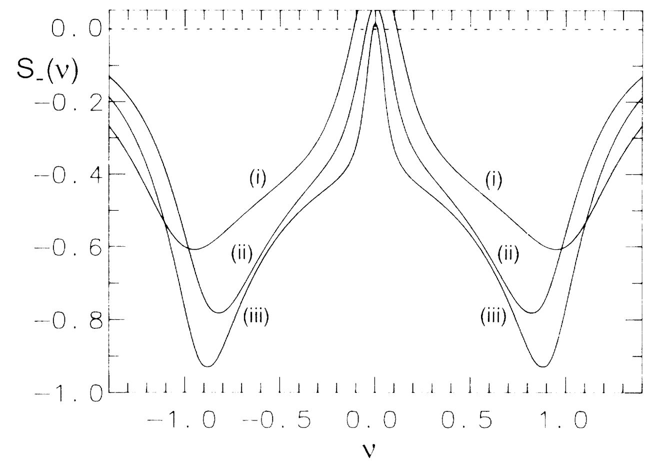

FIG. 2. Spectrum of squeezing S_- vs offset frequency $\nu = \Omega/\Omega_0$, with $\Omega_0 = g\sqrt{N}$. (i) $\mu = 13.5$, $C = 10$, $\Delta = 9.4$, $\theta = -0.47$, $x = 11$; (ii) $\mu = 13.5$, $C = 52$, $\Delta = -14.6$, $\theta = 0.86$, $x = 16.4$; and (iii) $\mu = 70$, $C = 200$, $\Delta = 74$, $\theta = -0.98$, $x = 92.8$.

specifically, we concentrate on the region around the crossing near $x = 12$ in Fig. 1. Since the regression of quantum fluctuations is governed by the same eigenvalue spectrum as are the normal modes, one might expect a phase-sensitive amplification and deamplification around the frequency Ω_0 that could lead to squeezing.

To translate this qualitative discussion into quantitative predictions for squeezed-state generation, a detailed analysis beginning with the Hamiltonian Eq. (1) must be carried out. Given the existing literature on the quantum statistical processes in optical bistability,[3,15] it is straightforward to arrive at an equation for the spectral density $A(\phi, \nu)$ describing the fluctuations of the quadrature amplitude

$$\hat{y}(t,\phi) = \hat{a}(t)e^{-i\phi} + \hat{a}^\dagger(t)e^{i\phi}$$

of the intracavity field without adiabatic elimination of cavity or field variables. Squeezing of the field emitted through the output mirror of the cavity is then described by the spectrum of squeezing $S(\phi,\nu) = 2\kappa A(\phi,\nu)$, where $\nu = \Omega/\Omega_0$.[16]

Our results for the spectrum of squeezing S are displayed in Fig. 2, with the phase ϕ optimized at each value of ν to maximize squeezing. The resultant spectrum is denoted by $S_-(\nu)$ and is defined such that $S_- = 0$ for the vacuum state and $S_- = -1$ for perfect squeezing. As anticipated, the frequency $\Omega = \Omega_0\nu$ for best squeezing is approximately given by the vacuum-field Rabi frequency $(\nu \approx 1)$ for each of the cases (i)-(iii). We also find that the value of the intracavity field x for optimum squeezing in each case is near a crossing as is shown in Fig. 1 for case (ii) and is such that $x < \Delta$. Note that substantial degrees of squeezing are predicted for relatively modest values of (C,x) relative to earlier treatments of squeezed-state generation with two-level atoms. Our studies indicate that squeezing persists over relatively broad regions of the parameter space $(C,\Delta,\theta,\mu,x,\Omega)$. For example, curve (iii) of Fig. 2 represents a factor of 2 squeezing over a frequency range $\Delta\Omega \cong 66\gamma$, which for $\gamma/2\pi = 10$ MHz gives $\Delta\Omega \cong 660$ MHz.

A diagram of the experimental arrangement that we have employed for squeezed-state generation based upon the analysis given above is shown in Fig. 3. The cavity is formed by a pair of mirrors of radius of curvature 1 m separated by 0.83 mm. The transmission coefficients of the two mirrors are $T_1 = 0.0075$ and $T_2 = 0.0002$. The measured cavity finesse is $F = 660 \pm 30$, while that inferred from the value of T_1 is $F_1 = 840$. Hence the ratio of output loss through m1 to loss by all other avenues is given by $\rho = F/F_1 = 0.79 \pm 0.06$, which implies a 21% reduction in squeezing as compared to an ideal single-

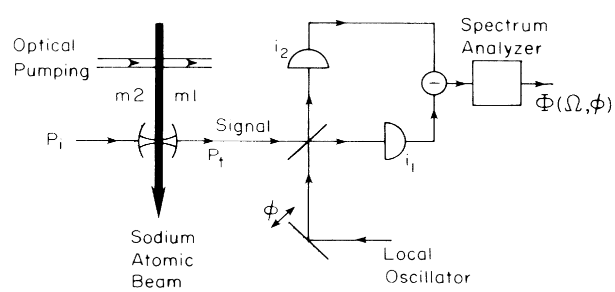

FIG. 3. Diagram of the essential elements of the experiment for generating and detecting squeezed states.

ended cavity. The intracavity medium is composed of three overlapping, optically prepumped beams of atomic sodium prepared in the $(3^2S_{1/2}, F=2, m_F=2)$ state and excited with circularly polarized light to the $(3^2P_{3/2}, F=3, m_F=3)$ state of the D_2 line. The beams are formed by three 0.5-mm apertures in the source oven lying along a line perpendicular to the plane of Fig. 3 and by a 0.3-mm aperture located 280 mm downstream from the oven and 15 mm upstream from the cavity waist. The maximum absorption αl for this configuration is $\alpha l = 0.4$. The fluorescence from the optical pumping beam (the frequency of which is offset by an acousto-optic modulator by -73 MHz relative to the input beam P_i) is used to lock the frequency of the principal exciting dye laser, such that $\Delta = -14.6$. A second independent dye laser overlaps the optical pumping region and transfers population from the $F=1$ to $F=2$ ground state via the $F=2$ excited state. The pumping fluorescence together with the recorded hysteresis cycle in absorptive optical bistability provide a measure of C during the experiments.[17]

Detection of the fluctuations in the quadrature amplitude of the signal beam emitted through the mirror m1 is accomplished with the balanced homodyne detector indicated in Fig. 3.[18] The photodiodes are EGG type FFD-060 with the glass windows removed and with the reflection from the diode surface collected and refocused onto the photodiode resulting in a quantum efficiency $\alpha = 0.85 \pm 0.04$. The homodyne efficiency is measured to be approximately $\eta = 0.93 \pm 0.07$ for each channel. Over the range 200–300 MHz the "shot noise" associated with the 2.5-mA dc photocurrent produced by the local oscillator exceeds the amplifier noise level by greater than 5 dB. That the local oscillator is indeed at the vacuum level and does not carry appreciable excess amplitude noise is confirmed by a comparison of the noise levels observed when the two photocurrents i_1 and i_2 are combined first with 0° and then with 180° phase shift. With the exception of coherent lines at multiples of the 83-MHz longitudinal mode spacing of the ion laser, we conclude that the local oscillator fluctuations are within $\pm 1\%$ of the vacuum level over the spectral range of interest in the current experiment. Furthermore, with the 180° phase shift actually employed in the squeezing measurements, any excess local oscillator noise is reduced by greater than 12 dB.

An example of our observation of noise reductions below the vacuum level is given in Fig. 4. The figure displays the spectral density of photocurrent fluctuations $\Phi(\phi, \Omega)$ (relative to the shot-noise level) at fixed frequency $\Omega/2\pi = 280$ MHz versus local oscillator phase ϕ. The trace marked (i) is the vacuum plus amplifier noise level obtained by either blocking the signal beam, blocking the atomic beams, or detuning the cavity. The trace labeled (ii) is with the signal beam present and clearly exhibits noise reductions below the vacuum level. Note

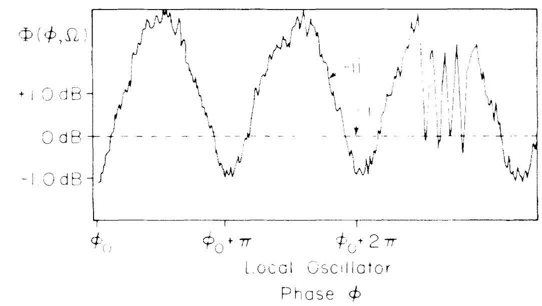

FIG. 4. Spectral density $\Phi(\phi, \Omega)$ of fluctuations of the difference photocurrent $i_1 - i_2$ vs local-oscillator phase ϕ at fixed analysis frequency $\Omega/2\pi = 280$ MHz. (i) Signal input blocked to define vacuum level; (ii) phase-sensitive fluctuations with signal beam present drop below the vacuum level. Operating conditions: $C = 52 \pm 8$, $\mu = 13.5$, $\Delta = -14.6 \pm 1$, $x = 25 \pm 10$. The sharp feature at the right side of the trace is due to the flyback of the piezoelectric driver used to scan the phase of the local oscillator. The time taken for the trace is 0.2 s.

that the noise reduction is achieved with incident laser power of only 500 μW. By constructing histograms of noise levels around the minima in a number of traces as in Fig. 4, we arrive at a figure of -1.0 ± 0.1 dB for the recorded noise reduction, which becomes -1.55 ± 0.18 dB after correction is made for the nonzero noise level of the amplifier. This figure represents a level of fluctuations $R_- = 0.70 \pm 0.03$ which is 30% below the level set by the vacuum state of the field at the signal port of the balanced homodyne detector. We have also explored the dependence of the phase-sensitive noise on offset frequency Ω. In qualitative terms the observed noise reductions extend over similar broad regions of frequency shown in Fig. 2.

By including the propagation loss $1 - T = 0.03$, detection quantum efficiency α, heterodyne efficiency η, and escape efficiency ρ from the cavity, we can relate the spectrum of squeezing S_- to the observed noise reduction R_-:

$$R_-(\Omega) = 1 + \rho T \alpha \eta^2 S_-(\Omega). \tag{2}$$

By separately measuring the quantities (ρ, T, α, η), we thus infer S_- from measurement of R_-. In the current arrangement, $R_- = 0.70 \pm 0.03$ corresponds to $S_- = -0.53 \pm 0.08$, or to a 53% decrease in fluctuations relative to the vacuum level before degradation by the various loss mechanisms associated with escape and detection. While this degree of squeezing is somewhat less than that predicted in Fig. 2(ii), a detailed comparison of experimental results with theoretical predictions is hampered at present by large uncertainties in the experimental parameters. We are currently striving to remedy this circumstance and to eliminate various possible noise sources such as stray absorption due to background sodium vapor in the vacuum chamber. We do stress that the observed frequency Ω for optimum noise reduction ($\Omega/2\pi = 280$ MHz) is in very good agreement with theoretical prediction ($\Omega/2\pi = 265$ MHz) for the values $C = 52$ and $\mu = 13.5$.

In summary, we have identified both theoretically and experimentally a new regime for squeezed-state generation associated with the coupling of a collection of two-level atoms to an optical cavity. The physical process responsible for the squeezing is a coupling-induced mode splitting in the eigenvalue spectrum of the system, which for weak fields and zero detunings is just the vacuum-field Rabi splitting. We have presented a theoretical analysis of the squeezing in this system based upon the formalism developed in Refs. 3, 8, 15, and 16, and have predicted that large degrees of squeezing should be attainable with rather modest values of atomic density and intracavity field. An experiment to confirm these ideas has been carried out and noise reductions of 30% relative to the vacuum level observed. Although our analysis has been restricted to the case of two-level atoms in a cavity, the idea of employing the composite structure formed from the coupling of a nonlinear medium to a cavity field should be of general applicability in the generation of squeezed states.

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