Stability Analysis of Networked Control Systems

by

Peter F. Al-Hokayem

B.E. Computer and Communications Engineering, American
University of Beirut, Lebanon, July 2001

THESIS

Submitted in Partial Fulfillment of the
Requirements for the Degree of

Master of Science
Electrical Engineering

The University of New Mexico

Albuquerque, New Mexico

August, 2003

Dedication

To my mentor, Professor Chaouki T. Abdallah

and my family for their endless support

إِلَى أُستَاذِي شَوقِي ط. عَبدالله

وَ عَائِلَتِي لِدَعِمِهِم أَلَّامُتَنَاهِي

Acknowledgments

First, and by far, I would like to express my gratitude for my mentor Professor Chaouki T. Abdallah who has been my advisor and friend, and whose enthusiasm for learning and mentoring is unsurpassed. He has guided me through various areas of research and instigated my intellectual curiosity into diverse areas of systems and controls. I would also like to thank Professor Peter Dorato for numerous discussions in the controls area, and for sharing his vast knowledge on the history and evolution of systems and controls. My thanks to Dr. Richard Scott Erwin for making the time to serve on my defense committee, in spite of his busy schedule.

There are many friends that have virtually contributed to the completion of this manuscript to whom I would like to extend my appreciation.

Finally, I dedicate this thesis to my family for their love and support, without which this work could not have been completed.

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ABSTRACT OF THESIS

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Abstract

Due to major advancements in the area of networking over the past decade, a new paradigm of control systems has emerged, namely Networked Control Systems. Such systems differ from classical control systems in that their control loops are closed around communication networks. Thus the need for new stability and performance guarantees arises.

In this research we aim to shed some light on the major advancements and inherent problems in Networked Control Systems. We also propose several methods to deal with such systems and characterize their performance when operating under severe communication limitations. We also present a case study pertaining to the bilateral teleoperation, a problem that has attracted a significant research effort for the past 15 years. We present several numerical examples with extended discussions in order to give insight into the theoretical results.

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[1]This section is abstracted from courses on *Stochastic Processes and Probability Theory*

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Chapter 1

Introduction

In a recent issue of the IEEE Control Systems Magazine [22], Prof. Ray DeCarlo[1] provided a list of everyday things that might provide a brief explanation about control engineering, suitable for novices. Two interesting answers were

  i. The left turn arrow appearing on a traffic light, upon sensing the presence of a car in the turn lane.

  ii. Cruise controls keeping your car at a desired speed up and down hills in rain, snow, and sleet!

While these answers might enlighten non-control engineers about control systems, we would like to propose another situation where *networked control systems* come into play.

Case 1 *Imagine two people trying to watch TV, one* (controller/ actuator) *on top of a six-floor building trying to change the orientation of the antenna and another* (observer) *on the first floor giving updates on the quality of reception. This process*

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[1]ECE Department, Purdue University

1

inevitably introduces delay *between the time when the person on the first floor 'shouts' better or worse command, and the time when the person on the 6th floor redirects the antenna and the action is observed on the first floor (time delay). On the other hand a better/worse command could be unheard due to a plane passing over the building (information loss). This interplay between issuing discrete observations that might be lost and issuing discrete commands that are not felt instantaneously is exactly what we mean by networked control systems!*

Note 1 *The situation in Case 1 could be expanded into a multiple antennas (nodes) setting, which is more interesting since the overall system becomes more complex with multiple nodes. It is also more realistic, since most of the time there are multiple actuators and observers trying to access the network.*

The above situation, though suitable as an informal example for novices, could be defined technically as follows.

Definition 1 *A Networked Control System (NCS) is a control system whose control loop is closed around a communications network.*

Note 2 *Throughout this thesis we only deal with control systems configured in a feedback fashion. Systems with only feedforward control configurations are trivial, since the corresponding controllers could be implemented locally, and hence the need for a network is not justified anymore.*

The problem of networked control systems has become a major issue that control engineers have to deal with due to the vast applications as will be seen in Chapter 2. It is interesting to note that NCS has been listed among top challenging problems in control systems today [117]. That is why we focus in this thesis on trying to pinpoint the main issues and solutions available, and propose several methods to deal with NCS.

1.1 Thesis Outline

The remainder of this thesis is divided into 6 chapters.

1.1.1 Chapter 2

The chapter is a an overview of recent results that deal with NCS. It is divided into two main sections the first dealing with existing models for networked systems and various methods used to deal with them. The second section points out the main problems encountered in NCSs such as packet loss, time delays, and limited communications.

1.1.2 Chapter 3

In this chapter we propose a new sampled-data model that deals with NCS. We study the stability and performance of such model through the application of Lyapunov -based techniques.

1.1.3 Chapter 4

This chapter deals with the effects of packet dropouts on the performance of discrete-time plants, when the control input is generated via a model plant. The network effect is manifested via the observation dropout rate between the plant sensors and the model plant.

1.1.4   Chapter 5

In this chapter we utilize results pertaining to the *uncertainty threshold principle* to deal with packet dropouts in the control input. The existence of an optimal control input, depends on the dropping rate of packets.

1.1.5   Chapter 6

Since the effects of the network are vital to networked systems, we present in this chapter a case study of teleoperated systems. Time delays and channel dynamics are discussed and a stability result is provided based on passivity theory.

1.1.6   Chapter 7

This chapter summarizes the main contributions of the thesis, provides very recent results in the area that the author became aware of by the time of completion of the thesis. We close with suggestions for future work and developments.

1.2   Contributions

The main contributions of this thesis could be summarized briefly as follows. Chapter 2 presents a comprehensive summary of NCSs which is vital for trying to tackle the area for the first time. We present a novel method to model time delays in NCS in Chapter 3. Chapter 4 combines for the first time the notion of model-based control systems with that of mean-square stability in order to address the issue of packet dropping between the sensors and the controller. Dropping packets in the control input is studied in Chapter 5, where we utilize the concept of uncertainty threshold

principle that was developed in the late 70's and specialize it into the context of packet dropping and our ability to derive an optimal control input based on the statistics of the dropping. In Chapter 6 we study for the first time the effects of time-delays and network dynamics on the performance of a special type on NCSs, namely bilaterally teleoperated systems.

Chapter 2

Recent Trends in the Stability of Networked Control Systems

2.1 Introduction

Over the past decade, major advancements in the area of communication and computer networks [87] have made it possible for control engineers to include them in feedback systems in order to achieve real-time requirements. This gave rise to a new paradigm in control systems analysis and design, namely Networked Control Systems. *Networked Control Systems (NCSs), are control systems whose control loop is closed around a communications network.* In such systems, the feedback is no longer instantaneous as in classical control systems [81]. Many systems fall under such classification and several examples of NCSs can be found in various areas such as: Automotive industry [47, 54, 85], teleautonomy [80, 102], teleoperation of robots [2, 34, 72, 74], and automated manufacturing systems [55]. Including the network into the design of such systems has made it possible to increase mobility, reduce the cost of dedicated cabling, ease upgrading of systems, and render maintenance easier

and cheaper. The drawback, however, is that the complexity of analysis and design of such systems increases many folds.

In order to achieve better performance in NCSs, several protocols were specifically developed to deal with the stringent demands of such control systems. The main objective of control networks was elegantly stated in [87]: *"Control networks must shuttle countless small but frequent packets among a relatively large set of nodes."* Many of these network protocols have been described in [43, 56, 78, 87], some of which are: ControlNet, DeviceNet, Ethernet, LonWorks, CAN, Profibus, P-Net, Archnet, EIB.

Developing new protocols is a vast field of study for computer scientists and engineers, however the main concern of controls engineers remains the twin issues of stability and performance. With such perspective in mind, a massive effort has been expanded into the study of stability and performance of NCSs (see [20]). In this chapter we are interested in a comprehensive presentation of existing stability results for NCSs, and various methods for addressing the issues of analysis and design of such systems in order to guarantee stability and achieve a performance level.

The chapter is divided into two major sections. Section 2.2 deals with several models that have been proposed to study the performance and various control schemes in NCSs. In Section 2.3 we address three challenges that face control engineers when designing NCSs, namely: Packet loss, time-delay, and limited communication.

For the remainder of this chapter we will refer to the following continuous-time state space representation of linear time-invariant (LTI) systems

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, A, B are real matrices of appropriate dimensions. The

corresponding discrete-time version of the state equation given by

$$
\begin{aligned}
x_{k+1} &= \Phi x_k + \Gamma u_k \\
\Phi &= e^{Ah} \\
\Gamma &= \int_0^h e^{As} ds \, B
\end{aligned}
\tag{2.2}
$$

where h is the fixed sampling period.

2.2 Existing Models

In this section we concentrate on presenting several models and control methods that have been utilized to deal with NCS.

2.2.1 Sampled-Data

The sampled-data model is the first step for design over networks due to our inability to transmit continuous signals over a digital communications network. The main objective of such technique is to obtain the longest sampling time h that can achieve stability, and consequently lower the usage of the network. In [43] the linear time-invariant system (2.1) was studied with an ideal uniform sampler $\mathcal{S}_h : v_k = v(t = kh), k \in \mathbb{Z}_+ = \{0, 1, 2, \dots\}$ and the corresponding zero-order hold $\mathcal{H}_T : v(t) = v_k, t \in [kh, (k+1)h), k \in \mathbb{Z}_+$, where v is a generic signal. A linear-quadratic-regulator (LQR) design with full-state feedback [25], for the continuous-time LTI system (2.1) with the performance index $V = \int_{t_0}^T \left(x^T Q x + u^T R u \right) dt$, yields the following *algebraic Riccati equation*

$$
A^T P + PA - PBR^{-1}B^T P + Q = 0
\tag{2.3}
$$

with $Q \geq 0$ and $R > 0$, whose solution $P > 0$ provides the feedback gain $K = -R^{-1}B^T P$. The solution P is used to construct the control Lyapunov function $V(x) = x^T P x$. The decay rate of V for the input $u = Kx$ is defined through the $\dot{V}(x, u) = -x^T L x$, where $L > 0$ and is given by

$$L \triangleq -(A + BK)^T P - P(A + BK) = Q - PBR^{-1}B^T P. \tag{2.4}$$

With such working frame, the goal is to characterize the stability of the sampled-data system with respect to the same control Lyapunov function and a decay rate close to the original closed-loop continuous-time system, i.e. characterize the couple $(P, \epsilon L), \epsilon > 0$, since the original system's stability is characterized by (P, L). The interested reader should consult [43] for stability results, which will be omitted here for brevity.

2.2.2 Model-Based NCS Control

The authors of [69] present an extended structural analysis of NCSs, using an eigenvalue approach. In their model (see Figure 2.1), the network resides between the sensors attached to the plant, and the actuators. The network is modelled as a fixed-rate sampling (h) of the continuous plant. They also construct a 'model' plant that provides state estimate $(\hat{x}(t))$, and the error between the actual plant and the model plant $(e(t) = x(t) - \hat{x}(t))$ is used to augment the state-vector, with the input being $u(t) = K\hat{x}(t)$. The idea behind this setting is to only transmit the measured state vector via the network periodically, and interpolate between the samples using the model plant. This reduces the communication bandwidth requirement of the networked system, hence lowering the utilization of the network. Then, stability analysis is applied to the following augmented system

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A + BK & -BK \\ \tilde{A} + \tilde{B}K & \hat{A} - \tilde{B}K \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \triangleq \Lambda \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \tag{2.5}$$

where $\tilde{A} = A - \hat{A}$ and $\tilde{B} = B - \hat{B}$, in order to obtain necessary conditions for guaranteeing stability of the closed-loop system. They analyze the performance of the system when the full state and partial state are available for feedback. Finally, they end with an analysis for discrete-time systems.
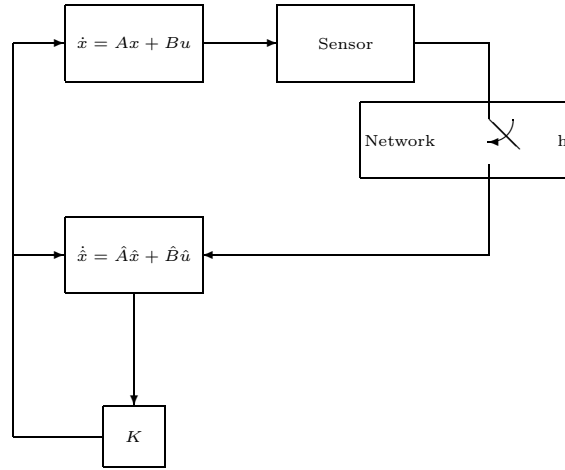
Figure 2.1: Model-Based NCS

2.2.3 Perturbation

The use of perturbation theory (see [49]) has been a very effective tool for studying the stability of NCSs in [10, 107, 108, 109, 110, 111, 115, 116]. The NCS is shown in Figure 2.2, where $\hat{u}(t)$ and $\hat{y}(t)$ are perturbed versions of the original signals $u(t)$ and $y(t)$, after passing through the network. The controller is designed for the network-free control system, and then a perturbation analysis is performed for the networked system. The error introduced by the network is

$$e(t) \triangleq \begin{bmatrix} \hat{y}(t) \\ \hat{u}(t) \end{bmatrix} - \begin{bmatrix} y(t) \\ u(t) \end{bmatrix}, \tag{2.6}$$
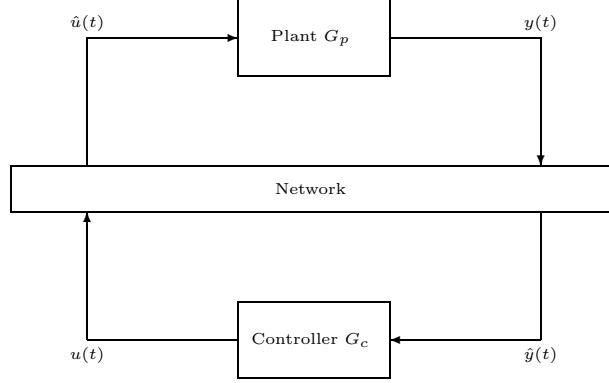
Figure 2.2: Perturbation Model

and the augmented system involves the plant $(x_p(t))$, controller $(x_c(t))$ and the error state vectors as follows:

$$\dot{z}(t) \triangleq \left[\begin{array}{c} \dot{x}(t) \\ \hline \dot{e}(t) \end{array} \right] \triangleq \left[\begin{array}{c} \dot{x}_c(t) \\ \dot{x}_p(t) \\ \hline \dot{e}(t) \end{array} \right] = \left[\begin{array}{cc} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{array} \right] z(t) = \Lambda z(t) \tag{2.7}$$

The matrices $\Lambda_{ij}; i, j \in 1, 2$ are defined in terms of the state-space models of the plant and controller in Figure 2.2. Since the controller is designed to stabilize the network-free system, then Λ_{11} is stable, i.e. there exists a matrix $P > 0$ such that

$$\Lambda_{11}^T P + P\Lambda_{11} = -I \tag{2.8}$$

A try-once-discard (TOD) protocol is introduced, where the next node[1] to transmit data on a multi-node network is decided on dynamically or statically based on the highest weighted error from the last transmission. The goal of analysis for this and other protocols is to find a maximum transmission interval that guarantees

---

[1]A node denotes every component of the system that is connected to the network, such as sensors and actuators

satisfactory stability performance. The maximum bound on the transmission time required to guarantee stability of the NCS is given through the following theorem.

Theorem 1 *[110, 111] An NCS with p nodes operating under TOD, or under static scheduling[2], is globally exponentially stable provided that the maximum allowable transfer time τ_m is given by*

$$
\begin{aligned}
\tau_m \quad < \quad \min\Bigg\{ & \frac{\ln(2)}{p||\Lambda||}, \frac{1}{4||\Lambda|| \left(\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}} + 1 \right) p(p+1)}, \\
& \frac{1}{8\lambda_{max}(P)\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}||\Lambda||^2 \left(\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}} + 1 \right) p(p+1)} \Bigg\}
\end{aligned}
\tag{2.9}
$$

where $\lambda_{max}(P), \lambda_{min}(P)$ are respectively the maximum and minimum eigenvalues of the matrix P defined in (2.8), and $||\Lambda||$ is the induced norm of the matrix Λ.

∎

Theorem 1 indicates that the networked system will maintain stability as long as the maximum time interval between successive transmissions of any node via the network does not exceed τ_m. Maintaining the maximum bound τ_m between successive transmissions, guarantees that the error for each node between transmissions does not exceed a certain threshold error, where the error of a node is calculated by taking the norm of the sub-vector $e_i(t)$, of (2.6) , representing sensors or actuators utilizing that node.

Similar results for nonlinear systems can be found in [107, 108, 109]. The bound given in Theorem 1 on τ_m was improved in [115, 116] to the following,

$$
\tau_m < \frac{\lambda_{\min}(Q)}{8\lambda_{max}(P)\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}||\Lambda||^2 \left(\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}} + 1 \right) p(p+1)},
\tag{2.10}
$$

---

[2]Typical polling or token ring structure

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of the positive definite symmetric matrix Q that satisfies the following Lyapunov stability equation

$$\Lambda_{11}^T P + P\Lambda_{11} = -Q \tag{2.11}$$

and Λ_{11} is given in (2.7).

2.2.4 Hybrid Systems

NCS have been cast into a hybrid systems context [9, 17, 115, 116]. The motivation behind this model is that with the presence of the network, a control systems becomes dependent on packet transmission and reception, hence the system's input changes/switches at each packet arrival. Consequently, the NCS has continuous dynamics (plant) and discrete dynamics (control-loop), yielding a hybrid system.

References [115, 116] utilize results previously derived for the stability of hybrid systems, to find bounds on the delay introduced by the network. In particular, [115] models the network as a constant delay τ introduced into the full state feedback as follows:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) - BK\hat{x}(t), \quad t \in [kh + \tau, (k+1)h + \tau] \\
\hat{x}(t^+) &= x(t - \tau), \qquad t \in [kh + \tau, k = 0, 1, 2, \dots]
\end{aligned} \tag{2.12}$$

where h is the sampling period. Then the trajectory of the delayed state vector $x(t - \tau)$ is solved for, in terms of $x(t)$ and $\hat{x}(t)$. The bound on the delay τ results from imposing Schur stability conditions on the following matrix.

$$H = \begin{pmatrix} e^{Ah} & -E(h)BK \\ e^{A(h-\tau)} & -e^{-A\tau}(E(h) - E(\tau))BK \end{pmatrix} \tag{2.13}$$

where for a given matrix M, $E(h)M \equiv \int_0^h e^{A(h-\eta)}M d\eta$.

Another approach that was presented in [9, 17], involved a specialization of the Witsenhausen hybrid-state continuous time model in [112]. The Witsenhausen hybrid model is fully characterized by $(M, \Omega, f, d, \omega, \mathcal{J})$, where

- M is the finite set of integers indexing the different partitions of the state space

- Ω is the discrete output set that result from applying ω to m_i

- $f : M \times \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n$, is the function governing the continuous dynamics of the system, i.e. $\dot{x}(t) = f(m_i, x(t), u(t))$, that is continuous in $x(t)$, for a fixed $m_i \in M$

- $d : M \times \mathcal{J} \to \mathcal{M}$, is the discrete state transition function that indicates the next discrete state, after the current state reaches any switching surface in the transition/switching set, $\mathcal{J}$

- $\omega : M \to \Omega$, i.e. for every discrete state we have $\omega(m_i) = o_i \in \Omega$

- $\mathcal{J}$ is the set of all the switching surfaces in the state space.

The Witsenhausen model was extended in [9, 17] to include N systems connected through a network, with the key assumption that the network induced delays are ignored. The N-systems model (see Figure 2.3) was modelled as $(M_i, V_i, \Omega_i, f_i, d_i, w_i, \mathcal{V}_i, v_i)$ for $1 \le i \le N$, where the extra terms are defined as:

- V_i is an discrete set describing the input to each plant

- $\mathcal{V}_i$ is an extra set of transition/switching surfaces

- $v_i : M \times \mathcal{V} \to M$, is the discrete-input discrete-state transition function that provides the next discrete state corresponding to the given input.
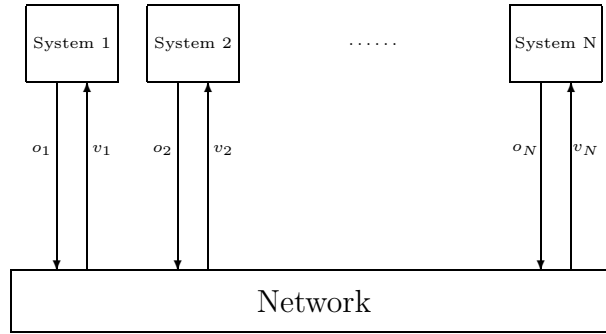
Figure 2.3: N Networked Systems

The Witsenhausen model was used for its simplicity in characterizing the NCS. Although not explored in [17], an optimal control analysis was carried out in [112], which could be extended to the NCS with N systems. The performance of the extended hybrid model was illustrated through a Heating Ventilating and Air Conditioning (HVAC) temperature control system for 3 rooms.

2.2.5 Decentralized Systems

A general definition of decentralized systems is found in [46] and can be stated as follows:

Definition 2 *Decentralized systems possess the property that local controllers only affect the response of specific system outputs. Such systems are also characterized by the restricted exchange of information between different groups of sensors and actuators.*

■

A specific version of such systems called the *switch box problem* was presented in [43, 44] and is mainly a protocol that governs the behavior of the network. The

problem involves two periodically varying controllers [3] $C_1(k)$ and $C_2(k)$ (see Figure 2.4), and the output signals of the plant $y_1(k)$, $y_2(k)$ going through a switch box (S) that only allows one signal to go through during each discrete-time interval. The dynamics of the switch are defined as follows

$$S_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad S_1 = \begin{bmatrix} 0 & I_{p2} \\ 0 & 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 & 0 \\ I_{p1} & 0 \end{bmatrix} \tag{2.14}$$

and $S(k)$ is a periodic switching sequence of (2.14) that relates the outputs $y_{1,2}$ to the controllers' inputs $v_{1,2}$ in the following manner

$$\begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} = S(k) \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} \tag{2.15}$$

The switch box is also periodically time-varying and the control problem can be formulated as follows.

Problem 1 *If the plant $G(s)$ is stabilitzable and detectable, and given a certain periodic switching sequence for the switch box to follow, does there exist periodically time-varying controllers $C_1(k)$ and $C_2(k)$ that make the closed-loop system exponentially stable.*

The solution to the Problem 1 involves the notions of completeness and weak completeness of a linear time-invariant system which are basically dependent on the rank of the following system matrix

$$M((A,B,C,D);z) = \begin{bmatrix} A - zI & B \\ C & D \end{bmatrix} \tag{2.16}$$

Several stability results were given in [43], in which the analysis was carried on through the use of the *lifting* technique (see [19] for definition) and assignability measures (see [43] for details).

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[3]Notation is different in this subsection to avoid confusion. k is the time-step and $1, 2$ indicate the signal number
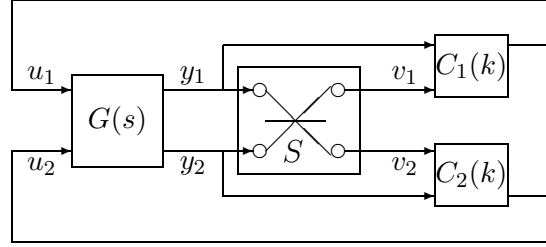
Figure 2.4: Switch box problem

2.2.6 Deadband

One control design for NCSs was proposed using *deadbands* [84]. When the system (2.2) has a new state value (x_k) to transmit over the network, it is compared to its prior value (x_{k-1}). If $|x_k - x_{k-1}| > \delta$, where δ is a threshold value, then $x(k)$ is transmitted, else no value is broadcast over the network. Several methods were used to find the value of δ that results in a stable system, since if δ increases above a threshold value, it may render the system unstable. The method was illustrated using PI, PID and deadbeat controllers. The main objective behind the deadband method is to only transmit data when needed, i.e. when the difference between two successive state values exceeds the threshold δ, hence decreasing the bit-rate needed to guarantee stability of the networked system.

2.3 NCS Inherent Issues

There are several issues that arise when dealing with NCS such as packet loss, time-delay, and limited communication. In the following sections we address each of these issues and present a fairly comprehensive survey of available results.

2.3.1 Packet Loss

Packet loss is an inherent problem with most computer networks (see [21]) due to several factors such as transmission time-outs, transmission errors and limited buffer size. To handle packet loss, transmission protocols are implemented with various error detection and correction schemes such as: parity bit, frame checksum, and cyclic redundancy check (CRC). To indicate that a packet has an error is however easier than correcting it. The previously mentioned methods might then fail, and hence the packet is deemed lost. There have been several attempts to study the effect of packet loss on the performance of the networked system [63, 95, 115, 116].

In [115, 116], a sufficient stability analysis was presented using Lyapunov method, where packet loss/no-loss was modelled as a switch and two closed-loop plant models were used,

- $z_{k+1} \triangleq \begin{bmatrix} x_{k+1} \\ x_{k+1}^{net} \end{bmatrix} = \begin{bmatrix} A & -BK \\ A & -BK \end{bmatrix} z_k = \tilde{A}_1 z_k$, if there is no packet loss

- $z_{k+1} \triangleq \begin{bmatrix} x_{k+1} \\ x_{k+1}^{net} \end{bmatrix} = \begin{bmatrix} A & -BK \\ 0 & I \end{bmatrix} z_k = \tilde{A}_2 z_k$, if there is packet loss

where the system dynamics are given by (2.2) and the control is a simple state feedback $u_k = -Kx_k^{net}$. The vector x_k^{net} is the state after the network/switch, which takes the value x_k when the packet is delivered or x_{k-1}^{net} if the packet is lost.

Theorem 2 *Consider the plant dynamics given by (2.2), and the state feedback $u_k = -Kx_k^{net}$. If the successful transmission rate is r and the non-networked closed-loop system is stable, then the networked systems retains stability as long as there exists a Lyapunov function $V(x_k) = x_k^T P x_k, P > 0$ and scalars η_1 and η_2 such that*

$$
\begin{aligned}
\eta_1^r \eta_2^{1-r} &> 1 \\
\tilde{A}_1^T P \tilde{A}_1 &\leq \eta_1^{-2} P \\
\tilde{A}_2^T P \tilde{A}_2 &\leq \eta_2^{-2} P.
\end{aligned}
$$

Moreover, if $0 < r \leq 1$ and the open-loop system is marginally stable, then the netwoked system is exponentially stable. And if the open-loop system is unstable, the closed-loop system is exponentially stable for all r such that

$$
\frac{1}{1 - \frac{\log[\lambda_{max}^2(A-BK)]}{\log[\lambda_{max}^2(A)]}} < r \leq 1
$$

where $\lambda_{max}(.)$ indicates the maximum eigenvalue of a matrix.

∎

In [63, 95] a stochastic approach was adopted and a Markovian jump system was examined to model packet loss. The idea is similar to that studied in [115, 116], however the process of dropping packets is modelled as of Bernoulli trials type[4]. The occurrence of the event that a packet is lost is indicated as follows

$$
\begin{aligned}
\mathbb{P}\{\theta_k = 0\} &= p \\
\mathbb{P}\{\theta_k = 1\} &= q = 1 - p
\end{aligned}
\tag{2.17}
$$

where $\theta_k = 1$ indicates that the packet arrives. In case a packet is dropped ($\theta_k = 0$), then the previous packet that arrived is used for control. In [63], the analysis starts

---

[4]see [105] for details of Bernoulli trials

with a continuous system, which is then discretized and a Linear Quadratic Regulator (LQR) analysis is used. However for simplicity, we will illustrate instead the analysis adopted in [95]. Let the system under study be given in discrete-time as

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k \\
y_k &= Cx_k
\end{aligned}
\tag{2.18}
$$

Using θ_k in the model we have

$$
y_k^{net} = \theta_k y_k + (1 - \theta_k) y_{k-1}^{net}
\tag{2.19}
$$

where y_k^{net} is the output of the plant after being transmitted through the network. Augmenting the state vector to include y_k^{net}, we have

$$
\begin{aligned}
\bar{x}_{k+1} \triangleq \begin{bmatrix} x_k \\ y_{k-1}^{net} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ \theta_k C & (1 - \theta_k)I \end{bmatrix} \bar{x}_k + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k \\
y_k^{net} &= \begin{bmatrix} \theta_k C & (1 - \theta_k)I \end{bmatrix} \bar{x}_k
\end{aligned}
\tag{2.20}
$$

For such Markovian jump system a state-space controller was designed, that is also Markovian jump system. The closed-loop system could be described by

$$
\underline{x}_{k+1} = \Gamma_{\theta_k} \underline{x}_k
\tag{2.21}
$$

where Γ_k^i is the closed-loop system matrix that evolves according to the outcome of the Markov chain at time step k, and $\underline{x}_k$ is the augmented state vector that involves the state of the plant $\bar{x}_k$, and that of the controller.

There are several notions of stability that are cited in [95], however we will only define the mean-square stability in order to state the stability result for (2.21). The system given in (2.21) with initial conditions $\underline{x}_0$ and θ_0 is mean-square stable if for every initial state we have $\lim_{k\to\infty} \mathbb{E}\left[||\underline{x}_k||^2\right] = 0$. Given that the transition probabilities from state i to state j of θ_k are given by $p_{ij} = p_j$ we have the following stability result.

Theorem 3 *[95] Given p_j, $j \in \{1, 2\}$, the system in (2.21) is mean-square stable if and only if there exists a matrix $P > 0$ such that $P - \sum_{j=1}^{2} p_j \Gamma_j^T P \Gamma_j > 0$.*

■

The result in Theorem 3 is a linear matrix inequality (LMI), and it is very similar to the idea of finding a common Lyapunov function for switching discrete-time systems.

2.3.2 Delay Analysis

A major problem in NCS is that the network introduces random, possibly unbounded[5], delays. An extensive study on time-delay systems can be found in [77]. Since the network is utilized to transmit packets of the sampled values of the output/state of the system, and several systems might be connected to the same network, delay is inevitable and might cause deterioration of the system performance[6]. There are several references that deal with the delay involved in NCSs [4, 13, 36, 38, 50, 78, 88, 114]. As seen in Figure 2.5 the general framework for the analysis of time delay in NCS involves a continuous plant $G(s)$, a sampler (S_h) with rate h, and a zero-order hold (H_h) device (ZOH), and a discrete controller $C(z)$. The network introduces two delays, the first τ_k^{sc} between the sensors and the controller , and another τ_k^{ca} between the controller and plant actuators. Like all sampled-data control systems, there is also an underlying computational delay τ^c representing the time the controller consumes to provide the desired input signal, however, this delay is usually absorbed into τ_k^{ca} (see [4]).

There are two inherent problems in NCS that result from network-induced delays, *message rejection* and *vacant sampling* [36, 88].

---

[5]Packet loss might be thought of as unbounded delay

[6]This is not always the case, since delay can be adequately utilized to stabilize the system as in [1]
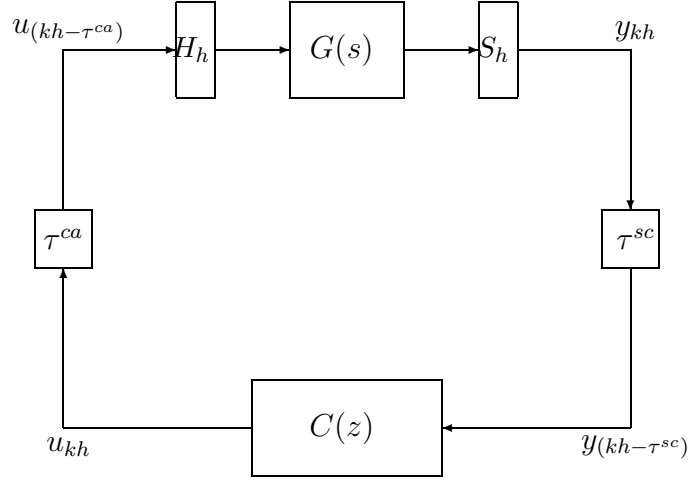
Figure 2.5: Delay Model

- Message rejection: When two or more samples from the sensors reach the controller between two sampling instants, then one of the messages is discarded.

- Vacant Sampling: When no data arrives to the controller during one sampling period, then the controller uses the previous sample.

The following proposition is then provided, the proof of which can be found in [36].

Proposition 1 *[36] The time-variant delays induced by the network, τ_k^{sc} and τ_k^{ca}, can be lumped together into one delay τ_k provided the following conditions hold:*

1. *Both sensor and controller have the same sampling period h*

2. *No message rejection or vacant sampling occur*

3. *$\tau(.) \geq 0$, a nonnegative function*

4. *$\tau_{(kh+\tau_k^{sc}+\tau_k^{ca})} = \tau_k^{sc} + \tau_k^{ca}$*

5. $\tau_{(kh+\epsilon)} \leq \epsilon$, $\forall \epsilon \in [\tau_k^{sc} + \tau_k^{ca}, h + \tau_k^{sc} + \tau_k^{ca})$

■

Assuming the conditions of Proposition 1 are satisfied and the two delays can be lumped, then 3 cases may arise:

- $\tau = \alpha h$, where $\alpha \in I\!N$

- $\tau < h$

- $\tau = (\beta - 1)h + \tau'$, where $\beta \in I\!N$, $\tau' < h$

We shall study each case separately and introduce the different techniques available for analysis and design.

<u>*Case #1:*</u> $\tau = \alpha h$, where $\alpha \in I\!N$

In such case where the delay is an integer multiple of the sampling period, it can be considered as a sensor-induced delay and a similar analysis can be followed as in [30]. Let y_k^i be the output of the plant subject to delay ih for $1 \leq i \leq \alpha$. Then the system can be studied by augmenting the state equations with the delayed measurements of the state, assuming the full state is available for feedback and the plant is represented in discrete time.

$$\begin{bmatrix} x_{k+1} \\ x_{k+1}^1 \\ \vdots \\ x_{k+1}^\alpha \end{bmatrix} = \begin{bmatrix} \Phi & 0 & \dots & 0 \\ I & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_k^1 \\ \vdots \\ x_k^\alpha \end{bmatrix} + \begin{bmatrix} \Gamma \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_k \qquad (2.22)$$

where Φ and Γ are defined in (2.2). The analysis can then be completed using the augmented matrix, since the augmented system is independent of delay.

Case #2: $\tau < h$

In this case the delay is less than the sampling period and the system is modelled as in [4].

$$
\begin{aligned}
x_{k+1} &= \Phi x_k + \Gamma_0 u_k + \Gamma_1 u_{k-1} \\
\Phi &= e^{Ah} \\
\Gamma_0 &= \int_0^{h-\tau} e^{A\eta} d\eta B \\
\Gamma_1 &= e^{A(h-\tau)} \int_0^{\tau} e^{A\eta} d\eta B
\end{aligned}
\tag{2.23}
$$

Case #3: $\tau = (\beta - 1)h + \tau'$, where $\beta \in \mathbb{N}$, $\tau' < h$

The discrete model here is combination of the previous two models and is represented as follows.

$$
x_{k+1} = \Phi x_k + \Gamma_0 u_{(k-\beta+1)} + \Gamma_1 u_{(k-d)}
\tag{2.24}
$$

where Φ, Γ_0 and Γ_1 are defined as in (2.23).

In [78] the case #2 model (2.23) was used with the addition of disturbances v_k and w_k

$$
x_{k+1} = \Phi x_k + \Gamma_0 u_k + \Gamma_1 u_{k-1} + v_k
\tag{2.25}
$$

$$
y_k = C x_k + w_k
\tag{2.26}
$$

where w_k and v_k are uncorrelated zero mean Gaussian white noise stochastic processes. With such setup, the delay τ_k is random and depends on the network load which is modelled as a Markov process (see Figure 2.6). For each state of the network load, *Low*, *Medium*, or *High*, there exist a corresponding distribution for the delay.
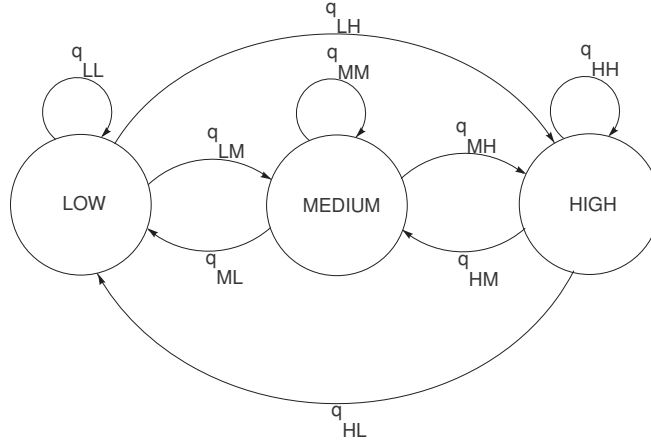
Figure 2.6: Markov model of the operation load of the communication network

The switching between different network states (r_k) is done probabilistically using transition probabilities

$$q_{ij} = \mathbf{P}\{r_{k+1} = i | r_k = j\}, \qquad i, j \in \{Low, Medium, High\} \tag{2.27}$$

Finally, an LQG optimal control problem is solved to generate a controller that guarantees stability. In [13] similar analysis as in Cases #2 and #3 was followed, and an augmented system with the system input being a part of the state vector was derived. Intervals for the allowable delay that maintains stability of the system were obtained for the integrator example.

Throughout the preceding time-delay analysis the sampling time of the sensors and controller were assumed identical, which is the usual case underlying most analysis and design methods in sampled-data and digital control systems. However, if that is not the case, an interesting analysis was presented in [62].

A rigorous study was performed in [50] to deal with random delays of integer multiples of the sampling period h. The network can introduce a random delay ranging from $0h$ to $d_1 h$ into the plant output, and from $0h$ to $d_2 h$ into the controller output, where

$d_1, d_2 \in I\!N$. The analysis was simplified to the single step delay case, i.e. $d_1 = 0$ and $d_2 = 1$. For this specific case, the system behaves as a Markovian jump system, and stability is intended in the Mean Square Sense. Stability was first studied for the single-step delay and two approaches were undertaken, deterministic structural perturbation and stochastic Lyapunov function analysis. Finally, the results were extended to the general multi-step delays.

In what follows we present a different approach to deal with network-induced delays using queues.

Queueing

One scheme that was presented in [18] involved equipping the system with a queue on the sensor side of the network as shown in Figure 2.7. The queue size is m, i.e. a
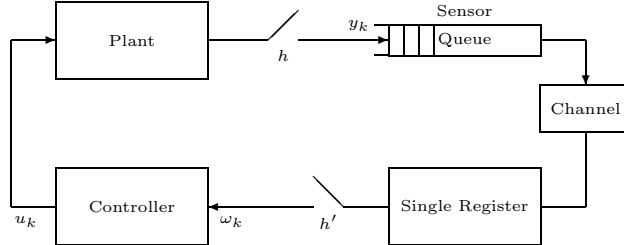
Figure 2.7: NCS with queue

maximum of m delayed measurements to be fed into the channel can be stored. The sampling times on both sides of the channel are equal $(h = h')$, and the controller input ω_k could be any of m values $\{y_k, \ldots, y_{k-m}\}$. The current size of the queue n is transmitted each time a transmission occurs for y_k. Depending on the value of the variable n, the delay index of y can be determined with precision ± 1, i.e. ω_k is

either y_{k-i} or y_{k-i+1} assuming $n = i$. The controller is then supposed to provide the best prediction of the actual non-delayed state x_k in the mean square sense, since there is a probability assignment to the events that $\omega_k = y_{k-i}$ or $\omega_k = y_{k-i+1}$, given that $n = i$. The above discussion assumes that the full state is available at the output ($y_k = x_k$). Another case was studied when the full state is not available for feedback, where the prediction was preformed in two stages, the first estimates the current state, and the second being a refinement of the estimation based on measured outputs.

2.3.3   Control and Communication

Recent developments in NCS have targeted the issue of limited communication available for control. By introducing the network into the control system, issues like channel/network capacity, encoding/decoding schemes and quantization arise. With regard to control systems, the capacity of the channel/network and its ability to convey a reasonable amount of information plays an important role in characterizing the stability of the system. Furthermore, the measured system outputs must be transmitted in a packet form over the network, hence the need for sampling, quantization and encoding/decoding.

A recent line of research in [71, 99, 100] aims to characterize the interplay between communications and control, and to explore the effect of channel coding/decoding schemes on the stability of control systems in the deterministic and stochastic sense. In the same vein, a new notion of capacity called *any-time capacity* was introduced in [91].

Another trend for studying NCS has focused on invoking new quantization schemes that reduce the number of bits to be transmitted over the channel/ network, hence providing the ability to use channels with limited capacities, as will be seen later.

Finally, we expose the reader to two concepts that sound promising to study the phenomenon of chaotic behavior of systems subject to quantization effects, namely *cell-to-cell mapping* and the *Frobenius-Perron operator*.

Information Theoretic Approach

In a leading work in [14], the concept of *attention* was introduced to the control of systems. The method involves solving an optimization problem to minimize an *attention functional* given by

$$\zeta = \int_\omega \psi \left(x, t, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial t} \right) dx dt \tag{2.28}$$

The outcome of this study is to obtain a measure for the control effort that has to be applied to the system to achieve stability. The more the input of the system varies with respect to time ($||\frac{\partial u}{t}||$) and the system state ($||\frac{\partial u}{\partial x}||$), the more attention we need for the system, and the more complex is the control law. This fact reflects on the NCS in the form of bit-rate; the more attention we need for the system the higher the bit-rate has to be. Furthermore, the bit-rate affects the capacity required from the network/channel in an information theoretic context which is the main focus in [7, 45, 68, 91, 99, 100, 104].

The extensive study in [99] has elegantly cast the control problem over networks into an information theoretic context. The network in an NCS problem was thought of as a channel with optimal encoding/decoding and maximum available bit-rate (see Figure 2.8). Both deterministic and stochastic[7] systems were controlled over the channel/network and the goal was to derive a bound on the bit-rate of the channel for different encoding and decoding schemes. The following two definitions were given for controllability and observability of networked systems in an information theoretic setting.

---

[7]Also see [100] for LQG setting
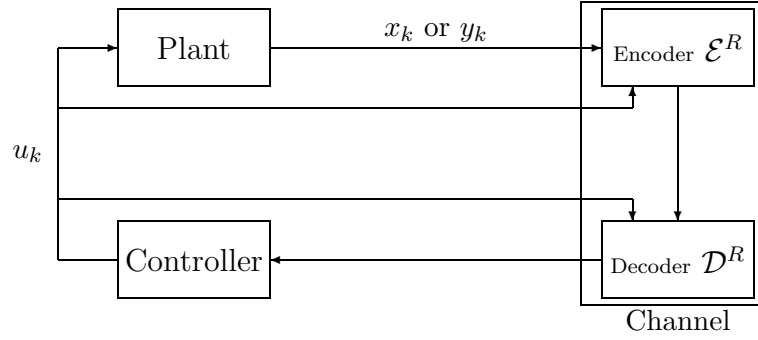
Figure 2.8: Control system with the channel in feedback loop

Definition 3 *[99] The discrete system in (2.2) is* **controllable** *with limited bit-rate if* $\forall x_0, x^\star \in \mathbb{R}^n$, $\forall \epsilon > 0$ *there exists a controller, encoder and decoder and* $k(\epsilon, x_0)$ *such that* $||x_k - x^\star|| \le \epsilon$.

∎

Definition 4 *[99] The discrete system in (2.2) is* **asymptotically observable** *if there exists a control sequence* $\{u_k\}$, *encoder and decoder such that*

1. $\forall \epsilon > 0$, $\exists \delta(\epsilon)$: $||x_0||_2 \le \delta(\epsilon) \Rightarrow ||x_k - \hat{x}_k||_2 \le \epsilon$, $\quad \forall k \ge 0$

2. $\forall \epsilon > 0, \delta > 0$, $\exists K(\epsilon, \delta)$: $||x_0||_2 \le \delta \Rightarrow ||x_k - \hat{x}_k||_2 \le \epsilon$, $\quad \forall k \ge K$

∎

For the controllability and observability notion in Definitions 3 and 4, and several classes of information patterns (channel encoding and decoding), that are defined in [99], the general lower bound on the bit-rate to achieve stability and/or asymptotic

observability was given by[8]

$$R > \sum_{i=1}^{l} \lambda_i^u(A) \tag{2.29}$$

where $\lambda_i^u(A)$ are the unstable eigenvalues of the matrix A, and $l \leq n$ is the corresponding number of unstable eigenvalues. Consequently, there is a clear connection between the unstable eigenvalues of the system under consideration (control setting) and the bit-rate required (information theoretic setting). Hence the information rate required depends on the amount of attention to be given to the system, since the stable eigenvalues do not require attention and they decay to the origin asymptotically.

This idea that the bit-rate required to stabilize a system over a network was revisited in [104], but in a quantization setting. The concept of a *trapping region* was also used to indicate that we can drive the system to an invariant region after which the control input can retain the state trajectory inside the invariant set.

The channel capacity was also related to the unstable eigenvalues of the control system to be stabilized in [7].

Theorem 4 *[7] Consider the following scalar discrete-time system*

$$x_{k+1} = ax_k + bu_k \tag{2.30}$$

that is the sampled version, with sampling period h, of the continuous-time system with transfer function $G(s) = \frac{\beta}{s-\alpha}$, i.e. $a = e^{\alpha h}$ and $b = \frac{\beta}{\alpha}(e^{\alpha h} - 1)$.
There exists as sampling period h and an finite set of admissible inputs of cardinality greater than $e^{\alpha h}$ and a regular[9] selection function $f : \mathbb{R}^n \to U$, that makes the closed-

---

[8]The same expression was derived in [**?**] in stochastic sense
[9]see [7] for complete definition of regularity

loop system boundable[10] *if and only if the data-rate satisfies the following condition*

$$R \geq \log e^{\alpha} \tag{2.31}$$

■

Note that the largest invariant set can be given by $\left[\frac{-bu_{max}}{a-1}, \frac{bu_{max}}{a-1} \right]$, where u_{max} is the largest admissible input level.

Anytime Capacity

When dealing with the channel/network in the feedback loop of systems, it was shown in [91] that the classical notion of channel capacity should be reexamined and described specifically for feedback systems. Along with the notions of classical capacity $\mathcal{C}$, which is the maximum data rate that can be transmitted over the channel with a specified small probability of error, and the zero-error capacity $\mathcal{C}_0$ [96], which is maximum data rate that can be transmitted over the channel with zero probability of error, the following novel notion of capacity was introduced.

Definition 5 *[91] The α-anytime capacity $\mathcal{C}_{anytime}(\alpha)$, is the maximum rate at which the channel can be used to transmit data with a probability of error that decays at a rate α, i.e. $\mathcal{C}_{anytime}(\alpha) = \sup\{R : \mathcal{E}^R, K > 0, \forall N, \exists \mathcal{D}_N^R, \quad \mathbb{P}_{error}(\mathcal{E}^R, \mathcal{D}_N^R) < K 2^{-\alpha N}\}$ where $\mathcal{E}^R$ is an encoder at rate R and $\mathcal{D}_N^R$ is the corresponding decoder with delay N.*

■

Having defined the any-time capacity $\mathcal{C}_{anytime}$, we consider the following unstable scalar system

$$x_{k+1} = ax_k + u_k + W_k, \qquad t \geq 0, a > 1 \tag{2.32}$$

---

[10]Being boundable is very similar to being containable [7], a concept defined in Section 2.3.3

where u_k is the input and $||W_k|| \leq \frac{w}{2}$ is a bounded discrete noise process. The main stability result that relates the any-time capacity to the unstable eigenvalues of the system is stated next.

Theorem 5 *[91, 92] Given the system (2.32), then it is stabilizable over a noisy channel if and only if* $\mathcal{C}_{anytime}(2 \log a) > \log a$ *for the channel with feedback.*

∎

Finally, it is understood that $\mathcal{C}_0 \leq \mathcal{C}_{anytime} \leq \mathcal{C}$.

Quantization Schemes

It is well known [4] that once quantization is introduced into the control loop, it leads to a complicated nonlinear analysis. It might also lead in specific cases to limit-cycle oscillations or chaotic behavior (to be studied in the next subsection). As in digital signal processing [82], quantization can be modelled as an additive white noise process, and its effect can then be studied in a linear fashion.

Over the past ten years, the quantization issue has become the subject for several studies for NCSs [15, 24, 27, 28, 37, 42, 43, 57, 58, 66], in order to describe its impact on the performance of the control system and introduce new quantization schemes that achieve lower bit-rates, which in turn decrease the use of the network in the feedback loop.

To the best of the authors knowledge, most of the aforementioned studies were influenced by the leading work in [24], which was not motivated from an NCS point of view. The main concern was to demonstrate the behavior of discrete-time linear systems subject to quantization and operating under state feedback control law. In order to state the results pertaining to [24], we need some preliminary notation.

Consider the following discrete-time linear system in (2.2) where $x(k) \in \mathbb{R}^n$, and let $\mathcal{J}$ be a finite or countable set. Let $\mathcal{Q} : \mathbb{R}^n \to \mathcal{J}$ be a uniform quantization mapping that results in the following quantization blocks/partitions of the state-space $\{U_j = \mathcal{Q}^{-1}(\{j\}) : j \in \mathcal{J}\}$. It is well known that the quantization operation introduces errors in the measured state, and $\mathcal{Q}(x), \forall x \in \mathbb{R}^n$ can be thought of as *"state measurements containing limited information"*. Under such limited information about the states, traditional asymptotic stability cannot be achieved as seen in the following proposition.

Proposition 2 *[24] Suppose that Φ in (2.2) has at least one unstable eigenvalue, and $0 \in U_0 = \mathcal{Q}^{-1}(\mathcal{Q}(0))$ and $U_0 \subset \mathbb{R}^n$ is a bounded partition. Then for every control input of the following type $u(k) = f^k(\mathcal{Q}(x_0), \ldots, \mathcal{Q}(x_k))$, the set of initial conditions that have trajectories tending to the origin as $k \to \infty$ has Lebesque[11] measure zero.*

∎

Since asymptotic stability can not be achieved in the traditional sense, the analysis in [24] proceeds to characterize an invariant *trapping set* $\mathcal{D}$ to which all the trajectories tend after some time N, depending on the initial condition x_0. The analysis then aimed at characterizing the behavior of the system once the trajectory is inside $\mathcal{D}$. Two behaviors are studied, deterministic and chaotic which will be seen in the next subsection, where the results are specialized for scalar systems.

A similar notion was developed to deal with the inability to achieve asymptotic stability in the traditional sense as seen in Proposition 2, namely *containability*.

Definition 6 *[113] A finite communication control[12] system on $\mathbb{R}^n$, is containable if for any sphere N centered at the origin, there exists an open neighborhood of the*

---

[11]See [35] for definition of Lebesque measure

[12]A different nomenclature that indicates an NCS with quantization, coding and decoding.

origin M and coding and feedback control laws such that any trajectory started in M remains in N for all time, with $M \subset N$.

■

Necessary and sufficient conditions were presented that provide containability of the closed-loop system.

- *Necessary:* The system $\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0$ and $y(t) = Cx(t)$ is containable only if

$$\tau^2 \leq D \tag{2.33}$$

  where $\tau \triangleq e^{\delta tr(A)}$, and δ is the transmission delay and we are using a D-ary codewords[13].

- *Sufficient:* Under the same conditions above, and using binary codewords, then the system is containable if

$$\tau_\infty^{2^n+1} < 2 \tag{2.34}$$

  where $\tau_\infty \triangleq ||e^{\delta A}||_\infty$.

Finally, necessary and sufficient conditions were given for scalar systems as such

Theorem 6 *[113] The system under consideration, restricted to the scalar case is containable if and only if*

$$\tau^2 \leq D$$

(notice that $\tau = \tau_\infty = e^\delta$).

---

[13]The coding alphabet contains D symbols, see [33] for more details on coding

■

The next step in the analysis of quantized NCS was presented in [15, 28]. The main idea in [15] was to make the sensitivity of a uniform quantizer time-variant and include its dynamics in the analysis. The existence of a quantization scheme and control input will be stated for continuous-time systems only, since they are paralleled for the discrete-time case. The idea was to utilize a zooming factor $z \in \{-1, 1\}$ that increases/decreases the quantization sensitivity Δ to allow sufficient qualitative indication of the state location that guarantees stability. With such approach the stability of the un-quantized state/output can be extended to the quantized version. The following two theorems put the discussion in perspective and can be extended to the discrete-time case and later to output feedback [15].

Theorem 7 *[15] Given a full state feedback gain K that renders the un-quantized system $\dot{x}(t) = (A - BK)x(t)$ stable. There exists a control policy of the form*

$$\begin{aligned} \Delta(t) &= G(z, \lfloor t/\tau \rfloor, q(x(\lfloor t/\tau \rfloor)), \Delta(\lfloor t/\tau \rfloor \tau)) \\ u(t) &= -KI_{[k_0\tau,\infty)}(t)\Delta(t)q(x(t)) \end{aligned}$$

where q is a uniform quantizer with sensitivity[14] $\Delta(t)$ and $k_0 > 0$, such that the solution of the closed-loop system

$$\begin{aligned} \dot{x}(t) &= Ax(t) - Bu(t), \qquad x_0 \in \mathbb{R}^n \\ \Delta(t) &= G(z, \lfloor t/\tau \rfloor, q(x(\lfloor t/\tau \rfloor)), \Delta(\lfloor t/\tau \rfloor \tau)), \qquad \Delta(0) = 0 \end{aligned}$$

approaches 0 as $t \to \infty$.

■

---

[14]Quantization sensitivity is the difference between two successive quantization levels

Theorem 8 *[15] Given a full state feedback gain K that renders the un-quantized system $\dot{x} = (A - BK)x$ stable. There exists a control policy of the form*

$$
\begin{aligned}
\Delta_i(t) &= G_i(z, \lfloor t/\tau \rfloor, q(x(\lfloor t/\tau \rfloor))) \qquad i = 1, \dots, n \\
u(t) &= H(t - \lfloor t/\tau \rfloor \tau, q(x(\lfloor t/\tau \rfloor \tau)))
\end{aligned}
$$

where q is a uniform quantizer with sensitivity $(\Delta_1(t), \dots, \Delta_n(t))$ and $z = 1$, such that the solution of the closed-loop system

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) - Bu(t), \qquad x_0 \in \mathbb{R}^n \\
\Delta_i(t) &= G_i(z, \lfloor t/\tau \rfloor, q(x(\lfloor t/\tau \rfloor))), \qquad \Delta(0) = 0
\end{aligned}
$$

approaches 0 as $t \to \infty$.

■

Notice that the scheme utilized with the zoom in and out retains convergence of the solution to the origin, hence stability is in the asymptotic sense.

In [28] however, a slightly different idea was proposed for the design of the quantizer. The objective was to quantize as coarsely as possible while maintaining stability. The measure of coarseness depends on the *quantization density* measure defined by

$$
\eta_g = \lim_{\epsilon \to \infty} \sup \frac{\#g[\epsilon]}{-\ln \epsilon} \tag{2.35}
$$

where g is a quantizer that stabilizes the system and $\#g[\epsilon]$ is the number of levels that g has in an interval $[\epsilon, 1/\epsilon]$. The coarsest quantizer corresponds to the quantizer with the smallest quantization density.

On the other hand, the analysis in [28] relied on Control Lyapunov Functions (CLF). It was shown that a quantizer which is coarsest with respect to a CLF $V(x)$ is of

logarithmic type, i.e. the levels are far apart when the state trajectory is far from the origin and get closer in a logarithmic fashion once close to the origin. The idea is that imprecise knowledge of the state is enough to steer the trajectory in the direction of the origin. Once close to the origin, more precise knowledge is required to reach the origin. The results here are also asymptotic contrary to those in Theorem 2.

The most interesting result in [28] is that of the interplay between sampling rate and quantization coarseness. The quantization density in (2.35) is redefined to include the effect of uniform sampling of period h as follows:

$$\eta_{g,h} = \frac{1}{h} \lim_{\epsilon \to \infty} \sup \frac{\#g[\epsilon]}{-\ln \epsilon} \tag{2.36}$$

This density measures the coarseness of the quantizer in space (quantization) as well as in time (sampling). The optimal sampling time satisfies the following equality

$$h^\star = \frac{\ln(1 + \sqrt{2})}{\sum_{i=1}^{k} \lambda_i^u(A)} \tag{2.37}$$

where $\lambda_i^u(A)$ corresponds to the unstable eigenvalues of the system matrix A. Furthermore, the base for the optimal logarithmic quantizer and the optimal sampling period are independent, hence they can be chosen separately.

The result in (2.37) can be compared to that given in [37], where a different analysis was followed, however, a similar result was obtained and is stated here for completeness,

$$h^\star = \frac{\ln(2)}{\sum_{i=1}^{k} \lambda_i^u(A)}. \tag{2.38}$$

The idea of zooming in and out-type quantization was also explored in [37] as well as in [58]. Specifically, in [58], the quantizer maintains a fixed number of partitions, but starts with large quantization partitions that shrink as the trajectory approaches the origin. The main result of the paper for quantized state-feedback is stated next, and the result can be paralleled for quantized output-feedback.

Theorem 9 *Global asymptotic stabilization of the system $\dot{x}(t) = Ax(t) + Bu(t)$, $x \in \mathbb{R}^n$ is possible if $N \geq 2^n$ and*

$$\max_{0 \leq t \leq \tau} ||e^{At}||_\infty < \lfloor \sqrt[n]{N} \rfloor \qquad (2.39)$$

where N is the number of quantization partitions, and $\lfloor . \rfloor$ is the floor function.

∎

With such results on quantization schemes being brought forward, several other extensions may be followed in a fairly involved study that appeared recently in [43]. Another interesting analysis appeared in [42] and assumes there are two quantizers one residing between the plant sensors and the controller, and another between the controller and the plant actuators. This procedure is the more general in the sense that the network resides on both sides of the plant/controller, not solely between the plant sensors and controller.

Chaotic Behavior & Cell-to-Cell Mapping

In [24], the final step of the analysis was to characterize the behavior of the quantized systems once the trajectory enters the invariant subspace $\mathcal{D}$ (see previous section). For scalar systems, there exists an invariant probability measure μ defined on $\mathcal{D}$ such that

$$\mu : \mathcal{D} \to \mathcal{D} \qquad (2.40)$$

Formally, define $(\mathcal{D}, \mathcal{A}, \mu)$ as a probability space, where $\mathcal{D}$ is the sample space, $\mathcal{A}$ is the minimal σ-algebra defined on $\mathcal{D}$, and μ is a probability measure on $\mathcal{D}$. Basically, the system inside $\mathcal{D}$ operates on a density f as an initial condition. Hence, the evolution of the state trajectory can be characterized through the use of the *Frobenius-Perron* operator, which is a special type of Markov operators (see [53]).

Definition 7 *[53] Given a probability space $(\mathcal{D}, \mathcal{A}, \mu)$, and $S : \mathcal{D} \to \mathcal{D}$ a nonsingular transformation. Then there exists a unique Frobenius-Perron (F-P) operator $\mathcal{P} : \mathcal{L}^1 \to \mathcal{L}^1$ corresponding to S and defined by:*

$$\int_{A \in \mathcal{A}} \mathcal{P}f(x)\mu(dx) = \int_{S^{-1}(A)} f(x)\mu(dx) \tag{2.41}$$

∎

The F-P operator posses several appealing properties such as

- *Linearity:* $\mathcal{P}(\alpha_1 f_1 + \alpha_2 f_2) = \alpha_1 \mathcal{P}f_1 + \mathcal{P}f_2$

- $\mathcal{P}f \geq 0, \qquad \forall f \geq 0, f \in \mathcal{L}^1$

- $||\mathcal{P}f|| = ||f||$

- $||\mathcal{P}|| = 1$

The main result in [24] is that under specific conditions, there exists an invariant density $f^\star$ (or fixed point of $\mathcal{P}$) defined as

$$\lim_{k \to \infty} ||\mathcal{P}^k f - f^\star|| \tag{2.42}$$

and hence characterization of the state can be achieved asymptotically according to the density $f^\star$.

An extension was presented in [94] for the use of the F-P operator to describe the statistical behavior of quantized systems. An autonomous quantized system can be represented by the evolution of the density defined on the state-space described by the F-P operator, and a *density discretizer* D_N, i.e.

$$
\begin{aligned}
f_{k+1}(x) &= \mathcal{P}f_k(x), \qquad \text{given} \quad f_0(x) \\
\mathbb{P}(\mathcal{Q}(x)|f_0(x)) &= D_N f_k(x)
\end{aligned}
\tag{2.43}
$$

where the discretizer D_N is defined by $D_N : \mathcal{D} \to \mathcal{W}^N$, which is obviously a projection of the density f onto an N-dimensional space, that represents the volume of the density resident in each quantization hyperbox. The method is illustrated through an enlightening example that involves scalar three quantization levels and density residing in all 3 levels. The results for autonomous systems were extended to non-autonomous systems through the use of the *Foias operator* [15].

Another promising method explored in [94] is the *cell-to-cell mapping.* For dynamical systems that are modelled by differential equations the continuous solution can usually be characterized through discrete points in the state-space, called point map or Poincaré map [40]. This is the approach used when dealing with numerical solutions of differential equations. Since quantization introduces partitions into the state-space, the cell-to-cell mapping is suggested to describe the evolution of the state trajectory at discrete times from one cell to the other in the state space. The idea is to divide each quantization hyperbox into smaller partitions and try to see where each partition maps into the state space so that the state trajectory of the quantized systems can be characterized. As we refine the partitions, the accuracy of characterization is increased and asymptotically the system is fully characterized.

2.4   Conclusions

This chapter has exposed the reader to many results pertaining to the analysis and design of Networked Control Systems. Several models that were utilized in the study of NCS such as sampled-data, model-based, and hybrid were presented briefly. Also some of the most critical problems in the design of NCS such as packet loss, network induced time-delays, and limited communication in the control loop were presented.

---

[15]See [53] for extended exposure to the Foias operator

Chapter 2. Recent Trends in the Stability of Networked Control Systems

In the next chapter, we present a novel sampled-data approach to deal with NCSs and several stability results based on Lyapunov argument.

Chapter 3

Networked Control Systems: A Sampled-Data Approach

In this chapter we present a novel modelling method for networked control systems, motivated from the sampled-data approach, discussed in Chapter 2. We study sufficient conditions that guarantee exponential stability for the closed-loop system and illustrate our results via a numerical example. The chapter deals with time-delays of fixed or random nature indirectly, hence addressing one of the main inherent issues in NCS, as mentioned in Chapter 2.

The contents of this chapter have been accepted for publication [39].

3.1   New Modelling of NCS

As seen in Chapter 2, there are several trends in modelling networked control systems. In this section we are going to introduce yet another modelling method and manipulate it to obtain a generalized LTI sampled-data system. The proposed model

allows us to avoid the tedious analysis of the effect of the delays introduced by the network. This is achieved through incorporating the delays into the model of the system, and rendering it sufficient to study the stability of the overall system, without explicitly addressing the actual value and nature of the delays. Before we introduce the new model, we present our assumptions:

I. The controller and actuators are directly attached to the plant, i.e. no transport delay exists between the controller and plant actuators.

II. The sensors are part of the plant model.

Proposition 3 *We model the network as a variable-rate ideal sampler (S_{τ_k}), between the plant (G) and the controller (C), and a corresponding zero-order hold (H_{τ_k}), as shown in Figure 3.1.*
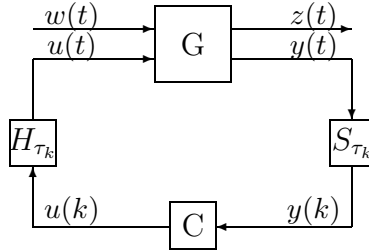

Figure 3.1: System Model

Consider the following plant model,

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\
z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\
y(t) &= C_2 x(t)
\end{aligned} \tag{3.1}
$$

43

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $w(t) \in \mathbb{R}^l$ is the vector of exogenous inputs, $z(t) \in \mathbb{R}^p$ is the vector of controlled outputs, and $y(t) \in \mathbb{R}^q$ is the vector of measurable outputs. Finally,

$$G = \left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & 0 & 0 \end{array} \right) = \left(\begin{array}{cc} G_{11} & G_{12} \\ G_{21} & G_{22} \end{array} \right) \tag{3.2}$$

We assume that $D_{21} = D_{22} = 0$, i.e. the transfer functions from the control input, $u(t)$, and from the exogenous input, $w(t)$, to the measured output, $y(t)$, are strictly proper. The latter condition provides continuity in the measured output vector [8], i.e. avoiding impulses in the output.

The above framework results in a time-varying system, that has both continuous and discrete signals, hence a hybrid system. The study of such systems is in general complex as discussed briefly in Chapter 2 , and a unified theory for such systems is not yet available [67]. For such reasons, we choose to manipulate the model in order to obtain a generalized LTI sampled-data system. In order to do so, we employ the *lifting* technique [8, 19], and incorporate the ideal sampler and hold devices into the plant model in the following manner:

$$
\begin{aligned}
\tilde{G} &= \left(\begin{array}{cc} L_{\tau_k} & 0 \\ 0 & S_{\tau_k} \end{array} \right) G \left(\begin{array}{cc} L_{\tau_k}^{-1} & 0 \\ 0 & H_{\tau_k} \end{array} \right) \\
&= \left(\begin{array}{cc} L_{\tau_k} G_{11} L_{\tau_k}^{-1} & L_{\tau_k} G_{12} H_{\tau_k} \\ S_{\tau_k} G_{21} L_{\tau_k}^{-1} & S_{\tau_k} G_{22} H_{\tau_k} \end{array} \right) \\
&= \left(\begin{array}{cc} \tilde{G}_{11} & \tilde{G}_{12} \\ \tilde{G}_{21} & \tilde{G}_{22} \end{array} \right)
\end{aligned} \tag{3.3}
$$

where $\tau_k = t_k - t_{k-1}$ is the variable sampling-rate, L_{τ_k} and $L_{\tau_k}^{-1}$ are the lifting and inverse lifting operators, respectively. The transformed system is shown in Figure 3.2.
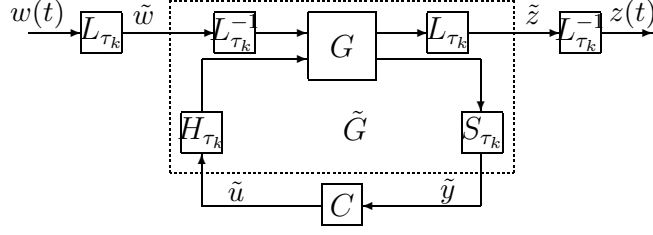
Figure 3.2: The Reconfigured NCS

Next we present the above transformations mathematically.

i. $\underline{G_{11} \to \tilde{G}_{11}}$

   The transfer function G_{11} relates $w(t)$ to $z(t)$, in continuous time. $\tilde{G}_{11}$ on the other hand relates $\tilde{w}$ to $\tilde{z}$ both being the lifted signals, corresponding to $w(t)$ and $z(t)$. Consequently the operator-valued entries of $\tilde{G}_{11}$ are given as follows:

$$
\begin{aligned}
\tilde{A} &= e^{A\tau_k} \\
\tilde{B}_1\tilde{w} &= \int_0^{\tau_k} e^{A(\tau_k-\eta)}B_1 w(\eta)d\eta \\
(\tilde{C}_1 x)(t) &= C_1 e^{At}x \\
(\tilde{D}_{11}\tilde{w})(t) &= D_{11}w(t) + C_1 \int_0^t e^{A(t-\eta)}B_1 w(\eta)d\eta
\end{aligned}
\tag{3.4}
$$

ii. $\underline{G_{12} \to \tilde{G}_{12}}$

   In a similar fashion, we transform B_{12} and D_{12} into B_{12} and D_{12}, respectively. And $\tilde{G}_{12}$ relates the discrete input u_k and the lifted output z_k.

$$
\begin{aligned}
\tilde{B}_2 &= \int_0^{\tau_k} e^{A\eta}d\eta B_2 \\
(\tilde{D}_{12}\tilde{u})(t) &= D_{12}\tilde{u} + C_1 \int_0^t e^{A\eta}d\eta B_2\tilde{u}\tilde{C}_2
\end{aligned}
\tag{3.5}
$$

iii. $\underline{G_{21} \to \tilde{G}_{21}}$ and $\underline{G_{22} \to \tilde{G}_{22}}$

   Both transformations follow from (6) and (7).

After applying the above transformations to (3.2) we obtain an LTI sampled-data system ($\tilde{G}$), which is shown in Figure 3.2. Then we refer back to the usual $\mathcal{H}^\infty$ (see [19]) design to obtain the controller (C). Assuming that the controller (C) has been designed, we present stability analysis results of the overall system in the next section.

3.2 Stability Analysis

In this section we study the stability of the model presented in the previous section. We shall start by deriving the closed-loop system that involves $\tilde{G}_{22}$ and the controller C. Note that we only need to stabilize $\tilde{G}_{22}$ due to the following theorem.

Theorem 10 *[8] The controller C internally stabilizes the hybrid system in Figure 3.2, if and only if it internally stabilizes the discrete-time system $\tilde{G}_{22}$ in (3.3).*

∎

The plant model of $\tilde{G}_{22}$ is described as follows,

$$
\begin{aligned}
x_{k+1} &= \tilde{A}x_k + \tilde{B}_2 u_k \\
y_k &= \tilde{C}_2 x_k = C_2 x_k
\end{aligned}
\tag{3.6}
$$

and the controller C is described by the following state-space realization

$$
\begin{aligned}
v_{k+1} &= A_c v_k + B_c y_k \\
u_k &= C_c v_k + D_c y_k
\end{aligned}
\tag{3.7}
$$

Combining (3.6) and (3.7) we get the following augmented state space representation

$$
\begin{aligned}
s_{k+1} &\doteq \begin{pmatrix} x_{k+1} \\ v_{k+1} \end{pmatrix} \\
&= \begin{pmatrix} \tilde{A} + \tilde{B}_2 D_c C_2 & \tilde{B}_2 C_c \\ B_c C_2 & A_c \end{pmatrix} \begin{pmatrix} x_k \\ v_k \end{pmatrix} \\
&\doteq H_k s_k
\end{aligned}
\tag{3.8}
$$

Notice that the above system does not take into account the effects of disturbances. Consequently, we shall introduce the effects of disturbances, through $w(t)$ in (3.1), into (3.8) as follows

$$
s_{k+1} = H_k s_k + \begin{pmatrix} \tilde{B}_1 \tilde{w} \\ 0 \end{pmatrix} \doteq H_k s_k + \Gamma_k
\tag{3.9}
$$

Before we plunge into the stability analysis, we shall present a general formal definition of *exponential stability* for discrete-time systems.

Definition 8 *The origin of the system $x_{k+1} = A_k x_k$ is exponentially stable if there exists an $\alpha > 0$, and for every $\epsilon > 0$ there exists a $\delta(\epsilon) > 0$, such that*

$$
||x_k|| \le \epsilon e^{-\alpha(t_k - t_0)} ||x_0||
\tag{3.10}
$$

whenever $||x_0|| < \delta(\epsilon)$ and $t_0 \ge 0$. If $\delta(\epsilon) \to \infty$ then the system is exponentially stable in the large.

■

The following theorem utilizes results in [41], and specializes them to solve the problem at hand.

Theorem 11 *The origin of the closed loop discrete-time system (3.8) is exponentially stable in the large provided,*

   i. $\sup_{\forall k \in N} \tau_k < \infty$

   ii. $||H_k|| < \frac{1}{\sqrt{2}}, \forall k \in N$

Proof. Given $||H_k|| < a < 1, \forall k \in N$, then there exist a symmetric matrix $P_k > 0$, such that $H_k^T P_k H_k - P_k = -I$. Then $||P_k|| \leq ||I|| + ||H_k^T P_K H_k|| \leq 1 + a^2 ||P_k|| \Rightarrow$ $1 \leq ||P_k|| \leq \frac{1}{1-a^2}$, since $0 < a < 1$.

Let $V(s_k) \doteq s(k)^T P_{k-1} s(k)$, then

$$
\begin{aligned}
\Delta V &\doteq V(s_{k+1}) - V(s_k) \\
&= s_{k+1}^T P_k s_{k+1} - s_k^T P_{k-1} s_k \\
&= s_k^T (H_k^T P_k H_k - P_k) s_k + s_k^T (P_k - P_{k-1}) s_k \\
&= -s_k^T I s_k + s_k^T (P_k - P_{k-1}) s_k \\
&\leq -||s_k||^2 + \left(\frac{a^2}{1-a^2} \right) ||s_k||^2 \\
&= \left(\frac{2a^2 - 1}{1-a^2} \right) ||s_k||^2
\end{aligned}
\tag{3.11}
$$

Since $||P_k - P_{k-1}||_{max} = \frac{1}{1-a^2} - 1 = \frac{a^2}{1-a^2}$. For the system to be stable, ΔV must be less than zero. Therefore, $\left(\frac{2a^2-1}{1-a^2} \right) < 0 \Rightarrow a < \frac{1}{\sqrt{2}}$.

The above result guarantees that the system (3.8) is stable. Still required to prove that it is exponentially stable. Since $V(s_k) \doteq s(k)^T P_{k-1} s(k)$ then

$$
||s_k||^2 \leq V(s_k) \leq \frac{1}{1-a^2} ||s_k||^2
\tag{3.12}
$$

Using (3.11), $V(s_{k+1}) \leq V(s_k) + \left(\frac{2a^2-1}{1-a^2} \right) ||s_k||^2 \leq (2a^2 - 2) V(s_k)$. But $||s_0||^2 \leq V(s_0) \leq \frac{1}{1-a^2} ||s_0||^2$ then

$$
V(s_k) \leq (2a^2)^k \cdot \left(\frac{1}{1-a^2} \right) ||s_0||^2
\tag{3.13}
$$

Combining (3.12) and (3.13) we get,

$$
||s_k|| \leq \sqrt{\frac{1}{1-a^2}} \cdot (\sqrt{2a^2})^k ||s_0||
\tag{3.14}
$$

Let $\alpha = min\{1, -\ln(\sqrt{2a^2})\}$ and $\epsilon = \sqrt{\frac{1}{1-a^2}}$, the result follows.   ∎

In the above analysis we have ignored the effect of the disturbances on the system. So we are going to extend the result of Theorem 11 to compensate for bounded and vanishing, state-bounded disturbances and in what follows.

Theorem 12 *(Bounded Disturbance) Given that the origin of the discrete-time system (3.8) is exponentially stable, and that $||\Gamma_k|| \leq \gamma < +\infty$ (bounded-input), then the system (3.9) has a bounded-state output.*

Proof.   The proof is simple through analyzing the time progression of the state-vector.

$x(k+1) = \prod_{i=0}^{k} H_i x(0) + \sum_{j=0}^{k} \left(\prod_{i=j+1}^{k} H_i \right) . \Gamma_j$. Taking the limit of k on both sides:

$\lim_{k \to \infty} (x(k+1)) = \lim_{k \to \infty} \left(\prod_{i=0}^{k} H_i . x(0) \right) + \lim_{k \to \infty} \left(\sum_{j=0}^{k} \left(\prod_{i=j+1}^{k} H_i \right) . \Gamma_j \right)$

$\Rightarrow ||x(\infty)|| \leq ||\Gamma|| . \left(\frac{1}{1-a} \right) = \frac{\gamma}{1-a} < \infty$. Since the first limit tends to zero as $k \to \infty$ and $||H_k|| < a < 1 \Rightarrow$ we take the maximum of $H_k = a$ and form a geometric progression whose answer is $\left(\frac{1}{1-a} \right)$.   ∎

Theorem 13 *(Vanishing Disturbance) The origin of the closed loop discrete-time system (3.9) is exponentially stable in the large provided,*

    i. $\sup_{\forall k \in N} \tau_k < \infty$

    ii. $||H_k|| < \frac{1}{\sqrt{2}}, \forall k \in N$

    iii. $||\Gamma_k|| < ||s_k||, \forall k \in N$

Proof. We follow a similar analysis as in Theorem 11. Let $||\Gamma_k|| < \gamma ||s_k||$, where $\gamma > 0$.

$$
\begin{aligned}
\Delta V &\doteq V(s_{k+1}) - V(s_k) \\
&= (H_k s_k + \Gamma_k)^T P_k (H_k s_k + \Gamma_k) - s_k^T P_{k-1} s_k \\
&= s_k^T (H_k^T P_k H_k - P_k) s_k + s_k^T (P_k - P_{k-1}) s_k \\
&\quad + 2 s_k^T H_k^T P_k \Gamma_k + \Gamma_k^T P_k \Gamma_k \\
&\leq \left(\frac{2a^2 - 1}{1 - a^2} \right) ||s_k||^2 \\
&\quad + \left(\frac{2a\gamma}{1 - a^2} + \frac{\gamma^2}{1 - a^2} \right) ||s_k||^2
\end{aligned}
\tag{3.15}
$$

And ΔV in (3.15) is always negative provided that $\gamma < 1$. The rest follows as in Theorem 11. ∎

3.3   Numerical Example

In this section we will consider a numerical example to illustrate the theoretical stability results derived in Section 3.2, specifically in Theorem 11.

Consider the following scalar continuous-time LTI plant model

$$
\begin{aligned}
\dot{x}(t) &= 0.5 x(t) + 10 u(t) \\
y(t) &= x(t)
\end{aligned}
\tag{3.16}
$$

whose discrete version is that described in (3.6). Consider also the following discrete-time LTI controller C

$$
\begin{aligned}
v_{k+1} &= 0.1 v_k - 0.5 y_k \\
u_k &= -0.5 v_k - y_k
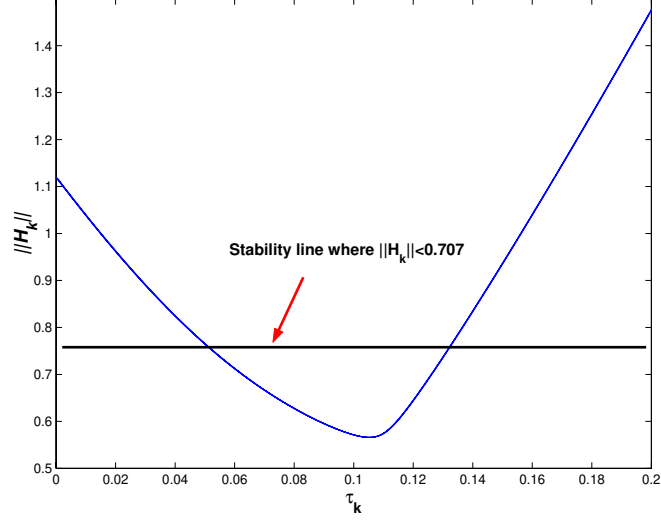\end{aligned}
\tag{3.17}
$$

Figure 3.3: Range of τ_k that corresponds to $||H_k|| < \frac{1}{\sqrt{2}}$

Consequently, the closed-loop system matrix H_k in (3.8), corresponding to (3.16) and (3.17), is given by

$$H_k = \left(\begin{array}{c|c} \dfrac{e^{0.5\tau_k} - 20(e^{0.5\tau_k} - 1)}{-0.5} & \dfrac{-10(e^{0.5\tau_k} - 1)}{0.1} \end{array} \right) \tag{3.18}$$

By Theorem 11, we need to keep the norm of H_k less than $\frac{1}{\sqrt{2}}$. Since we fixed the values for the controller parameters, we can vary τ_k to meet the required condition on H_k. The range of τ_k for which the induced Euclidean norm[1] of H_k is less than $\frac{1}{\sqrt{2}}$ is shown in Figure 3.3, where

$$0.061 < \tau_k < 0.126. \tag{3.19}$$

In order to fully understand the implications of varying the sampling time τ_k on the stability of the system, we will first study the behavior of the closed loop system in (3.18) given a constant τ_k.

---

[1]The induced Euclidean norm of any matrix M is given by $\left[\lambda_{max}(M^T M) \right]^{1/2}$, where λ_{max} denotes the maximum eigenvalue.

The response of the closed-loop system at the boundary of the range given in (3.19), i.e. $\tau_k = 0.126$, is shown in Figure 3.4 where the system retains its stability.
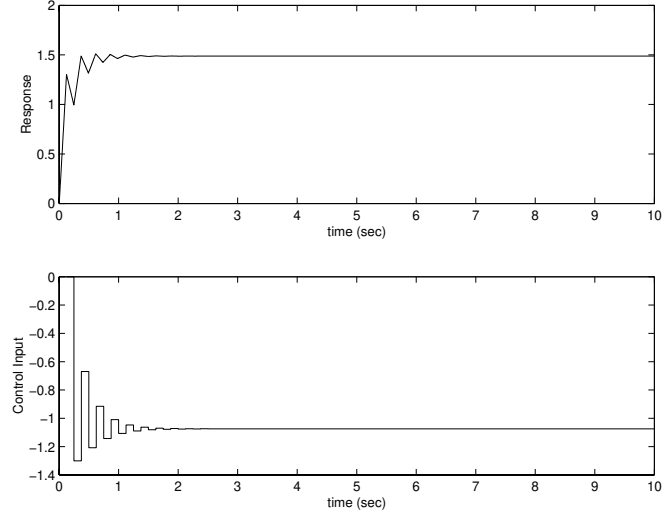

Figure 3.4: System response for $\tau_k = 0.126$

We further increase the value to τ_k beyond 0.126 until we hit the first instability point. As seen in Figure 3.5, the response of the closed loop system diverges for $\tau_k = 0.164$. This conveys the conservativeness of the stability analysis, since the results are sufficient but not necessary.

Finally, we test the system response for a variable sampling time given by

$$\tau_k = 0.126 + \epsilon \times U \tag{3.20}$$

where U is a uniformly distributed random number between 0 and 1, and $\epsilon \in \mathbb{R}$. This representation of τ_k allows us to see how far can we sample randomly beyond the theoretical bound and still maintain stability. As seen in Figure 3.6, the system diverges for $\epsilon = 0.076$.

It is interesting to compare the two results presented in Figures 3.5 and 3.6. For the random case, the value of τ_k depends on the outcome of the random number
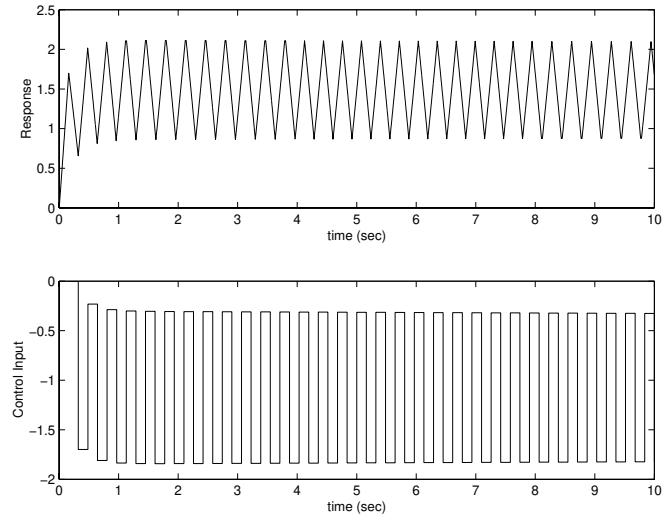
52

Figure 3.5: System response for $\tau_k = 0.164$

U given in (3.20), whose mean is 0.038 for the simulation in Figure 3.6. Hence, average$(\tau_k) = 0.126 + $ average$(\epsilon \times U) = 0.164$ which is the same as the fixed τ_k in Figure 3.5. Consequently, the random τ_k behaves like the fixed one on average.

3.4 Conclusions

In this chapter we have presented a new method for modelling Networked Control Systems. The main idea is to view NCS as a variable-rate, sampled-data system. Then, we utilized some results pertaining to the stability of such sampled-data systems and extended them to the problem at hand.

In the next chapter, we present a stability analysis for NCSs operating under observation dropouts, i.e. packet drops in the state observations.
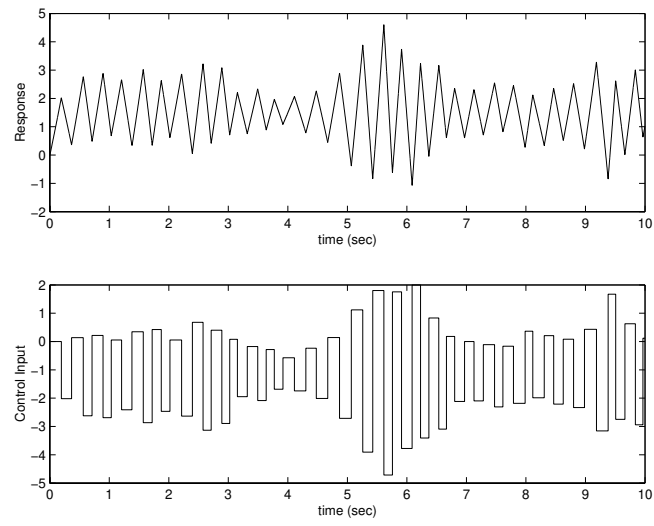
Figure 3.6: System response for $\tau_k = 0.126 + \epsilon \times U$

Chapter 4

Discrete-Time Model-Based Control with Observation Dropouts

In this chapter we will employ various concepts such as model-based control, Markovian Jump linear Systems, and Linear Matrix Inequalities as a numerical analysis tool for studying the effect of packet dropouts on the stability and performance of a networked system. Note that packet dropouts is one of the main inherent issues in NCS, as discussed in Chapter 2.

We start by providing preliminary results needed in the analysis in Section 4.1. In Section 4.2, we state the problem and the analysis of the proposed solution. We present a numerical example in Section 4.3, with some discussion of the results observed. Finally, we conclude in Section 4.4.

4.1 Preliminaries

4.1.1 Markov Chains [1]

A probability space is represented by $(\Omega, \mathcal{F}, \mathbb{P})$, where

- Ω is a sample or phase space

- $\mathcal{F}$ is a σ-algebra of subsets of Ω, i.e

  i. $\varnothing$ and Ω belong to $\mathcal{F}$

  ii. $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$

  iii. $A_1, A_2, \cdots \in \mathcal{F} \Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$

- $\mathbb{P}$ is a probability measure such that $\forall A \in \mathcal{F}$ we have $\mathbb{P}(A) \in [0, 1]$, i.e. $\mathbb{P} : A \to [0, 1]$.

Definition 9 *Let $\{\theta_k\}$ be a sequence of random variables that take values in Ω, then $\{\theta_k\}$ is a Markov Chain if and only if*

$$\mathbb{P}\left(\theta_{k+1} = j | \theta_k = i, \theta_{k-1} = i_{k-1}, \ldots, \theta_0 = i_0\right) = \mathbb{P}\left(\theta_{k+1} = j | \theta_k = i\right) =: p_k(i, j) \quad (4.1)$$

for all $j, i, i_{k-1}, \ldots, i_0 \in \Omega$ and $\forall k$, where $p_k(i, j)$ is the transition probability from state i to state j at moment k. In other words, the state of the Markov chain depends only on the previous state and not the whole history of the chain.

■

---

[1] This section is abstracted from courses on *Stochastic Processes and Probability Theory* by Professor Vladimir Koltchinskii, Mathematics and Statistics Department, University of New Mexico.

Definition 10 *A Markov chain $\{\theta_k\}$ is said to stationary if and only if $p^n(i,j) = p(i,j) \forall n$*

∎

Let $\mathcal{P} = [p(i,j)]_{i,j \in \Omega}$ denote the transition probability matrix of a Markov chain $\{\theta_n\}$, then a very important property called *Irreducibility* can be deduced from the structure of $\mathcal{P}$.

Definition 11 *A Markov chain $\{\theta_k\}$ is irreducible, if there exist only one equivalent class, i.e. any 2 states in Ω commute, where a state i commutes with another state j ($i \leftrightarrow j$), if and only if $\exists n : p^n(i,j) > 0$ and $p^n(j,i) > 0$.*

∎

If the matrix $\mathcal{P}$ is not composed of block diagonal entries, then $\{\theta_k\}$ is irreducible. To adapt all these concepts into the context of packet dropping, we consider a Markov chain $\{\theta_k\}$ with a binary phase space $\Omega = \{0, 1\}$, governed by the following transition probabilities

$$
\begin{aligned}
\mathbb{P}\left(\theta_{k+1} = 0 | \theta_k = 0\right) &= \mathbb{P}\left(\theta_{k+1} = 0 | \theta_k = 1\right) = p \\
\mathbb{P}\left(\theta_{k+1} = 1 | \theta_k = 0\right) &= \mathbb{P}\left(\theta_{k+1} = 1 | \theta_k = 1\right) = 1 - p
\end{aligned}
\tag{4.2}
$$

Then the state transition probability matrix is given by

$$
\mathcal{P} = \begin{pmatrix} p & p \\ 1-p & 1-p \end{pmatrix}
\tag{4.3}
$$

which is not block diagonal, and hence the chain is irreducible, i.e. the probability of either state occurring at time k is never zero. This can also be seen from the fact that $\mathcal{P} = \mathcal{P}^n$. Hence, Definition 11 is satisfied.

4.1.2   Mean Square Stability

Consider the unforced discrete-time stochastic state equation

$$x_{k+1} = \Lambda_{\theta_k} x_k \tag{4.4}$$

where $\theta_k \in \Omega = \{0, 1\}$ is a Markov chain, with transition probabilities given by (4.2), that determines the structure of the state matrix Λ at each time-step k, hence the system is called a Markovian Jumping system. Although there are various definitions of stochastic stability for such systems[2], we are interested in mean-square stability.

Definition 12 *[11] The Markovian Jumping system (4.4) is asymptotically mean -square stochastically stable (MSS), if for all finite initial conditions x_0, θ_0 we have*

$$\lim_{k \to \infty} \mathbb{E}\{||x_k||^2\} = 0 \tag{4.5}$$

∎

The following two Theorems provide conditions for mean-square stability for the system given in (4.4).

Theorem 14 *[23] The system (4.4), with transition probabilities given by (4.2), is mean-square stable (MSS) if and only if there exists a matrix $G > 0$ that satisfies the following Lyapunov-like equation*

$$G - p\Lambda_0^T G \Lambda_0 - (1-p)\Lambda_1^T G \Lambda_1 > 0 \tag{4.6}$$

∎

---

[2]See [52] for general stochastic stability definitions or [103, 23] for specific ones.

Theorem 15 *[23] The system (4.4), with transition probabilities given by (4.2), is mean-square stable (MSS) if and only if $\lambda_{max}(\mathcal{A}) < 1$, where*

$$
\begin{aligned}
\mathcal{A} &\triangleq VH \\
V &\triangleq \mathcal{P} \otimes I_4 \\
H &\triangleq \begin{pmatrix} \bar{\Lambda}_0 \otimes \Lambda_0 & 0 \\ 0 & \bar{\Lambda}_1 \otimes \Lambda_1 \end{pmatrix}
\end{aligned}
\tag{4.7}
$$

■

The $\otimes$ indicates the Kronecker product of two matrices and is defined as follows

Definition 13 *Let A be an $n \times p$ matrix and B an $m \times q$ matrix. The $mn \times pq$ matrix*

$$
A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \dots & a_{1p}B \\ a_{21}B & a_{22}B & \dots & a_{2p}B \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1}B & a_{n2}B & \dots & a_{np}B \end{pmatrix}
\tag{4.8}
$$

is called the Kronecker product of A and B.

Note 3 *Since Theorems $14 - 15$ are necessary and sufficient for MSS, then both are equivalent and we will explore both conditions for stability in Section 4.3.*

4.1.3   Linear Matrix Inequalities

In this section we briefly define Linear Matrices Inequalities (LMIs). As mentioned in [12, 93], the idea of LMIs goes back to the work of A.M. Lyapunov, when he was studying the stability of dynamical systems. A sufficient condition for stability

of a dynamical system of the following form $\dot{x}(t) = Ax(t)$, is that the following inequalities be satisfied

$$P > 0$$
$$A^T P + PA < 0. \tag{4.9}$$

Formally, see [12], an LMI is an inequality of the form

$$F(x) \triangleq F_0 + \sum_{i=1}^{m} x_i F_i \geq 0 \tag{4.10}$$

where $x = [x_1 \quad x_2 \quad \ldots \quad x_m]^T \in \mathbb{R}^m$ is called the decision vector to be calculated, and the symmetric matrices $F_i \in \mathbb{R}^{n \times n}$ are given, and $F : \mathbb{R}^n \to \mathbb{R}^{n \times n}$ is an affine function in the variable x.

One important property that we will explore in Section 4.2, is that multiple LMIs $(F_1(x) > 0, \ldots, F_k(x) > 0)$ can be converted into

$$\mathbb{F}(x) \triangleq \begin{pmatrix} F_1(x) & 0 & 0 & \ldots & 0 \\ 0 & F_2(x) & 0 & \ldots & 0 \\ 0 & 0 & F_3(x) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & F_k(x) \end{pmatrix} > 0 \tag{4.11}$$

which is again an LMI. Hence, the problem of solving multiple LMIs is equivalent to solving a single augmented LMI.

4.2 Problem Formulation

As seen in Chapter 2, *model-based control* has been a useful tool in studying networked control systems. The main idea is to place the network between the sensors,

that are directly attached to the plant, and the controller. In [69], the idea was explored extensively and various stability conditions were obtained for continuous and discrete plants. In this chapter we concentrate on the discrete version and study the effects of random packet drops on the performance of the closed-loop system under state feedback.

Consider the following discrete-time plant model

$$x_{k+1} = Ax_k + Bu_k \tag{4.12}$$

with a discrete-time model plant, that does not match exactly the plant model due to inaccuracy in modelling or identification, given by

$$\hat{x}_{k+1} = \hat{A}\hat{x}_k + \hat{B}u_k. \tag{4.13}$$

It provides the state value for state feedback at each sample time for both the plant and the model plant, through the following input

$$u_k = K\hat{x}_k. \tag{4.14}$$

Define the error vector between the plant and the model as $e_k = x_k - \hat{x}_k$, then along with (4.12) and (4.13), the following closed-loop system results

$$z_{k+1} \triangleq \begin{pmatrix} x_{k+1} \\ e_{k+1} \end{pmatrix} = \begin{pmatrix} A + BK & -BK \\ \Delta A + \Delta BK & (1 - \theta_k)\hat{A} - \Delta BK \end{pmatrix} \begin{pmatrix} x_k \\ e_k \end{pmatrix}$$
$$\triangleq \Lambda_{\theta_k} z_k \tag{4.15}$$

where $\Delta A = A - \hat{A}$, $\Delta B = B - \hat{B}$ indicate the error in model matching and $\theta_k \in \{0, 1\}$ is a Markov chain that indicates reception ($\theta_k = 1$) or loss ($\theta_k = 0$) of a packet containing the state measurement x_k. If a packet is received, then it is used as an initial condition for the next time step in the model plant, otherwise the previous state of the model plant is used.

Given the setting above, we have two situations that might result based on the state of θ_k

$$
\begin{aligned}
\Lambda_1 &= \begin{pmatrix} A + BK & -BK \\ \Delta A + \Delta BK & -\Delta BK \end{pmatrix}, & \text{if} \quad \theta_k = 1 \\
\Lambda_0 &= \begin{pmatrix} A + BK & -BK \\ \Delta A + \Delta BK & \hat{A} - \Delta BK \end{pmatrix}, & \text{if} \quad \theta_k = 0.
\end{aligned}
\tag{4.16}
$$

Hence, for the model described in (4.15), MSS means that there exists a symmetric matrix P , such that the following two LMI's are satisfied

$$
\begin{aligned}
&P > 0 \\
&P - p\Lambda_0^T P \Lambda_0 - (1 - p)\Lambda_1^T P \Lambda_1 > 0
\end{aligned}
\tag{4.17}
$$

where p is the probability of dropping a packet. According to (4.11) we can transform the 2 LMIs in (4.17) into a single LMI to study stability.

4.3   Example

In what follows we consider a scalar example that illustrates the analytical results in Section 4.2. Consider the following plant

$$
x_k = 1.5x_k + u_k
\tag{4.18}
$$

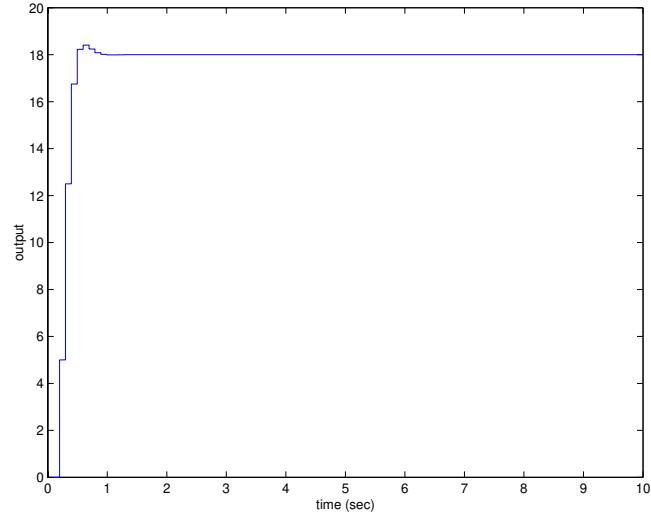and the model plant

$$
\hat{x}_k = 1.4\hat{x}_k + 0.8u_k
\tag{4.19}
$$

Figure 4.1: System response for $p = 0\%$

let $K = -1$, the resulting closed loop matrices

$$
\Lambda_1 = \begin{pmatrix} 0.5 & 1 \\ -0.1 & 0.2 \end{pmatrix} \qquad \text{with} \qquad \lambda_{1,2}^1 = 0.35 \pm 0.278\mathrm{i}
$$

$$
\Lambda_0 = \begin{pmatrix} 0.5 & 1 \\ -0.1 & 1.6 \end{pmatrix} \qquad \text{with} \qquad \lambda_{1,2}^0 = 0.6, 1.5 \tag{4.20}
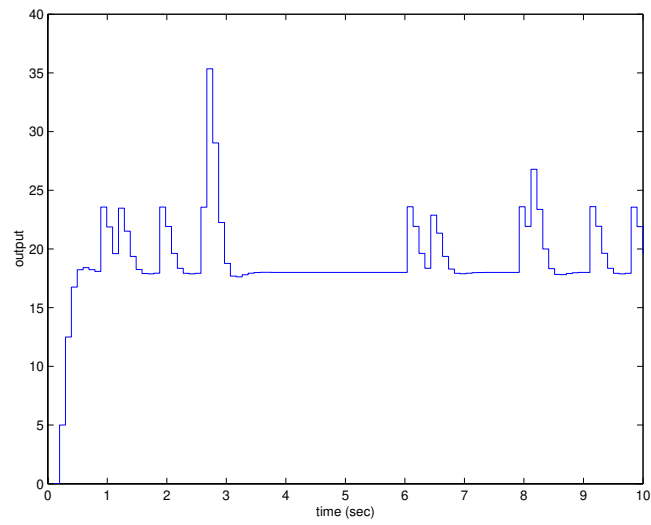$$

Figure 4.2: System response for $p = 10\%$

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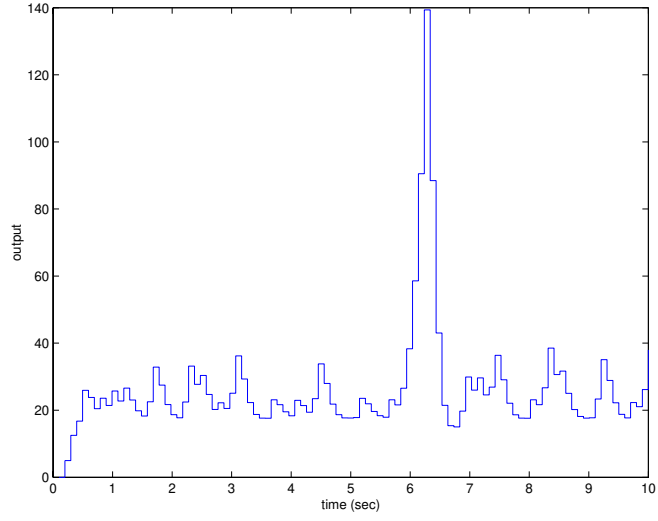
Figure 4.3: System response for $p = 43\%$

where $\lambda_{1,2}^i$ are the eigenvalues of $\Lambda_i, i \in \{0, 1\}$. Note that the matrix Λ_1 has eigenvalues inside the unit circle in the complex plane, hence stable. Meanwhile, Λ_0 has an eigenvalue outside the unit circle, and hence the probability of receiving a packet
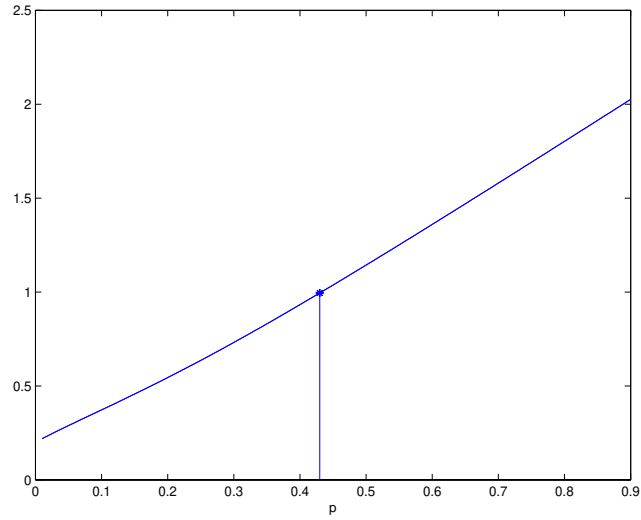

Figure 4.4: $\lambda_{max}(\mathcal{A})$ versus p

$(1 - p)$ or dropping it (p) will place a weighting factor on the interplay between a stable system matrix Λ_1 and an unstable one Λ_0 in (4.17). This will affect our ability to obtain a symmetric matrix $P > 0$, since if the probabilistically dominant matrix is unstable the LMI is not feasible.

Using the LMI [32] toolbox in MATLAB, we get that the maximum feasible dropping rate is $p = 43\%$ and the corresponding value of

$$P = \begin{pmatrix} 120.7 & -420.8 \\ -420.8 & 4369.6 \end{pmatrix} \tag{4.21}$$

To verify the numerical results obtained through LMI's numerical solver, we simulate the closed-loop system using Simulink. The system response with no packet losses is given in Figure 4.1. Notice that due to discrepancy between the plant and the model plant, more time steps are required to reach steady-state, i.e. the response is slower than direct state-feedback for the plant.

To recognize the effect of packet dropping we simulate the same system for a small dropping rate of $p = 10\%$, as seen in Figure 4.2, where the system retains its stability in mean-square sense. Once a packet is lost the response grows, hence the peaks. Consequently, the system requires some time constant to redirect the response back to steady-state level. As also seen, the peaks vary in amplitude with the highest being attained as a consequence of several consecutive drops. Therefore, the higher the drop rate the larger is the divergence from the steady-state level. The response for a $p = 43\%$ drop rate is shown in Figure 4.4 where the system is still stable.

It is very interesting to explore the stability condition given in Theorem 15, in order

to realize that it is equivalent to the LMI stability condition. For $p = 43\%$ we have

$$\mathcal{A} = \begin{pmatrix} 0.1075 & 0.215 & 0.215 & 0.43 & 0.1425 & 0.285 & 0.285 & 0.57 \\ -0.0215 & 0.344 & -0.043 & 0.688 & -0.0285 & 0.057 & -0.057 & 0.57 \\ -0.0215 & -0.043 & 0.344 & 0.688 & -0.0285 & -0.057 & 0.057 & 0.114 \\ 0.0043 & -0.0688 & -0.0688 & 1.1008 & 0.0057 & -0.0114 & -0.0114 & 0.0228 \\ 0.1075 & 0.215 & 0.215 & 0.43 & 0.1425 & 0.285 & 0.285 & 0.57 \\ -0.0215 & 0.344 & -0.043 & 0.688 & -0.0285 & 0.057 & -0.057 & 0.114 \\ -0.0215 & -0.043 & 0.344 & 0.688 & -0.0285 & -0.057 & 0.057 & 0.114 \\ 0.0043 & -0.0688 & -0.0688 & 1.1008 & 0.0057 & -0.0114 & -0.0114 & 0.0228 \end{pmatrix} \tag{4.22}$$

and $\lambda_{max}(\mathcal{A}) = 0.995 < 1$ which is stable. This result illustrates the equivalence between Theorems 14 - 15.

4.4 Conclusions

In this chapter we have investigated the effects of packet drops on the stability of networked control systems. We utilized a model-based technique, that coupled with Markovian Jump Systems stability results, gave us necessary and sufficient results for the stability of networked systems operating via lossy channels. A simple example was presented to clarify the presentation and allow the reader to gain insight into the connotations of theoretical results.

In the next chapter, we exploit the uncertainty threshold principle to study the performance of a networked system operation under packet losses in the plant input.

Chapter 5

Optimal Control under Input Uncertainty

In this Chapter we utilize results pertaining to the *uncertainty threshold principle* [5, 51], in order to address the existence of an optimal state-feedback control input, when the packets are being dropped between the controller output and the plant actuators, which was one of the main inherent issues in NCS, as discussed in Chapter 2.

5.1   Problem Statement

Consider the following plant model

$$x_{k+1} = Ax_k + \theta_k B u_k \tag{5.1}$$

operating under the state-feedback control law

$$u_k = -G_k x_k \tag{5.2}$$

where θ_k is a Markov chain that takes values in the phase space $\Omega = \{0, 1\}$, as in Chapter 4. Our objective is to study the existence of an optimal feedback control law $u_k^\star$ that minimizes a certain quadratic cost functional. A more general problem is that of the *uncertainty threshold principle* [5, 51], that can be easily simplified to solve our problem (see also [48]).

5.2 Uncertainty Threshold Principle

Let γ_k, θ_k, and ξ_k be random sequences with the corresponding mean values μ_γ, μ_θ, and $\mu_\xi = 0$ and variance values σ_γ^2, σ_θ^2, and σ_ξ^2, respectively. Consider the following discrete-time dynamical system

$$x_{k+1} = \gamma_k A x_k + \theta_k B u_k + \xi_k \tag{5.3}$$

where $\sigma_{\gamma\theta}^2$ is the cross correlation between the uncertainty in the system matrix A and input random sequences, and ξ is independent of γ and θ.

Our objective is to derive an optimal state-feedback control input $u_k^\star$ that minimizes the following cost functional

$$\mathcal{J} = \mathbb{E}\left(\frac{1}{N} \sum_{s=0}^{N} x_s^T Q x_s + u_s^T R u_s \right) \tag{5.4}$$

where $Q \geq 0$, $R > 0$, and the pair $(Q^{1/2}, A)$ is observable.

Using stochastic dynamical programming [3], the optimal input is described by

$$u_k^\star = - \left(\left[R + (\mu_\theta^2 + \sigma_\theta^2) B^T P_{k+1} B \right]^{-1} (\mu_\gamma \mu_\theta + \sigma_{\gamma\theta}^2) B^T P_{k+1} A \right) x_k \tag{5.5}$$

where P_k is described recursively backward in time through the Riccati-like equation

$$\begin{aligned} P_k = \quad & (\mu_\gamma^2 + \sigma_\gamma^2) A^T P_{k+1} A + Q \\ & - (\mu_\gamma \mu_\theta + \sigma_{\gamma\theta}^2)^2 A^T P_{k+1} B \left[R + (\mu_\gamma^2 + \sigma_\gamma^2) B^T P_{k+1} B \right]^{-1} B^T P_{k+1} A \end{aligned} \tag{5.6}$$

with $P_N = 0$.

With the above setting we have the following theorem that states the uncertainty threshold principle, which guarantees the convergence of the iterative equation (5.6), and consequently the boundedness of the cost function (5.4).

Theorem 16 *[51] Under state-feedback, an optimal infinite horizon solution for the system (5.3) exists if and only if*

$$\max_i |\lambda_i(A)| < \frac{1}{\beta} \tag{5.7}$$

where $\beta^2 \triangleq \mu_\gamma^2 + \sigma_\gamma^2 - \frac{(\mu_\gamma \mu_\theta + \sigma_{\gamma\theta}^2)^2}{\mu_\theta^2 + \sigma_\theta^2} \geq 0.$

∎

The requirement throughout the proof of Theorem 16 is that the matrix $B \in \mathbb{R}^{n \times n}$ be invertible (see [48, 51]), which is a major assumption on the structure of the system. However, as remarked in [51], it is believed that this requirement is just necessary and can be eliminated through more detailed analysis of the convergence of (5.6).

5.3 Numerical Example

Consider the system (5.1) again. In order to apply the results from Theorem 16, we need to calculate the first and second order statistics of the Markovian random variable θ_k.

$$\mu_\theta = \mathbb{E}(\theta) = \sum_{i=1}^{2} i \times \mathbb{P}(\theta = i) = 1 - p \tag{5.8}$$

$$\sigma_\theta^2 = \mathbb{E}(\theta - \mu_\theta)^2 = \mathbb{E}(\theta^2) - \mu_\theta^2 = (1 - p) - (1 - p)^2 = p(1 - p) \tag{5.9}$$
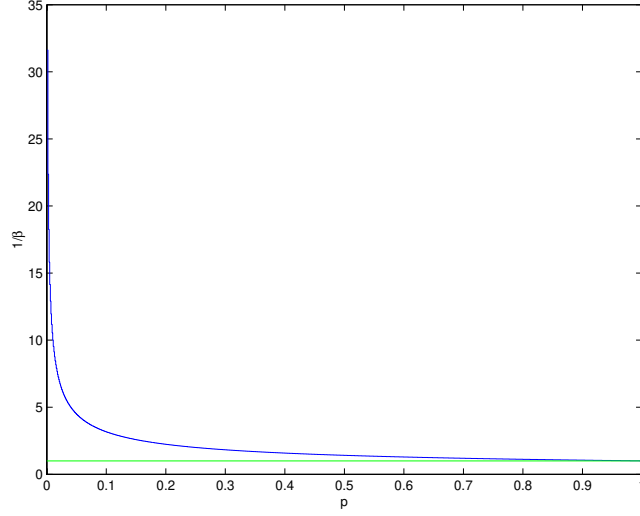
Figure 5.1: $\frac{1}{\beta}$ v/s probability of dropping a packet p

And the resulting value for β is

$$\beta = \sqrt{1 - \frac{\mu_\theta^2}{\mu_\theta^2 + \sigma_\theta^2}} = \sqrt{\frac{\sigma_\theta^2}{\mu_\theta^2 + \sigma_\theta^2}} = \sqrt{p} \Rightarrow \frac{1}{\beta} = p^{-1/2} \tag{5.10}$$

The plot for the maximum allowable eigenvalue of A versus the probability of dropping a packet is shown in Figure 5.1. As the value of p approaches 1, the system behaves like an open-loop with no control input available, and hence the maximum eigenvalue of A that can be tolerated is $\lambda_{max}(A) < 1$, i.e. an already stable system. On the other hand, if p approaches 0, i.e. the probability of dropping a packet is very low, virtually any maximum eigenvalue of A can be tolerated as long as the pair (A, B) is controllable.

To illustrate the point for a specific example, consider again the system (5.1) with $A = 1.5$ and $B = 1$. According to Theorem 16 and (5.10), we have $p < 0.444$ as an upper bound on the dropping rate of packets under which we can still guarantee the existence of an optimal control input. The values of P_k for $N = 100$ that solve (5.6) recursively are shown in Figure 5.3. As observed, the solution of (5.6) diverges
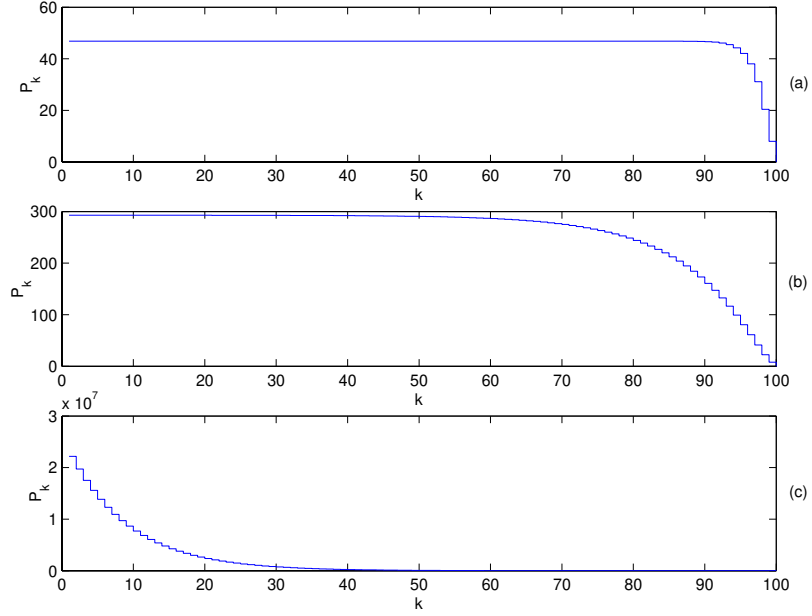
Figure 5.2: P_k for different values of p: (a) $p = 0.2$ (b) $p = 0.4$ (c) $p = 0.5$

between $p = 0.4$ and $p = 0.5$, as predicted.

5.4   Conclusions

In this chapter we have utilized the uncertainty threshold principle to address the issue of information loss in the control input, when the drop is characterized by a Markov chain with finite first- and second-order statistics.

In the next chapter, we present a case study of teleoperated systems taking into consideration the delays and possible dynamics of the underlying network.

Chapter 6

Impact of Network Dynamics on the Stability of Teleoperation

This chapter aims at shedding some light on the effects of a dynamic channel with delay, on the performance of a teleoperated system under force feedback. The network in this case is a dynamical channel with delays, which extends current results that model the network as a pure time delay. Hence, this case study of teleoperated systems is vital for the general understanding of the role of network or channel dynamics and its effect on the networked system's performance. It is interesting to note though that in this chapter the analysis assumes that the network is solely dedicated to the teleoperated system. Also note that the chapter partially addresses the notion of network-induced time-delays, that was introduced in Chapter 2.

Partial contents of this chapter have been published in [72], and the full content is currently under review for publication [73].

6.1 Introduction

The increasing interest in controlling dynamical systems via networks make the analysis and design of communication networks an important problem. Since communication means data transfer from at least one source node to one destination node, the communication delays (network-induced, propagation, round trip-time) in the channel, as well as possible dynamics, become important in characterizing the performance of the overall scheme (oscillations, instability, packet loss).

In this context, we are interested in characterizing the way that delays and dynamics affect communication channels connecting, for example, the master and slave sites (bilateral teleoperation) through the Internet. It is well known that the passivity of the channel (see, e.g.[2, 29, 75, 76]) may be used to guarantee desirable characteristics for the closed-loop schemes (see also [64]). The techniques proposed to perform such an analysis use the scattering transformation [2] or the waves variable transformation [75, 76], if the delays are assumed constant. The case of time-varying delays was considered in [65, 74] using the wave transformation approach, but under some assumptions on the delay variation.

Assume now that the communication channel is shared not only by the master and slave, but also by various source-destination pairs: the information of the remote and/or local sites is transmitted in small packets and is routed in real-time through a finite, but possibly large number of intermediate nodes. In such a situation, the network traffic will also impact the communication delay [106] and it seems difficult to assume that the delay between the master and the slave is constant through the channel. As remarked in [65] or [74], a more realistic assumption is to consider time-varying delays. Note however that their analysis of the communication channel is based on the property that the delay is a $\mathcal{C}^1$ function, a hypothesis quite difficult to satisfy in the Internet environment.

Based on the comments above, a reasonable way to handle such a problem is to consider a distributed time-varying delay in the communication channel satisfying two assumptions: first, that the corresponding kernels (see section 2) are $\mathcal{L}_2$, and second that the information delay from master to slave or from slave to master is known to fall within certain known bounds.

While the first assumption is a standard one, the second is suggested by some of the (standard) congestion control algorithms (see [98] and the references therein), where packet losses induce a retransmission procedure if some acknowledgements have not arrived in due time, called retransmit time-out. The time-out values depend on the actual status of the network (for example, the corresponding round-trip time) and some "reference" values (minimal or some average round-trip time). In our case, one assumes that these values have been fixed at the beginning of the connection to some reasonable values with respect to the traffic load and the service guarantees in the channel.

In our opinion, these seem to be the minimal assumptions required on the communication channel if transport or propagation delays in the network are to be considered. Note that we are not interested in this chapter to analyze the overall system's dynamics, which is the main focus of ongoing studies. The analysis section in this chapter can be traced back to an earlier co-work by the author in [72].

Before we start our analysis we would like to point out some recent advances in the area of teleoperation. First a new configuration was presented in [83] where instead of the velocity and force being communicated along the channel, they transmit the position and integral of the force. Moreover they prove that the system retains the passivity property under the new signals being communicated. Several related studies have come to the attention of the author by the time of completion of this chapter, namely in the latest issue of the IEEE Proceedings [26, 90, 79].

In Section 6.2 we present the proof of passivity of the dynamic channel with time-delay. Simulations that illustrate the results are shown in Section 6.3 with some discussion related to the results in Section 6.3.1. And we conclude in Section 6.4.
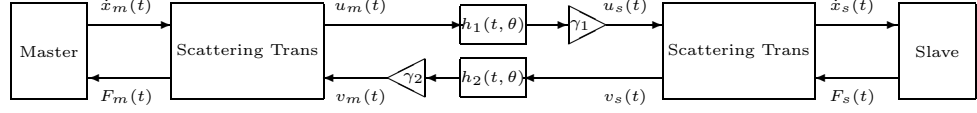

Figure 6.1: The controlled system.

6.2   Passivity Analysis with Dynamic Channel

Consider a standard bilateral teleoperation system [2], and the wave variable transformation to preserve passivity. Based on [76], the velocity $\dot{x}$ and the force F are transformed into the wave domain via:

$$
\begin{aligned}
u_m(t) &= \frac{F_m(t) + b\dot{x}_m(t)}{\sqrt{2b}} \qquad v_m(t) = \frac{F_m(t) - b\dot{x}_m(t)}{\sqrt{2b}} \\
u_s(t) &= \frac{F_s(t) + b\dot{x}_s(t)}{\sqrt{2b}} \qquad v_s(t) = \frac{F_s(t) - b\dot{x}_s(t)}{\sqrt{2b}}
\end{aligned}
\tag{6.1}
$$

where $b > 0$ is the so-called characteristic wave impedance (a trade-off between force and velocity), subscript m denotes the master and subscript s the slave, respectively, see Figure 6.1.

The key idea of the transformation is to include velocity and force information which make the system relatively robust to interactions with unknown environments. Further comments can be found in the references cited above. Let us consider that the

Chapter 6. Impact of Network Dynamics on the Stability of Teleoperation

communication channel includes some distributed delays, that is:

$$u_s(t) = \gamma_1 \int_{-\tau_1}^{0} h_1(t,\theta)u_m(t+\theta)d\theta, \tag{6.2}$$

$$v_m(t) = \gamma_2 \int_{-\tau_2}^{0} h_2(t,\theta)v_s(t+\theta)d\theta, \tag{6.3}$$

where τ_1 and τ_2 are known, and the kernels h_1, h_2 are $\mathcal{L}_2$ (see the arguments in the introduction). The gains γ_1 and γ_2 will be computed latter, and are used to render the communication channel passive with respect to the considered wave transformation.

As in [76], let us compute the overall power input in the communication channel:

$$P_{input}(t) = \dot{x}_m(t)F_m(t) - \dot{x}_s(t)F_s(t). \tag{6.4}$$

Using the wave transformations (6.1), one gets:

$$\begin{aligned}
\mathcal{E}_{stored} &= \int_0^t P_{input}(\xi)d\xi \\
&= \frac{1}{2}\int_0^t \left[u_m(\xi)^T u_m(\xi) - \gamma_2^2 v_m(\xi)^T v_m(\xi) - \gamma_1^2 u_s(\xi)^T u_s(\xi) + v_s(\xi)^T v_s(\xi) \right] d\xi,
\end{aligned} \tag{6.5}$$

that leads to:

$$\begin{aligned}
\mathcal{E}_{stored} &= \frac{1}{2}\int_0^t \left(u_m(\xi)^T u_m(\xi) + v_s(\xi)^T v_s(\xi) \right) d\xi \\
&\quad -\frac{\gamma_1^2}{2}\int_0^t \left(\int_{-\tau_1}^{0} h_1(\xi,\theta)u_m(\xi+\theta)d\theta \right)^T \cdot \left(\int_{-\tau_1}^{0} h_1(\xi,\theta)u_m(\xi+\theta)d\theta \right) d\xi \\
&\quad -\frac{\gamma_2^2}{2}\int_0^t \left(\int_{-\tau_2}^{0} h_2(\xi,\theta)v_s(\xi+\theta)d\theta \right)^T \cdot \left(\int_{-\tau_2}^{0} h_2(\xi,\theta)v_s(\xi+\theta)d\theta \right) d\xi
\end{aligned} \tag{6.6}$$

if ones uses the definition (6.2)-(6.3) of the delays in the communication channel.

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Let us apply Hölder's inequality [89] for the channel from the master to the slave:

$$\left(\int_{-\tau_1}^{0} h_1(\xi, \theta) u_m(\xi + \theta) d\theta \right)^T \cdot \left(\int_{-\tau_1}^{0} h_1(\xi, \theta) u_m(\xi + \theta) d\theta \right)$$
$$\leq \left(\int_{-\tau_1}^{0} h_1(\xi, \theta)^T h_1(\xi, \theta) d\theta \right) \cdot \left(\int_{-\tau_1}^{0} u_m(\xi + \theta)^T u_m(\xi + \theta) \right), \tag{6.7}$$

and the similar one for the channel from the slave to the master, respectively.

Denote :

$$\alpha_1(\xi) = \left(\int_{-\tau_1}^{0} h_1(\xi, \theta)^T h_1(\xi, \theta) d\theta \right), \tag{6.8}$$

$$\alpha_2(\xi) = \left(\int_{-\tau_2}^{0} h_2(\xi, \theta)^T h_2(\xi, \theta) d\theta \right). \tag{6.9}$$

Then we have:

$$\mathcal{E}_{stored} \geq \frac{1}{2} \int_0^t \left[u_m(\xi)^T u_m(\xi) - \gamma_1^2 \int_{-\tau_1}^{0} \alpha_1(\xi) u_m(\xi + \theta)^T u_m(\xi + \theta) d\theta \right] d\xi$$
$$+ \frac{1}{2} \int_0^t \left[v_s(\xi)^T v_s(\xi) - \gamma_2^2 \int_{-\tau_2}^{0} \alpha_2(\xi) v_s(\xi + \theta)^T v_s(\xi + \theta) d\theta \right] d\xi \tag{6.10}$$

under zero initial conditions and standard bilateral teleoperation diagram (see Figure 6.1).

Using the Fubini Theorem [89], we will have:

$$\mathcal{E}_{stored} \geq \frac{1}{2\tau_1} \int_{-\tau_1}^{0} \left[\int_0^t u_m(\xi)^T u_m(\xi) - \alpha_1(\xi) \gamma_1^2 \tau_1 u_m(\xi + \theta)^T u_m(\xi + \theta) d\xi \right] d\theta$$
$$+ \frac{1}{2\tau_2} \int_{-\tau_2}^{0} \left[\int_0^t v_s(\xi)^T v_s(\xi) - \alpha_2(\xi) \gamma_2^2 \tau_2 v_s(\xi + \theta)^T v_s(\xi + \theta) d\xi \right] d\theta \tag{6.11}$$

Then, we have the following result:

Proposition 4 *The communication channel defined above is passive under the wave transformation (6.2)-(6.3) if the gains γ_i, $i = \overline{1,2}$ in the communication channel satisfy the condition:*

$$
\gamma_i \;\; < \;\; \frac{1}{\sqrt{\tau_i \sup\limits_{t>0} \displaystyle\int_{-\tau_i}^{0} h_i(t,\theta)^T h_i(t,\theta)d\theta}},
\tag{6.12}
$$

under zero initial condition.

Proof: The notations (6.8)-(6.9) and the condition (6.12) imply that:

$$
\gamma_1^2 \tau_1 \alpha_1(t) \;\; < \;\; 1,
\tag{6.13}
$$
$$
\gamma_2^2 \tau_2 \alpha_2(t) \;\; < \;\; 1,
\tag{6.14}
$$

for all $t > 0$.

Using such corrections in (6.11) on the corresponding products $\alpha_1(\xi)\tau_1\gamma_1$ and $\alpha_2(\xi)\tau_2\gamma_2$, it follows that a lower bound on the stored energy has the form :

$$
\mathcal{E}_{stored} \geq \frac{1}{2} \int_{-\tau_1}^{0} \left(\int_{t+\theta}^{t} u_m(\xi)^T u_m(\xi)d\xi \right) d\theta + \frac{1}{2} \int_{-\tau_2}^{0} \left(\int_{t+\theta}^{t} v_s(\xi)^T v_s(\xi)d\xi \right) d\theta
\tag{6.15}
$$

which is positive since each term in the corresponding integrals is positive. This ends the proof. ∎

Remark 1 *Since $h_1(\cdot,\cdot)$ and $h_2(\cdot,\cdot)$ are $\mathcal{L}_2$ kernels, it follows that both gains γ_1 and γ_2, respectively are well-defined.*

6.3 Numerical Example

In this section we will present a numerical example to illustrate the effects of the channel on the behavior of the teleoperation system. Consider the first-order dynamical system with delay

$$H_i(s,\theta) = \frac{e^{-s\theta}}{s+\rho} \tag{6.16}$$

which translates into the following $\mathcal{L}_2$ kernel

$$h_i(t,\theta) = e^{-\rho(t-\theta)} \tag{6.17}$$

And the corresponding gain to the kernel (6.17) is specified by

$$
\begin{aligned}
\gamma_i \;\; &< \;\; \frac{1}{\sqrt{\tau_i \sup_{t>0} \int_{-\tau_i}^{0} \left(e^{-\rho(t-\theta)}\right)^T \left(e^{-\rho(t-\theta)}\right) d\theta}} = \\
&= \;\; \frac{1}{\sqrt{\tau_i \sup_{t>0} \frac{e^{-2\rho t}(1-e^{-2\rho\tau_i})}{2\rho}}} = \\
&= \;\; \sqrt{\frac{2\rho}{\tau_i(1-e^{-2\rho\tau_i})}} \tag{6.18}
\end{aligned}
$$

The teleoperation system that we shall study is shown in Figure 6.2. It consists of a first order dynamical system with a PI controller with respect to velocity. The same
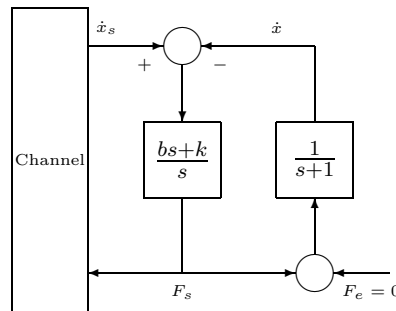

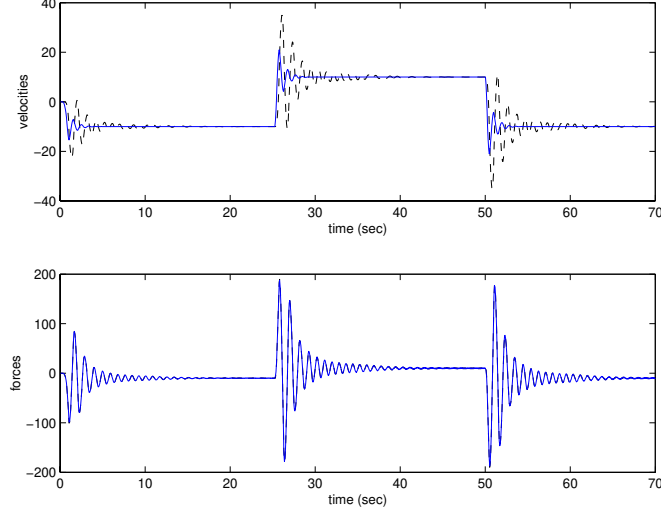Figure 6.2: Slave's side of the system

Figure 6.3: $H_i(s, \theta) = 1$ and $\gamma_i = 1$

system is found on the Master's end of the channel. F_e is the contact force induced by the environment when the system faces an obstacle, and is assumed equal to zero in the simulations.

In order to avoid reflections in the channel, we put the teleoperation system under force control as in [75]. This is possible since the transformations (6.1) can be always defined as long as one wave signal and one power signal are supplied as inputs [75]. We also match the proportional gain of the controller to that of the channel. The virtual model at the Master side of the channel is assumed to be exactly like the actual model on the Slave side. We simulated the teleoperation system operating under different channels and the results are summarized in the following.

- The response of the system with no-delay and inactive channel ($h_i(t, \theta) = 1, \gamma_i = 1$) is shown in Figure 6.3. As seen the Slave's velocity converges to the Master's velocity.

- Another example is seen in Figure 6.4, where a constant delay of 2 seconds was
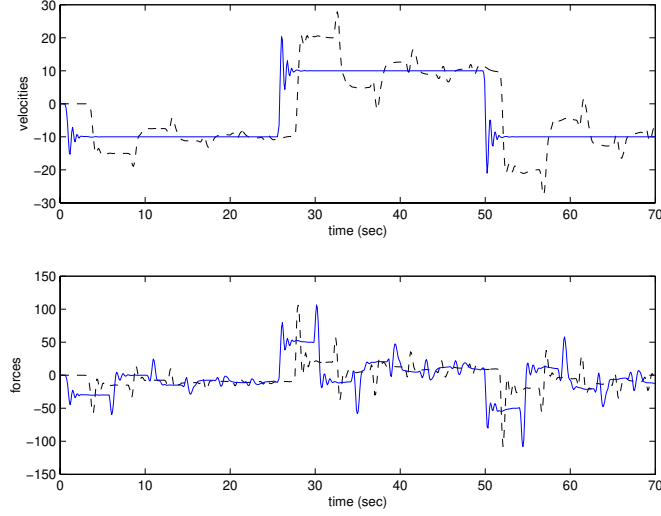
Figure 6.4: $H_i(s, \theta) = e^{-2s}$ and $\gamma_i = 1$

added to the channel on both channels, i.e. $\theta = 2$. The Slave velocity does not track the Master's velocity anymore, although it stays bounded. Decreasing the gain to $\gamma_i = 0.5$, we obtain a better response as shown in Figure 6.5. However, the Slave's response still does not converge to the Master's response.

- Finally, we introduce a dynamic channel with delay that produces the response shown in Figure 6.6. The response is unstable. Designing the gains as in (6.18) for $\rho = 0.5$ and $\tau_i = 2$, we have $\gamma < 0.76$. The response with a gain of $\gamma = 0.7$ is shown in Figure 6.7, where the system regains its stability.

6.3.1   Discussion

As seen in Section 6.3 there are different perspectives in teleoperated systems when a dynamic channel is considered. When the delay is introduced, it distorts the response of the system on the Slave's side and the Slave feels a delayed effect of the reference velocity on the Master's side and vice versa. However, when the gains are adjusted,
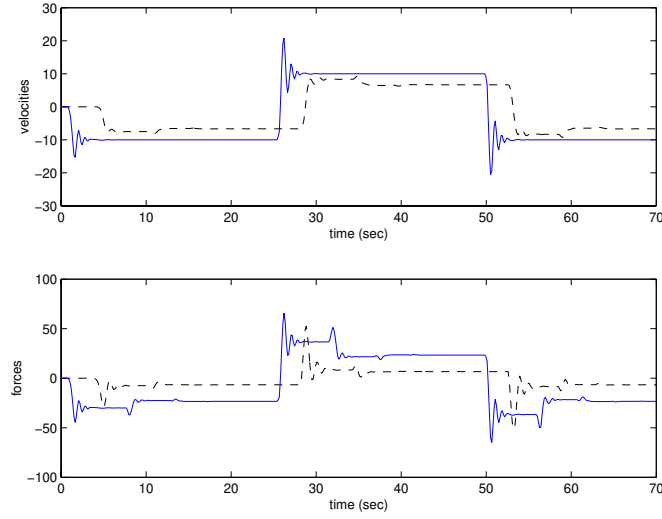
Figure 6.5: $H_i(s, \theta) = e^{-2s}$ and $\gamma_i = 0.5$

the wave signals inside the channel are diminished and hence the growing effect that was introduced by the delay is attenuated by the gains. However, the systems cannot see the channel as a transparent media anymore and will converge to different steady-state levels. On the other hand, the introduction of the dynamic channel with delay rendered the overall system unstable. This was due to the fact that the chosen pole of the channel transfer function caused it to behave like an integrator, hence magnifying the wave signals and in turn the reference velocities. The gains soothed the effect of this magnification by attenuating at the same rate, hence balancing the effect of the channel. Note finally, that a dynamic channel might change the impedance matching that we had in the pure delay case and further investigation of the matter is required.
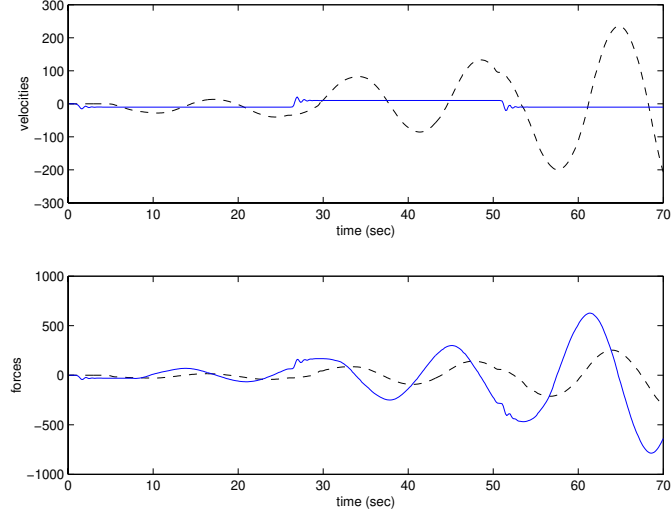
Figure 6.6: $H_i(s, \theta) = \frac{e^{-2s}}{s+0.5}$ and $\gamma_i = 1$

6.4 Conclusions

In this chapter we have studied the effect of network dynamics on the performance of teleoperated systems. This effect was obvious in the simulations and a passivity analysis was utilized to compensate for it.

In the next chapter, we present our conclusion of the thesis, including the most recent results that were not included in the manuscript due to time constraints, and state our recommendations for future extensions of the results stated in the thesis.
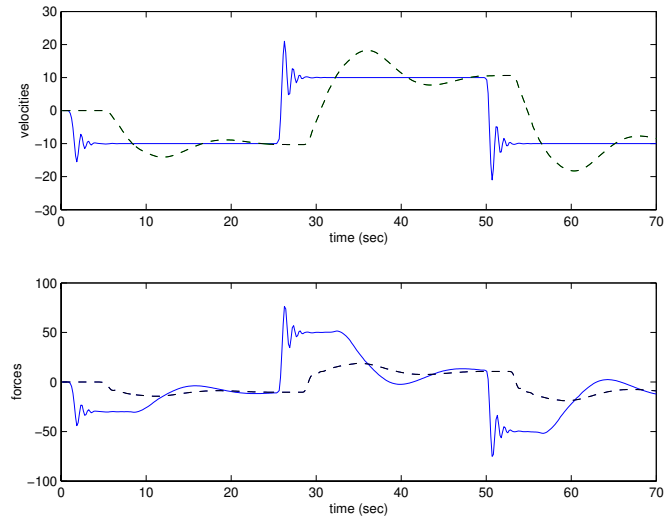
Figure 6.7: $H_i(s,\theta) = \frac{e^{-2s}}{s+0.5}$ and $\gamma_i = 0.7$

Chapter 7

Conclusions and Future Work

The objective of this thesis was to present a systematic overview of the main problems inherent to networked control systems, and provide new methods to tackle these problems. We presented a comprehensive survey of the literature in Chapter 2, that can serve as a tutorial for engineers starting to explore the area of networked systems. A new sampled-data model was proposed to study such systems accompanied with sufficient Lyapunov stability arguments. The issue of packet drops was addressed in a probabilistic fashion and the solution was provided through the theory of Markovian jump systems and the uncertainty threshold principle. A case study of teleoperated systems through networks was discussed and inherent problems such as channel dynamics and time delay were investigated.

7.1   Recent Advances

There are several papers that have recently appeared, by the time of submission of the thesis, hence we only mention them briefly.

In [70] variable sampling times were used in model-based networked control systems,

to provide extensions to results previously obtained in [69]. New results pertaining to packet dropouts were provided in [6, 60, 61, 97]. A nonlinear input-output stability analysis has been developed in [101]. Quantization issues were addressed in [16, 59]. The effects of communication constraints on the design of networked control systems was presented in [31]. In [86] a fault-tolerant networked control systems approach was taken through the study of time-triggered protocol (TTP) for communication.

7.2 Future Work

The area of networked control systems is still a raw arena for research efforts, and constitutes a new paradigm for control engineers to explore. Hence, our work in this thesis has the potential to be further expanded in several directions.

The stability results derived for the sampled-data model in Chapter 3, may yet be expanded to become necessary and sufficient for the sampling time. This is due to the fact that Lyapunov analysis provides only sufficient results.

Further exploration of the stochastic nature of packet dropouts is in order, to take into consideration the type of protocols and network dynamics. Our approach was to model the dropout as a Markov chain, which did not take into account retransmissions utilized in some protocols. Even though retransmissions are hard to analyze in terms of control systems, there is a possibility that they might improve the system performance when an observer-type analysis is utilized.

The case study of teleoperated systems is a vital problem that is of interest to several engineering communities, and several problems need to be addressed such as channel dynamics and delays. Chapter 6 has constituted our first steps into the area of teleoperation and further studies are being currently pursued to relate the control design on both sides of the channel taking into account the knowledge about the

channel dynamics and inherent delays.

Throughout the thesis we have dealt with linear systems, however we would investigate how our results might be extended to nonlinear systems. This might be possible in the case of the sampled-data analysis presented in Chapter 3, through extending the Lyapunov-type results to nonlinear systems.

Finally, we would like to point out that the analysis presented in this thesis is general in the sense that we did not consider a specific underlying network type, structure, or operating protocol. Hence, the results can be applied to wired or wireless type of networks, with the knowledge that several models were devised for the latter.

References

[1] C.T. Abdallah, P. Dorato and J. Benitez-Read, "Delayed Positive Feedback Can Stabilize Oscillatory Systems," *Proceedings of the IEEE American Control Conference*, San Francisco, California USA, pp. 3106-3107, June 1993.

[2] R.J. Anderson and M.W. Spong, "Bilateral Control of Teleoperators with Time Delay," *IEEE Transactions on Automatic Control*, AC-34, no.5, pp. 494-501, May 1989.

[3] M. Aoki, *Optimization of Stochastic Systems, 2nd Edition*, Academic Press, CA, 1989.

[4] K.J. Åstrom and B. Wittenmark, *Computer Controlled Systems: Theory and Design, 2nd Edition*, Prentice Hall, Englewood Cliffs, NJ, 1990.

[5] M. Athans, R. Ku and S.B. Gershwin, "The Uncertainty Thershold Principle: Some Fundamental Limitations of Optical Decision Making Under Dynamic Uncertainty," *IEEE Transactions on Automatic Control,* vol. 22, no. 3, pp. 491-495, June 1977.

[6] B. Azimi-Sadjadi, "Stability of Networked Control Systems in the Presence of Packet Losses," Submitted to the IEEE Conference on Decision and Control, 2003.

[7] J. Baillieul, "Feedback Coding for Information-based Control: Operating Near the Data-Rate Limit," *Proceedings of the IEEE Conference on Decision and Control*, Las Vagas, Nevada USA, pp. 3229-3236, Dec. 2002.

[8] B.A. Bamieh and J.B. Pearson, "A General Framework for Linear Periodic Systems with Applications to $\mathcal{H}^\infty$ Sampled-Data Control," *IEEE Transactions on Automatic Control,* vol. 37, pp. 418-435, April 1992.

References

[9] O.V. Beldiman, *Control Networks*, Department of Electrical and Computer Engineering, Duke University, 1998.

[10] O. Beldiman, L.G. Bushnell, G.C. Walsh, H.O. Wang and Y. Hong, "Perturbations in Networked Control Systems," Proceedings ASME-IMECE DSCD, New York, NY, November 2001.

[11] K. Benjelloun and E.K. Boukas, "Mean Square Stochastic Stability of Linear Time-Delay System with Markovian Jumping Parameters," *IEEE Transactions on Automatic Control,* vol. 43, no. 10, pp. 1456-1460, October 1998.

[12] S. Boyd, L. El-Ghaoui, E. Feron, V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory,* SIAM Studies in Applied Mathematics, vol. 15, Philadelphia, Pennsylvania, 1994.

[13] M.S. Branicky, S.M. Phillips and W. Zhang, "Stability of Networked Control Systems: Explicit Analysis of Delay," *Proceedings of the IEEE American Control Conference*, Chicago IL, vol. 4, pp. 2352-2357, June 2000.

[14] R.W. Brockett, "Minimum Attention Control," *Proceedings of the IEEE Conference on Decision and Control*, San Diego, California USA, December 1997.

[15] R.W. Brockett and D. Liberzon, "Quantized Feedback Stabilization of Linear Systems," *IEEE Transactions on Automatic Control*, vol.45, no.7, pp.1279-1289, July 2000.

[16] F. Bullo and D. Liberzon, "On Quantized Control and Geometric Optimization," Submitted to the IEEE Conference on Decision and Control, 2003.

[17] L. Bushnell, O. Beldiman and G. Walsh, "An Equivalence between a Control Network and a Switched Hybrid System," Lecture Notes in Computer Science, v. 1386, p.64-80, Springer-Verlag 1998.

[18] H. Chan and Ü. Özgüner, "Closed-loop control of systems with communication network with queues," *International Journal of Control*, vol. 62, no. 3, pp. 493-510, 1995.

[19] T. Chen and B.A. Francis, *Optimal Sampled-Data Control Systems,* Springer, London, 1995.

[20] M-Y Chow and Y Tipsuwan, "Network-Based Control Systems: Tutorial," *The 27th Annual Conference of the IEEE Industrial Electronics Society, IECON '01*, vol. 3, pp. 1593-1602, 2001.

References

[21] D.E. Comer, *Computer Networks and Internets*, International Edition, Prentice Hall International Inc., Upper Saddle River, NJ, 1999.

[22] *IEEE Control Systems Magazine,* vol. 22, no. 5, pp. 112, October 2002.

[23] O.L.V. Costa and M.D. Fragoso, "Stability Results for Discrete-Time Linear Systems with Markovian Jumping Parameters," *Journal of Mathematical Analysis and Applications,* vol. 179, pp.154-178, 1993.

[24] D.F. Delchamps, "Stabilizing a Linear System with Quantized State Feedback," *IEEE Transactions on Automatic Control*, vol. 35, no. 8, pp. 916-924, August 1990.

[25] P. Dorato, C.T. Abdallah and V. Cerone, *Linear Quadratic Control: An Introduction*, Reprint Edition, Krieger Publishing Company, Florida, 2000.

[26] I. Elhajj, N. Xi, W. K. Fung, Y.-H. Liu, Y. Hasegawa, T. Fukuda, "Supermedia-Enhanced Internet-Based Telerobotics," *Proceedings of the IEEE,* vol. 91, issue 3, pp. 396-421, March 2003.

[27] N. Elia and S.K. Mitter, "Quantization of Linear Systems," *Proceedings of the IEEE Conference on Decision and Control,* Phoenix, Arizona USA, pp. 3428-3433, December 1999.

[28] N. Elia and S.K. Mitter, "Stabilization of Linear Systems With Limited Information," *IEEE Transactions on Automatic Control*, vol. 46, no. 9, pp. 1384-1400, Sept. 2001.

[29] A. Eusebi and C. Melchiori "Force reflecting telemanipulators with time-delay: Stability analysis and control design," *IEEE Transactions on Robotics & Automation,* vol. 14, pp. 635-640, 1998.

[30] G.F. Franklin, J.D. Powell and M. Workman, *Digital Control of Dynamic Systems,* 3rd Edition, Addison Wesley Longman Inc., 1998.

[31] H. Fujioka and K. Ito, "Performance Optimization for a Class of Networked Control Systems with Communication Contstraints," *Proceedings of the IEEE American Control Conference*, Denver, Colorado, pp. 248-253, June 2003.

[32] P. Gahinet, A. Nemirovski, A. J. Laub and M. Chilali, "LMI Control Toolbox Manual for Control Analysis and Design", available from Mathworks: www.mathworks.com

[33] R.G. Gallager, *Information Theory and Reliable Communication*, John Wiley and Sons, NY, 1968.

References

[34] F. Göktas, J.M. Smith and R. Bajcsy, "Telerobotics Over Communication Networks," *Proceedings of the IEEE Conference on Decision and Control*, San Diego, Calif. USA, vol. 3, pp. 2399-2404, Dec 1997.

[35] P.R. Halmos, *Measure Theory,* D. Van Nostrand Company, Inc., NY, 1950.

[36] Y. Halevi and A. Ray, "Integrated Communication and Control Systems: Part I-Analysis," *Journal of Dynamic Systems, Measurement, and Control*, vol. 110, pp. 367-373, December 1988.

[37] J. Hespanha, A. Ortega and L. Vasudevan, "Towards The Control of Linear Systems with Minimum Bit-Rate," *Proceedings of the International Symposium on the Mathematical Theory of Networks and Systems*, August 2002.

[38] A. Hörjel, *Bluetooth in Control*, MS Thesis, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 2001.

[39] P.F. Hokayem, C.T. Abdallah, "Networked Control Systems: A Sampled-Data Approach", Accepted at the IEEE International Symposium on Intelligent Control, Houston, Texas, USA, October 5-8, 2003.

[40] C.S. Hsu, *Cell-to-Cell Mapping: A Method of Global Analysis for Nonlinear Systems*, Applied Mathematical Sciences, vol. 64, Springer-Verlag, New York, 1987.

[41] Hu, B. and Michel, A.N., "Stability Analysis of Digital Feedback Control Systems with Time-Varying Sampling Periods," *Automatica*, vol. 36, pp. 890-905, 2000.

[42] H. Ishii and T. Basar, "Remote Control of LTI systems over networks with state quantization," *Proceedings of the IEEE Conference on Decision and Control*, Las Vagas, Nevada USA, pp. 830-835, Dec. 2002.

[43] H. Ishii and B.A. Francis, *Limited Data Rate in Control Systems with Networks*, Lecture Notes in Control and Information Sciences, vol. 275, Springer-Verlag, Berlin, Germany, 2002.

[44] H. Ishii and B.A. Francis, "Control over Networks," *Proceedings of the 38th Annual Allerton Conference on Communication, Control, and Computing*, University of Illinois, Urbana-Champaign, IL, October, 2000.

[45] R. Jain, T. Simsek and P. Varaiya, "Control under communication constraints," *Proceedings of the IEEE Conference on Decision and Control*, Las Vagas, Nevada USA, pp.3209-3216, Dec. 2002.

References

[46] M. Jamshidi, *Large-Scale Systems: Modeling and Control,* Elsevier Science Publishing Co., 1983.

[47] R.K. Jurgen, "Coming from Detroit: networks on wheels", *IEEE Spectrum*, vol. 23, no. 6, pp. 53-59, 1986.

[48] T. Katayama, "On the Matrix Riccati Equation for Linear Systems with a Random Gain," *IEEE Transactions on Automatic Control,* vol. 21, no. 5, pp. 770-771, October 1976.

[49] H.K. Khalil, *Nonlinear Systems*, 3rd Edition, Prentice Hall, Upper Saddle River, NJ, 2002.

[50] R. Krtolica, Ü. Özgüner, H. Chan, H. Göktaş, J. Winkelman and M. Liubakka, "Stability of Linear Feedback Systems with Random Communication Delays," *International Journal of Control*, vol. 59, no. 4, pp. 925-953, 1994.

[51] R.T. Ku and M. Athans, "Further Results on the Uncertainty Principle," *IEEE Transactions on Automatic Control,* vol. 22, no. 5, pp. 866-868, October 1977.

[52] H.J. Kushner, *Stochastic Stability and Control,* Mathematics in Science and Engineering vol. 33, Academic Press, NY, 1967.

[53] A. Lasota and M.C. Mackey, *Chaos, Fractals, and Noise: Stochastic Aspects of Dynamics,* 2nd Edition, Applied Mathematical Sciences, vol. 97, Springer-Verlag, New York, 1994.

[54] G. Leen and D. Heffernan, "Expanding Automotive Electronic Systems," *IEEE Computer*, vol. 35, no. 1, pp. 88-93, January 2002.

[55] F-L Lian, J.R. Moyne and D.M. Tilbury, "Implemetation of Networked Machine Tools in Reconfigurable Manufacturing Systems," *Proceedings of the 2000 Japan-USA Symposium on Flexible Automation*, Ann Arbor, MI, July 2000.

[56] F-L Lian, J.R. Moyne and D.M. Tilbury, "Performance Evaluation of Control Networks: Ethernet, ControlNEt, and DeviceNet," Technical Report UM-MEAN-99-02, Ann Arbor, MI, Febuary 1999.

[57] D. Liberzon, "Stabilization by Quantized State or Output Feedback: A Hybrid Control Approach," *Proceedings of the 15th IFAC World Congress*, Barcelona, Spain, July 2002.

[58] D. Liberzon, "A note on stabilization of linear systems with limited infromation," *Proceedings of the IEEE Conference on Decision and Control*, Las Vagas, Nevada USA, pp. 836-841, December 2002.

References

[59] D. Liberzon, "Stabilizing a Nonlinear System with Limited Information Feedback," Submitted to the Conference of Decision and Control, 2003.

[60] Q. Ling and M.D. Lemmon, "Optimal Dropout Compensation in Networked Control Systems," Submitted to the IEEE Conference on Desicion and Control, 2003.

[61] Q. Ling and M.D. Lemmon, "Soft Real-Time Scheduling of Networked Control Systems with Dropouts Governed by a Markov Chain," *IEEE American Control Conference*, Denver, Colorado, pp. 4845-4850, June 2003.

[62] L.-W. Liou and A. Ray, "Integrated Communication and Control Systems: Part III-Nonindentical Sensor and Controller Sampling," *Journal of Dynamic Systems, Measurement, and Control*, vol. 112, pp. 357-364, September 1990.

[63] X. Liu and A. Goldsmith, "Wireless Communication Tradeoffs in Distributed Control," Submitted to the *First IEEE International Workshop on Sensor Network Protocols and Applications* (SNPA 2003).

[64] R. Lozano, B. Brogliato, O. Egeland and B. Maschke, *Dissipative systems analysis and control. Theory and applications*, CES, Springer, London, 2000.

[65] R. Lozano, N. Chopra and M.W. Spong, "Passivation of force reflecting bilateral teleoperation with time varying delay," *Proceedings of the 8th Mechatronics Forum International Conference*, Enschede, Netherlands, June 2002.

[66] J. Lunze, "Qualitative Modelling of Linear Dynamical Systems with Quantized State Measurements," *Automatica*, vol. 30, no. 3, pp. 417-431, 1994.

[67] Malmborg, J., *Analysis and Design of Hybrid Control Systems,* Ph.D. dissertation, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1998.

[68] A.S. Matveev and A.V. Savkin, "Optimal LQG Control via Limited Capacity Communication Networks," *Proceedings of the IEEE Conference on Decision and Control*, Las Vagas, Nevada USA, pp.4047-4052, Dec. 2002.

[69] L.A. Montestruque and P.J. Antsaklis, *Model-Based Networked Control System - Stability,* ISIS Technical Report ISIS-2002-001, Notre Dame, IN, January 2001.

[70] L.A. Montestruque and P.J. Antsaklis, "Stochastic Stability for Model-Based Networked Control Systems," Proceedings of the American Control Conference, Denver, Colorado, pp. 4119-4124, June 2003.

References

[71] G.N. Nair and R.J. Evans, "Mean Square Stability of Stochastic Linear Systems with Data Rate Constraints," *Proceedings of the IEEE Conference on Decision and Control*, Las Vagas, Nevada USA, pp. 1632-1637, Dec. 2002.

[72] S-I Niculescu, C.T. Abdallah and P.F. Hokayem, "Some Remarks on the Wave Transformation Approach for Telemanipulators with Time-Varying Distributed Delay," 4th *Asian Control Conference*, Singapore, 2002.

[73] S-I Niculescu, C.T. Abdallah and P.F. Hokayem, "Effects of Channel Dynamics on the Stability of Teleoperation," Submitted to IFAC Workshop on Time-Delay Systems INRIA, Rocquencourt, France, September 8-10, 2003.

[74] G. Niemeyer and J.-J. E. Slotine, "Towards Force-Reflecting Teleoperation Over the Internet," *Proceedings of the 1998 International Conference on Robotics & Automation*, Leuven, Belgium, pp. 1909-1915, May 1998.

[75] G. Niemeyer and J.-J. E. Slotine, "Stable adaptive teleoperation," *IEEE Journal on Oceanic Engineering,* vol. 16, pp. 152-162, 1991.

[76] G. Niemeyer and J.-J. E. Slotine, "Designing force reflecting teleoperators with large time delays to appear as virtual tools," *Proceedings of the 1997 IEEE ICRA*, Albuquerque, NM, pp. 2212-2218, 1997.

[77] S-I Niculescu, *Delay Effects on Stability: A Robust Control Approach*, Lecture Notes in Control and Information Systems, vol. 269, Springer-Verlag, London, 2001.

[78] J. Nilsson, *Real Time Control Systems with Delays,* Ph.D. dissertation, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1998.

[79] R. Oboe, "Force-Reflection Teleoperation over the Internet: The JBIT Project," *Proceedings of the IEEE,* vol. 91, issue 3, pp. 449-462, March 2003.

[80] M. Oda, T. Doi and K. Wakata, "Tele-Manipulation of a satellite mounted robot by an on-ground astronaut", *Proceedings of the IEEE International Conference on Robotics & Automation*, Seoul, Korea, pp. 1891-1896, May 2001.

[81] H. Ogata, *Modern Control Engineering*, 3rd Edition, Prentice Hall International Inc., Upper Saddle River, NJ, 1997.

[82] A.V. Oppenheim, R.W. Schafer and J.R. Buck, *Discrete-Time Signal Processing,* 2nd Edition, Prentice Hall, Upper Saddle River, NJ, 1999.

References

[83] R. Ortega, N. Chopra and M.W. Spong, "A new passivity formulation for bilateral teleoperation with time delay," in *Advances in Time-Delay Systems*, workshop CNRS-NSF, Paris France, January 22-24, 2003.

[84] P. Otanez, *"Performance Optimization of Networked Control Systems*, M.S. Thesis, University of Michigan, April 2002.

[85] Ü. Özgüner, H. Göktaş, H. Chan, J. Winkelman, M. Liubakka and R. Krtolica, "Automotive Suspension Control Through a Computer Communication Network," *IEEE Conference on Control Applications*, vol. 2, pp. 895-900, 1992.

[86] R. Patankar, "A Model for Fault-Tolerant Networked Control System Using TTP/C Communication," *Proceedings of the IEEE American Control Conference,* Denver, Colorado, pp. 533-537, June 2003.

[87] R.S. Raji, "Smart networks for control," *IEEE Spectrum*, pp. 49-55, June 1994.

[88] A. Ray and Y. Halevi, "Integrated Communication and Control Systems: Part II-Design Considerations," *Journal of Dynamic Systems, Measurement, and Control*, vol. 110, pp. 374-381, Dec 1988.

[89] W. Rudin, *Real and complex analysis*, McGraw Hill, New York, 1966.

[90] R. Šafarič, S. Šinjur, B. Žalik and R.M. Parkin, "Control of Robot Arm with Virtual Environment via the Internet," *Proceedings of the IEEE,* vol. 91, issue 3, pp. 422-429, March 2003.

[91] A. Sahai, "Evaluating Channels for Control: Capacity Reconsidered," *Proceedings of the American Control Conference,* vol. 4, pp. 2358-2362, 2000.

[92] A. Sahai, ""Any-time" Capacity and A Separation Theorem For Tracking Unstable Processes," ISIT2000, Sorrento, Italy, June 25-30, 2000.

[93] C. Scherer and S. Weiland, *Linear Matrix Inequalities in Control,* Lecture Notes, Mechanical Engineering Systems and Control Group, Delft University of Technology, The Netherlands, October 2000 (available at http://www.ocp.tudelft.nl/sr/personal/Scherer/lmi.pdf).

[94] J. Schröder and J. Lunze, "Representation of Quantized Systems by the Frobenius-Perron Operator," *Hybrid Systems: Computation and Control*, pp. 473-486, 2001.

[95] P. Seiler and R. Sengupta, "Analysis of Communication Losses in Vehicle Control Problems," *Proc. of the American Control Conference*, Arlington, VA, pp. 1491-1496, June 25-27, 2001.

References

[96] C.E. Shannon, "The Zero Error Capacity of a Noisy Channel," *IEEE Transactions on Information Theory,* vol. 2, no. 3, pp. 8-19, September 1956.

[97] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla and S. Sastry, "Kalman Filtering with Intermittent Observation," Submitted to the Conference on Decision and Control, 2003.

[98] W. Stevens, *TCP/IP illustrated*, Addison Wesley, Reading, MA, 1994.

[99] S.C. Tatikonda, *Control Under Communication Constraints*, Ph.D. Thesis, Massachusetts Institute of Technology, Sept. 2000.

[100] S. Tatikonda, A. Sahai and S. Mitter, "Control of LQG Systems Under Communication Constraints," *Proceedings of the IEEE American Control Conference*, San Diego, Calif., pp. 2778-2782, June 1999.

[101] A.R. Teel and D. Nešić, "Input-Output Stability Properties of Networked Control Systems," Submitted to the IEEE Conference on Decision and Control, 2003.

[102] M. Trivedi, B. Hall, G. Kogut and S. Roche, "Web-based Teleautonomy and Telepresence," 45th *SPIE Optical Science and Technology Conference*, Applications and Science of Neural Networks, Fuzzy Systems, and Evolutionary Computation III, vol. 4120, San Diego, August 2000.

[103] J.B.R. do Val, C. Nespoli and Y.R.C. Zúñiga, "Stochastic Stability for Markovian Jump Linear Systems Associated with a Finite Number of Jump Times," *Proceedings of the American Control Conference*, Anchorage, AK, pp. 334-339, May 2002.

[104] E. Verriest and M. Egerstedt, "Control with Delayed and Limited Information: A First Look," *Proceedings of the IEEE Conference on Decision and Control*, Las Vagas, Nevada USA, pp. 1231-1236, Dec. 2002.

[105] R.E. Walpole, R.H. Myers and S.L. Myers, *Probability and Statistics for Engineers and Scientists*, 6th Edition, Prentice Hall International Inc., Upper Saddle River, NJ, 1998.

[106] J. Walrand and P. Varaiya, *High-performance communication networks*, Morgan Kaufmann Publ., San Francisco, CA, 1996.

[107] G. Walsh, O. Beldiman and L.G. Bushnell, "Asymptotic behavior of nonlinear networked control systems," *IEEE Transactions on Automatic Control*, vol. 46, no. 7, pp. 1093-1097, July 2001.

References

[108] G.C. Walsh, O. Beldiman and L. Bushnell, "Asymptotic Behavior of Networked Control Systems," *Proceedings of the IEEE International Conference on Control Applications*, Kohala Coast-Island of Hawai'i, Hawai'i, USA, August 22-27, 1999.

[109] G.C. Walsh, O. Beldiman and L. Bushnell, "Error Encoding Algorithms for Networked Control Systems," *Proceedings of the IEEE Conference on Decision and Control*, Phoenix, Arizona, USA, pp. 4933 -4938, December 1999.

[110] G.C. Walsh, H. Ye and L. Bushnell, "Stability Analysis of Networked Control Systems," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 3, pp. 438-446, May 2002.

[111] G.C. Walsh, H. Ye and L. Bushnell, "Stability Analysis of Networked Control Systems," *Proceedings of the IEEE American Control Conference*, San Diego, CA, pp. 2876-2880, June 1999.

[112] H.S. Witsenhausen, "A Class of Hybrid-State Continuous-Time Dynamic Systems," *IEEE Transactions on Automatic Control*, vol. 11, no. 2, pp. 161-167, April 1966.

[113] W.S. Wong and R.W. Brockett, "Systems with Finite Communication Bandwidth Constraints II: Stabilization with Limited Information Feedback", *IEEE Transactions on Automatic Control*, vol. 44, no. 5, pp. 1049-1053, May 1999.

[114] J.K. Yook, D.M. Tilbury and N.R. Soparkar, "A Design Methodology for Distributed Control Systems to Optimize Performance in the Presence of Time Delays," International Journal of Control, vol. 74, no. 1, pp. 58-76, 2001.

[115] W. Zhang, *Stability Analysis of Networked Control Systems,* Ph.D. dissertation, Department of Electrical Engineering and Computer Science, Case Western Reserve University, Cleveland, OH USA, August 2001.

[116] W. Zhang, M.S. Branicky and S.M. Phillips, "Stability Analysis of Networked Control Systems," *IEEE Control Systems Magazine,* pp. 84-99, February 2001.

[117] http://www.cds.caltech.edu/ murray/topten/