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Squeezing and entanglement delay using slow light

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We examine the interaction of a weak probe with N atoms in a Λ-level configuration under the conditions of electromagnetically induced transparency (EIT). In contrast to previous works on EIT, we calculate the output state of the resultant slowly propagating light field while taking into account the effects of ground state dephasing and atomic noise for a more realistic model. In particular, we propose two experiments using slow light with a nonclassical probe field and show that two properties of the probe, entanglement and squeezing, characterizing the quantum state of the probe field, can be well-preserved throughout the passage.

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The coherent and reversible storage of the quantum state of a light field is an important issue for the realization of many protocols in quantum-information processing. Recently much work has been done to address this issue by utilizing the phenomenon of electromagnetically induced transparency (EIT) [1–10]. In this paper, we demonstrate that under the conditions of EIT, the quantum state of the stored light field can be well preserved even in the presence of dephasing and noise.

In a conventional EIT setup, a strong, coherent field ("control field") is used to make an otherwise opaque medium transparent near an atomic resonance. A second, weak field ("probe") with a restricted bandwidth about this resonance can then propagate without absorption and with a substantially reduced group velocity compared to a pulse in vacuum, thus delaying (or effectively "storing") the light field within the atomic cloud for a duration equal to the delay time of the light pulse caused by the EIT medium.

In this paper we concentrate on two representative quantities characterizing the amount of quantum information of a light field: squeezing, representing a subquantum noise level of fluctuation in the observable of one beam; and entanglement, where the subquantum noise fluctuation occurs in the correlation between two beams. We calculate the effect of the atom-light interaction on each quantity and show that the slowing of the light need not significantly degrade the information carried. Previous works on photon storage have indicated that in the absence of dephasing between the two ground states of the lambda system, and ignoring the Langevin noise operators arising from atomic coupling to a vacuum reservoir, the quantum state of light field is well preserved after traversing the EIT medium [2,3,11]. Here we further highlight the robustness of storage using EIT and show that even *with* dephasing and noise taken into account, entanglement and squeezing of the pulse at the exit of the medium need not differ significantly from that of the input pulse under experimentally realizable parameter regimes.

We follow the model outlined in [2] and use a quasi-one-dimensional model, consisting of two co-propagating beams passing through an optically thick medium of length L consisting of three-level atoms. The atoms have two metastable lower states $|b\rangle$ and $|c\rangle$ interacting with the two optical fields $\hat{\mathcal{E}}(z,t)$ and Ω_c as shown in Fig. 1. $\hat{\mathcal{E}}(z,t)$ is a weak quantum field that couples the ground state $|b\rangle$ and excited state $|a\rangle$, and is related to the positive frequency part of the electric field by

$$\hat{E}^{+}(z,t) = \sqrt{\frac{\hbar\omega_{ab}}{2\epsilon_0 V}}\hat{\mathcal{E}}(z,t)e^{i(\omega_{ab}/c)(z-ct)},$$

where $\omega_{\mu\nu}=(E_\mu-E_\nu)/\hbar$ is the frequency of the $|\mu\rangle\leftrightarrow|\nu\rangle$ transition. V is the quantization volume of the electromagnetic field, which is taken to be the interaction volume. The $|c\rangle\rightarrow|a\rangle$ transition is driven resonantly by a classical coherent control field with Rabi frequency Ω_c. We consider the case of copropagating fields to minimize the effects of Doppler shift.

To perform a quantum analysis of the light-matter interaction it is useful to introduce locally-averaged atomic operators. Assuming a length interval Δz contains $N_z \gg 1$ atoms over which the slowly-varying amplitude $\hat{\mathcal{E}}(z,t)$ does not change much, we can introduce the locally-averaged, slowly-varying atomic operators

$$\hat{\sigma}_{\mu\nu}(z,t) = \frac{1}{N_z}\sum_{z_j\in N_z}\hat{\sigma}^{j}_{\mu\nu}(t)e^{i(\omega_{\mu\nu}/c)(z-ct)}, \qquad (1)$$

where $\hat{\sigma}^{j}_{\mu\nu}(t)=|\mu^j(t)\rangle\langle\nu^j(t)|$ for the jth atom.

Going to the continuum limit, the interaction Hamiltonian can be written in terms of the locally-averaged atomic operators as

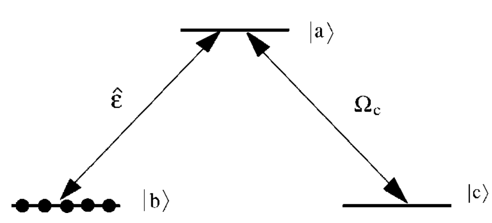

FIG. 1. Λ level structure of the atoms.

$$\hat{\mathcal{H}} = -\int \frac{N\hbar}{L}[g\hat{\sigma}_{ab}(z,t)\hat{\mathcal{E}}(z,t) + \Omega_c(z,t)\hat{\sigma}_{ac}(z,t) + \text{H.c.}]dz, \tag{2}$$

where $g=d_{ba}\sqrt{\omega_{ab}/2\epsilon_0 V\hbar}$ is the atom-field coupling constant, d_{ba} is the atomic dipole moment for the $|b\rangle \leftrightarrow |a\rangle$ transition, and L is the cell length. The equations of motion are then given by

$$\dot{\hat{\sigma}}_{bb} = \gamma_b\hat{\sigma}_{aa} + \gamma_{bc}(\hat{\sigma}_{cc} - \hat{\sigma}_{bb}) - ig\hat{\mathcal{E}}\hat{\sigma}_{ab} + ig^*\hat{\mathcal{E}}^\dagger\hat{\sigma}_{ba} + \hat{F}_{bb},$$

$$\dot{\hat{\sigma}}_{cc} = \gamma_c\hat{\sigma}_{aa} + \gamma_{bc}(\hat{\sigma}_{bb} - \hat{\sigma}_{cc}) - i\Omega_c\hat{\sigma}_{ac} + i\Omega_c^*\hat{\sigma}_{ca} + \hat{F}_{cc},$$

$$\dot{\hat{\sigma}}_{ba} = -\gamma_{ba}\hat{\sigma}_{ba} + ig\hat{\mathcal{E}}(\hat{\sigma}_{bb} - \hat{\sigma}_{aa}) + i\Omega_c\hat{\sigma}_{bc} + \hat{F}_{ba},$$

$$\dot{\hat{\sigma}}_{bc} = -\gamma_{bc}\hat{\sigma}_{bc} - ig\hat{\mathcal{E}}\hat{\sigma}_{ac} + i\Omega_c^*\hat{\sigma}_{ba} + \hat{F}_{bc},$$

$$\dot{\hat{\sigma}}_{ac} = -\gamma_{ac}\hat{\sigma}_{ac} - ig^*\hat{\mathcal{E}}^\dagger\hat{\sigma}_{bc} + i\Omega_c^*(\hat{\sigma}_{aa} - \hat{\sigma}_{cc}) + \hat{F}_{ac},$$

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}} = ig^*N\hat{\sigma}_{ba}, \tag{3}$$

where we have included the decays of the atomic dipole operators γ_μ and the associated Langevin noise operators which describe the effect of spontaneous decay caused by the coupling of atoms to all the vacuum field modes. The random decay process adds noise to individual atomic operators represented by the single atom Langevin noise operators $\hat{F}^i_{\mu\nu}(t)$. The continuous Langevin noise operators are related to the single-atom noise operators by the same relation as Eq. (1)

$$\hat{F}_{\mu\nu}(z,t) = \frac{1}{N_z}\sum_{z_j\in N_z}\hat{F}^i_{\mu\nu}(t)e^{i(\omega_{\mu\nu}/c)(z-ct)}.$$

The decay rate γ_{bc} of the coherence between the two ground states is of critical importance, and arises chiefly from atomic collisions and atoms drifting out of the interaction region. For rubidium vapor cells with a buffer gas, typically $\gamma_{bc} \approx 1$ kHz although it is possible to attain $\gamma_{bc} \approx 160$ Hz [7].

In order to solve the propagation equations we make the usual assumption that the quantum field intensity is much less than that of the classical control field Ω_c [2,12]. Assuming all the atoms are initially in the state $|b\rangle$ we can solve Eq. (3) perturbatively to first order in $g\hat{\mathcal{E}}/\Omega_c$ to obtain a set of three closed equations

$$\dot{\hat{\sigma}}_{ba} = -\gamma_{ba}\hat{\sigma}_{ba} + ig\hat{\mathcal{E}} + i\Omega_c\hat{\sigma}_{bc} + \hat{F}_{ba}, \tag{4}$$

$$\dot{\hat{\sigma}}_{bc} = -\gamma_{bc}\hat{\sigma}_{bc} + i\Omega_c^*\hat{\sigma}_{ba} + \hat{F}_{bc}, \tag{5}$$

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}} = ig^*N\hat{\sigma}_{ba}. \tag{6}$$

To solve these equations we Fourier transform to the frequency domain via

$$\widetilde{F}(z,\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{F}(z,t)e^{i\omega t}dt,$$

where $\omega=0$ corresponds to the carrier frequency ω_{ab} in the interaction picture. We solve Eqs. (4) and (5) for $\hat{\sigma}_{ba}$ in terms of $\hat{\mathcal{E}}$, substitute into Eq. (6), and perform formal integration over z to obtain the field at the exit of the cell after interaction with the EIT medium. We find

$$\widetilde{\mathcal{E}}(L,\omega) = e^{-\Lambda(\omega)L}\widetilde{\mathcal{E}}(0,\omega) + \frac{g^*N}{c}\int_0^L e^{-\Lambda(\omega)(L-s)} \times \left[\frac{-\Omega_c\widetilde{F}_{bc}(s,\omega) + (\omega + i\gamma_{bc})\widetilde{F}_{ba}(s,\omega)}{(\gamma_{ab} - i\omega)(\gamma_{bc} - i\omega) + |\Omega_c|^2}\right]ds, \tag{7}$$

with

$$\Lambda(\omega) = \frac{\frac{|g|^2N}{c}(\gamma_{bc} - i\omega)}{(\gamma_{ba} - i\omega)(\gamma_{bc} - i\omega) + |\Omega_c|^2} - \frac{i\omega}{c}. \tag{8}$$

Equation (7) can be interpreted as follows: The amplitude of the field operator is attenuated and phase shifted according to the function $\Lambda(\omega)$, the values of which depend on the actual frequency component of the field. Expanding $\Lambda(\omega)L$ about the carrier frequency $\omega=0$ gives

$$\Lambda(\omega)L = KL - \frac{i\omega L}{v_g} + \frac{\omega^2}{\delta\omega^2} + O(|\omega|^3), \tag{9}$$

where

$$K = \frac{N|g|^2\gamma_{bc}}{c(\gamma_{ba}\gamma_{bc} + |\Omega_c|^2)},$$

$$v_g = \frac{c}{1 + \frac{N|g|^2(|\Omega_c|^2 - \gamma_{bc}^2)}{(\gamma_{ba}\gamma_{bc} + |\Omega_c|^2)^2}},$$

$$\delta\omega^2 = \frac{c(\gamma_{ba}\gamma_{bc} + |\Omega_c|^2)^3}{N|g|^2L(|\Omega_c|^2(2\gamma_{bc} + \gamma_{ba}) - \gamma_{bc}^3)}.$$

The zeroth order term represents attenuation due to the finite coherence lifetime between the two ground states which is proportional to the dephasing rate γ_{bc}. Note that it also prevents perfect transparency even at resonance and will probably be the ultimate limitation on storage using EIT type techniques. However, it is possibly to significantly reduce γ_{bc} by using Bose-Einstein condensates, for example.

The first order term in ω describes a modification of the group velocity from

$$c \to v_g = \frac{c}{1 + \frac{N|g|^2(|\Omega_c|^2 - \gamma_{bc}^2)}{(\gamma_{ba}\gamma_{bc} + |\Omega_c|^2)^2}}$$

which can be many orders of magnitudes smaller than c. For $|\Omega_c| \gtrsim 2\gamma_{bc}$ the effect of the nonzero γ_{bc} on the group veloc-

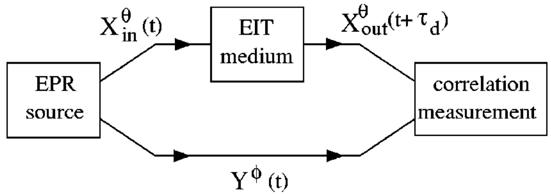

FIG. 2. Setup to measure whether entanglement of slowed light is preserved. $\hat{X}$ and $\hat{Y}$ are two entangled beams where $\hat{X}$ passes through an EIT medium and is consequently delayed by time τ_d while $\hat{Y}$ travels in vacuum. The angle θ and ϕ denotes the specific quadrature to be interrogated.

ity is hardly noticeable. The nonzero dephasing rate does, however, prevent v_g from reaching zero. In fact, the group velocity increases towards c as $|\Omega_c|$ decreases below $\sim 2\gamma_{bc}$, although significant absorption due to the breakdown of EIT will have already occurred once $|\Omega_c|$ becomes comparable to γ_{bc} and thus it is difficult to speak meaningfully of a group velocity.

The quadratic term in Eq. (9) represents absorption of the high frequency components that fall outside the EIT transparency window $\delta\omega$ [2]. This justifies the Taylor's expansion of $\Lambda(\omega)$, since to prevent significant pulse distortion, one would choose the bandwidth of the input pulse $\Delta\omega$ to satisfy $\Delta\omega/\delta\omega \ll 1$. For $|\Omega_c| \leq \gamma_{ab}$ and small γ_{bc}, $\delta\omega$ is approximately a linearly decreasing function of γ_{bc}, a tool which could be used experimentally to estimate the value of γ_{bc}.

Returning to Eq. (7), the second term on the right-hand side represents vacuum noise added to the probe field as it interacts with the atoms. We now determine under what conditions this noise contribution is small.

As we are interested in whether the quantum properties of the input field survive the passage throught the EIT setup, we consider two quantities of interest: one relating to entanglement and the other to squeezing. Here we propose some possible experiments that could be performed to discern the effect of slow light on entanglement and squeezing, and show how the presence of the dephasing and decay mechanisms that we have included in our model manifest themselves in the output beam for each type of experiment.

To investigate the effect of slow light on squeezing, we consider an experiment where the amount of squeezing before and after passing through the EIT medium are compared [8]. For an entanglement measurement, we propose a setup shown in Fig. 2. Starting with a pair of entangled beams, which could be produced, for example, from parametric down conversion, one beam passes through an EIT medium of length L and is consequently delayed by a time τ_d while the other beam passes through the same length of vacuum. At the output, we perform a measurement to quantify the degree of entanglement between the field that was slowed and the field that was not.

To quantify squeezing and entanglement, it is usual to consider the field quadrature operator which has a Fourier transform in the temporal domain of

$$\tilde{X}_{\rm out}^{\theta}(\omega) = \hat{\mathcal{E}}(L,\omega)e^{i\theta} + \tilde{\mathcal{E}}^{\dagger}(L,-\omega)e^{-i\theta}. \tag{10}$$

Using the solution (7), the output quadrature operator is related to the input via the relation

$$\begin{aligned}\tilde{X}_{\rm out}^{\theta}(\omega) &= \tilde{X}_{\rm in}^{\theta}(\omega)e^{-\Lambda(\omega)L} - \frac{Ng}{c}\int_0^L e^{-\Lambda(\omega)(L-s)} \\ &\quad\times \frac{\Omega_c\tilde{F}_{bc}(s,\omega)e^{i\theta} + \Omega_c^*\tilde{F}_{bc}^{\dagger}(s,-\omega)e^{-i\theta}}{(\gamma_{ba}-i\omega)(\gamma_{bc}-i\omega)+|\Omega_c|^2}ds \\ &\quad+ \frac{Ng}{c}\int_0^L e^{-\Lambda(\omega)(L-s)}(\omega+i\gamma_{bc}) \\ &\quad\times \frac{\tilde{F}_{ba}(s,\omega)e^{i\theta} - \tilde{F}_{ba}^{\dagger}(s,-\omega)e^{-i\theta}}{(\gamma_{ba}-i\omega)(\gamma_{bc}-i\omega)+|\Omega_c|^2}ds,\end{aligned} \tag{11}$$

where we take g to be real for simplicity.

To calculate the degradation of squeezing after passing through the EIT medium, we calculate the output squeezing flux spectrum defined by

$$\mathcal{S}_{\rm out}(\omega)\delta(\omega+\omega') = \frac{c}{L}\langle\tilde{X}_{\rm out}(\omega)\tilde{X}_{\rm out}(\omega')\rangle. \tag{12}$$

In order to compute Eq. (12) using Eq. (11) it is necessary to calculate the correlation functions of the Langevin noises involved. These can be derived using the generalized Einstein relations. In the space-time domain, the generalized Einstein relation for the single atom operators can be written as [13,14]

$$\begin{aligned}\langle\hat{F}_{\mu\nu}^i(t_1)\hat{F}_{\alpha\beta}^j(t_2)\rangle &= \langle\mathcal{D}(\hat{\sigma}_{\mu\nu}^j\hat{\sigma}_{\alpha\beta}^i) - \mathcal{D}(\hat{\sigma}_{\mu\nu}^i)\hat{\sigma}_{\alpha\beta}^i - \hat{\sigma}_{\mu\nu}^i\mathcal{D}(\hat{\sigma}_{\alpha\beta}^j)\rangle \\ &\quad\times \delta(t_1-t_2)\delta_{ij},\end{aligned} \tag{13}$$

where the notation $\mathcal{D}(\hat{\sigma}_{\mu\nu}^j)$ denotes the deterministic part of the Heisenberg equation of motion for $\hat{\sigma}_{\mu\nu}^j$, that is the equation for $\dot{\hat{\sigma}}_{\mu\nu}^j$ with the Langevin noise terms omitted. The Dirac delta function in Eq. (13) represents the short memory of the vacuum reservoir modes while the Kronecker delta occurs because we assume each atom couples only to its own reservoir.

Using the definition of the locally-averaged Langevin force operators in terms of their single atom counterpart, we derive the following for the nonzero correlations of the continuous Langevin correlations. After transforming to the frequency domain, these are

$$\begin{aligned}\langle\tilde{F}_{ba}(z_1,\omega_1)\tilde{F}_{ab}^{\dagger}(z_2,\omega_2)\rangle &= \frac{\delta(z_1-z_2)\delta(\omega_1+\omega_2)}{n\mathcal{A}}(\gamma_{ba}\langle\hat{\sigma}_{aa}\rangle \\ &\quad+ 2\gamma_{ba}\langle\hat{\sigma}_{bb}\rangle - \gamma_{bc}\langle\hat{\sigma}_{bb}-\hat{\sigma}_{cc}\rangle),\end{aligned} \tag{14}$$

$$\langle\tilde{F}_{ba}^{\dagger}(z_1,\omega_1)\tilde{F}_{bc}(z_2,\omega_2)\rangle = \frac{\delta(z_1-z_2)\delta(\omega_1+\omega_2)}{n\mathcal{A}}\gamma_{bc}\langle\hat{\sigma}_{ac}\rangle, \tag{15}$$

$$\langle\tilde{F}_{bc}^{\dagger}(z_1,\omega_1)\tilde{F}_{ba}(z_2,\omega_2)\rangle = \frac{\delta(z_1-z_2)\delta(\omega_1+\omega_2)}{n\mathcal{A}}\gamma_{bc}\langle\hat{\sigma}_{ca}\rangle, \tag{16}$$

$$\langle \tilde{F}_{bc}(z_1,\omega_1)\tilde{F}^{\dagger}_{bc}(z_2,\omega_2)\rangle = \frac{\delta(z_1-z_2)\,\delta(\omega_1+\omega_2)}{n\mathcal{A}}(\gamma_{ba}\langle\hat{\sigma}_{aa}\rangle + \gamma_{bc}\langle\hat{\sigma}_{cc}+\hat{\sigma}_{bb}\rangle), \tag{17}$$

$$\langle \tilde{F}^{\dagger}_{bc}(z_1,\omega_1)\tilde{F}_{bc}(z_2,\omega_2)\rangle = \frac{\delta(z_1-z_2)\,\delta(\omega_1+\omega_2)}{n\mathcal{A}}(\gamma_{ba}\langle\hat{\sigma}_{aa}\rangle + \gamma_{bc}\langle\hat{\sigma}_{cc}+\hat{\sigma}_{bb}\rangle), \tag{18}$$

where n is the atomic density, $\mathcal{A}$ is the cross section area of the beam and we have taken $\gamma_b=\gamma_c=\gamma_{ba}=\gamma_{ca}$ in Eq. (3) for convenience.

We substitute Eq. (11) into Eq. (12) and simplify using Eqs. (14)–(18). In accordance with the weak probe assumption we also set $\langle\hat{\sigma}_{aa}\rangle\approx\langle\hat{\sigma}_{cc}\rangle\approx\langle\hat{\sigma}_{ac}\rangle\approx 0$. The output squeezing spectrum (normalized with shot noise at 1) is

$$\mathcal{S}_{\mathrm{out}}(\omega) = \mathcal{S}_{\mathrm{in}}(\omega)e^{-2\,\mathrm{Re}\{\Lambda(\omega)\}L} + \frac{N|g|^2}{c}\left[\frac{1-e^{-2\,\mathrm{Re}\{\Lambda(\omega)\}L}}{2\,\mathrm{Re}\{\Lambda(\omega)\}}\right] \times \frac{(\omega^2+\gamma_{bc}^2)(2\gamma_{ba}-\gamma_{bc})+2|\Omega_c|^2\gamma_{bc}}{|(\gamma_{ba}-i\omega)(\gamma_{bc}-i\omega)+|\Omega_c|^2|^2}. \tag{19}$$

Before discussing how much the squeezing in the probe beam degrades due to the slow light propagation, we first consider how the entanglement between two beams degrades after one of the fields passes through an EIT medium. If we define the difference operator between the quadratures of the two beams $\hat{X}^{\theta}_{\mathrm{in}}(t)$ and $\hat{Y}^{\phi}_{\mathrm{in}}(t)$ as

$$\hat{Z}_{\mathrm{in}}(\theta,\phi,t) = \hat{X}^{\theta}_{\mathrm{in}}(t) - \hat{Y}^{\phi}_{\mathrm{in}}(t) \tag{20}$$

then the two beams are said to be entangled when both of the combinations involving non-commuting observables for one of the beams $\hat{Z}_{\mathrm{in}}(\theta,\phi,t)$ and $\hat{Z}_{\mathrm{in}}(\theta+\pi/2,\phi-\pi/2,t)$ are squeezed [15,16]. Note that squeezing in both is required to constitute an Einstein-Podolsky-Rosen (EPR) paradox [17], and that squeezing in just one variable is insufficient.

At the output of the EIT medium, we account for the effects of slow light by looking for squeezing in the time adjusted variable

$$\hat{Z}_{\mathrm{out}}(\theta,\phi,t) = \hat{X}^{\theta}_{\mathrm{out}}(t+\tau_d) - \hat{Y}^{\phi}_{\mathrm{out}}(t), \tag{21}$$

where $\tau_d=L(1/v_g-1/c)$ is the delay due to the EIT effect compared to light that had travelled distance L in vacuum. Again, for a nonclassical correlation between the two beams, we require squeezing in $\hat{Z}_{\mathrm{out}}(\theta,\phi,t)$ and $\hat{Z}_{\mathrm{out}}(\theta+\pi/2,\phi-\pi/2,t)$. Fourier transforming Eq. (21) we obtain $\tilde{Z}_{\mathrm{out}}(\theta,\phi,\omega)=\tilde{X}^{\theta}_{\mathrm{out}}(\omega)e^{-i\omega\tau_d}-\tilde{Y}^{\phi}_{\mathrm{out}}(\omega)$ with $\tilde{Y}^{\phi}_{\mathrm{out}}=\tilde{Y}^{\phi}_{\mathrm{in}}e^{i\omega L/c}$, the phase shift arising from free evolution of the light field that has propagated through a distance L in vacuum at speed c.

Defining the flux of entanglement spectrum for $\tilde{Z}_{\mathrm{out}}(\theta,\phi,\omega)$ in a way analogous to Eq. (12)

$$\mathcal{A}_{\mathrm{out}}(\theta,\phi,\omega)\,\delta(\omega+\omega') = \frac{c}{L}\langle\tilde{Z}_{\mathrm{out}}(\theta,\phi,\omega)\tilde{Z}_{\mathrm{out}}(\theta,\phi,\omega')\rangle, \tag{22}$$

the entanglement criteria described above (called Duan's inseparability criteria [15]) can be recast using Duan's inseparability measure $\mathcal{I}(\omega)$ into the form [15,16]

$$\mathcal{I}(\omega) = \sqrt{\mathcal{A}_{\mathrm{out}}(\theta,\phi,\omega)\mathcal{A}_{\mathrm{out}}(\theta+\pi/2,\phi-\pi/2,\omega)} < 1. \tag{23}$$

Evaluating the right-hand side of Eq. (22) using Eq. (11) and (14)–(18) we get

$$\begin{aligned}\frac{L}{c}\delta(\omega+\omega')\mathcal{A}_{\mathrm{out}}(\theta,\phi,\omega) &= \langle\tilde{X}^{\theta}_{\mathrm{in}}(\omega)\tilde{X}^{\theta}_{\mathrm{in}}(\omega')\rangle e^{-2\,\mathrm{Re}\{\Lambda(\omega)\}L} - \langle\tilde{X}_{\mathrm{in}}(\omega)^{\theta} \\ &\quad\times\tilde{Y}^{\phi}_{\mathrm{in}}(\omega')\rangle e^{-[\Lambda(\omega)+i\omega/v_g]L} - \langle\tilde{Y}^{\phi}_{\mathrm{in}}(\omega)\tilde{X}^{\theta}_{\mathrm{in}}(\omega')\rangle \\ &\quad\times e^{-[\Lambda(-\omega)-i\omega/v_g]L} + \langle\tilde{Y}^{\phi}_{\mathrm{in}}(\omega)\tilde{Y}^{\phi}_{\mathrm{in}}(\omega')\rangle \\ &\quad+\delta(\omega+\omega')\frac{N|g|^2}{c}\left[\frac{1-e^{-2\,\mathrm{Re}\{\Lambda(\omega)\}L}}{2\,\mathrm{Re}\{\Lambda(\omega)\}}\right] \\ &\quad\times\frac{(\omega^2+\gamma_{bc}^2)(2\gamma_{ba}-\gamma_{bc})+2|\Omega_c|^2\gamma_{bc}}{|(\gamma_{ba}-i\omega)(\gamma_{bc}-i\omega)+|\Omega_c|^2|^2}.\end{aligned} \tag{24}$$

Note that the noise addition represented by the last term in Eq. (24) is identical to the one that appeared in the output squeezing spectrum, a clear consequence of our choice of entanglement measure. Also from this term, we see that the destructive effects are independent of the particular quadrature considered. In other words, the imperfections in the setup adversely affect the phase and amplitude correlations by the same amount.

For a clearer insight into how the entanglement is affected, it is instructive to utilize the Taylor's expansion for $\Lambda(\omega)$ in Eq. (9) and throw away the quadratic and higher order terms on the grounds that the probe bandwidth is well inside the transparency window. Then under the parameter regime when the absorption is low ($KL\ll 1$), the entanglement at the output is

$$\mathcal{A}_{\mathrm{out}}(\theta,\phi,\omega) \approx \mathcal{A}_{\mathrm{in}}(\theta,\phi,\omega)e^{-KL} + \frac{N|g|^2}{c}\left[\frac{1-e^{-2KL}}{2K}\right] \times\frac{(\omega^2+\gamma_{bc}^2)(2\gamma_{ba}-\gamma_{bc})+2|\Omega_c|^2\gamma_{bc}}{|(\gamma_{ba}-i\omega)(\gamma_{bc}-i\omega)+|\Omega_c|^2|^2}. \tag{25}$$

As an indication of the amount of degradation in squeezing and entanglement, we consider a cell of length 3.5 cm with atomic density of 1×10^{12} atoms per cm^3, $\gamma_{ba}=6\pi$ MHz, $\gamma_{bc}=10$ Hz, and $\Omega_c=30\pi$ MHz. The modified group velocity is 3100 ms^{-1} corresponding to a delay time of 11 μs and a transparency window of 6.5 MHz. For normalized noise variance and choosing Duan's inseparability measure as defined in Eq. (23) to be initially 0.4 at 1 MHz (1.0 is the standard quantum limit), the output variance characterizing squeezing retained in the slowed light is 0.43, and the output entanglement is 0.45. In contrast, keeping all other parameters the same but choosing $\gamma_{bc}=5$ kHz, the normalized noise variance and entanglement measure at the output are 0.49 and 0.53 respectively, with no significant changes in group velocity or transparency window. Thus we see that preservation of quantum properties relies crucially on having small values of γ_{bc} as well as the probe bandwidth staying well inside the transparency window, as can be seen from the presence of ω^2 type terms in Eqs. (19) and (24). Mechanisms for reducing γ_{bc} via buffer gas in vapor cells have been investigated experimentally [7,18].

In conclusion, we have demonstrated that both entanglement and squeezing of the probe field can be almost perfectly preserved under slow light setup, even when quantum noise due to atom-light interactions are taken into account, provided the ground state decoherence rate γ_{bc} is sufficiently small. In many experimental situations, however, there may be other mechanisms, for example, coupling to states outside the Λ system, which are the dominant contribution to losses in EIT [19]. Also, we have only considered the slow light scenario while the key to true storage of light using EIT relies on dynamically controlling Ω_c as outlined in [2]. Nevertheless, our results still underline the robustness of quantum-information delay using an atomic lambda system, allowing the possibility of using slow light for squeezing or entanglement experiments.

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