plant-model matching conditions with unparallel b_i in B, the asymptotic state tracking is achieved by the adaptive control scheme. At the time instant when one actuator fails, there is a transient response in the state tracking errors because of the system actuation structure change. As time goes on, with the help of controller adaptation, the tracking errors become smaller and go to zero. At the time instant when one actuator fails, the controller parameters (e.g., k_3 in Fig. 2) also have a transient behavior, and then go to some constant values. These values are not necessarily to be the true matching parameters. All signals in the adaptive control system are bounded, and stability and convergence are ensured.

VI. Concluding Remarks

In this note, we derived a set of new necessary and sufficient conditions for actuator failure compensation for linear time-invariant system with actuator failures characterized by unknown input signals stuck at some unknown fixed values at unknown time instants, for state tracking with state feedback. It is shown that the number of active actuators and the actuation structure are crucial for compensation designs. With more than one actuator active, the necessary and sufficient conditions for actuator failure compensation design are much less restrictive than those conditions with only one actuator active. Such conditions are required for both the nominal design with system knowledge and the adaptive design without system knowledge. An adaptive actuator failure compensation control scheme based on relaxed system actuation conditions is developed for systems with unknown dynamics parameters and actuator failure parameters including failure values, times and patterns. For the developed adaptive control scheme, the stability of the closed-loop system and asymptotic state tracking properties are ensured. Simulation results for the linearized Boeing 747 model (lateral motion) verified the desired system performance with failure compensation.

References

[1] F. Ahmed-Zaid, P. Ioannou, K. Gousman, and R. Rooney, "Accommodation of failures in the F-16 aircraft using adaptive control," *IEEE Contr. Syst. Mag.*, vol. 11, pp. 73–78, Jan. 1991.

[2] M. Bodson and J. E. Groszkiewicz, "Multivariable adaptive algorithms for reconfigurable flight control," *IEEE Trans. Contr. Syst. Technol.*, vol. 5, pp. 217–229, Mar. 1997.

[3] J. D. Boskovic and R. K. Mehra, "Stable multiple model adaptive flight control for accommodation of a large class of control effector failures," in *Proc. 1999 Amer. Control Conf.*, pp. 1920–1924.

[4] J. D. Boskovic, S.-H. Yu, and R. K. Mehra, "A stable scheme for automatic control reconfiguration in the presence of actuator failures," in *Proc. 1998 Amer. Control Conf.*, pp. 2455–2459.

[5] M. A. Demetriou and M. M. Polycarpou, "Incipient fault diagnosis of dynamical systems using online approximators," *IEEE Trans. Automat. Contr.*, vol. 43, pp. 1612–1617, Nov. 1998.

[6] P. M. Frank, "Enhancement of robustness in observer-based fault detection," *Int. J. Control*, vol. 59, no. 4, pp. 955–981, 1994.

[7] G. F. Franklin, J. D. Powell, and A. Emani-Naeini, *Feedback Control of Dynamic Systems*, 3rd ed. Reading, MA: Addison-Wesley, 1994.

[8] M. Gopinathan, J. D. Boskovic, R. K. Mehra, and C. Rago, "A multiple model predictive scheme for fault-tolerant flight control design," in *Proc. 37th IEEE Conf. Decision Control*, 1998, pp. 1376–1381.

[9] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Upper Saddle River, NJ: Prentice-Hall, 1996.

[10] W. D. Morse and K. A. Ossman, "Model-following reconfigurable flight control system for the AFTI/F16," *J. Guid., Control Dyna.*, vol. 13, no. 6, pp. 969–976, 1990.

[11] M. M. Polycarpou and A. B. Trunov, "Learning approach to nonlinear fault diagnosis: Detectability analysis," *IEEE Trans. Automat. Contr.*, vol. 45, pp. 806–812, Apr. 2000.

[12] G. Tao, S. M. Joshi, and X. L. Ma, "Adaptive state feedback and tracking control of systems with actuator failures," *IEEE Trans. Automat. Contr.*, vol. 46, pp. 78–95, Jan. 2001.

[13] A. T. Vemuri and M. M. Polycarpou, "Robust nonlinear fault diagnosis in input–output systems," *Int. J. Control*, vol. 68, no. 2, pp. 343–360, 1997.

[14] H. Wang and S. Daley, "Actuator fault diagnosis: An adaptive observer-based technique," *IEEE Trans. Automat. Contr.*, vol. 41, pp. 1073–1078, July 1996.

[15] H. Wang, Z. J. Huang, and S. Daley, "On the use of adaptive updating rules for actuator and sensor fault diagnosis," *Automatica*, vol. 33, no. 2, pp. 217–225, 1997.

Stability Analysis and Observer Design for Neutral Delay Systems

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***Abstract*—This note is concerned with the observer design problem for a class of linear delay systems of the neutral-type. The problem addressed is that of designing a full-order observer that guarantees the exponential stability of the error dynamic system. An effective algebraic matrix equation approach is developed to solve this problem. In particular, both observer analysis and design problems are investigated. By using the singular value decomposition technique and the generalized inverse theory, sufficient conditions for a neutral-type delay system to be exponentially stable are first established. Then, an explicit expression of the desired observers is derived in terms of some free parameters. Furthermore, an illustrative example is used to demonstrate the validity of the proposed design procedure.**

***Index Terms*—Algebraic matrix equation, exponential stability, neutral systems, observer design, time-delay systems.**

I. Introduction

In the past few decades, the stability analysis and feedback stabilization problems for neutral-type delay systems have attracted the attention of many authors, see, e.g., [2], [3], [6], [7], [9], [12], [14], [19], [20], and the references therein. The systems that can be described by neutral-type systems include, but are not limited to, lumped parameter networks interconnected by transmission lines, systems of a turbojet engine, infeed grinding, and continuous induction heating of a thin moving body [9]. A special class of neutral systems is retarded systems that include applications in chemical reactors, rolling mill, ship stabilization, manual control, microwave oscillator, population immune response, and distribution of albumin in the blood stream.

In [9], it has been shown that the reactor in a chemical engineering system can sometimes be described by a linear neutral delay equation

$$\dot{x}(t) - J\dot{x}(t-d) = Ax(t) + A_d x(t-d).$$

Note that the presence of a retarded argument in the state derivatives makes the investigation of such equations more complicated than equa-

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tions with delays only in the states. The major difficulty results from the fact that neutral systems almost always have infinite spectrum, called a neutral root chain, in a vertical strip of the complex plane (see, e.g., [3], [14], and [20]). Thus, in most existing literature, the authors assumed that either there is no unstable neutral root chain or they can first use derivative feedback to assign the unstable neutral root chain to the left-hand side of the complex plane. In the stochastic setting, the neutral stochastic delay systems have been introduced in [8], and the asymptotic stability and exponential stability of such kind of systems have been studied in [8] and [11], respectively.

On the other hand, the problem of observer design has been well studied for more than three decades in various branches of science and engineering. The celebrated Luenberger observer theory provides a solution to this problem, but it no longer holds for linear neutral systems. We call an observer an exponential one if the dynamics of the estimation error is exponentially stable. It is noted that the design of exponentially fast observers for linear and nonlinear stochastic systems is also an attractive research topic, see, e.g., [15]–[17], and references therein. So far, the exponential observer design problem for *neutral-type* delay systems has not yet been fully investigated in the literature, and remains to be important and challenging.

In this note, we address the observer design problem for a class of linear neutral systems. Here, attention is focused on the design of a linear observer such that the dynamics of the estimation error is exponentially stable, independent of the time delay. Sufficient conditions are proposed to guarantee the existence of a desired exponential observer, which is derived in terms of the solutions to several algebraic matrix equations. Unlike most existing work, by using the present approach, we do not have to perform a spectral analysis, that is, to consider the effect due to the unstable neutral root chain. We demonstrate the usefulness and applicability of the developed theory by means of a numerical example.

Notation: The notations used in this note are fairly standard. $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ and $\mathbb{R}^+$ denote the n-dimensional Euclidean space, the set of all $n \times m$ real matrices and the set of all positive scalars, respectively. $\mathbb{N}$ is the set of all natural numbers. The superscript "T" denotes the transpose and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are real symmetric matrices, means that $X - Y$ is positive–semidefinite (respectively, positive definite). I is the identity matrix with compatible dimension. We let $h > 0$ and $C([-h, 0]; \mathbb{R}^n)$ denote the family of continuous functions φ from $[-h, 0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-h\leq\theta\leq 0} |\varphi(\theta)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a matrix, denote by $\|A\|$ its operator norm, i.e., $\|A\| = \sup\{|Ax|: |x| = 1\} = \sqrt{\lambda_{\max}(A^TA)}$ where $\lambda_{\max}(\cdot)$ [respectively, $\lambda_{\min}(\cdot)$] means the largest (respectively, smallest) eigenvalue of A. A^+ stands for the Moore–Penrose inverse of A. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

II. Problem Formulation and Assumptions

Consider the following linear continuous-time state-delayed system of the neutral-type:

$$\dot{x}(t) - J\dot{x}(t-h) = Ax(t) + A_d x(t-h) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^p$ is the measurement output. h denotes the constant time-delay which appears in both the state and the derivative term of the system equation. The initial data $x(t)$ satisfies $x(t) = \varphi(t)$ for $t \in [-h, 0]$ and $\varphi := \{\varphi(s): -h \leq s \leq 0\} \in C([-h, 0]; \mathbb{R}^n)$. A, J, A_d, E_1, C, E_2 are known constant matrices with appropriate dimensions.

As discussed in the introduction, the system in (1) and (2) is of the neutral type and can thus represent certain important kinds of physical systems. In this note, we consider the following full-order linear observer:

$$\dot{\hat{x}}(t) - J\dot{\hat{x}}(t-h) = A\hat{x}(t) + A_d\hat{x}(t-h) + K[y(t) - C\hat{x}(t)] \tag{3}$$

where the constant matrix K is the observer parameter vector to be designed.

Let the error state be

$$e(t) = x(t) - \hat{x}(t) \tag{4}$$

then it follows from (1)–(3) that:

$$\dot{e}(t) - J\dot{e}(t-h) = A_c e(t) + A_d e(t-h) \tag{5}$$

where

$$A_c := A - KC. \tag{6}$$

Assumption 1: The matrix J satisfies $J \neq 0$ and $\|J\| < 1$.

Now, observe the error dynamic system (5) and let $e(t;\xi)$ denote the state trajectory from at time t corresponding to the initial data $e(\theta) = \xi(\theta)$ on $-h \leq \theta \leq 0$ in $C([-h, 0]; \mathbb{R}^n)$. Clearly, the system (5) admits a trivial solution $e(t; 0) \equiv 0$ corresponding to the initial data $\xi = 0$ (see [8] and [11]). Also, since $J \neq 0$, it follows from [7, Th. 7.2] that the solution $e(t)$ of (5) exists and is unique.

Definition 1: Given $\xi \in C([-h, 0]; \mathbb{R}^n)$, the corresponding trivial solution of the system (5) is asymptotically stable if

$$\lim_{t\to\infty} |e(t;\xi)| = 0 \tag{7}$$

and is exponentially stable if there exist constants $\alpha > 0$ and $\beta > 0$ such that

$$|e(t;\xi)| \leq \sqrt{\alpha}\, e^{-\beta t/2} \sup_{-h\leq\theta\leq 0} |\xi(\theta)|. \tag{8}$$

Definition 2: The observer (3) is said to be an exponential (respectively, asymptotic) observer if, for every $\xi \in C([-h, 0]; \mathbb{R}^n)$, the corresponding error dynamics system (5) is exponentially (respectively, asymptotically) stable.

The primary objective of this note is to design an exponential observer for linear neutral time-delay system (1)–(2). To be specific, we shall focus on the design of the observer parameter, K, such that the error dynamic system (5) is exponentially stable, independent of the time-delay h.

III. Stability Analysis

In this section, we tackle the observer analysis problem. Suppose that the observer parameter, K, is given. We shall establish sufficient conditions under which linear neutral delay system (5) is exponentially stable.

To begin with, we give a lemma that will be frequently used in deriving our main results.

Lemma 1: Let $f \in \mathbb{R}^n$, $g \in \mathbb{R}^n$ and $\varepsilon > 0$. Then, we have $f^Tg + g^Tf \leq \varepsilon f^Tf + \varepsilon^{-1}g^Tg$.

Proof: The proof follows from the inequality $(\varepsilon^{1/2}f - \varepsilon^{-1/2}g)^T(\varepsilon^{1/2}f - \varepsilon^{-1/2}g) \geq 0$ immediately. ■

The next theorem will show that the exponential stability of the system (5) is related to the existence of the positive definite solution to an algebraic matrix equation, and, therefore, offer a key for solving the addressed observer design problem.

Theorem 1: Let the observer parameter K be given, $R > 0$ be a positive–definite matrix and $\sigma > 0$ be a sufficiently small scalar. If there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ such that the following matrix equation:

$$A_c^T P + PA_c + \varepsilon_2 A_c^T A_c + \varepsilon_1 P^2 + \left(\varepsilon_2^{-1} + \varepsilon_3^{-1}\right) J^T P^2 J + \left(\varepsilon_1^{-1} + \varepsilon_3\right) A_d^T A_d + \sigma I + R = 0 \quad (9)$$

has a solution $P > 0$, then (5) is exponentially stable.

Proof: Fix $\xi \in C([-\tau, 0]; \mathbb{R}^n)$ arbitrarily and write $e(t;\xi) = e(t)$. For $(e(t), t) \in \mathbb{R}^n \times \mathbb{R}_+$, we define the following Lyapunov function candidate:

$$V(e, t) = (e(t) - Je(t-h))^T P(e(t) - Je(t-h)) + \int_{t-h}^{t} e^T(s) Q e(s)\, ds \quad (10)$$

where $P > 0$ is a solution of matrix equation (9) and $Q > 0$ is defined by

$$Q := \left(\varepsilon_2^{-1} + \varepsilon_3^{-1}\right) J^T P^2 J + \left(\varepsilon_1^{-1} + \varepsilon_3\right) A_d^T A_d + \sigma I. \quad (11)$$

The derivative of V along a given trajectory is obtained as

$$\begin{aligned}\frac{d}{dt} V(e, t) &= e^T(t) A_c^T P e(t) + e^T(t) P A_c e(t) + e^T(t) P A_d e(t-h) \\ &\quad + e^T(t-h) A_d^T P e(t) - e^T(t) A_c^T P J e(t-h) \\ &\quad - e^T(t-h) J^T P A_c e(t) - e^T(t-h) A_d^T P J e(t-h) \\ &\quad - e^T(t-h) J^T P A_d e(t-h) + e^T(t) Q e(t) \\ &\quad - e^T(t-h) Q e(t-h). \end{aligned} \quad (12)$$

Let $\varepsilon_1, \varepsilon_2, \varepsilon_3$ be positive scalars. It then follows from Lemma 1 that:

$$\begin{aligned}&e^T(t) P A_d e(t-h) + e^T(t-h) A_d^T P e(t) \\ &\quad \le \varepsilon_1 e^T(t) P^2 e(t) + \varepsilon_1^{-1} e^T(t-h) A_d^T A_d e(t-h)\end{aligned} \quad (13)$$

$$\begin{aligned}&-e^T(t) A_c^T P J e(t-h) - e^T(t-h) J^T P A_c e(t) \\ &\quad \le \varepsilon_2 e^T(t) A_c^T A_c e(t) + \varepsilon_2^{-1} e^T(t-h) J^T P^2 J e(t-h)\end{aligned} \quad (14)$$

$$\begin{aligned}&-e^T(t-h) A_d^T P J e(t-h) - e^T(t-h) J^T P A_d e(t-h) \\ &\quad \le \varepsilon_3 e^T(t-h) A_d^T A_d e(t-h) \\ &\qquad + \varepsilon_3^{-1} e^T(t-h) J^T P^2 J e(t-h).\end{aligned} \quad (15)$$

For simplicity, we denote

$$\Pi := A_c^T P + PA_c + \varepsilon_2 A_c^T A_c + \varepsilon_1 P^2 + Q \quad (16)$$

where Q is defined in (11), and then (9) and (11) indicate that $\Pi = -R < 0$.

Substituting (11), (13)–(15) into (12) yields

$$\begin{aligned}\frac{d}{dt} V(e, t) &\le e^T(t) \Pi e(t) - \sigma e^T(t-h) e(t-h) \\ &= \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix}^T \begin{bmatrix} \Pi & 0 \\ 0 & -\sigma \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix} \\ &\le -\min(\lambda_{\min}(-\Pi), \sigma) \left\| \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix} \right\|^2 \\ &\le -\min(\lambda_{\min}(-\Pi), \sigma) |e(t)|^2 < 0 \end{aligned} \quad (17)$$

which implies from Assumption 1 and [6] that the system (5) is asymptotically stable.

In order to show the exponential stability, we need to make some standard manipulations on the relation (17) by utilizing the technique developed in [11] and [13]. The details are along the similar line of the proof of [11, Th. 2.1], and are thus omitted. Here, we just mention that, for the exponential stability of (5), the required constant $\beta > 0$ in (8) is the unique root of the equation

$$\min(\lambda_{\min}(-\Pi), \sigma) - \beta \lambda_{\max}(P) - \beta h \lambda_{\max}(Q) e^{\beta h} = 0 \quad (18)$$

and the required constant $\alpha > 0$ can be determined by

$$\alpha := \lambda_{\min}^{-1}(P) \Big[\lambda_{\max}(P) + h \lambda_{\max}(Q) \Big(1 + h e^{\beta h} \Big) \Big].$$

This completes the proof of Theorem 1. ■

Remark 1: The use of the matrix $R > 0$ is just to ensure that $\Pi < 0$. In general, this positive–definite matrix should be chosen sufficiently small in a matrix norm sense.

Remark 2: In Theorem 1, the analysis results for the exponential stability of the error dynamic system (5) are established. Note that the results also hold for general linear neutral-type delay systems. It is shown that the addressed observer design problem is solvable if the solution to an algebraic matrix equation is known to exist. The results in Theorem 1 may be conservative due to the use of the inequalities in (13)–(15). However, the conservatism can be significantly reduced in a matrix norm sense by appropriate choices of the parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$. For the relevant discussion and corresponding optimization algorithm, we refer the reader to [18] and the references therein.

Remark 3: As mentioned in Section I, different from most existing literature, we do not need to conduct a spectral analysis and reassign the unstable neutral root chain when testing the stability of a neutral system. Theorem 1 shows that the stability of a neutral system is only related to the solution of an algebraic Riccati-like equation, and thus provides us with a more convenient way to deal with the stability of the neutral system.

IV. Observer Design

We shall focus on the observer design problem in this section. Note that since the pioneering work of Luenberger, significant advances have been made in the observer theory. Owing to its utility and its intimate connection with fundamental system concepts, the observer theory has long been one of the cornerstones of modern system theory, and has been substantially developed in many different directions, such as system monitoring and regulation, fault identification and detection.

Based on Theorem 1, the task of this section can be divided into two parts. The first is to derive the conditions under which there exists an observer gain matrix, K, such that the matrix equation (9) holds for some positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and positive definite matrix $R > 0$. The second is to parameterize the desired observer gains, if they exist.

In the sequel, the following lemma is needed.

Lemma 2 [5]: Let $X \in \mathbb{R}^{m \times n}$ and $Y \in \mathbb{R}^{m \times p}$ $(m \le p)$. There exists a matrix V that satisfies simultaneously

$$Y = XV \quad VV^T = I$$

if and only if

$$XX^T = YY^T.$$

In this case, a general solution for V can be expressed as

$$V = V_X \begin{bmatrix} I & 0 \\ 0 & U \end{bmatrix} V_Y^T, \quad U \in \mathbb{R}^{(n - r_X) \times (p - r_X)}, \qquad UU^T = I \quad (19)$$

where V_X and V_Y come from the singular value decomposition of X and Y, respectively

$$X = U_X \begin{bmatrix} Z_X & 0 \\ 0 & 0 \end{bmatrix} V_X^T = [\,U_{X1} \quad U_{X2}\,] \begin{bmatrix} Z_X & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{X1}^T \\ V_{X2}^T \end{bmatrix} \tag{20}$$

$$Y = U_Y \begin{bmatrix} Z_Y & 0 \\ 0 & 0 \end{bmatrix} V_Y^T = [\,U_{Y1} \quad U_{Y2}\,] \begin{bmatrix} Z_Y & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{Y1}^T \\ V_{Y2}^T \end{bmatrix} \tag{21}$$

and $r_X = \operatorname{rank}(X)$, $U_X = U_Y$, $Z_X = Z_Y$.

Now, to obtain the conditions for the existence of a desired observer gain, K, we can rearrange (9) as follows:

$$A^T P + PA + \varepsilon_1 P^2 + \varepsilon_2 A^T A + \left(\varepsilon_2^{-1} + \varepsilon_3^{-1}\right) J^T P^2 J + \left(\varepsilon_1^{-1} + \varepsilon_3\right) A_d^T A_d + R + \sigma I + \Delta(K) = 0 \tag{22}$$

where

$$\begin{aligned}\Delta(K) &= -(KC)^T (P + \varepsilon_2 A) - (P + \varepsilon_2 A)^T (KC) + (KC)^T (\varepsilon_2 I)(KC) \\ &= \left[\varepsilon_2^{1/2} (KC)^T - \varepsilon_2^{-1/2} \left(P + \varepsilon_2 A^T\right)\right] \\ &\quad \cdot \left[\varepsilon_2^{1/2} (KC)^T - \varepsilon_2^{-1/2} \left(P + \varepsilon_2 A^T\right)\right]^T \\ &\quad - \varepsilon_2^{-1} \left(P + \varepsilon_2 A^T\right) \left(P + \varepsilon_2 A^T\right)^T \end{aligned} \tag{23}$$

and (22) can be equivalently written by

$$\begin{aligned}&\left(\varepsilon_2^{-1} - \varepsilon_1\right) P^2 - \left(\varepsilon_1^{-1} + \varepsilon_3\right) A_d^T A_d - \left(\varepsilon_2^{-1} + \varepsilon_3^{-1}\right) J^T P^2 J - R - \sigma I \\ &= \left[\varepsilon_2^{1/2} (KC)^T - \varepsilon_2^{-1/2} \left(P + \varepsilon_2 A^T\right)\right] \\ &\quad \cdot \left[\varepsilon_2^{1/2} (KC)^T - \varepsilon_2^{-1/2} \left(P + \varepsilon_2 A^T\right)\right]^T. \end{aligned} \tag{24}$$

Notice that the right-hand side of (24) is positive–semidefinite. Obviously, one of the necessary conditions for the existence of a desired observer gain is

$$\Omega := \left(\varepsilon_2^{-1} - \varepsilon_1\right) P^2 - \left(\varepsilon_1^{-1} + \varepsilon_3\right) A_d^T A_d - \left(\varepsilon_2^{-1} + \varepsilon_3^{-1}\right) J^T P^2 J - R - \sigma I \geq 0. \tag{25}$$

Now, assume that (25) is true and let $\Omega^{1/2}$ be the square root of Ω. Then, (9) or (24) becomes

$$\left[\varepsilon_2^{1/2} (KC)^T - \varepsilon_2^{-1/2} \left(P + \varepsilon_2 A^T\right)\right] \cdot \left[\varepsilon_2^{1/2} (KC)^T - \varepsilon_2^{-1/2} \left(P + \varepsilon_2 A^T\right)\right]^T = \left(\Omega^{1/2}\right)\left(\Omega^{1/2}\right)^T. \tag{26}$$

Lemma 2 implies that (26) holds if and only if there exists an orthogonal matrix V ($V \in \mathbb{R}^{n \times n}$) satisfying

$$\varepsilon_2^{1/2} (KC)^T - \varepsilon_2^{-1/2} \left(P + \varepsilon_2 A^T\right) = \Omega^{1/2} V \tag{27}$$

or

$$C^T K^T = \varepsilon_2^{-1/2} \Omega^{1/2} V + \varepsilon_2^{-1} \left(P + \varepsilon_2 A^T\right). \tag{28}$$

It follows from [1] that, there exists an orthogonal matrix V such that (28) has a solution for K, if and only if there exists an orthogonal matrix V such that

$$\left[I - C^T \left(C^T\right)^+\right]\left[\varepsilon_2^{-1/2} \Omega^{1/2} V + \varepsilon_2^{-1} \left(P + \varepsilon_2 A^T\right)\right] = 0 \tag{29}$$

where $(C^T)^+$ denotes the Moore–Penrose inverse of C^T.

By defining

$$X := \varepsilon_2^{-1/2} \left[I - C^T \left(C^T\right)^+\right] \Omega^{1/2} \tag{30}$$

$$Y := -\varepsilon_2^{-1} \left[I - C^T \left(C^T\right)^+\right] \left(P + \varepsilon_2 A^T\right) \tag{31}$$

we can easily rewrite (29) as

$$XV = Y \tag{32}$$

which is, again by Lemma 2, equivalent to

$$XX^T = YY^T. \tag{33}$$

Since $I - C^T (C^T)^+$ is symmetric, (33) can be expressed as

$$\left[I - C^T \left(C^T\right)^+\right] \left[\Omega - \varepsilon_2^{-1} \left(P + \varepsilon_2 A^T\right) \left(P + \varepsilon_2 A^T\right)^T\right] \cdot \left[I - C^T \left(C^T\right)^+\right] = 0 \tag{34}$$

where Ω is defined in (25).

We can see from the above derivation that, given scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, $\sigma > 0$ and a matrix $R > 0$, the solvability problem for an observer gain matrix K to satisfy (9) is equivalent to that for a matrix $P > 0$ to satisfy both (25) and (34).

To this end, we sum up the above results in the following theorem that offers the conditions for the existence of a observer gain matrix K such that matrix equation (9) holds.

Theorem 2: There exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \sigma$, a matrix $R > 0$ and an observer gain matrix K such that (9) holds, if and only if there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \sigma$ and matrix $R > 0$ such that (25) and (34) have a solution $P > 0$.

Next, prior to characterizing the set of the desired observer gains, we introduce the singular value decompositions (20) and (21), where X, Y are defined in (30), (31), respectively.

Suppose now that the conditions of Theorem 2 are satisfied. It follows from [1] that a general solution to (28) is given by

$$K = \left\{ \left(C^T\right)^+ \left[\varepsilon_2^{-1/2} \Omega^{1/2} V + \varepsilon_2^{-1} \left(P + \varepsilon_2 A^T\right)\right] + \left[I - \left(C^T\right)^+ C^T\right] Z \right\}^T \tag{35}$$

where $Z \in \mathbb{R}^{p \times n}$ is arbitrary, V is any orthogonal matrix satisfying $XV = Y$ and can be expressed, by Lemma 2, as

$$V = V_X \begin{bmatrix} I & 0 \\ 0 & U \end{bmatrix} V_Y^T \quad U \in \mathbb{R}^{(n - r_X) \times (p - r_X)} \tag{36}$$

where the matrix U is arbitrary orthogonal, $r_X = \operatorname{rank}(X)$.

We are now ready to characterize all desired observer gains satisfying (9) by substituting (36) into (35).

Theorem 3: Assume that the conditions of Theorem 2 are satisfied. Then the set of all observer gains satisfying (9) is parameterized by

$$\mathcal{K} = \left\{ K \colon K = \left[\left(C^T\right)^+ \left[\varepsilon_2^{-1/2} \Omega^{1/2} V_X \begin{bmatrix} I & 0 \\ 0 & U \end{bmatrix} V_Y^T + \varepsilon_2^{-1} \left(P + \varepsilon_2 A^T\right)\right] + \left[I - \left(C^T\right)^+ C^T\right] Z \right]^T \right\} \tag{37}$$

where $P > 0$ is a solution to (25) and (34), Ω is defined in (25), V_X and V_Y come from the singular value decomposition of X and Y in (20) and (21), $Z \in \mathbb{R}^{p \times n}$ is arbitrary, and $U \in \mathbb{R}^{(n - r_X) \times (p - r_X)}$ is arbitrary orthogonal, $r_X = \operatorname{rank}(X)$.

Finally, our main results, which are easily deduced from Theorems 1, 2, and 3, are summarized in the following corollary.

Corollary 1: Consider linear neutral delay system (1)–(2). If there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \sigma$ and matrix $R > 0$ such that (25) and (34) have a solution $P > 0$, then corresponding to the observer (3) whose gain matrix is determined by (37), the estimation error dynamic system (5) is exponentially stable.

Remark 4: It is worth mentioning that, the set of the desired observer gains, when it is not empty, must be very large because of the *free* design parameters in the expression of observer gains, such as U, Z. This yields much design freedom that offers the possibility for directly achieving further performance requirements on the estimation process, such as the transient property, H_2-norm constraint and reliability behavior. In particular, for the application of the freedom (in choosing an orthogonal matrix) contained in the parameterization of a set of filters, we refer the reader to [10]. This probably gives a special feature to the results obtained in the present note.

Remark 5: In practice, we often wish to solve (25) and (34), and then construct the desired observer gains from (37) directly. In general, this can be done as follows. First, the scalar parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$ can be determined by using the optimization approach proposed in [18] and the references therein, respectively, in order to reduce the possible conservatism that may result from the inequalities (13)–(15). Then, we can solve the following linear matrix inequality (LMI)

$$\Omega_1 := \left(\varepsilon_2^{-1} - \varepsilon_1\right) P^2 - \left(\varepsilon_1^{-1} + \varepsilon_3\right) A_d^T A_d - \left(\varepsilon_2^{-1} + \varepsilon_3^{-1}\right) J^T P^2 J - \sigma I > 0 \quad (38)$$

for P^2 by using the powerful Matlab LMI toolbox, and hence obtain $P > 0$. Next, we can solve the linear matrix equation (34) for $R > 0$ satisfying $R \leq \Omega_1$ or (25) (in the simplest case, we could take $R = \Omega_1$), and obtain a set of desired observer gains from (37).

V. Numerical Example

In this section, an example is presented to demonstrate the effectiveness and flexibility of the proposed observer design approach.

Consider linear neutral delay system (1)–(2) with

$$A = \begin{bmatrix} 2.5 & -0.5 \\ 0 & -3 \end{bmatrix} \quad A_d = \begin{bmatrix} 0.1 & -0.05 \\ 0.03 & 0.1 \end{bmatrix}$$
$$C = [1 \quad 0] \quad J = 0.1 I_2$$

Set $\sigma = 0.0022$. Using the method discussed in Remark 5 of Section IV, we may choose the appropriate parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$, and obtain P and Ω_1 as follows:

$$\varepsilon_1 = 0.3284 \quad \varepsilon_2 = 0.2072, \qquad \varepsilon_3 = 6.1159$$
$$P = \begin{bmatrix} 2.4886 & 0.0002 \\ 0.0002 & 0.7052 \end{bmatrix} \quad \Omega_1 = \begin{bmatrix} 27.4447 & 0.0212 \\ 0.0212 & 2.0953 \end{bmatrix}.$$

Assume that the positive–definite matrix R has the form $R = [r_{ij}]_{2\times 2}$ $(i, j = 1, 2)$. The condition (34) indicates that $r_{22} = 2.01$. Furthermore, subject to another constraint (25), we can select other elements of R as $r_{11} = 4$, $r_{12} = r_{21} = 0.05$, and thus obtain the matrix R, and subsequently matrices Ω, V_X, V_Y as the following:

$$R = \begin{bmatrix} 4.0000 & 0.0500 \\ 0.0500 & 2.0100 \end{bmatrix} \quad \Omega = \begin{bmatrix} 23.4447 & -0.0288 \\ -0.0288 & 0.0853 \end{bmatrix}$$
$$V_X = \begin{bmatrix} -0.0192 & 0.9998 \\ 0.9998 & 0.0192 \end{bmatrix} \quad V_Y = \begin{bmatrix} -0.7776 & 0.6287 \\ 0.6287 & 0.7776 \end{bmatrix}.$$

Let us now consider the analytical expression (37). In this case, since $I - (C^T)^+ C^T = 0$, the matrix Z does not affect the solution. Hence, substituting $U = 1$ and $U = -1$ into this expression leads to the following two desired observer gains:

$$K_1 = \begin{bmatrix} 21.3658 \\ 8.1347 \end{bmatrix} \quad K_2 = \begin{bmatrix} 7.9928 \\ -8.4055 \end{bmatrix}.$$

If we choose $R = \operatorname{diag}\{3, 2.01\}$, then the desired observer gains corresponding to $U = 1$ and $U = -1$ are, respectively

$$K_1 = \begin{bmatrix} 7.8062 \\ -8.5446 \end{bmatrix} \quad K_2 = \begin{bmatrix} 21.4627 \\ 8.3463 \end{bmatrix}.$$

It is not difficult to test that, with all obtained four observer gains, the prespecified exponential stability constraint on the estimation process is met.

VI. Conclusion

This note has studied the observer design problem for a class of continuous-time neutral delay systems. A modified algebraic matrix equation approach has been developed to construct linear full-order observers assuring exponential stability for the estimation error system, irrespective of the time delay. By using the generalized inverse theory and singular value decomposition technique, we have derived both the existence conditions and the analytical expression of the desired observers. A numerical example has demonstrated the effectiveness and flexibility of the present design approach.

We have emphasized that there exist much design freedom that can be used to directly meet other performance requirements, such as the constraints on the H_2 norm of the transfer function from possible noise input to estimation error output. The main results can also be extended to the parameter uncertain systems. These will be the subjects of further investigations.

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References

[1] A. Ben-Israel and T. N. E. Greville, *Generalized Inverses: Theory and Applications*. New York: Wiley, 1974.
[2] I. D. Clarkson and D. P. Goodall, "On the stabilizability of imperfectly known nonlinear delay systems of the neutral type," *IEEE Trans. Automat. Contr.*, vol. 45, pp. 2326–2331, Dec. 2000.
[3] Y. A. Fiagbedzi, "Feedback stabilization of neutral systems via the transformation technique," *Int. J. Control*, vol. 59, pp. 1579–1589, 1994.
[4] P. Gahinet, A. Nemirovsky, A. J. Laub, and M. Chilali, "LMI control toolbox: For use with Matlab," The MATH Works, Inc., Natick, MA, 1995.
[5] K. Glover, "All optimal Hankel-norm approximations of linear multivariable systems and their L^∞-error bounds," *Int. J. Control*, vol. 39, pp. 1115–1193, 1984.
[6] J. K. Hale, *Theory of Functional Differential Equations*. New York: Springer-Verlag, 1977.
[7] J. K. Hale and S. M. Verduyn Lunel, *Introduction to Functional Differential Equations*. New York: Springer-Verlag, 1993.
[8] V. B. Kolmanovskii and V. R. Nosov, *Stability and Periodic Modes of Control Systems With Aftereffect*. Moscow, Russia: Nauka, 1981.
[9] ——, *Stability of Functional Differential Equations*. New York: Academic, 1986.

[10] L. Li, L. Xie, W.-Y. Yan, and Y. C. Soh, "Design of low-order linear-phase IIR filters via orthogonal projection," *IEEE Trans. Signal Processing*, vol. 47, pp. 434–448, Feb. 1999.

[11] X. X. Liao and X. Mao, "Exponential stability in mean square of neutral stochastic differential difference equations," *Dyna. Contin., Discrete Impulsive Syst.*, vol. 6, pp. 569–586, 1999.

[12] H. Logemann and S. Townley, "The effect of small delays in the feedback loop on the stability of neutral systems," *Syst. Control Lett.*, vol. 27, pp. 267–274, 1996.

[13] X. Mao, *Stochastic Differential Equations and Applications*. U.K.: Horwood, 1997.

[14] T.-J. Tarn, T. Yang, X. Zeng, and C. Guo, "Periodic output feedback stabilization of neural systems," *IEEE Trans. Automat. Contr.*, vol. 41, pp. 511–521, Apr. 1996.

[15] Z. Wang and K. J. Burnham, "Robust filtering for a class of stochastic uncertain nonlinear time-delay systems via exponential state estimation," *IEEE Trans. Signal Processing*, vol. 49, pp. 794–804, Apr. 2001.

[16] Z. Wang, J. Zhu, and H. Unbehauen, "Robust filter design with time-varying parameter uncertainty and error variance constraints," *Int. J. Control*, vol. 72, pp. 30–38, 1999.

[17] Z. Wang and B. Huang, "Robust H_2/H_∞ filtering for linear systems with error variance constraints," *IEEE Trans. Signal Processing*, vol. 48, pp. 2463–2467, Aug. 2000.

[18] L. Xie and Y. C. Soh, "Robust Kalman filtering for uncertain systems," *Syst. Control Lett.*, vol. 22, pp. 123–129, 1994.

[19] S. Xu, J. Lam, and C. Yang, "H_∞ and positive real control for linear neutral delay systems," *IEEE Trans. Automat. Contr.*, vol. 46, pp. 1321–1326, Aug. 2001.

[20] R. T. Yanushevsky, "Optimal control of linear differential-difference systems of neutral type," *Int. J. Control*, vol. 49, pp. 1835–1850, 1989.

An Adaptive Compensator for a Class of Linearly Parameterized Systems

Jeng-Tze Huang

***Abstract*—A compensation design for a class of linearly parameterized systems is presented. The compensator consists of a typical linearizing control and an adaptive observer for online estimation of the system's parameters. The proposed method achieves the asymptotic stability of the tracking and the estimation error dynamics, provided the basis functions in the regressor vector are linearly independent in terms of the desired system states. No persistent excitation and measurement of the highest derivatives of the system states are required. A numerical example is given to demonstrate the validity of the proposed design.**

***Index Terms*—Adaptive observers, basis function, linear independence.**

I. Introduction

Parameter uncertainty, which may arise due to a lack of precise knowledge of the system parameters and/or external structured disturbances, is often encountered in the control of a dynamical system and degrades the tracking performance. When bounds of uncertainty are available, robust control methods provide simple and straightforward solutions to the issue for guaranteeing the practical stability of the tracking error dynamics [1]. However, the system may exhibit unwanted chattering behavior due to conservative bounds of uncertainty. For structured linear-in-parameter uncertainty, standard adaptive control methodologies apply to ensure the asymptotic stability of the tracking error dynamics. However, exact parameter estimation can only be obtained under the condition of persistent excitation, which is impossible to verify its fulfillment in advance for typical feedback designs [2]. Therefore, more efficient designs releasing such criteria are in demand for applications requiring online parameter identification as well.

Specifically, it is first noted that the adaptive observer in [3] fulfilled that need for a servo system with Coulomb friction. The author extended the results to compensate friction modeled by Coulomb plus a linear viscous friction in [4]. Comparing to typical adaptive control schemes as in [5], these designs are superior in that the friction is identified correctly in addition to asymptotic tracking performance without relying on persistent excitation. However, they are useful exclusively for the linear friction cases, occupying only a minor fraction of applications demanding the same goals. For further extensions, an adaptive compensation design for a class of linearly parameterized systems is proposed in this note. The design consists of a linearizing control and an adaptive nonlinear observer for estimating the actual parameters. It achieves the asymptotic stability of the tracking and the estimation error dynamics provided the basis functions in the regressor vector are linearly independent in terms of the desired states. In contrast to standard adaptive algorithms, such conditions can be verified in advance. Moreover, no persistent excitation and measurement of the highest derivatives of the system states are required. Hence, it is appealing to practical applications. A case study via simulation is undertaken to demonstrate its validity.

The paper is organized as follows. A statement about the concerned problem is given in Section II. The proposed compensation design is described in Section III. To demonstrate the validity of the proposed design, a case study of a one-dimensional servo system with friction is undertaken in Section IV. Finally, conclusions are made in Section V.

II. Problem Statement

Consider a class of linearly parameterized systems described by

$$\begin{aligned}\dot{x}_1 &= x_2\\ &\vdots\\ \dot{x}_{n-1} &= x_n\\ \dot{x}_n &= u - F(\theta,\, x)\end{aligned} \tag{1}$$

where $x = [x_1,\, x_2,\, \ldots,\, x_{n-1},\, x_n]^T \in R^n$ is the system state vector; $u \in R$ is the control input; $F(\theta,\, x) \in R$ is the unknown structured uncertainty and $\theta \in R^r$ is the unknown parameter vector. In the rest of this note, $F(\theta,\, x)$ is assumed to be expressible as a linear combination of a set of *known* basis functions, i.e.,

$$F(\theta,\, x) = \theta^T \phi(x) = \sum_{i=1}^{r} \theta_i \phi_i(x) \tag{2}$$

where $\phi_i(x),\ i = 1,\, \ldots,\, r$ are the corresponding basis functions of the regressor vector $\phi(x)$. Meanwhile, $\phi(x)$ is assumed to be continuously differentiable in x and is bounded for bounded states. It is noted that the expression in (2) covers a wide variety of applications.

Given a desired trajectory $x^d(t) = [x_1^d,\, \ldots,\, x_{n-1}^d,\, x_n^d]^T$, the control objective is to drive $x(t) \to x^d(t)$ as $t \to \infty$ and identify θ correctly. A typical linearizing control for $u(t)$ in (1) is adopted

$$u(t) = \dot{x}_n^d + k^T e + \hat{\theta}^T(t)\phi(x) \tag{3}$$

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